Multirate Hybrid CT/DT Cascade $\Sigma\Delta$ Modulators with Decreasing OSR of Back-end DT stages

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Abstract—This paper presents novel architectures of multirate hybrid cascade continuous-time/discrete-time $\Sigma\Delta$ modulators that take advantage of the potentially faster operation of the continuous-time part of the circuit, while keep a reduced sampling operation of the back-end discrete-time stages. Compared to conventional multirate $\Sigma\Delta$ modulators, the proposed architectures use a higher sampling rate in the front-end (continuous-time) stage of the modulator, whereas the back-end (discrete-time) stages operate at a lower rate. It is demonstrated that the intrinsic aliasing signal can be cancelled in the digital domain, with no additional analog hardware required. The resulting $\Sigma\Delta$ topologies are potentially faster than conventional multirate $\Sigma\Delta$ modulators, more power efficient than hybrid monorate architectures and more robust than cascade continuous-time implementations. The combination of these features results in a new class of $\Sigma\Delta$ modulators, very suited for the implementation of analog-to-digital converters in the next generation of broadband wireless telecom systems.

I. INTRODUCTION

$\Sigma\Delta$ Modulators ($\Sigma\Delta$ Ms) have been commonly used in the last years for the implementation of Analog-to-Digital Converters (ADCs) in a number of telecom systems implemented in mainstream nanometer CMOS technologies. Although Discrete-Time (DT) implementations – normally using Switched-Capacitor (SC) circuits – have been used by most reported state-of-the-art Integrated Circuits (ICs), the need of higher and higher data rates demands for unaffordable dynamic requirements of the operational amplifiers, what forces using low values of the OverSampling Ratio (OSR) – one of the main ingredients contributing to the performance of $\Sigma\Delta$ Ms [1]. Alternatively, Continuous-Time (CT) $\Sigma\Delta$ Ms can provide faster rates than their DT counterparts with relatively lower power consumption. However, CT- $\Sigma\Delta$ Ms are more sensitive than DT- $\Sigma\Delta$ Ms against some circuit non-idealities, such as element tolerances and clock jitter [2].

The mentioned limitations can be partially solved by using the so-called Hybrid CT/DT $\Sigma\Delta$ Ms (H- $\Sigma\Delta$ Ms), in which some DT parts of the $\Sigma\Delta$ – usually the front-end blocks – are replaced with CT circuits, thus providing faster operation and embedded anti-aliasing filtering, with reduced power consumption [3]–[10]. However, in spite of the mentioned advantages, the majority of reported H- $\Sigma\Delta$ ICs [5]–[10] do not really exploit the capability of CMOS CT circuits to operate up to the GHz range with reasonable linearity [11], and hence to significantly increase the digitized signal bandwidth with medium-high values of OSR. This is due to the fact that the

maximum sampling rate of H- $\Sigma\Delta$ Ms is indeed limited by the DT part of the system. A possible solution to alleviate this restriction could be using a different sampling frequency for each part (either CT or DT) of the H- $\Sigma\Delta$, i.e. applying *multirating* [12].

The idea of using MultiRate $\Sigma\Delta$ Ms (MR- $\Sigma\Delta$ Ms) is not new and has been applied to both DT- and CT- $\Sigma\Delta$ Ms [12][13]. However, in both cases, the strategy consists of using a lower OSR in the front-end blocks – where most of the power is consumed – and a higher OSR in the subsequent stages or blocks of the modulator – where the dynamic requirements can be relaxed. This approach has been recently applied to cascade H- $\Sigma\Delta$ Ms, by using double-sampling technique to implement the DT back-end stage more efficiently [14].

