Rotational motions in homogeneous anisotropic elastic media

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ABSTRACT

Rotational motions in homogeneous anisotropic elastic media are studied under the assumption of plane wave propagation. The main goal is to investigate the influences of anisotropy in the behavior of the rotational wavefield. The focus is on P-waves that theoretically do not generate rotational motion in isotropic media. By using the Kelvin–Christoffel equation, expressions are obtained of the rotational motions of body waves as a function of the propagation direction and the coefficients of the elastic modulus matrix. As a result, the amplitudes of the rotation rates and their radiation patterns are quantified and it is concluded that (1) for strong local earthquakes and typical reservoir situations quasi P-rotation rates induced by anisotropy are significant, recordable, and can be used for inverse problems; and (2) for teleseismic wavefields, anisotropic effects are unlikely to be responsible for the observed rotational energy in the P coda.

INTRODUCTION

Seismological studies are traditionally based on the observation, processing, and inversion of three orthogonal components of translational ground motions (displacement, velocity, or acceleration). In theory, to fully describe the motion of the solid volume around a point, one needs to consider not only three components of translation but also six components of strain and three components of rotations (see Aki and Richards, 2002; Stein and Wysession, 2003; Cochard et al., 2006). Although geophysical studies associated with translational motions and strain have attained a large success, the full benefits of rotational motions are still under investigation, and this type of signal has been ignored for a long time. This is mainly because sen-

sors for rotational motions with sufficient resolution were not available until recently. After advances in laser technology, instruments that consistently record rotational ground motions around one axis or all three orthogonal axes have already been developed and tested (McLeod et al., 1998; Pancha et al., 2000; Schreiber et al., 2006, 2009; Wassermann et al., 2009). As a consequence, studies that aim to exploit the additional information given by collocated measurements of translational and rotational ground motion are growing rapidly (e.g., Igel et al., 2005, 2007; Cochard et al., 2006; Suryanto et al., 2006; Pham et al., 2009a, 2009b; Ferreira and Igel, 2009; Fichtner and Igel, 2009; Bernauer et al., 2009). For example, several recent studies covering many aspects of rotational ground motions have been presented in a special issue on Rotational Seismology and Engineering Applications of the *Bulletin of the Seismological Society of America* in May 2009 (Lee et al., 2009b). In particular, a review by Lee et al. (2009a) envisions the emergence of a new branch in the fields of observational seismology and earthquake engineering.

Once the technology for its measurement has become available, the key question is: What kind of information do we expect to retrieve from carrying out joint measurements of translations and rotations — compared with just translational observations? Early studies (Takeo and Ito, 1997) indicate that earthquake rupture histories can be better constrained with measurements of rotational motions. Furthermore, the multicomponent point measurements of rotational and translational motions allow the estimation of wavefield properties (e.g., phase velocities, propagation direction; Igel et al., 2007; Cochard et al., 2006) and allow the recovery of shear-wave structure without traveltimes information (Fichtner and Igel, 2009; Bernauer et al., 2009) as well as constraining the scattering properties of the near-receiver crustal structure (Pham et al., 2009b).

An aspect that has been missing in previous studies is the role that the elastic anisotropy might play in the rotational measurements. It is well known that anisotropy is a ubiquitous phenomenon in the

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earth's crust and upper mantle. Many authors (e.g., Crampin, 1981, 1984; Crampin et al., 1982, 1984; Sams et al., 1993; Larner, 1993) report the influence of anisotropic effects on seismic processing and interpretation. Tsvankin (1996) emphasized that progress in seismic inversion and processing in anisotropic media depend on our ability to relate different seismic signatures to the anisotropy parameters. In reservoir and drilling engineering, the simultaneous determination of the elastic coefficients and anisotropy parameters helps in predicting flow paths for improved oil recovery, designing a hydraulic fracturing scheme, determining mud weight and selecting drill bit, and preventing hole collapse during and after drilling (Wang, 2002). At present, the anisotropy parameters can only be retrieved using laboratory measurements or arrays of standard translational seismometers deployed, at least, along a line.

