Analysis of Steady-State Common-Mode Response in Differential LC-VCOs

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Abstract— This paper analyzes the common-mode response of LC voltage-controlled oscillators (VCOs) in DC and periodic steady state regimes. The dependence of the common-mode voltage (v_{cm}) on the oscillation amplitude is theoretically studied. Closed and simple expressions for v_{cm} suitable for the VCO design and optimization are derived. The agreement with transistor level simulations has been verified in a 1.2V low-power 90nm CMOS case of study.

Key words: RF, LC-VCO, Common-Mode Voltage, VCO design and optimization.

I. INTRODUCTION

Integrated LC voltage-controlled oscillators (VCOs) are common functional blocks in modern radio frequency communication applications and one of the most critical sub-systems in the RF section. During the past decade, the VCO design towards low phase-noise and low-power consumption has been covered in several publications [1]-[5]. A deep theoretical comprehension of the oscillation mechanisms and source of degradations is achieved. This includes the non-linear nature of the amplitude self-maintained sustenance [2], analysis of phase-noise [3] and VCO transient response [4]-[5].

In the case of LC-VCOs, attention has been basically paid to the differential mode [1]-[5]. The impact of the common-mode at the output of the VCO (v_{cm}) was not relevant since enough voltage margin was found in process nodes above 90nm. With technology scaling and trends towards low-voltage designs, the situation has changed and some new effects should be considered in the design and optimization process [6]. Actually, for a particular design, v_{cm} could suffer a voltage drop from 0 up to 100mV depending on the oscillation amplitude (A). If this variation is not well controlled, the VCO performance could be degraded by extra phase noise (since more harmonics become relevant for noise folding-down) and it could become more sensitive to voltage supplies.

Additionally, any voltage variation should be considered in the circuit connected at the output of a VCO, since v_{cm} defines its input common-mode. For instance, this is the typical case for I/Q phase generation using a DC-coupled divider-by-two (DIV2) [7]. Consequently, a good characterization of v_{cm} is fundamental for the VCO-DIV2 optimization.

In this work a theoretical study of the common-mode voltage is performed. The dependence of v_{cm} on A is analytically justified. Contents in the paper are distributed as follows. In Sect. II, the proposed LC-VCO model and followed notation are introduced. Section III analyzes the non-linear behavior of the VCO active section. A first order transistor model is considered to provide closed-form intuitive relationships. Both common and differential modes are covered for completeness. In Sect. IV, a real VCO design in 90nm with active DIV2 load is presented for validation. Finally, conclusions are drawn.

II. PROPOSED MODEL FOR LC-VCO ANALYSIS

Fig.1a shows a simplified schematic of a well-known CMOS LC-VCO. A case of study with complementary cross-coupled active section and PMOS current source (I_B) is considered for illustration purpose (results herein presented can be similarly derived for other VCO topologies). An equivalent model of the circuit, which describes both differential and common-mode, is depicted in Fig.1b. The proposed model is constituted by a memory-less active block [8] connected to a lossy resonator tank (L_p, C_p, R_p). This model implicitly assumes three basic hypotheses. These are: a) non quasi-static effects are negligible [9], and hence, the oscillation frequency ω_0 is well below the transistor f_T , b) the circuit has a differential topology symmetry, and therefore, the currents through terminals v_1 and v_2 are balanced, $I(v_1) = -I(v_2) = i_d$, and the tank common-mode current is negligible, and c) parasitics due to transistors and passive devices can be grouped on the tank elements, being their values,

$$\begin{aligned} L_p &= L \quad ; \quad C_p = C + C_{load} + 1 / 2(C_{MOS,P} + C_{MOS,N}) \\ R_p &= (G_{pL} + G_{pC} + g_{dsp} / 2 + g_{dsn} / 2)^{-1} \\ C_{MOS} &= 4C_{gd} + C_{gs} + C_{gb} + C_{db} \end{aligned} \quad (1)$$

where G_{pL} and G_{pC} are the parasitic parallel conductance of tank-inductor and tank-capacitor respectively. The rest of parameters correspond to MOS transistors.

Figure 1. a) Simplified LC-VCO schematic; b) proposed model.