This paper presents a new concept of multirate cascade $\Sigma\Delta$ Ms that benefits of using a hybrid CT/DT implementation. The proposed cascade H- $\Sigma\Delta$ Ms use a higher OSR in the CT front-end stage whereas the remaining parts of the modulator – implemented using DT circuits – operate at lower values of OSR, thus relaxing the speed requirements of the corresponding SC integrators. The main drawback of this approach is that the *downsampling* operation of the back-end stages causes *aliasing*, what forces using an inter-stage Anti-Aliasing Filter (AAF). However, as demonstrated in this paper, the operation of the AAF can be translated into the digital domain, thus simplifying the analog part of the circuit. As a result, innovative multirate cascade H- $\Sigma\Delta$ Ms are synthesized, that can potentially achieve higher sampling rates than previously reported ones with no additional analog hardware required.

II. CONVENTIONAL MULTIRATE CASCADE H- $\Sigma\Delta$ MS

Fig. 1 shows the conceptual implementation of a cascade 2-stage¹ MultiRate Hybrid $\Sigma\Delta$ (MR H- $\Sigma\Delta$ Ms). The circuit nature of the different modulator blocks, either CT or DT, as well as their corresponding sampling frequencies, f_{si} , are highlighted in the figure. The most common situation is that the front-end stage – operating at f_{s1} – is implemented using CT circuits, whereas the remaining i -th stages – sampled at $f_{si} > f_{s1}$ – are based on DT circuits. The use of a complete CT stage (instead of just the front-end integrator) maximizes the embedded AAF [9], whereas takes advantage of both the relaxed dynamic requirements of the front-end CT circuitry as compared to their DT counterparts, and benefits from increasing

¹ For the sake of simplicity, the architectures under study in this paper focus on 2-stage cascade topologies. However, as will be shown in Section III, this analysis can be extended to more general cascade N -stage $\Sigma\Delta$ Ms.

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Figure 1. Block diagram of a conventional cascade MR H-ΣΔM.

values of OSR in the (DT) back-end stages – where the speed requirements are less demanding than in the front-end ones [12]-[14].

The operation behind the modulator in Fig. 1 is conceptually the same as in a conventional *monorate* cascade ΣΔM. All stage outputs are combined by the Digital Cancellation Logic (DCL) transfer functions so that ideally only the quantization error of the last stage remains, and it is shaped by a Noise Transfer Function (NTF) whose order, L , is the sum of the orders of all stages in the cascade, L_i . Thus, assuming a linear model for the quantizers, it can be shown that the In-Band Noise (IBN) power at the output of an N -stage cascade MR ΣΔM is given by [12]:

$$\text{IBN}_{\text{US_H-MR}} \approx \frac{\Delta^2 \pi^{2L}}{12(2L+1)} \frac{1}{\text{OSR}_1^{2L_1} \text{OSR}_2^{2L_2} \dots \text{OSR}_N^{2L_N+1}} \quad (1)$$

where $\text{OSR}_i = f_{s1}/(2B_w)$ stands for the oversampling ratio of the i -th stage, and B_w is the input signal bandwidth. Note that considering that all OSR_i are equal to OSR , the expression in (1) becomes the same as in the case of a monorate cascade ΣΔM. The benefit of using lower values of OSR in the front-end CT stage is useful when the subsequent DT stages can operate at high enough OSRs. However, in spite of their less demanding speed requirements, the use of high values of OSR in the back-end stages is not practical when digitizing wideband signals, in the order of 10-100 MHz. In such a case,

Figure 2. Proposed *downsampling* cascade (2-stage) MR H-ΣΔMs. (a) Conceptual block diagram. (b) Including digital AAF.

sampling rates in the order of GHz may be necessary in order to digitize such signals with low-medium effective resolutions (8-10 bits). In such a case, the use of back-end SC stages may not be practical because of the prohibitive values of the required Gain BandWidth (GBW) of the corresponding operational amplifiers. An alternative solution could be implementing a complete CT cascade MR ΣΔM [13]. However, this is less robust in practice as a consequence of the sensitivity of cascade CT-ΣΔMs to circuit element tolerances, which is particularly critical in the case of nanometer CMOS implementations [4].

