Insensitivity of Leptogenesis with Flavor Effects to Low Energy Leptonic *CP* Violation

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If the baryon asymmetry of the Universe is produced by leptogenesis, *CP* violation is required in the lepton sector. In the seesaw extension of the standard model with three hierarchical right-handed neutrinos, we show that the baryon asymmetry is *insensitive* to the Pontecorvo-Maki-Nagakawa-Sakata phases: thermal leptogenesis can work for any value of the observable phases. This result was well known when there were no flavor effects in leptogenesis; we show that it remains true when flavor effects are included.

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Introduction.—*CP* violation is required to produce the puzzling excess of matter (baryons) over antimatter (antibaryons) observed in the Universe [1]. If this baryon asymmetry of the Universe (BAU) was made via leptogenesis [2], then *CP* violation in the lepton sector is needed. So any observation thereof, for instance in neutrino oscillations, would support leptogenesis by demonstrating that *CP* is not a symmetry of the leptons. It is interesting to explore whether a stronger statement can be made about this tantalizing link between low-energy observable *CP* violation and the BAU.

In this Letter, we address a phenomenological question: "Is the baryon asymmetry *sensitive* to the phases of the lepton mixing matrix (PMNS matrix)?" Electroweak precision data were said to be sensitive to the top mass, meaning that a preferred range for m_t could be extracted from the data. Here, we ask a similar question, assuming the baryon asymmetry is generated, via leptogenesis, from the decay of the lightest "right-handed" (RH) neutrino: Given the measured value of the baryon asymmetry, can an allowed range for the PMNS phases be obtained?

It was shown in [3] that the BAU produced by thermal leptogenesis in the type 1 seesaw, without "flavor effects," is insensitive to PMNS phases. That is, the PMNS phases can be zero while leptogenesis works, and the *CP* asymmetry of leptogenesis can vanish for arbitrary values of the PMNS phases. In fact, the "unflavored" asymmetry is controlled by phases from the RH sector only, and it would vanish were this sector *CP* conserving. However, it was recently realized that lepton flavor matters in leptogenesis [4–6]: In the relevant temperature range $10^9 \to 10^{12}$ GeV, the final baryon asymmetry depends separately on the lepton asymmetry in τs, and on the lepton asymmetry in muons and electrons. So in this Letter, we revisit the question addressed in [3], but with the inclusion of flavor effects. Our analysis differs from recent discussions [7] (2RHN model), [8,9] (*CP* as a symmetry of the N sector), [10] (sequential N dominance) in that we wish to do a bottom-up analysis of the three generation seesaw. Ideally, we wish to express the baryon asymmetry in terms of observables, such as the light neutrino masses and PMNS matrix, and free parameters. Then, by inspection, one could determine whether fixing the baryon asymmetry constrained the PMNS phases.

Notation and review.—We consider a seesaw model [11], where three heavy ($M \gtrsim 10^9$ GeV) Majorana neutrinos N_I are added to the standard model (SM). The Lagrangian at the N_I mass scale is

$$\mathcal{L} = \bar{e}_R^j \mathbf{Y}_{\mathbf{e}\,ij} H_d \ell^i + \bar{N}^J \lambda_{iJ} H_u \ell^i + \bar{N}^J \frac{\mathbf{M}_{JK}}{2} N^{cK} + \text{H.c.}, \tag{1}$$

where the flavor index order on the Yukawa matrices $\mathbf{Y_e}$, λ is left-right, and $H_u = i\sigma_2 H_d^*$.

There are six phases among the 21 parameters of this Lagrangian. We can work in the mass eigenstate basis of the charged leptons and the N_I, and write the neutrino Yukawa matrix as

$$\lambda = V_L^\dagger D_\lambda V_R, \tag{2}$$

where D_λ is real and diagonal, and V_L, V_R are unitary matrices, each containing three phases. So at the high scale, one can distinguish *CP* violation in the left-handed doublet sector (phases that appear in V_L) and in the right-handed singlet sector (phases in V_R). Leptogenesis can work when there are phases in either or both sectors.