At the onset of this study, there are several open questions: (1) How does anisotropy affect rotational ground motions? (2) Can rotational motions help to constrain anisotropic properties? (3) What instrument configurations are necessary to extract the relevant information? We will attempt to give preliminary answers to these questions in the following sections.

The main goal of this study is to estimate the amplitudes of the rotational motions that we can expect in various geophysical problems and their dependence on the anisotropic properties of the medium and the propagation direction. We first approach the problem in an analytical way, focusing on the solution for plane waves in linear elastic anisotropic media. Moreover, we provide estimates of the expected amplitudes of rotational motions induced by P-waves as a function of the degree of anisotropy. Last but not least, this study confirms the fact that anisotropy alone cannot account for the P coda vertical rotational motions observed on a ring laser at the Fundamental station in Wettzell, Germany, as reported by Igel et al. (2007) and Pham et al. (2009b).

FUNDAMENTAL THEORY

In the framework of classical elasticity, a general plane-harmonic-wave solution for the displacement vector $\mathbf{u}$ of body waves is

$$\mathbf{u} = A \mathbf{n} \exp[i(\omega t - \boldsymbol{\kappa} \cdot \mathbf{x})], \quad (1)$$

where A is the ground displacement peak amplitude; $\mathbf{n} = n_1 \mathbf{e}_1 + n_2 \mathbf{e}_2 + n_3 \mathbf{e}_3$ is the unit vector denoting the direction of the particle displacement (i.e., the wavefield polarization; $\mathbf{e}_i$ ($i = 1, 2, 3$) are the unit vectors along the coordinate axes); $n_i = u_i / \sqrt{u_1^* u_1 + u_2^* u_2 + u_3^* u_3} = \|u_i\|/A$ are the direction cosines of $\mathbf{n}$ and i is the imaginary unit; superscript $*$ represents complex conjugate; and $\|z\|$ stands for magnitude of the complex number z . Furthermore, the wavenumber vector is $\boldsymbol{\kappa} = \omega/v(\ell_1 \mathbf{e}_1 + \ell_2 \mathbf{e}_2 + \ell_3 \mathbf{e}_3) = \omega/v \mathbf{l}$ and the quantities ω , v , and $\mathbf{l} = \ell_1 \mathbf{e}_1 + \ell_2 \mathbf{e}_2 + \ell_3 \mathbf{e}_3$ are, respectively, the angular frequency, the phase velocity, and the unit vector denoting the propagation direction of the plane wave. Finally, the direction cosines of $\mathbf{l}$ are denoted ℓ_i .

The rotational motions $\boldsymbol{\Omega}$ generated by the plane wave expressed by equation 1 can be calculated by taking half of the curl of the displacement field (Igel et al., 2005; Cochard et al., 2006)

$$\begin{aligned} \boldsymbol{\Omega} &= \frac{1}{2}(\nabla \times \mathbf{u}) = -\frac{A}{2} i \boldsymbol{\kappa} \times \mathbf{n} \exp[i(\omega t - \boldsymbol{\kappa} \cdot \mathbf{x})] \\ &= -\frac{A}{2} \frac{i \omega}{v} \begin{pmatrix} \ell_2 n_3 - \ell_3 n_2 \\ \ell_3 n_1 - \ell_1 n_3 \\ \ell_1 n_2 - \ell_2 n_1 \end{pmatrix} \exp\left[i \omega \left(t - \frac{1}{v} \mathbf{l} \cdot \mathbf{x}\right)\right]. \end{aligned} \quad (2)$$

The corresponding rotation rates $\dot{\boldsymbol{\Omega}}$ are then

$$\dot{\boldsymbol{\Omega}} = \partial_t \boldsymbol{\Omega} = \frac{A}{2} \frac{\omega^2}{v} \begin{pmatrix} \ell_2 n_3 - \ell_3 n_2 \\ \ell_3 n_1 - \ell_1 n_3 \\ \ell_1 n_2 - \ell_2 n_1 \end{pmatrix} \exp\left[i \omega \left(t - \frac{1}{v} \mathbf{l} \cdot \mathbf{x}\right)\right]. \quad (3)$$

