Fitting von Bertalanffy growth curves in short-lived fish species. A new approach

by

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INTRODUCTION

There are several mathematical expressions to describe growth in fishes. The most commonly used is the von BERTALANFFY (1938) growth equation, which has in its favor, under a biological point of view, the fact of being derived from phisiological concepts.

We will assume that von BERTALANFFY growth equation describes well all stages of growth in fishes. The present paper tries to show that, in short-lived species of least, this equation, as fitted under the regular procedures, does not describe well the true growth pattern of the species. A new approach is presented in order to correct this error as much as possible.

MEANING OF THE PARAMETERS OF THE VON BERTALANFFY EQUATION

Von BERTALANFFY (1938) equation as stated originally reads:

$$L_t = L_\infty - (L_\infty - L_0) e^{-Kt}$$

The precise meaning of the parameters of this equation is as follows:

a) L_∞: maximum or asimptotic length that an average fish may reach given certain conditions. This parameter depends on both anabolic and catabolic processes.

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b) K represents one third of the rate of mass destruction per unit of time.
   It has then a very precise catabolic meaning.

c) L_0 is the length of the individuals when $t = 0$. We should interpret this
   as the length of the larvae at hatching.

As it can be seen each of those parameters has a well defined biological
meaning. A rearrangmeent of this equation can be made (BEVERTON and HOLT,
1957) resulting:

$$L_t = L_\infty (1 - e^{-K(t-t_0)})$$

in which the parameter L_0 dissappears and a new one t_0 appears, and it has
a very specific meaning that is: an age t_0 at which the organism with the
same growth pattern as that observed in later life would have been of zero
length.

We should like to stress that t_0 exists because L_0 exists, that is: the ordinate
in the origin is not zero but L_0. If the organism really grows according to the
von BERTALANFFY equation, L_0 should have to be the length at hatching. Thus
t_0 is not simply a correction factor as it is considered in some works, but it
has a definite meaning and depends, or should depend, on L_0. Under these
circumstances, if the species in consideration grows in accordance with the
von BERTALANFFY, equation, t_0 should have to be negative and not depart much
from 0, taking into account the small size of the fish larvae at hatching, in
most of the cases.

USUAL WAY OF FITTING VON BERTALANFFY GROWTH CURVES

In most of the cases von BERTALANFFY growth curves are fitted to the fish-
able period of life of the species population. We think that this procedure
gives a very bad representation of the real growth pattern of that particular
species-population, specially within the first year of life. We feel that the
problem is worse in the case of short-lived species. Parameters K and t_0 have
absolutelly not any biological meaning when curves are fitted in this way.

Table 1 shows the result of the fits of von BERTALANFFY growth curves to
several species of clupeoids. The resultant values of t_0, approaching in many
cases -2 years, clearly show the inadequacy of those expressions to represent
the real growth pattern of those species-populations. Such inadequacy is more
clearly shown in Figure 1 where it can be seen that the y— intercept in most
of the cases would give a L_0 value of 70-80 mm. If L_0 is the length at hatching
we can see that those fits have no biological sense, hence this applies to the
value of the parameter K of those curves.

TABLE 1

Species	Author	Fishery	K	L_∞*	t_0
Engraulis mordax	Spratt (1975)	S. California	0.299	165.52	−1.7144
Sardinops melanosticta	Nakai & Hayashi (1962)	Japan	0.81	224	−0.58
Sardinops ocellata	Baird (1970)	S. Africa	0.2247	306	−1.505
Sardinella jessieu	Nassef (1961)	Suez	0.83	160	−0.64
Sardina pilchardus	Larrañeta (1965)	Castellón	0.3065	203	−1.54
Sardina pilchardus	Larrañeta (1975)	Alicante	0.286	223.8	−2.154
Sprattus sprattus	Iles & Johnson (1962) (a)	Wash	0.53	164.3	0.4**
Sprattus sprattus	Iles & Johnson (1962) (b)	Wash	0.46	140	−0.2**
Sprattus sprattus	López Veiga (1977)	Galicia	0.37	179.8	−1.46

* Length in mm.
** (a) and (b) correspond to two groups of the same year class differing in growth characteristics.