At energies accessible to experiment, well below the N_I mass scale, the light (LH) neutrinos acquire an effective Majorana mass matrix [12]:

$$[m] = \lambda M^{-1} \lambda^T v^2 = U D_m U^T, \tag{3}$$

where $v = 174$ GeV is the Higgs vacuum expectation value, D_m is diagonal with real eigenvalues, and U is the PMNS matrix. There are nine parameters in $[m]$, which is "in principle" experimentally accessible. Two mass differences and two angles of U are measured, leaving the mass scale, one angle, and three phases of U unknown.

From the above we can write

$$D_m = U^\dagger V_L^\dagger D_\lambda V_R D_M^{-1} V_R^T D_\lambda V_L^* U^* v^2, \tag{4}$$

so we see that the PMNS matrix will generically have phases if V_L and/or V_R are complex. Like leptogenesis, it receives contributions from CP violation in the LH and RH sectors. Thus it seems "probable," or even "natural," that there is some relation between the CP violation of leptogenesis and of the PMNS matrix. However, the notion of relation or dependence is nebulous [13], so we address the clearer and simpler question of whether the baryon asymmetry is *sensitive* to PMNS phases. By this we mean: If the total baryon asymmetry is fixed, and we assume to know all the neutrino masses and mixing angles, can we predict ranges for the PMNS phases?

We suppose that the baryon asymmetry is made via leptogenesis, in the decay of the lightest singlet N_1, with $M_1 \sim 10^{10}$ GeV. Flavor effects are relevant in this temperature range [4–6,14]. N_1 decays to leptons ℓ_α, an amount $\epsilon_{\alpha\alpha}$ more than to antileptons $\bar{\ell}_\alpha$, and this lepton asymmetry is transformed to a baryon asymmetry by SM processes (sphalerons). We will further suppose that the partial decay rates of N_1 to each flavor are faster than the expansion rate of the Universe H. This implies that N_1 decays are close to equilibrium, and there is a significant washout of the lepton asymmetry due to N_1 interactions (strong washout regime); we discuss later why this assumption does not affect our conclusions.

Flavor effects are relevant in leptogenesis [4–6] because the final asymmetry cares which leptons ℓ are distinguishable. N_1 interacts only via its Yukawa coupling, which controls its production and destruction. The washout of the asymmetry, by decays, inverse decays, and scatterings of N_1, is therefore crucial for leptogenesis to work, because otherwise the opposite sign asymmetry generated at early times during N_1 production would cancel the asymmetry produced as they disappear. To obtain the washout rates (for instance, for $\ell + H_u \to N_1$), one must know the initial state particles, that is, which leptons are distinguishable.

At $T \sim M_1$, when the asymmetry is generated, SM interactions can be categorized as much faster than H, of order H, or as much slower. Interactions that are slower than H can be neglected. H^{-1} is the age of the Universe and the time scale of leptogenesis, so the faster interactions should be resummed—for instance into thermal masses. In the temperature range $10^9 \lesssim T \lesssim 10^{12}$ GeV, interactions of the τ Yukawa are faster than H, so the ℓ_τ doublet is distinguishable (has a different "thermal mass") from the other two lepton doublets. The decay of N_1 therefore produces asymmetries in $B/3 - L_\tau$, and in $B/3 - L_o$, where ℓ^o ("other") is the projection in ℓ^e and ℓ^μ space, of the direction into which N_1 decays [16]: $\hat{\ell}_o = (\lambda_{\mu 1}\hat{\mu} + \lambda_{e1}\hat{e})/\sqrt{|\lambda_{\mu 1}|^2 + |\lambda_{e1}|^2}$. Following [6], we approximate these asymmetries to evolve independently. In this case, the baryon to entropy ratio can be written as the sum over flavor of the flavored CP asymmetries $\epsilon_{\alpha\alpha}$ times a flavor-dependent washout parameter $\eta_\alpha < 1$, which is obtained by solving the relevant flavored Boltzmann equations [4–6]:

$$Y_B \simeq \frac{12}{37}\frac{1}{3g_*}(\epsilon_{\tau\tau}\eta_\tau + \epsilon_{oo}\eta_o), \tag{5}$$

where $g_* = 106.75$ counts entropy, and $12/37$ is the fraction of a $B - L$ asymmetry which, in the presence of sphalerons, is stored in baryons.