Looking at equation 3, one can recognize that, in homogeneous unbounded isotropic media, P-waves do not generate any rotational motion. This is because, for P-waves, the vectors $\mathbf{l}$ and $\mathbf{n}$ are identical, in opposition to S-waves, which have perpendicular $\mathbf{l}$ and $\mathbf{n}$ and hence produce rotation motion. Hence, in unbounded isotropic media, rotations can only be the consequence of an S-wave. However, this is no longer the case in anisotropic elastic media in which the phase velocity v and the direction cosines n_i depend on the wave propagation direction ℓ_i . Hence, P-waves in anisotropic media can, in general, produce rotational motions. As a consequence, the measurement of significant rotational signals during the passage of a P-wave can be regarded as an indicator of anisotropy.

To understand the behavior of the rotational motions in anisotropic elastic media, we must first realize that the phase velocity v , the polarization n_i , and the material properties are bound by the Kelvin–Christoffel equation (see e.g., Musgrave, 1970; Carcione, 2001, p. 11)

$$(\boldsymbol{\Gamma} - \rho v^2 \mathbf{I}_3) \cdot \mathbf{u} = 0, \quad (4)$$

where ρ is the material density — a constant isotropic parameter; $\mathbf{I}_3$ is the 3×3 identity matrix; $\boldsymbol{\Gamma}$ is the symmetric Kelvin–Christoffel matrix

$$\boldsymbol{\Gamma} = \mathbf{L} \cdot \mathbf{C} \cdot \mathbf{L}^T, \quad (5)$$

$$\mathbf{L} = \begin{pmatrix} \ell_1 & 0 & 0 & 0 & \ell_3 & \ell_2 \\ 0 & \ell_2 & 0 & \ell_3 & 0 & \ell_1 \\ 0 & 0 & \ell_3 & \ell_2 & \ell_1 & 0 \end{pmatrix}, \quad (6)$$

and $\mathbf{C}$ is the elastic modulus matrix and is given by

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{pmatrix}. \quad (7)$$

Whenever a material possesses more than two independent parameters defining the entries of $\mathbf{C}$, the seismic wave velocities differ depending on the propagation direction and the material is called “anisotropic.” A material that has all 21 coefficients in $\mathbf{C}$ independent is called a material of the triclinic symmetry class, or simply tri-

clinic material. This is the most general case of anisotropy and includes as special cases all of the other crystalline symmetry classes (i.e., monoclinic, trigonal, tetragonal, orthorhombic, hexagonal, cubic, and isotropic). One of the most important anisotropy symmetry classes for seismological purposes is transverse isotropy. When the symmetry axis coincides with $\mathbf{e}_3$ the elastic modulus matrix $\mathbf{C}_{\text{TI}}$ of a transversely isotropic material is (see e.g., Thomsen, 1986; Carcione, 2001, p. 6)

$$\mathbf{C}_{\text{TI}} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \quad 2c_{66} = c_{11} - c_{12}. \quad (8)$$

The isotropic case can be considered as a special case of anisotropy in which $c_{11} = c_{22} = c_{33} = \lambda + 2\mu$, $c_{12} = c_{13} = c_{23} = \lambda$, $c_{44} = c_{55} = c_{66} = \mu$ and all of the other coefficients are equal to zero. Here λ and μ are the Lamé coefficients of the material.