A. General VCO Description

The memory-less active block in Fig.1b is in practice a four-terminal device. Bias v_a and ground nets must be explicitly included to describe the common-mode behavior. The response of this block will be generally given by a set of functions of the current and voltages at the different nets,

$$\mathbf{f}(i_d, I_B, v_1, v_2, v_a) = 0 \quad (2)$$

This set of functions is suitable for all instants in the VCO evolution, since the active-block does not have memory elements. In addition to this constrain, the circuit model in Fig.1b must satisfy the Kirchhoff's current law (KCL) for charge conservation, and hence,

$$i_d + i_L + i_C + v_d / R_p = 0; \quad i_C = C_p \frac{dv_d}{dt}; \quad v_d = L_p \frac{di_L}{dt} \quad (3)$$

Taking the derivative of the first term in (3) and substituting the inductor and capacitor laws, we arrive at,

$$\frac{d^2 v_d}{dt^2} + \frac{1}{R_p C_p} \frac{dv_d}{dt} + \frac{1}{L_p C_p} v_d + \frac{1}{C_p} \frac{di_d}{dt} = 0 \quad (4)$$

Given the initial capacitor and inductor conditions, the VCO dynamic response is fully characterized by (2), (4), and the definition $v_d = v_1 - v_2$. This implies solving an ordinary differential equation (ODE) joint with a non-differential non-linear equation set.

B. Particulation for Differential VCO Topologies

For differential VCO topologies, such as the example of Fig.1a, the previous general analysis can be drastically simplified. Actually, due to the active section symmetry, we propose to express the constitutive relationships in (2) as an uncoupled set of the following three functions,

$$\mathbf{f}(i_d, I_B, v_1, v_2, v_a) = 0 \Leftrightarrow \begin{cases} i_d = f_d(v_d, I_B) \\ v_{cm} = f_{cm}(v_d, I_B) \\ v_a = f_a(v_{cm}) \end{cases}, \quad \begin{cases} v_1 = v_{cm} + v_d / 2 \\ v_2 = v_{cm} - v_d / 2 \end{cases} \quad (5)$$

The details of the uncoupling method will be developed in Sect. III. Applying this equivalence to (4), it follows,

$$\frac{d^2 v_d}{dt^2} + \frac{1}{C_p} \left(\frac{1}{R_p} + \frac{\partial f_d}{\partial v_d} \right) \frac{dv_d}{dt} + \omega_0^2 v_d = 0; \quad \omega_0 = \frac{1}{\sqrt{L_p C_p}} \quad (6)$$

which is now a non-linear ODE in v_d that can be solved independently. Having its solution, the other variables are directly obtained by substitution in (5).

Traditional studies have exclusively focused on the differential part [1]-[5] and do not worry about the common-mode dependence [6]. For practical VCO designs verifying the start-up condition [8], $1/R_p < -(df_d/dv_d)|_{v_d=0}$, the stationary solution of (6) will be a periodical signal, generally given by,

$$v_d(t) = \sum_{n \geq 1} A_n \cos(n \omega_0 t + \phi_n) \quad (7)$$

where A_n and ϕ_n are the amplitude and phase parameters for the different Fourier's series terms. If the quality factor of the tank is much higher than one as usual, $Q = \omega_0 R_p C_p \gg 1$, a nearly sinusoidal is found in the stationary situation,

$$v_d(t) \approx A \cos(\omega_0 t + \phi) \quad (8)$$

In [4]-[5], closed-form equations for the amplitude A and oscillation frequency ω_0 using level-1 transistor models are derived. These relationships provide a very interesting information and intuitive view for the VCO design. In this work, we apply the same formalism to the analysis of the common-mode, that is, to obtain the curve $v_{cm} = f_{cm}(v_d, I_B)$. For completeness, the expressions of $v_d = f_d(v_d, I_B)$ and $v_a = f_a(v_{cm})$ will be also derived.