In the limit of hierarchical RH neutrinos, the CP asymmetry in the decay $N_1 \to \ell_\alpha H$ can be written as

$$\epsilon_{\alpha\alpha} \simeq -\frac{3M_1}{16\pi v^2 [\lambda^\dagger\lambda]_{11}} \mathrm{Im}\{[\lambda]_{\alpha 1}[m^\dagger\lambda]_{\alpha 1}\}, \tag{6}$$

where m is defined in Eq. (3).

In the case of "strong washout" for all flavors, which corresponds to $\Gamma(N_1 \to \ell_\alpha H_u) > H_{(T=M_1)}$ for $\alpha = \tau$, o, the washout factor is approximately [6,17]

$$\eta_\alpha \simeq 1.3\left(\frac{m_*}{6A_{\alpha\alpha}\tilde{m}_{\alpha\alpha}}\right)^{1.16} \to \frac{m_*}{5A_{\alpha\alpha}\tilde{m}_{\alpha\alpha}}, \tag{7}$$

where there is no sum on α, $m_* \simeq 10^{-3}$ eV, and $A_{\tau\tau} \simeq A_{oo} \sim 2/3$ [6,16,18]. The (rescaled) N_1 decay rate is

$$\tilde{m} = \sum_\alpha \tilde{m}_{\alpha\alpha} = \sum_\alpha \frac{|\lambda_{\alpha 1}|^2}{M_1} v^2. \tag{8}$$

An equation.—Combining Eqs. (5)–(7), we obtain $Y_B \propto \epsilon_{\tau\tau}/\tilde{m}_{\tau\tau} + \epsilon_{oo}/\tilde{m}_{oo}$, where ($\alpha$ not summed)

$$\frac{\epsilon_{\alpha\alpha}}{\tilde{m}_{\alpha\alpha}} = \frac{3M_1}{16\pi v^2 \tilde{m}} \sum_\beta \mathrm{Im}\{\hat{\lambda}_\alpha m_{\alpha\beta}\hat{\lambda}_\beta\}\frac{|\lambda_\beta|}{|\lambda_\alpha|} \tag{9}$$

and the Yukawa couplings of N_1 have been written as a phase factor times a magnitude: $\hat{\lambda}_\alpha|\lambda_\alpha| = \lambda_{\alpha 1}^*$. So the baryon asymmetry can be approximated as

$$Y_B \simeq Y_B^{bd}\left(\frac{\mathrm{Im}\{\hat{\lambda}_\tau m\hat{\lambda}_\tau\}}{m_{\mathrm{atm}}} + \frac{\mathrm{Im}\{\hat{\lambda}_o m\hat{\lambda}_o\}}{m_{\mathrm{atm}}} + \frac{\mathrm{Im}\{\hat{\lambda}_\tau m\hat{\lambda}_o\}}{m_{\mathrm{atm}}}\left[\frac{|\lambda_o|}{|\lambda_\tau|} + \frac{|\lambda_\tau|}{|\lambda_o|}\right]\right)\frac{1}{A_{\tau\tau}}. \tag{10}$$

The prefactor of the parentheses $Y_B^{bd} = \frac{12}{37}\frac{M_1 m_{\mathrm{atm}}}{16\pi v^2}\frac{m_*}{5g_*\tilde{m}}$ is the upper bound on the baryon asymmetry, which would be obtained in the strong washout case by neglecting flavor effects. Recall that this equation is only valid in strong washout for all flavors.

This equation reproduces the observation [6] that: (i) for equal asymmetries and equal decay rates of all distinguishable flavors, flavor effects increase the upper bound on the baryon asymmetry by $\sum_a A_{aa}^{-1} \sim 3$. (ii) More interestingly, having stronger washout in one flavor can increase the baryon asymmetry (via the term in brackets). So models in which the Yukawa coupling $\lambda_{\tau 1}$ is significantly different from $\lambda_{\mu 1}$, λ_{e1}, can have an enhanced baryon asymmetry (with cooperation from the phases).

Finally, this equation is an attractive step toward writing the baryon asymmetry as a real function of real parameters (Y_B^{bd}, depending on M_1 and $\tilde{m}_1$), times a phase factor [19]. In this case, the phase factor is a sum of three terms, depending on the phases of the N_1 Yukawa couplings, light neutrino mass matrix elements normalized by the heaviest mass, and a (real) ratio of Yukawas.

