BED SHEAR STRESS IN TURBULENT WAVE-CURRENT BOUNDARY LAYERS

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ABSTRACT

An approach for calculating turbulent flows in a wave-current boundary layer over a slowly varying bed is presented. Waves are periodic in time with several harmonics. In this paper, we adopt a time invariant eddy viscosity model, in which the eddy viscosity is linearly proportional to the distance from the bed. The boundary-layer flow field is solved analytically in terms of Fourier components. The approach allows fast computations and can be easily included in a phase resolving wave propagation model. As a part of the results, bottom shear stress and the spatial variation of the boundary layer thickness are also obtained. Present results compare well with experimental data and can explain the asymmetries in the bottom shear stress under sawtooth shaped waves.

Keywords: Turbulent boundary layer, bottom friction, eddy viscosity.

INTRODUCTION

As ocean waves propagate into shallow water, bottom friction can dissipate wave energy and affects wave transformation. The bottom shear stress is one of the main mechanisms responsible for sediment transport. Moreover, in coastal region wind-driven currents often interact with waves and therefore can also modify the bottom shear stress. Different approaches have been developed over the last three decades to study turbulent boundary-layer flows with wave-current interactions considered (see e.g., Soulsby et al. (1993)).

Recently, Liu and Orfila (2004) presented an analytical expression for the bottom shear stress with a constant laminar (or eddy) viscosity under a transient wave train. The bottom

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shear stress formula is written as a convolution integral of the near-bed flow acceleration. Liu and Orfila's approach can not be applied to the "steady state" condition (i.e., zero acceleration). Moreover, their approach requires that the process begins from a quiescent state. Liu (2006) extended Liu and Orfila's approach to investigate wave turbulent boundary layer flows where the eddy viscosity is a power function of the distance from the bottom. Again, all these approaches excluded the steady current in their formulations.

The effects of combined waves and currents in a turbulent boundary layer have usually been studied using the mixing length concept. For example, Grant and Madsen (1979), hereinafter GM, using a time invariant eddy viscosity model, developed a theory for the combined monochromatic wave and current where the bed shear stress is expressed in terms of the square of the near bottom velocity. In GM's model, the current above the boundary layer feels a larger resistance due to the presence of the monochromatic wave ("apparent roughness").

In this paper a new approach is developed to calculate the bottom stress under a turbulent boundary layer in a combined wave-current periodic system where the wave field consists in several harmonics. Following Kajiura (1964) or GM, we adopt a time invariant eddy viscosity model

$$\nu_t = \kappa u_{*c} \zeta, \quad (1)$$

with $\kappa \approx 0.40$ being the von Karman constant, u_{*c} is a characteristic frictional velocity, and ζ the coordinate normal to the bed. We remark here that in the present work the bed slope is assumed to be mild and thus the unit normal at the bed is very close to the vertical direction. Furthermore, unlike GM, we only require the information on the free stream velocity for both wave and current.

Introducing δ as the characteristic thickness of the boundary layer, (1) can be written as

$$\nu_t = \frac{\nu_0 \zeta}{\delta}, \quad (2)$$

with

$$\nu_0 = \kappa \delta u_{*c}, \quad (3)$$

being the characteristic eddy viscosity (ν_t at $\zeta = \delta$). In the above equation there are three physical quantities to be specified: δ , ν_0 and u_{*c} . The objective of the paper is to develop an approach to find the relationship between δ and u_{*c} by solving the linearized momentum equation inside the turbulent boundary layer under waves and currents. In addition, we shall focus on a periodic wave system in which ω is the fundamental wave frequency. Hence, the characteristic boundary-layer thickness can be related to the characteristic eddy viscosity as

$$\delta = \sqrt{\nu_0 / \omega}, \quad (4)$$

which constitutes the third equation for three unknown physical quantities.

The paper is structured as follows. In the next section the boundary layer equations for the rotational velocity component are presented and solved in the frequency domain, including the zeroth harmonic, corresponding to the steady current component. The relationship between the bed shear stress and the boundary-layer characteristic thickness is then presented in the following section. This approach is applied to 1D monochromatic waves with/without currents. New results are discussed and compared with experimental data taken in the wave-current system.