III. PROPOSED MULTIRATE CASCADE H-ΣΔMS

The mentioned problems of MR H-ΣΔMs, can be alleviated if the concept of *multirating* is redefined just in the opposite way as was conceived, i.e going from an *upsampling* multirate system – in which the front-end stage rate is increased in the subsequent stages – to a *downsampling* multirate system – in which the front-end (CT) stage operates at the highest sampling rate, thus benefiting from their potentially faster operation.

A. From upsampling to downsampling MR H-ΣΔMs

Fig. 2(a) shows a conceptual block diagram of the proposed *downsampling* cascade 2-stage MR H-ΣΔM. As the back-end (DT) stage operates at a lower rate than the front-end (CT) stage, an inter-stage (analog) AAF is needed in order to avoid aliasing. In practice, this block can be embedded in a SC implementation of the back-end integrators as suggested in [15]. Taking advantage of the DT-CT equivalence [2], the analysis of Fig.2(a) can be done starting from a complete DT- implementation of the MR ΣΔM. The resulting hybrid CT/DT architecture can be easily synthesized by using a DT-to-CT transformation method [3][4]. Thus, assuming an ideal AAF transfer function, $H_{LP}(z)$ – i.e a complete cancellation of the aliased signal components – the Z-transform of the modulator output is given by:

$$Y(z) \approx STF_1(z)DCL_1(z)X(z) + NTF_1(z)DCL_1E_1(z) - STF_2(z^{1/r})H_{LP}(z)DCL_2(z)E_{1,DS-US} + NTF_2(z^{1/r})DCL_2E_{2,US} \quad (2)$$

where $r \equiv f_{s2}/f_{s1} \equiv \text{OSR}_2/\text{OSR}_1$; $STF_{1,2}(z)$ and $NTF_{1,2}(z)$ stand for the Signal- and Noise Transfer Functions of the first- and second-stages of the modulator, respectively; $E_{1,2}(z)$ represents the Z-transform of the quantization error; and the subscripts DS and US denote respectively that an operation of *DownSampling* and *UpSampling* have been applied to $E_{1,2}(z)$.

Note from (2) that, as a consequence of both DS and US operations performed over $E_1(z)$, this error cannot be completely cancelled since, theoretically, $E_1(z) \neq E_{1,DS-US}(z)$. This problem can be solved by applying the same signal processing to $E_1(z)$ in both the analog and the digital domain, as illustrated in Fig. 2(b). In this case, the Z-transform of the modulator output can be written as:

$$Y(z) \approx STF_1(z)H_{LP1}(z)DCL_1(z)X_{DS-US}(z) + NTF_1(z)H_{LP1}(z)DCL_1E_{1,DS-US} - STF_2(z^{1/r})H_{LP2}(z)DCL_2(z)E_{1,DS-US} + NTF_2(z^{1/r})DCL_2E_{2,US} \quad (3)$$

It is shown from (3) that $E_1(z)$ can be cancelled if

$$H_{LP1}(z) = H_{LP2}(z); \quad DCL_1(z) = STF_2(z^{1/r}); DCL_2(z) = NTF_1(z) \quad (4)$$

Assuming $STF_{1,2}(z) = z^{-L_{1,2}}$ and $NTF_{1,2}(z) = (1 - z^{-1})^{L_{1,2}}$, it can be shown that the IBN power at the output of Fig. 2(b) is given by:

$$\text{IBN}_{2\text{-stage_DS_H-MR}} \approx \frac{\Delta^2}{12} \frac{\pi^{2(L_1+L_2)}}{r^{2L_2} \text{OSR}_1^{2(L_1+L_2)+1} [2(L_1+L_2)+1]} \quad (5)$$

which results in the same expression as a monorate case when $r = 1$.