SOURCES OF ERROR

In our understanding, two may be the major sources of error when fitting von BERTALANFFY growth curves. The first one may be dut to an improper setting of the birthday of the population not making it coincident with the peak of the spawning season.

Also it is observed in most of the cases that the mean length at age 1, has normally a rather big variance. It may occur that mean length at that age is biased towards the larger sizes due to the fact that those are the first entering the fishery. Also the existence of several broods within an annual class may occur, as it is the case in the sprat in Galician waters (NW Spain) (LÓPEZ VEIGA, 1977); if otoliths are used for ageing it would be very difficult to differenciate individuals belonging to each one of the broods and all of them will be given the same origin when they probably have a different one. This may affect, again, the proper setting of a birthday for the population.

AN APPROACH TO CALCULATE VON BERTALANFFY GROWTH CURVE

We are now proposing a new method to calculate a more realistic growth equation. In order to apply it we will need first the growth equation resultant of the fit, as it is made usually.

Von BERTALANFFY growth equation can be written, applying logarithms:

$$L_n (L_\infty - l_t) = L_n L_\infty + Kt_0 - Kt$$

The method implies to have an idea of the length at hatching that is $l_t = l_0$ when t = 0, and other two mean lengths at age, derived from the previous fit,

let them be l_1 and l_2 when $t = 1$ and $t = t_1 + n$ respectively. Then a system of three equations with three unknowns has to be solved:

$$Ln\,(L_\infty - l_0) = Ln\,L_\infty + Kt_0 \tag{a}$$

$$Ln\,(L_\infty - l_1) = Ln\,L_\infty + Kt_0 - Kt_1 \tag{b}$$

$$Ln\,(L_\infty - l_2) = Ln\,L_\infty + Kt_0 - K(t_1 + n) \tag{c}$$

which can be easily solved sustracting (a) — (b) and (b) — (c) obtaining

$$Ln\,(L_\infty - l_0) - Ln\,(L_\infty - l_1) = Kt_1 \tag{1}$$

$$Ln\,(L_\infty - l_1) - Ln\,(L_\infty - l_2) = Kn \tag{2}$$

Dividing (1)/(2) we obtain:

$$\frac{Ln\left(\dfrac{L_\infty - l_0}{L_\infty - l_1}\right)}{Ln\left(\dfrac{L_\infty - l_1}{L_\infty - l_2}\right)} = \frac{t_1}{n}$$

and from here:

$$\left(\frac{L_\infty - l_0}{L_\infty - l_1}\right) = \left(\frac{L_\infty - l_1}{L_\infty - l_1}\right)^{(t_1/n)} \tag{3}$$

Equation (3) may be easily solved by iteration or, if we choose l_1 and l_2 such that $t_1/n = 1$ which occurs when $(t_1 + n) = 2\,t_1$, equation (3) may be solved and:

$$L_\infty = \frac{l_1^2 - l_2\,l_0}{2\,l_1 - l_0 - l_2} \tag{4}$$

and once L_∞ is known we may obtain from (1)

$$K = \frac{Ln\left(\dfrac{L_\infty - l_0}{L_\infty - l_1}\right)}{t_1} \tag{5}$$

and from (a) the value of t_0 may be obtained

$$t_0 = \frac{Ln\left(\dfrac{L_\infty - l_0}{L_\infty}\right)}{K}$$

TABLE 2

Species	Author	Fishery	K	L_∞*	t_0
Engraulis mordax	Spratt (1975)	S. California	0.49	158.19	−0.05
Sardinops melanosticta	Hakai & Hayashi (1962)	Japan	1.15	222.00	−0.02
Sardinops ocellata	Baird (1970)	S. Africa	0.42	271.15	−0.04
Sardinella jessieu	Nassef (1961)	Suez	1.13	158.30	−0.02
Sardina pilchardus	Larrañeta (1965)	Castellón	0.53	190.67	−0.04
Sardina pilchardus	Larrañeta (1975)	Alicante	0.58	208.35	−0.03
Sprattus sprattus	Iles & Johnson (1962) (a)	Wash	0.43	168.50	−0,06**
Sprattus sprattus	Iles & Johnson (1962) (b)	Wash	0.49	139.17	−0,06**
Sprattus sprattus	López Veiga (1977)	Galicia	0.60	172.95	−0.04

* Length in mm.
** See footnote in table 1.