TURBULENT VELOCITY IN A COMBINED WAVE-CURRENT BOUNDARY LAYER

The velocity field inside a boundary layer can be decomposed into rotational and irrotational components, $\mathbf{u}_r$ and $\mathbf{u}_i = \nabla \Phi$, respectively, i.e., $\mathbf{u} = \mathbf{u}_r + \nabla \Phi$. Assuming that $2\pi/\omega$ is the fundamental period of the periodic wave system, we can further decompose the velocity field at a fixed point in a Fourier series in time as

$$\begin{pmatrix} \mathbf{u}_r(t) \\ \nabla \Phi(t) \end{pmatrix} = \frac{1}{2} \sum_{n=-\infty}^{\infty} \begin{pmatrix} \mathbf{u}_{r,n} \\ \nabla \Phi_n \end{pmatrix} \exp(in\omega t) = \frac{1}{2} \begin{pmatrix} \mathbf{u}_{r,0} \\ \nabla \Phi_0 \end{pmatrix} + \sum_{n>0} \Re \left\{ \begin{pmatrix} \mathbf{u}_{r,n} \\ \nabla \Phi_n \end{pmatrix} \exp(in\omega t) \right\}, \quad (5)$$

where $\Re$ denotes “the real part of” and the Fourier components $\mathbf{u}_{r,n}$ and Φ_n depend on spatial coordinates.

The linearized momentum equation for each Fourier component of the rotational velocity inside the boundary layer can be written in the following form:

$$in\mathbf{u}_{r,n} = \delta \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \mathbf{u}_{r,n}}{\partial \xi} \right), \quad (6)$$

where ξ is the vertical coordinate pointing upwards. Note that, for a mild slope, $\zeta \simeq \xi$, where ζ denotes the coordinate normal to the bed.

The boundary conditions for the rotational velocity component are slightly different for wave and current components. For wave components, $n \neq 0$, the rotational velocity component diminishes away from the bottom (i.e., outside the boundary layer), i.e.,

$$\mathbf{u}_{r,n} \rightarrow 0, \quad \text{as } \xi \rightarrow \infty. \quad (7)$$

and the no-slip condition is applied at a displacement elevation, ξ_0 , characterizing the hydraulic roughness of the bed,

$$\mathbf{u}_{r,n} = -\nabla \Phi_n, \quad \text{at } \xi = \xi_0. \quad (8)$$

Following the open channel theory (Graf and Altinakar 1998), the displacement thickness can be approximated as,

$$\xi_0(t) = \frac{k_s}{30} + \frac{\nu}{9.2 |\mathbf{u}_*|}, \quad (9)$$

where k_s is the equivalent roughness, ν the kinematic viscosity of water and $\mathbf{u}_*$ the friction velocity, defined so as

$$\tau_b = \rho |\mathbf{u}_*| \mathbf{u}_*. \quad (10)$$

For a rough bed (i.e. $k_s|\mathbf{u}_*|/\nu \gg 1$) equation (9) can be simplified as,

$$\xi_0 = \frac{k_s}{30}, \quad (11)$$

which is the same expression used in Orfila et al. (?). In the present approach, we assume that ξ_0 has a quasi permanent behavior, i.e.,

$$\xi_0 = \frac{k_s}{30} + \frac{\nu}{9.2u_{*c}}, \quad (12)$$

in which u_{*c} is a constant characteristic frictional velocity and is to be determined.

The solution to (6) subject to boundary conditions (7) and (8) was obtained first by Kaijura (1964),

$$\mathbf{u}_{r,n}(\xi) = -\nabla \Phi_n(\xi_0) \Xi_n^0(\xi/\delta), \quad (13)$$

where $\Xi_n^0(\xi/\delta)$ is given in terms of Kelvin functions as

$$\Xi_n^j(\xi/\delta) = \frac{K_j\left(2\sqrt{in\xi/\delta}\right)}{K_0\left(2\sqrt{in\xi_0/\delta}\right)}. \quad (14)$$

The real and imaginary parts for the first harmonic $n = 1$ of the rotational velocity are shown in Figure 1. The rotational velocity essentially vanishes at $\xi \simeq 2\delta$. We remark here that, since the solution includes ξ in the form “ $n\xi$ ”, the rotational velocity diminishes faster for higher harmonics.