B. Practical implementation of the proposed architecture

Note from (4) that, in order to get a correct performance of the modulator in Fig. 2(b), a perfect matching is required between the analog and the digital versions of $H_{LP}(z)$, i.e. $H_{LP1}(z)$ and $H_{LP2}(z)$, respectively. This matching requirement cannot be obtained in practice, thus resulting in a residual error that degrade the effective resolution of the modulator. This problem can be solved by using the proposed cascade MR H- $\Sigma\Delta$ M shown in Fig. 3(a). In this case, as the inter-stage (analog) AAF has been removed, the error signal, E_1 – which is sampled at the second-stage – will have aliased components at multiples of f_{s2} . This can be expressed in the Z-domain as:

$$E_1(z) = r \sum_{k=0}^{1/r-1} E_1(z^r e^{j2\pi k r}) \quad (6)$$

Note that, the aliased components are indeed additional quantization error components, and therefore, they can be also cancelled by properly designing the digital functions $H_{1,2}(z)$ for a given value of r . Thus, in the case of $r = 1/2$ and $r = 1/4$, it can be shown that both E_1 and its corresponding alias signal can be cancelled if the following condition is satisfied:

$$H_1 = H_2 = \begin{cases} NTF_1(-z) & (r = 1/2) \\ NTF_1(ze^{j\pi/2})NTF_1(-z)NTF_1(ze^{j3\pi/2}) & (r = 1/4) \end{cases} \quad (7)$$

Following the same procedure as in previous sections, it can be shown that the IBN at the output of the modulator in Fig. 3(a), is given by (5). However, on the contrary to the modulator in Fig. 2(b), the modulator in Fig. 3(a) does not need any additional analog block as compared to a conventional cascade H- $\Sigma\Delta$ M. Moreover, the cancellation condition in (7) is completely performed in the digital domain, instead of the analog domain as in (4), thus making the implementation of Fig. 3(a) more robust in practice.

Figure 3. Proposed downsampling MR H- $\Sigma\Delta$ M without analog AAF. (a) 2-stage cascade. (b) Generic N -stage cascade topology.

C. Extension to N -stages and synthesis of DS MR H- $\Sigma\Delta$ Ms

The concept of *downsampling* multirate H- $\Sigma\Delta$ Ms can be extended to a cascade of N stages as illustrated in Fig. 3(b), where the front-end (CT) stage is sampled at f_{s1} and the remaining $N-1$ stages are sampled at $f_{s2} < f_{s1}$. Thus, in the more general case, the quantization error of the front-end stage (including its aliasing component) can be completely cancelled if $H_i(z)$ are all equal and satisfy the condition in (7). In such a case, and assuming $NTF_1(z) = (1 - z^{-1})^{L_1}$, eq. (7) transforms into

$$H_1(z) = H_2(z) = \dots = H_N(z) = \left(\sum_{k=0}^{1/r-1} z^{-k} \right)^{L_1} \quad (8)$$

and the IBN at the output of the modulator in Fig. 3(b) is given by:

$$\text{IBN}_{\text{DS-H-MR}} \approx \frac{\Delta^2 \pi^{2L}}{12(2L+1)} \frac{1}{r^{2(L-L_1)+1} \text{OSR}_1^{2L+1}} \quad (9)$$

It is interesting to compare the above expression with the one obtained for a conventional multirate – given in (1). Thus, assuming that $\text{OSR}_i = \text{OSR}_2$ for $N \geq i > 2$ it can be shown that:

$$\frac{\text{IBN}_{\text{DS-H-MR}}}{\text{IBN}_{\text{US-H-MR}}} \approx \frac{1}{r_{\text{DS}}^{2(L-L_1)+1} r_{\text{US}}^{2L_1}} \left(\frac{\text{OSR}_{2\text{US}}}{\text{OSR}_{1\text{DS}}} \right)^{2L+1} \quad (10)$$