C. Common-Mode Analysis

Under our proposal, the function $v_{cm} = f_{cm}(v_d, I_B)$ describes the common-mode response $v_{cm}(t)$ at any time. In practice, only the stationary time average of this wave, V_{cm}^{ST} , is relevant, since current source I_B is usually decoupled from the active block, at any frequency, for correct VCO operation (C_{byp} in Fig. 1a). It is worth noticing that the v_{cm} voltage at quiescent state (v_{cm} at $v_d = 0$) could significantly vary from the value at steady state V_{cm}^{ST} [6]. Actually, up to 100mV difference respect to the expected quiescent point value could be found in current CMOS 90nm process, and therefore, the existing drop should be considered during design. This phenomenon can be theoretically evaluated using the function $v_{cm} = f_{cm}(v_d, I_B)$ under the oscillation voltage in (7). The output signal will be also periodic, but in this case with a half period due to even symmetry of f_{cm} function (see Sect. III). The resulting value in steady state condition, V_{cm}^{ST} , is given by the Fourier's series coefficient of order zero as,

$$V_{cm}^{ST} = \langle v_{cm}(t) \rangle = \frac{\omega_0}{\pi} \int_{t_0}^{t_0 + \pi/\omega_0} f_{cm}(v_d(t)) dt \quad (9)$$

which using (8) for nearly sinusoidal VCOs simplifies to,

$$V_{cm}^{ST}(A, I_B) = \frac{1}{\pi} \int_0^\pi f_{cm}(A \sin \theta, I_B) d\theta = \frac{2}{\pi} \int_0^{\pi/2} \frac{f_{cm}(x, I_B)}{\sqrt{A^2 - x^2}} dx \quad (10)$$

explicitly showing the dependence of the common-mode on the oscillation amplitude A .

III. MEMORY-LESS ACTIVE BLOCK DESCRIPTION

We have stated before that the memory-less active block is completely characterized by the non-linear functions f_d , f_{cm} and f_a . Obtaining this information requires, in general, the use of electrical simulations and numerical methods due to the complexity of the compact i - v relations of MOS transistors. Since f_d , f_{cm} and f_a are functions of a memory-less block, we can get the relationships between voltages and currents in static behavior, and then extend them for any regime. Test set-up for dc-sweep simulations consist of exciting the active block with a dc current source, i_d , between the block input terminals v_1 and v_2 . Then, voltages at nodes v_1 , v_2 (or v_d , v_{cm}) and v_a are taken for each i_d value considering bias current I_B as a fixed parameter. Finally, results are expressed according to (5) following the next general procedure.

A. General Derivation of the Active Block Functions

The derivation of the memory-less block constitutive function requires an analysis of the transistor current relationships,

$$I_B = i_{p1}^{(1)} + i_{p2}^{(2)} = i_{n1}^{(3)} + i_{n2}^{(4)} \quad ; \quad i_d = i_{n2}^{(3)} - i_{p2}^{(4)} = i_{p1}^{(1)} - i_{n1}^{(3)} \quad (11)$$

where i_{pk} is the drain output current of PMOS M_{pk} , and i_{nk} is the drain input current of NMOS M_{nk} in Fig. 1a. Taking into account the dependence of the MOS drain current on its terminal voltages, we can proceed as follows to get the wanted expressions. Considering the equality (2) in (11),

$$\Phi_c(v_1, v_2, I_B) = I_B - i_{n1}(v_1, v_2) - i_{n2}(v_2, v_1) = 0 \quad (12)$$

and substituting, $v_1 = v_{cm} + v_d/2$, $v_2 = v_{cm} - v_d/2$, the common-mode function f_{cm} is obtained by working out v_{cm} as,

$$\Phi_c(v_{cm}, v_d, I_B) = 0 \Rightarrow v_{cm} = f_{cm}(v_d, I_B) \quad (13)$$

The topological symmetry in this part of the active block implies that a change of sign in v_d will not change the value of v_{cm} , and therefore f_{cm} must be an even function of v_d . Applying the same substitutions in the equalities (3)-(4) in (11), we arrive at,

$$\begin{aligned} \Phi_a = i_{p1}(v_a, v_1, v_2) - i_{n1}(v_1, v_2) + i_{p2}(v_a, v_2, v_1) - i_{n2}(v_2, v_1) &= 0 \\ \Downarrow \\ \Phi_a(v_a, v_{cm}, v_d) = 0 &\Rightarrow v_a = f_a(v_{cm}) \end{aligned} \quad (14)$$

where the inverse of f_{cm} , $v_d = f_{cm}^{-1}(v_{cm}, I_B)$, is applied to get the relationship between v_a and v_{cm} . To conclude, the dependence of i_d on the differential voltage is derived from equality (4) in (11) considering the previous results, in the form,

$$i_d = i_{p1}(v_a, v_1, v_2) - i_{n1}(v_1, v_2) \Rightarrow i_d = f_d(v_d, I_B) \quad (15)$$

B. Closed-form Derivation using Level-1 Transistor Model

Solving the previous set of operations for MOS advanced level models could be a hard task and surely approximations by numerical methods have to be applied. Nevertheless, simple and closed forms can be derived when a level-1 MOS model is considered. As demonstrated below, this approach is good enough to describe an actual LC-VCO implementation whichever advanced level model it employs.