The current field is given by the zeroth harmonic and the momentum equation (6) reduces to

$$0 = \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \mathbf{u}_{r,0}}{\partial \xi} \right). \quad (15)$$

Since the mean flow is purely rotational, hereinafter $\mathbf{u}_{r,0} = \mathbf{u}_0$. Applying the no-slip condition and specifying $\mathbf{u}_0(\xi_a)$ as twice the mean free stream velocity at the outer edge of the boundary

layer (i.e., at $\xi = \xi_a$, where ξ_a is order δ), the solution for (15)

$$\mathbf{u}_0(\xi) = \mathbf{u}_{r,0}(\xi) = \mathbf{u}_0(\xi_a) \frac{\ln(\xi/\xi_0)}{\ln(\xi_a/\xi_0)}. \quad (16)$$

BED SHEAR STRESS AND BOUNDARY LAYER THICKNESS

For each harmonic, the bed shear stress is expressed as

$$\tau_{b,n} = \rho \nu_t \left. \frac{\partial \mathbf{u}_{r,n}}{\partial \xi} \right|_{\xi=\xi_0}. \quad (17)$$

Introducing the solutions for the velocity field, (13) and (16), into (17) and considering the contributions from all harmonics, we obtain the following expression for the bottom shear stress

$$\tau_b = \rho \omega \delta \left[\frac{1}{2} \frac{\mathbf{u}_0(\xi_a)}{\ln(\xi_a/\xi_0)} + \sum_{n>0} \Re \left\{ \nabla \Phi_n(\xi_0) \Xi_n^1(\xi_0/\delta) \sqrt{in\xi_0/\delta} \exp(in\omega t) \right\} \right], \quad (18)$$

where the first term accounts for the stress induced by steady current and the second for those by different wave harmonics. We remark here that the bottom shear stress (18) is a function of the unknown boundary layer thickness that is part of the solution.

On the other hand, with $\delta = \sqrt{\nu_0/\omega}$, equation (3) becomes

$$\delta = \frac{\kappa u_{*c}}{\omega}. \quad (19)$$

Therefore, we need to relate u_{*c} to bed shear stress so as to close the problem. Three different models are examined. Recalling the expression (10), GM used $u_{*c} = \sqrt{|\tau_{b,\max}|/\rho}$ (herein, model I). Moreover, the idea to use the mean shear stress to determine the characteristic friction velocity $u_{*c} = \sqrt{\langle |\tau_b| \rangle / \rho}$ is also reasonable (model II), in which $\langle \cdot \rangle$ stands for the time average over a wave period. Following Kajiura (1964), the characteristic friction velocity u_{*c}

can be determined by the concept of equivalent energy dissipation (model III). Thus,

$$u_{*c} = \frac{\langle \tau_b \cdot \mathbf{u}_b \rangle}{\rho \langle \mathbf{u}_* \cdot \mathbf{u}_b \rangle}, \quad (20)$$

where $\mathbf{u}_b$ is the free stream velocity.

All three models can be solved using the same iterative procedure. For each model, initial guess for u_{*c} is first made, and the corresponding guesses for δ and ξ_0 are also obtained by using (19) and (13), respectively. The time history of the bed shear stress can then be computed through (18). The value of u_{*c} is updated from τ_b according to the model used, and the above procedure is repeated until a convergence criteria is satisfied. We note that, in this procedure, we need to specify the fundamental wave frequency ω , bed roughness k_s , the free stream mean velocity $\mathbf{u}_0(\xi_a)$, the elevation ξ_a and the free stream velocity components $\nabla \Phi_n(\xi_0)$. Furthermore, we note that because the length scale of the boundary layer, as long as ξ_a is in the order of δ we can assume $\nabla \Phi_n(\xi_0) \approx \nabla \Phi_n(\xi_a)$, so that the free stream velocity $\mathbf{u}_b$ can be written as

$$\mathbf{u}_b = \frac{\mathbf{u}_0(\xi_a)}{2} + \sum_{n>0} \Re \{ \nabla \Phi_n(\xi_a) \exp(in\omega t) \}. \quad (21)$$

The present approach allows fast computations of the bed shear stress for multi-harmonic waves with currents not necessarily co-linear, and for smooth, transitional and rough cases. To illustrate the capability of the present approach we will describe a monochromatic wave without current and a case where wave and current are co-linear. In the first case, analytical results are provided for models I and II (model III is solved numerically).

FRICITION COEFFICIENT FOR 1D MONOCHROMATIC WAVES

Consider a 1D monochromatic wave with zero mean, $u_0 = 0$, $u_1 = \partial \Phi_1 / \partial x$ and $\partial \Phi_n / \partial x = 0$ for $n > 1$. The bottom shear stress and its time mean of the absolute value can be readily obtained from (18)

$$\tau_b = \rho \omega \delta \Re \left\{ u_1 \Xi_1^1(\xi_0/\delta) \sqrt{i\xi_0/\delta} \exp(i\omega t) \right\}; \quad (22)$$

and therefore u_{*c} is, for models I and II respectively,

$$u_{*c} = \left(\frac{\sqrt{|\tau_{b,\max}|/\rho}}{\sqrt{\langle |\tau_b| \rangle / \rho}} \right) = \left(\frac{1}{\sqrt{2/\pi}} \right) \sqrt{\omega \delta u_1 |\Xi_1^1(\xi_0/\delta)| \sqrt{\xi_0/\delta}}, \quad (23)$$

so that, from (19), the characteristic boundary layer thickness is

$$\delta = \left(\frac{1}{2/\pi} \right) \frac{\kappa^2 u_1 |\Xi_1^1(\xi_0/\delta)| \sqrt{\xi_0/\delta}}{\omega}, \quad (24)$$

for models I and II, respectively.