Explicitly, the components of the Kelvin–Christoffel matrix are

$$\begin{aligned} \Gamma_{11} &= c_{11}\ell_1^2 + c_{66}\ell_2^2 + c_{55}\ell_3^2 + 2c_{56}\ell_2\ell_3 + 2c_{15}\ell_3\ell_1 \\ &\quad + 2c_{16}\ell_1\ell_2 \\ \Gamma_{22} &= c_{66}\ell_1^2 + c_{22}\ell_2^2 + c_{44}\ell_3^2 + 2c_{24}\ell_2\ell_3 + 2c_{46}\ell_3\ell_1 \\ &\quad + 2c_{26}\ell_1\ell_2 \\ \Gamma_{33} &= c_{55}\ell_1^2 + c_{44}\ell_2^2 + c_{33}\ell_3^2 + 2c_{34}\ell_2\ell_3 + 2c_{35}\ell_3\ell_1 \\ &\quad + 2c_{45}\ell_1\ell_2 \\ \Gamma_{12} &= c_{16}\ell_1^2 + c_{26}\ell_2^2 + c_{45}\ell_3^2 + (c_{46} + c_{25})\ell_2\ell_3 \\ &\quad + (c_{14} + c_{56})\ell_3\ell_1 + (c_{12} + c_{66})\ell_1\ell_2 \\ \Gamma_{13} &= c_{15}\ell_1^2 + c_{46}\ell_2^2 + c_{35}\ell_3^2 + (c_{45} + c_{36})\ell_2\ell_3 \\ &\quad + (c_{13} + c_{55})\ell_3\ell_1 + (c_{14} + c_{56})\ell_1\ell_2 \\ \Gamma_{23} &= c_{56}\ell_1^2 + c_{24}\ell_2^2 + c_{34}\ell_3^2 + (c_{44} + c_{23})\ell_2\ell_3 \\ &\quad + (c_{36} + c_{45})\ell_3\ell_1 + (c_{25} + c_{46})\ell_1\ell_2. \end{aligned} \quad (9)$$

It is clear that equation 4 is an eigenequation for the eigenvalues $(\rho v^2)_m$ and eigenvectors $(\mathbf{u})_m$, $m = 1, 2, 3$. The dispersion relation is expressed by

$$\det(\mathbf{\Gamma} - \rho v^2 \mathbf{I}_3) = 0, \quad (10)$$

from which we can obtain the three phase velocities v_m ($m = 1, 2, 3$) as a function of the direction cosines ℓ_i and the independent coefficients of the elastic modulus matrix $\mathbf{C}$. These velocities correspond to the three body wave modes propagating in an unbounded homogeneous medium: the highest value of v_m corresponds to a quasi-P-wave (qP), whereas the other two correspond to quasi-S-waves qS1 and qS2, usually ordered in descending v_m value. For each solution v , we obtain from equation 4 an associated eigenvector $\mathbf{u}$; hence, the polarizations n_i are fully determined. As a consequence, finding the rotation rates produced by any plane wave is possible for a given an-

isotropic material. First, we set a propagation direction and wave mode, so that we can find the values v and n_i using equations 10 and 4. Substituting these values into equation 3 and setting a value for the wave's peak displacement A and frequency ω , one finally obtains the three components of the rotation rate $\dot{\Omega}$.

We can now use the aforementioned process in an effort to visualize the influence that anisotropy has in the behavior of the rotational wavefield. In Figure 1, we use a plotting similar to that used by de la Puente (2008) to depict the variability of the phase velocities and peak rotation rates depending on the propagation direction. In particular, we show the values of peak rotation rates around two axes θ and ϕ generated by plane waves in a transversely isotropic (TI) medium. The three axes $\mathbf{l}$ (corresponding to the propagation direction), θ , and ϕ of an orthogonal Cartesian coordinate system are defined in Appendix A. For the particular case of a wave propagating in the x direction, we have $\mathbf{l} = \mathbf{e}_1$, $\theta = \mathbf{e}_3$ and $\phi = \mathbf{e}_3$, so that the three directions coincide with the Cartesian axes x , y , and z . The rotation rates in all cases are obtained by rotating the three components of the rotation rates that we obtained using equation 3, as shown in Appendix B. We assume plane harmonic waves of peak displacement $A = 10^{-5}$ m and period $T = 1$ s propagating in Mesaverde clay shale. The elastic parameters of the material are taken from Thomsen (1986) and are presented in Table 1. The plots show the highest values (phase velocity or rotation rates around θ and ϕ components) for each propagation direction. Our calculations for this particular material reveal that, for all qP-, qS1-, and qS2-waves, no rotational motion around the wave-propagation direction $\mathbf{l}$ is generated. The same result (i.e., the component of the rotation rate vector in the propagation direction $\mathbf{l}$ is zero) is also obtained for the monoclinic, orthorhombic, and tri-