Let us assume, for further simplification and compactness of expressions, a common absolute value of the threshold voltages, equal effective transconductance, and no channel length modulation for NMOS and PMOS transistors,

$$V_T = V_{T,N} = -V_{T,P} \quad ; \quad \beta = k'_N \frac{W_N}{L_N} = k'_P \frac{W_P}{L_P} \quad ; \quad \lambda_N = \lambda_P = 0 \quad (16)$$

After analyzing all possible cases of operation regions for transistors in Fig. 1a, the complete behavior of the VCO can be summarized as a function of the differential voltage v_d excursion, and classified according to the parameter $V_B = \sqrt{(2I_B/\beta)}$. That is presented in Table I, where results for $v_d < 0$ are omitted due to their even symmetry. Two different situations depending on the parameter V_B are distinguished, being the case $V_B < V_T$ the most interesting in low-voltage low-power applications [10]. We focus on this situation.

Any couple of transistors diagonally opposite, (M_{P1}, M_{N2}) and (M_{P2}, M_{N1}) are always in the same operation region. This relationship is justified because gate and drain to source voltages, $|v_{GS}|$, $|v_{DS}|$, are always equal in these transistors. Attending to the voltage v_d value, three cases of study can be distinguished. In Case 1, v_d is close to 0 below V_B and all transistors remain in saturation. VCO quiescent state belongs to this case. When increasing v_d up to V_T , the circuit enters in the Case 2. In this situation, (M_{P2}, M_{N1}) maintains its operation, and (M_{P1}, M_{N2}) pass to cut-off region, since their voltage gates (v_1, v_2) increase and decrease, respectively. Finally, in Case 3, with v_d above V_T , the couple (M_{P2}, M_{N1}) enters in ohmic region while (M_{P1}, M_{N2}) continues in cut-off.

TABLE I. OPERATION REGIONS FOR TRANSISTORS IN ACTIVE BLOCK

Case	$V_B < V_T$		$V_B > V_T$	
	M_{P2}, M_{N1}	M_{P1}, M_{N2}	M_{P2}, M_{N1}	M_{P1}, M_{N2}
1: $0 < v_d < V_B$	saturation	saturation	saturation	saturation
2: $V_B < v_d < V_T$	saturation	cut-off	ohmic	saturation
3: $V_T < v_d < V_{DD}$	ohmic	cut-off	ohmic	cut-off

After this qualitative analysis, a mathematical description of the different operation conditions could be performed. Let us show the analysis of the Case 1 with $V_B < V_T$ as an example. In this case, if $0 \leq |v_d| \leq V_B$, all the transistors are in saturation. Considering a level-1 transistor model for this region in the procedure of Sect. III.A, we arrive at,

$$\begin{aligned} V_B^2 = (v_1 - V_T)^2 + (v_2 - V_T)^2 &\Rightarrow v_{cm} = V_T + \sqrt{2V_B^2 - v_d^2} / 2 \\ (v_1 - V_T)^2 + (v_2 - V_T)^2 &= (v_a - v_1 - V_T)^2 + (v_a - v_2 - V_T)^2 \Rightarrow v_a = 2v_{cm} \quad (17) \\ i_d = \frac{\beta}{2} ((v_a - v_1 - V_T)^2 - (v_1 - V_T)^2) &= -\frac{\beta}{2} v_d \sqrt{2V_B^2 - v_d^2} \end{aligned}$$