The friction coefficient f_w introduced by Jonsson (1967), is defined as

$$f_w = \frac{2\tau_{b,\max}}{\rho u_{\max}^2}. \quad (25)$$

Recalling expression (22) and recognizing that $u_1 = u_{\max} = a_b \omega$, where a_b is the near bottom semi-exursion, we can rewrite the friction coefficient, for models I and II, respectively, as

$$f_w = \left(\frac{2}{4/\pi} \right) \frac{\kappa^2 \xi_0}{\delta} |\Xi_1^1(\xi_0/\delta)|^2 = \left(\frac{2}{\pi} \right) \frac{\delta^2}{\kappa^2 a_b^2}. \quad (26)$$

It is clear that f_w depends on both roughness, a_b/k_s , and Reynolds number, $a_b u_1/\nu$. In the rough case, when $a_b u_1/\nu \rightarrow \infty$, Reynolds number is not important. Figure 2 shows $f_w(a_b/k_s)$ for the rough case using models I, II and III. The same Figure displays the experimental expression by Nielsen (1992) and the theoretical expression by Fredsoe and Deigaard (1992) (recommended only for $a_b/k_s > 50$), which are respectively

$$f_w = \exp(5.5(k_s/a_b)^{0.2} - 6.3); \quad (27)$$

$$f_w = 0.04(a_b/k_s)^{-0.25} \quad \text{for } a_b/k_s > 50. \quad (28)$$

Finally, experimental data by Sleath (1987) ($\square$), Riedel (1972) (+) and Jensen (1989) ($\triangle$)

for turbulent oscillatory flow over rough beds are also plotted.

From Figure 2, three models show similar results in the analysis of monochromatic waves over a rough bed, and for $a_b/k_s \gtrsim 10$ these curves also compare well with experimental data. Model I, which is similar to GM, yields slightly bigger values for f_w , while model III (equivalent to Kajiura (1964) for waves only) remains between models I and II. All three models provide smaller values of the friction coefficient than those of available data for $a_b/k_s < 10$.

BOTTOM SHEAR STRESS UNDER WAVES AND CURRENTS

In this section we apply our approach to a combined wave-current boundary layer system. In order to show the capability of the present models in dealing with waves with multiple harmonics, let us first consider one-dimensional waves with a mean current so that the time history of the free stream velocity (at $\xi = \xi_a$) is given as

$$u_b = -1 + \frac{3}{4} \sum_{n=1}^{20} \Re \left\{ \frac{\exp(i(n-1/2)\pi)}{2^{n-1}} \exp\left(\frac{in\pi t}{5}\right) \right\}. \quad (29)$$

The above sawtooth shape velocity is displayed in Figure 3. The steady current has been chosen so that the free stream velocity is negative all the time. Assuming that k_s is $5 \cdot 10^{-4}$ m and $\xi_a = 5 \cdot 10^{-2}$ m, the resulting time histories of the bottom stress using the present models are also plotted. The trends are similar for all three models. Moreover, relative differences are in the same order of magnitude as those shown in Figure 2 (where logarithmic scales have been used). From Figure 3, it is clear that the bed stress can be a positive value even though the free stream velocity is negative. We remark here that the traditional quadratic bottom stress model, in which the bottom stress is in phase with the free stream velocity, can not provide correct prediction in the present situation.

To compare existing models and experimental data for monochromatic waves with cur-

rents, Soulsby et al. (1993) introduced the following dimensionless variables:

$$\chi \equiv \frac{\tau_c}{\tau_c + \tau_w}; \quad \gamma \equiv \frac{\langle \tau_b \rangle}{\tau_c + \tau_w}; \quad \Gamma \equiv \frac{\tau_{b,\max}}{\tau_c + \tau_w}, \quad (30)$$

where τ_c and τ_w are the bed stress corresponding to the current component u_0 alone and the maximum bed stress for the monochromatic wave component alone respectively. Moreover, $\langle \tau_b \rangle$ and $\tau_{b,\max}$ are the mean and maximum bottom stress corresponding to the total velocity (wave and current). Note that without considering wave and current interaction in the boundary layer would give $\gamma = \chi$ and $\Gamma = 1$ (Soulsby et al. 1993). Departures from the above trivial solutions are related to the wave and current interaction.