Figure 1. Example of isosurfaces of the peak phase velocity in km/s (left column) and peak rotation rate around two axes θ and ϕ (middle and right columns) in nrad/s as a function of the wave propagation direction. The plots shown correspond to qP- (top row), qS1- (middle row), and qS2- (bottom row) waves. A plane harmonic wave of peak displacement 10^{-5} m and period $T = 1$ s propagating in a TI material (Mesaverde clay-shale) was assumed. The propagation direction $\mathbf{l}$ of the plane wave together with the θ and ϕ axes create an orthogonal Cartesian axis system. The vertical propagation direction in this example coincides with the symmetry axis of the TI material. For all three wave types, no rotational motion around the propagation direction $\mathbf{l}$ is generated.

clinic materials for which the elastic coefficients were mentioned in [de la Puente \(2008\)](#). In fact, this result follows directly from equation 3 if we project the value of the rotation rate on the $\mathbf{l}$ direction, which results in $\mathbf{l} \cdot \dot{\boldsymbol{\Omega}} = 0$, regardless of the material's properties.

Figure 1 shows us that the phase velocity and rotation rates strongly depend on the wave's propagation direction. The anisotropic behavior is clearly revealed by the apparition of rotational motions caused by qP-waves, which should be zero for all directions in homogeneous isotropic materials, as we mentioned previously. This again suggests that qP rotational motions contain information on anisotropic properties of the medium. It should be noted that there is more than one order of magnitude difference between the rotational amplitude of qP-waves and that of qS-waves in this example. Thus, to know if qP rotational motions can be used for inversion, it is fundamental to investigate the magnitude of the rotation rates associated with qP for different levels of anisotropy.

Although there are several symmetry classes in anisotropy, the elastic anisotropy of rocks is usually moderate and, in most cases, a TI symmetry is sufficient to describe its effects in the wavefield (e.g., [Tsvankin, 1996](#); [Guéguen and Sarout, 2009](#)). Moreover, transverse isotropy is also acknowledged as the anisotropic case of broadest geophysical applicability ([Thomsen, 1986](#)). Thus, hereafter in this study our investigations focus only on the rotational motions caused by qP-waves in the particular case of a $(\mathbf{e}_1, \mathbf{e}_3)$ plane of a TI medium with $\mathbf{e}_3$ as the axis of symmetry.

Before going further, we attempt to verify the analytical equations that we have. We will compare in the following section the peak rotation rates of qP-waves obtained using this analytical solution with the results obtained from numerical simulations using the ADER-DG method (the combination of a Discontinuous Galerkin finite element method and an Arbitrary high-order DERivative time integration approach developed by [Dumbser and Käser, 2006](#) and extended to anisotropic materials by [de la Puente et al., 2007](#)).

qP ROTATIONAL MOTIONS IN TI MEDIA

When considering plane P-waves propagating in the $(\mathbf{e}_1, \mathbf{e}_3)$ plane ($\ell_2 = 0, n_2 = 0$) of a TI medium whose symmetry axis is $\mathbf{e}_3$, equation 3 can be rewritten as

$$\dot{\boldsymbol{\Omega}} = \frac{A}{2} \frac{\omega^2}{v} \begin{pmatrix} 0 \\ \ell_3 n_1 - \ell_1 n_3 \\ 0 \end{pmatrix} \exp \left[i\omega \left(t - \frac{1}{v} \mathbf{l} \cdot \mathbf{x} \right) \right], \quad (11)$$

and the Kelvin-Christoffel equation 4 can be simplified as

$$\begin{pmatrix} c_{11}\ell_1^2 + c_{55}\ell_3^2 - \rho v^2 & 0 & (c_{13} + c_{55})\ell_1\ell_3 \\ 0 & c_{66}\ell_1^2 + c_{55}\ell_3^2 - \rho v^2 & 0 \\ (c_{13} + c_{55})\ell_1\ell_3 & 0 & c_{55}\ell_1^2 + c_{33}\ell_3^2 - \rho v^2 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 0. \quad (12)$$