CP violation.—In this section, we use Eq. (10) to show that the baryon asymmetry is insensitive to the PMNS phases. The parameters of the lepton sector can be divided into "measurables," which are the neutrino and charged lepton masses, and the three angles and three phases of the PMNS matrix U. The remaining nine parameters are unmeasurable. We want to show that for any value of the PMNS phases, there is at least one point in the parameter space of the unmeasurables where a large enough baryon asymmetry is obtained. The approximations leading to Eq. (10) are only valid in a subset of the unmeasurable parameter space, but if we can find points in this subspace, we are done. We first show analytically that such points exist, then we do a parameter space scan to confirm that leptogenesis can work for any value of the PMNS phases.

If the phases of the $\lambda_{\alpha 1}$ were independent of the PMNS phases, and a big enough Y_B could be obtained for some value of the PMNS phases, then our claim is true by inspection: for any other values, the phases of the $\lambda_{\alpha 1}$ could be chosen to reproduce the same Y_B. However, there is in general some relation between the phases of m and those of $\lambda_{\alpha 1}$, so we proceed by looking for an area of parameter space where the phases of the $\lambda_{\alpha 1}$ can be freely varied without affecting the measurables. Then we check that a large enough baryon asymmetry can be obtained.

Such an area of parameter space can be found using the R matrix parametrization of Casas-Ibarra [20], where the complex orthogonal matrix R is defined such that $\lambda v \equiv U D_m^{1/2} R D_M^{1/2}$. Taking a simple R of the form

$$R = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \qquad (11)$$

and parametrizing $U = VP$, where V is a CKM-like unitary matrix with one "Dirac" phase $e^{-i\delta}$ appearing with $\sin\theta_{13}$, and $P = \mathrm{diag}\{e^{i\varphi_1/2}, e^{i\varphi_2/2}, 1\}$, gives

$$\frac{\lambda_{\tau 1} v}{\sqrt{M_1 m_3}} = U_{\tau 1}\sqrt{\frac{m_1}{m_3}}\cos\phi + U_{\tau 3}\sin\phi \simeq \frac{\sin\phi}{\sqrt{2}}, \qquad (12)$$

$$\frac{\lambda_{\mu 1} v}{\sqrt{M_1 m_3}} = U_{\mu 1}\sqrt{\frac{m_1}{m_3}}\cos\phi + U_{\mu 3}\sin\phi \simeq \frac{\sin\phi}{\sqrt{2}}, \qquad (13)$$

$$\frac{\lambda_{e1} v}{\sqrt{M_1 m_3}} = U_{e1}\sqrt{\frac{m_1}{m_3}}\cos\phi + U_{e3}\sin\phi, \qquad (14)$$

where we took hierarchical neutrino masses. We neglect λ_{e1} because its absolute value is small. With this choice of the unknown R, the phases of the $\lambda_{\alpha 1}$ are effectively independent of the PMNS phases. So for any choice of PMNS phases that would appear on the m of Eq. (10), the phases of the Yukawa couplings can be chosen independently, to ensure enough CP violation for leptogenesis.

We now check that a large enough baryon asymmetry can be obtained in this area of parameter space. The parentheses of Eq. (10) can be written explicitly as

$$\mathrm{Im}\left\{\frac{\sin^2\phi^*}{|\sin\phi|^2}(m_{\tau\tau} + m_{\mu\mu} + 2m_{\mu\tau})\right\}\frac{1}{m_{\mathrm{atm}}}. \qquad (15)$$

Writing $\phi^* = \rho - i\omega$, the final baryon asymmetry can be estimated from Eq. (10) as

$$\frac{Y_B}{10^{-10}} \simeq -\left(\frac{M_1}{10^{11}\ \mathrm{GeV}}\right)\frac{\sin\rho\cos\rho\sinh\omega\cosh\omega}{(\sin^2\rho\cosh^2\omega + \cos^2\rho\sinh^2\omega)^2}, \qquad (16)$$

which can equal the observed $8.7^{+0.3}_{-0.4} \times 10^{-11}$ [21] for $M_1 \sim \mathrm{few} \times 10^{10}$ GeV, and judicious choices of ρ and ω.