Using a similar handling for the rest of cases (omitted for space limitation), the constitutive functions of the active block with $V_B = \sqrt{(2I_B/\beta)}$ are finally derived as:

$$\begin{aligned} \bullet \quad v_{cm} = f_{cm}(v_d, I_B) = & \begin{cases} V_T + \sqrt{2V_B^2 - v_d^2} / 2, & \text{if } 0 \leq |v_d| \leq V_B \\ V_T + V_B - |v_d| / 2, & \text{if } V_B \leq |v_d| \leq V_T \\ V_T - |v_d| / 2 + \sqrt{(|v_d| - V_T)^2 + V_B^2}, & \text{if } V_T \leq |v_d| \leq V_{DD} \end{cases} \quad (18) \end{aligned}$$

$$\bullet \quad i_d = f_d(v_d, I_B) = \begin{cases} -\beta v_d \sqrt{2V_B^2 - v_d^2} / 2, & \text{if } 0 \leq |v_d| \leq V_B \\ -I_B \text{sign}(v_d), & \text{if } V_B \leq |v_d| \leq V_{DD} \end{cases} \quad (19)$$

$$\bullet \quad v_a = f_a(v_{cm}) = 2v_{cm}, \quad \forall |v_d| \leq V_{DD}$$

In order to validate the theoretical predictions using (10) and (18), a VerilogA transistor level-1 model was implemented under Cadence FrameWork II. Figure 2 shows the comparison with Spectre RF results for both the DC and transient steady state simulations of a LC-VCO implementation. The value of v_{cm} versus the differential voltage v_d and the V_{cm}^{ST} dependence on the oscillation amplitude A are depicted, respectively. This implementation uses $V_{DD}=1.2V$, $I_B=320\mu A$, $V_T=0.38V$ and $\beta=65.17mA/V^2$, $L=1nH$, $C=1pF$ and $C_{load}=0F$. The oscillation amplitude has been controlled assuming different parasitic parallel resistances of the tank, $R_p \in [0.22, 2.95]k\Omega$. The discrepancy between theoretical and

Figure 2. Comparison between theoretical predictions and Spectre RF results using a VerilogA model ($\lambda = 0$ to $0.5V^{-1}$ in steps of $0.1V^{-1}$).

simulation results is almost negligible even if channel length modulation is considered, $\lambda \in [0, 0.5]V^{-1}$.

IV. AN EXAMPLE IN A RF 90NM CMOS PROCESS

In this section, the theoretical analysis of the common-mode is validated in a real VCO design in a RF 90nm CMOS process. The VCO, depicted in Fig.3, uses a 1nH differential inductor and MIM capacitor array with 1pF total capacitance. A DIV2 based on two source-coupled latches (SCL) is considered as active load. Figure 4 shows the transient evolution of the VCO common-mode and the corresponding oscillation voltage. As expected from (18), v_{cm} varies from the initial quiescent state as a function of the oscillation amplitude A . This behavior could degrade the performance of the VCO due to the additional phase noise, but it becomes critical for the DIV2 operation, since v_{cm} defines its input level in a DC-coupled scheme. The effect has been traditionally overcome by extra design margin, since VCO-DIV2 co-simulation was only performed at the final design steps. Early co-simulation with Spectre RF results is not time-effective. Actually, more than $\frac{1}{2}$ hour (Intel Core II 2.6GHz CPU) is need for a single iteration on the VCO design in nominal conditions.

In our work this simulation time has been reduced in several orders of magnitude without a significant accuracy penalty, and hence, making VCO-DIV2 co-design feasible early in the design phase. Figure 5 repeats the simulation in Fig. 2 and it includes: a) the characteristic of the common-mode f_{cm} using a simple DC-sweep simulation, b) the dynamic response evaluated in SpectreRF from 50 periodic steady state analysis, c) our theoretical prediction using (10) over the curve a). The computation time for the theoretical curve is below 1s. Notice that the error between b) and c) is in the worst case below 18mV.

Figure 3. Simplified schematic of the LC-VCO with DIV2 load.

Figure 4. Transient evolution of v_{cm} showing the dependence on v_d .

Figure 5. Comparison between theoretical predictions and Spectre RF results for the 90nm CMOS case of study.

This discrepancy is justified by the drastic topological reduction of the circuit in Fig. 1a respect to the model in Fig. 1b.

V. CONCLUSIONS

A theoretical study of the stationary common-mode in LC VCOs is done in this work. The dependence of the common-mode voltage, v_{cm} , on the oscillation amplitude is theoretically studied. Closed and simple expressions for v_{cm} suitable for the VCO design and optimization are derived. They provide an accurate prediction of the v_{cm} drops which minimizes risk in current low-voltage low-power applications at an early design phase due to the reduction of voltage room.

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