Equation 12 shows the dispersion relation for coupled waves (qP and qSV)

$$(c_{11}\ell_1^2 + c_{55}\ell_3^2 - \rho v^2)(c_{55}\ell_1^2 + c_{33}\ell_3^2 - \rho v^2) - (c_{13} + c_{55})^2 \ell_1^2 \ell_3^2 = 0, \quad (13)$$

that allows estimating the phase velocities v of qP-waves as (see e.g., [Tsvankin, 1995a](#); [Carcione, 2001](#))

$$v = (2\rho)^{-1/2} \sqrt{c_{11}\ell_1^2 + c_{33}\ell_3^2 + c_{55} + D}, \quad (14a)$$

D

$$= \sqrt{[(c_{11} - c_{55})\ell_1^2 + (c_{55} - c_{33})\ell_3^2]^2 + 4[(c_{13} + c_{55})\ell_1\ell_3]^2}. \quad (14b)$$

The polarization of qP can be inferred from equations 12 and 13

$$\frac{n_1}{n_3} = \frac{u_1}{u_3} = s \sqrt{\frac{c_{55}\ell_1^2 + c_{33}\ell_3^2 - \rho v^2}{c_{11}\ell_1^2 + c_{55}\ell_3^2 - \rho v^2}}, \quad (15)$$

where $s = -1$ if $\ell_1\ell_3(c_{11}\ell_1^2 + c_{55}\ell_3^2 - \rho v^2) \geq 0$; $s = 1$ if $\ell_1\ell_3(c_{11}\ell_1^2 + c_{55}\ell_3^2 - \rho v^2) < 0$.

In the $(\mathbf{e}_1, \mathbf{e}_3)$ plane $u_2 = 0$, using $n_1^2 + n_3^2 = 1$ and equation 15 we obtain

$$n_1 = s \sqrt{\frac{c_{55}\ell_1^2 + c_{33}\ell_3^2 - \rho v^2}{c_{11}\ell_1^2 + c_{33}\ell_3^2 + c_{55} - 2\rho v^2}}, \quad (16)$$

$$n_3 = \sqrt{\frac{c_{11}\ell_1^2 + c_{55}\ell_3^2 - \rho v^2}{c_{11}\ell_1^2 + c_{33}\ell_3^2 + c_{55} - 2\rho v^2}}. \quad (17)$$

Substituting equations 14a, 16, and 17 into equation 11, the rotation rates induced by plane P-waves propagating in the $(\mathbf{e}_1, \mathbf{e}_3)$ plane of a TI material with $\mathbf{e}_3$ as the axis of symmetry can be expressed as a function of the propagation direction as

$$\dot{\boldsymbol{\Omega}} = A\omega^2 \sqrt{\frac{\rho}{2(c_{11}\ell_1^2 + c_{33}\ell_3^2 + c_{55} + D)}} \begin{pmatrix} 0 \\ \ell_3 n_1 - \ell_1 n_3 \\ 0 \end{pmatrix} \times \exp \left[i\omega \left(t - \frac{1}{v} \mathbf{l} \cdot \mathbf{x} \right) \right], \quad (18a)$$

$$n_1 = s \sqrt{\frac{D - (c_{55} - c_{11})\ell_1^2 - (c_{33} - c_{55})\ell_3^2}{2D}}, \quad (18b)$$

$$n_3 = \sqrt{\frac{D + (c_{55} - c_{11})\ell_1^2 + (c_{33} - c_{55})\ell_3^2}{2D}}, \quad (18c)$$

$$s = -1 \quad \text{if} \quad \ell_1\ell_3[(c_{11} - c_{55})\ell_1^2 + (c_{55} - c_{33})\ell_3^2 - D] \geq 0, \quad (18d)$$

Table 1. Elastic coefficients for the TI material (Mesaverde clay shale), given in $[10^9 \text{ N} \cdot \text{m}^{-2}]$.