A similar argument can be made if the light neutrino mass spectrum is inverse hierarchical.

The scatter plots of Fig. 1 show that a large enough baryon asymmetry can be obtained for any value of the PMNS phases.

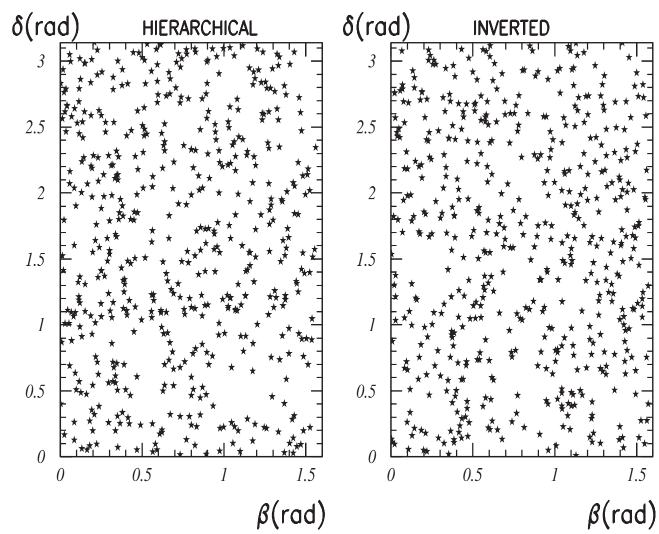

FIG. 1. A random selection of points where the baryon asymmetry is large enough, for some choice of the unmeasurable parameters of the seesaw. The light neutrino masses are taken nondegenerate, and the Majorana phase of the smallest one can be neglected. The Dirac phase δ is defined such that $U_{e3} = \sin\theta_{13}e^{-i\delta}$, and β is the majorana phase of $m_2 = |m_2|e^{2i\beta}$. The baryon asymmetry arises in the decay of N_1 of mass $M_1 = 10^{10}$ GeV.

The plots are obtained by fixing $M_1 = 10^{10}$ GeV and the measured neutrino parameters to their central values. To mimic the possibility that β and δ could be determined $\pm 15°$, β-δ space is divided into 50 squares. In each square, the program randomly generates values for β, δ, $0.001 < \theta_{13} < 0.2$, the smallest neutrino mass $< \sqrt{\Delta m^2_{\rm sol}}/10$, and the three complex angles of the R matrix. It estimates the baryon asymmetry from the analytic approximations of [6], and puts a cross if it is big enough. The program is a proto–Monte Carlo–Markov chain, preferring to explore parameter space where the baryon asymmetry is large enough.

Parametrizing with the R matrix imposes a particular measure (prior) on parameter space. This could mean we explore only a class of models. This is okay because the aim is only to show that, for any PMNS phases, a large enough asymmetry *can* be found.

Discussion.—The relevant question, in discussing the ‘‘relation’’ between CP violation in the PMNS matrix and in leptogenesis, is whether the baryon asymmetry is *sensitive* to the PMNS phases. The answer was ‘‘no’’ for unflavored leptogenesis in the standard model seesaw [3]. This was not surprising; the seesaw contains more phases than the PMNS matrix, and many unmeasurable real parameters which can be adjusted to obtain a big enough asymmetry. In this Letter, we argue that the answer does not change with the inclusion of flavor effects in leptogenesis: For any value of the PMNS phases, it is possible to find a point in the space of unmeasurable seesaw parameters such that leptogenesis works. This flavored asymmetry can be written as a function of PMNS phases, and unmeasurables as entered in the unflavored calculation. These can still be adjusted to get a big enough asymmetry. In view of this discouraging conclusion, it is maybe worthwhile to emphasize that CP violation from both the left-handed and right-handed neutrino sectors contributes both to the PMNS matrix and the baryon asymmetry. Moreover, the answer to this question in a minimal supergravity framework, with additional information from lepton flavor violating observables [22], is still a work in progress.

In the demonstration that the baryon asymmetry (produced via thermal leptogenesis) is insensitive to PMNS phases, we found an interesting approximation for the ‘‘phase of leptogenesis’’ [see Eq. (10)], when all lepton flavors are in strong washout.

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