AN EXAMPLE: *SPRATTUS SPRATTUS* OFF GALICIA (NW SPAIN)

The growth equation for *Sprattus sprattus* off Galicia (NW Spain) has been calculated by LÓPEZ VEIGA (1977):

$$l_t = 179.8 \, (1 - e^{-0.37(t+1.46)})$$

Fitting the line directly to the observed mean lengths at age by the Beverton's method (RICKER, 1975). Table 3 shows the mean lengths at age resulting from this equation, as well as from the ones derived by the new method, depending on the values fo t_1 and t_2 chosen. It seems to us looking at table 3 that the most appropiate values are achieved by using t_1 and t_2: 3 and 6 years respectively.

TABLE 3

	Original growth curve	2 and 4	3 and 6	2 and 6	1 and 5	1 and 6	2 and 5
K	0.37	0.79	0.60	0.70	1.04	0.99	0.68
L_∞	179.80	162.80	172.95	170.91	164.22	168.86	173.40
t_0	−1.46	−0.03	−0.04	−0.03	−0.02	−0.02	−0.03
Age							
1	107.44	90.43	80.51	88.09	107.44	107.44	87.49
2	129.82	129.82	122.38	129.82	144.10	145.98	129.82
3	145.28	147.76	145.28	150.52	157.09	160.34	151.30
4	155.95	155.95	157.81	160.79	161.70	165.68	162.19
5	163.33	159.67	164.67	165.89	163.33	167.68	167.71
6	168.42	161.77	168.42	168.42	163.91	168.42	170.52
7	171.94	162.15	170.47	169.67	164.11	168.70	171.94

There is a big difference between the mean length at age 1 in the originally fitted curve and the one obtained by the new method: 107.44 and 80.51 mm respectively, but we have some reasons to believe that 107.44 is overestimating the mean length at age 1. Observed data show that at the entrance to the exploited phase the mean length for 0 age group is then 93.8 mm, corresponding to an age of 0.86 year. The same data show (LÓPEZ VEIGA, 1977) that mean length at age 1, ranges from 90.15 to 104.75 mm, with a mean value of 99.01 mm. Those values have been taken into account when fitting the growth curve. We believe that it suggests that 107.44 mm obtained from the curve for age 1 is biased towards bigger mean lengths at age values.

If we assume that there is a tendency to overestimate the mean lengths at the younger stages, due to the fact that the bigger fish are always recruited earlier to the fishery, then it may happen that 80.51 mm, is representing the *true* mean length at age 1 for the population.

DISCUSSION

The method may be applied to other related species. Table 2 shows the results of the application of the method to the species of table 1. In general it can be seen that K values tend to increase by the application of the new method, whereas L_∞ values tend to decrease and t_0 becomes rather homogeneous and small for all species. It has been assumed for all of them that $l_0 = 4$ mm. Figure 1 shows the resultant curves of the application of both, the usual and the new method.

It has to be discussed the validity of the adjusted curve (new method) and the unadjusted one (usual method). The ajusted curve, at least in short lived species, certainly describes better the true growth pattern of the fish, so under a biological point of view there must be no doubt that this modification should have to be applied preferently. In long lived species we do not have enough criteria to decide if the application of this method will mean any substantial improvement.