Figure 4 shows $\gamma(\chi) (\leq 1)$ and $\Gamma(\chi) (\geq 1)$ for the rough bed case with $a_b/k_s = 10^4$ (as shown in Figure 7 in Soulsby et al. (1993)) using the three models introduced.

Experimental data for $\gamma(\chi)$ collected from Soulsby *et al.* (1993) are also shown in Figure 4. Although in the experimental data the values for the current stress τ_c and for the wave induced stress τ_w were not found experimentally –but through traditional friction expressions–, model II shows good agreement with the experimental data.

Experimental data for the rough bed case and $a_b/k_s \gg 1$ is scarce. Lodahl *et al.* (1998) conducted experiments in a smooth pipe, so that the data are for laminar and smooth turbulent conditions (only the later are used for comparison). For each experiment, the cross sectional mean velocity averaged over a period and the maximum velocity in time at the centerline are given. Moreover, the time averaged and the maximum wall shear stresses were measured using a hot film. In order to compare the experimental data with the present analysis, the logarithmic profile in expression (16) was assumed valid throughout the cross section.

Figure 5 shows the comparison between experimental and computed time averaged shear stress. As depicted in Figure 5, the interaction between waves and currents is an important issue, well captured by all three models. Moreover, the results of the different models are

the same order of magnitude.

Similarly, Figure 6 shows the comparison for the maximum shear stress. Notice that differences between all three models are smaller than the usual errors in the friction expressions.

CONCLUDING REMARKS

A boundary layer analysis for waves and currents has been introduced assuming a time independent eddy viscosity. Three different models are proposed with different choices of the characteristic bottom frictional velocity. In particular, model I recovers the results by Grant and Madsen (1979) for the case of monochromatic waves only.

Decomposing the flow into Fourier components, the formulation takes into account the mean flow and the strength of wave harmonics to obtain the bed shear stress time history. The friction coefficient given by the formulation for monochromatic waves compares well with available experimental numbers for $a_b/k_s > 10$ for all three models. For the cases of wave and current interaction, important differences appear in the result of these three different models. The lack of comprehensive set of experimental data prevents us from making a recommendation for one of these three models.

The simplicity of the formulation allows us to introduce it in a more complex wave propagation models like the Boussinesq type to study the effects of the boundary layer on the damping of waves in shallow water.

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NOTATION

The following symbols are used in this paper:

a_b = near bottom semi-excursion;

f_w = friction coefficient;

k_s = bottom equivalent roughness;

$\mathbf{u}$ = horizontal velocity;

$\mathbf{u}_*$ = friction velocity;

u_{*c} = characteristic friction velocity;

$\mathbf{u}_0$ = horizontal current velocity;

$\mathbf{u}_r$ = horizontal rotational velocity;

$\mathbf{u}_{r,n}$ = n^{th} harmonic of the horizontal rotational velocity;

δ = boundary layer characteristic thickness;

κ = von Karman's constant (≈ 0.4)

ν = molecular kinematic viscosity;

ν_t = eddy viscosity;

ν_0 = characteristic eddy viscosity;

ω = characteristic frequency;

Φ = velocity potential;

Φ_n = n^{th} harmonic of the velocity potential;

ρ = density of water;

τ_b = bottom shear stress;

$\tau_{b,n}$ = n^{th} harmonic of the bottom shear stress;

τ_c = bottom shear stress due to currents;

τ_w = bottom shear stress due to waves;

ξ = vertical coordinate;

ξ_a = vertical coordinate where the mean velocity is given;

ζ = normal to the bottom coordinate.

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FIG. 1. Solution of the rotational velocity for $\partial\Phi_1/\partial x = 1$.

FIG. 2. Friction coefficient $f_w(a_b/k_s)$ for monochromatic waves: rough case. Symbols correspond to experimental data by Sleath (1987) ($\square$), Riedel (1972) (+) and Jensen (1989) ($\triangle$)

FIG. 3. Bed shear under sawtooth wave and current.

FIG. 4. Bed stress under rough waves and currents according to Figure 1 in Malarkey and Davies (1993). Symbols correspond to experimental data.

FIG. 5. Mean wall shear stress for smooth waves and currents: experimental data by Lodahl et al. (1998) against theoretical results.

FIG. 6. Maximum wall shear stress for smooth waves and currents: experimental data by Lodahl et al. (1998) against theoretical results.