where $r_{\text{DS,US}}$ and $\text{OSR}_{1\text{DS,2US}}$ stand for the value of r and $\text{OSR}_{1,2}$ for the *downsampling* and *upsampling* MR H- $\Sigma\Delta$ M cases, respectively². Assuming $\text{OSR}_{1\text{DS,2US}} = \text{OSR}_{2\text{US}}^{2(2L_1-L-1/2)}$ and $r_{\text{DS}} = 1/r_{\text{US}}$, it is shown that expression in (10) reduces to $r_{\text{DS}}^{4L_1}$, meaning that the proposed architecture is better than the conventional one if $L_1 > L/2 + 1$. However, if we consider $\text{OSR}_{1\text{DS}} = k_{\text{OSR}} \text{OSR}_{2\text{US}}$ with $k_{\text{OSR}} = 1/r_{\text{DS}} = r_{\text{US}} > 1$, eq. (10) gives $r_{\text{DS}}^{4L_1} < 1$. That is, the proposed architecture achieves a better effective resolution than a conventional multirate architecture if we assume that the maximum value of OSR, i.e the OSR of the CT stage, is larger than the maximum OSR of the conventional one. This condition is indeed very common in practice since the CT front-end stage is potentially faster than the SC ones.

IV. EXAMPLES AND SIMULATION RESULTS

As an illustration of the proposed methodology, Fig. 4 shows the block diagram of two DS MR cascade H- $\Sigma\Delta$ Ms: 2-1 (Fig. 4(a)) and 2-2 (Fig. 4(b)). In all cases, the modulators have been synthesized starting from a complete DT topology and applying a DT-to-CT transformation to the front-end stage. These modulators have been simulated using SIMSIDES, a time-domain behavioral simulator for $\Sigma\Delta$ Ms [16] in order to verify the proposed methodology and architectures. As an illustration, Fig. 5 depicts the output spectra of the modulators in Fig. 4 for different values of r , showing the variation of the notch frequency associated to the multirate operation.

Fig. 6 represents IBN vs. OSR for different values of r and the number of bits of the internal quantizers (B), showing a good agreement between simulations and theoretical predictions. The impact of main circuit errors has been also considered. As an example, Fig. 7 shows a 500-sample Montecarlo simulation of the modulators in Fig. 4 for different values of r , considering a standard deviation of 1% in all analog coefficients. The IBN degradation is similar to the one obtained in conventional cascade $\Sigma\Delta$ Ms [1].

CONCLUSIONS

A new concept of multirate hybrid (CT/DT) cascade $\Sigma\Delta$ M has been presented. Contrary to conventional multirate architectures, the proposed $\Sigma\Delta$ M topologies use the highest sampling rate at the (CT)

² Note from the definition of r that $r_{\text{DS}} < 1$ and $r_{\text{US}} > 1$.

front-end stage of the modulator instead of the back-end one. This approach exploits the potential capability of CT circuits to operate at higher frequencies, whereas relaxes the speed requirements of the DT parts of the modulator. The main drawback comes from the aliasing due to the implicit downsampling operation that takes place between the front-end and subsequent stages of the modulator. This problem is circumvented by properly designing the digital cancellation logic of the modulator, without needing any extra analog circuits as compared to conventional cascade $\Sigma\Delta$ Ms. The resulting H- $\Sigma\Delta$ Ms are potentially faster and more power efficient than conventional multirate modulators, being a promising approach to digitize 10-100MHz signals with medium-high resolutions.

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Figure 4. Examples of the proposed DS MR cascade H- $\Sigma\Delta$ Ms:
(a) 2-1 architecture. (b) 2-2 architecture.

Figure 5. Output spectra of the modulators in Fig. 4: (a) 2-1. (b) 2-2.

Figure 6. IBN vs. OSR for different values of r and B :
(a) Modulator in Fig. 4(a). (b) Modulator in Fig. 4(b).

Figure 7. MonteCarlo Simulation of Fig. 4 for:
(a) $OSR_1=8$, $r=1/2$. (b) $OSR_1=16$, $r=1/4$.