The unadjusted curve yields values of length at age closer to what the fishery takes, except for the youngest ages, than the adjusted one. Under a management point of view, if the recruitment to the fishery takes place at a late stage, as it is usual with gadoids, for example, it may be more practical to use the unadjusted curve. If the recruitment to the fishery takes place at an early stage, as it happens in short-lived species, as clupeoids, the use of one or other curve, may produce differences when analitical models are applied. For example in *Sprattus sprattus* of Galicia using the modified Beverton and Holt (1966) equation, F_{max} was 1.22, using the unadjusted groth curve, and 1.06 using the adjusted one (LÓPEZ VEIGA, 1977).
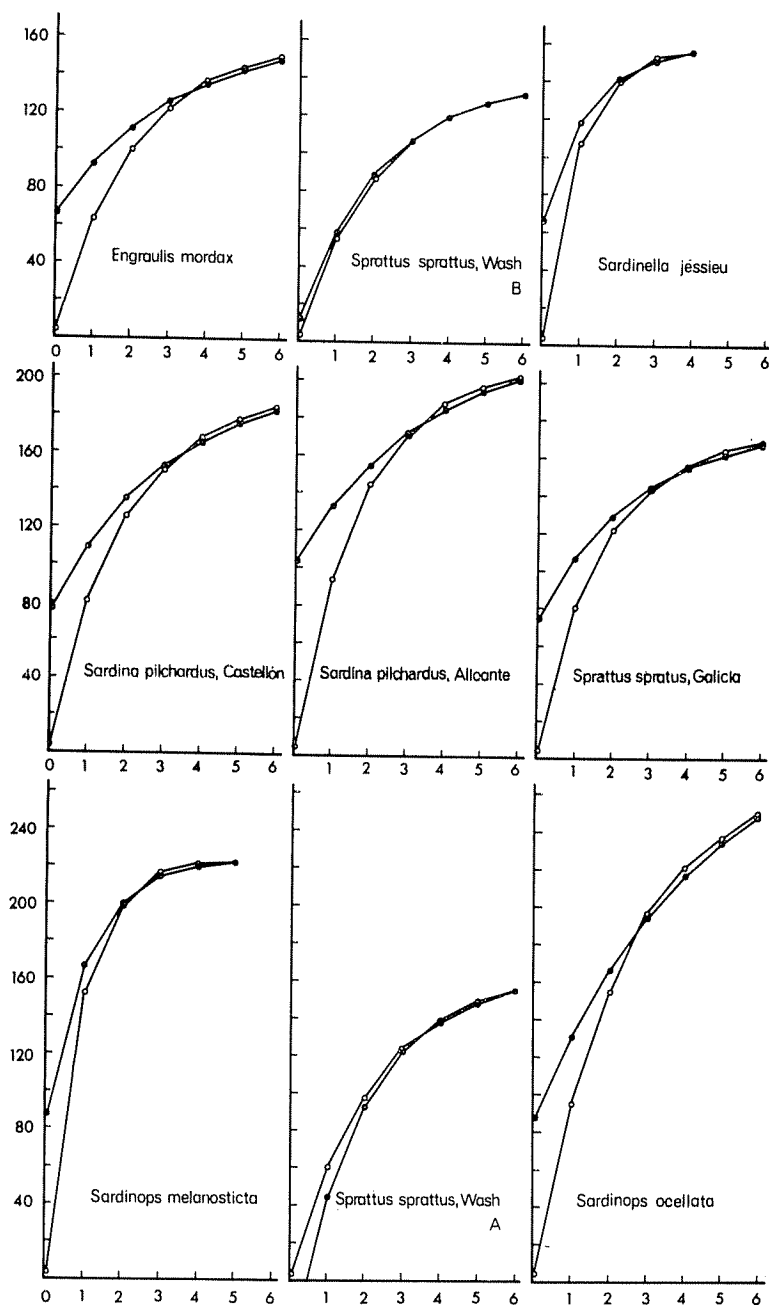
FIG. 1. Unadjusted (black circles) and adjusted (open circues) growth curves, for the species in tables 1 and 2.

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SUMMARY

In the present paper the usual way of adjusting the von Bertalanffy's growth equation are critiziced, with special reference to short-lived species. A new approach to adjust this equation, once that the usual methods have been applied, is proposed in order to get a more realistic expression of the real growth patterns of such species.

RESUMEN

EL AJUSTE DE LA ECUACIÓN DE CRECIMIENTO DE VON BERTALANFFY EN ESPECIES DE VIDA CORTA. UN NUEVO MÉTODO. — En el presente trabajo se hace una crítica de los métodos normales de ajuste de la ecuación de von Bertalanffy al crecimiento en los peces, con especial referencia a las especies de vida corta. Se describe un nuevo método para aproximar dicha ecuación, una vez ajustada la misma por los métodos normales, en orden a conseguir una expresión que se ajuste al crecimiento real de dichas especies.

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