ρ	c_{11}	c_{12}	c_{13}	c_{22}	c_{23}	c_{33}	c_{44}	c_{55}	c_{66}
2590	66.6	19.7	39.4	66.6	39.4	39.9	10.9	10.9	23.45

The material density ρ is given in $[\text{kg} \cdot \text{m}^{-3}]$. All of the other coefficients are zero.

$$s = 1 \quad \text{if} \quad \ell_1 \ell_3 [(c_{11} - c_{55})\ell_1^2 + (c_{55} - c_{33})\ell_3^2 - D] < 0. \quad (18e)$$

Assuming that ψ is the angle between the wave propagation direction $\mathbf{l}$ and the symmetry axis $\mathbf{e}_3$ of the TI medium, we have

$$\ell_1 = \sin \psi \quad (19a)$$

$$\ell_3 = \cos \psi. \quad (19b)$$

This substitution allows us to calculate the rotation rates induced by plane P-waves propagating in the $(\mathbf{e}_1, \mathbf{e}_3)$ plane of a TI material as a function of ψ .

To evaluate the correctness of the analytical solution, we model the synthetic seismograms created by a plane P-wave of dominant period of 1 s propagating upward through a TI medium that the symmetry axis belongs to the $(\mathbf{e}_1, \mathbf{e}_3)$ plane. A model of 30,000 m length, 30,000 m width, and 20,000 m depth is used. The material chosen is Mesaverde clay shale (Table 1). To check the effect of different incidence angles, we rotate the symmetry axis of the material a full circle inside the $(\mathbf{e}_1, \mathbf{e}_3)$ plane with steps of 5° . Such rotation of the material, having fixed the propagation direction of the plane wave, is equivalent to a test in which the material is fixed and the wave propagation direction $\mathbf{l}$ is rotated. In total, we investigate 72 different cases characterized by the angle between the material's symmetry axis and the vertical axis, ranging from 0° to 360° .

Six-component seismograms (three components of rotation rates and three components of translational velocities) are calculated using the ADER-DG method. The modeling parameters are detailed in Table 2.

The simulation results show that there is only rotational signal around the $\mathbf{e}_2$ axis, as we expected from theory (see equation 18). We superimpose the normalized peak rotation rates around the $\mathbf{e}_2$ axis obtained from the simulations and the analytic approach as a function of the angle ψ between the wave propagation direction and the symmetry axis of the material in a polar coordinate system. The excellent fit between both results presented in Figure 2 indicates the consistency of the analytical and numerical solutions.

We summarize that, for homogeneous anisotropic elastic media, we can estimate the rotational motions caused by a certain plane wave as a function of propagation direction and elastic coefficients. The anisotropic behavior is clearly revealed by the appearance of qP rotational motions. In the following, we extend the analytical equa-

tions to be able to infer the magnitude of the rotation rates associated with qP depending on the degree of anisotropy of the material.

MAGNITUDE OF qP ROTATIONS IN TERMS OF THOMSEN PARAMETERS

As reported by Thomsen (1986), for the case of transverse isotropy, an alternative and more descriptive set of parameters can be used to fully describe the medium's properties in substitution of the five elastic coefficients presented in equation 8. These parameters are the two elastic equivalent quantities α_0 and β_0 and three coefficients γ , ϵ , and δ^* (usually called Thomsen parameters) expressed as follows

$$\alpha_0 = \sqrt{\frac{c_{33}}{\rho}}, \quad (20)$$

$$\beta_0 = \sqrt{\frac{c_{55}}{\rho}}, \quad (21)$$

$$\epsilon = \frac{c_{11} - c_{33}}{2c_{33}}, \quad (22)$$

$$\gamma = \frac{c_{66} - c_{55}}{2c_{55}}, \quad (23)$$

Figure 2. Variations of the normalized peak rotation rate induced by a qP-wave of period $T = 1$ s propagating in a vertical plane of a TI medium (Mesaverde clay shale) as a function of the angle ψ between the wave propagation direction and the symmetry axis of the medium. The results are presented in a polar coordinate system. Solid line = analytic results, dash-dot line = numerical simulation results using the ADER-DG method.

Table 2. The modeling parameters used in this study.

Mesh type	Hexahedral
Element edge length	1000 m
Total number of elements	18,000
Polynomial degree inside elements	3
Number of processors	64
Length of seismograms	5 s
Boundary conditions	Absorbing (top), inflow (bottom), periodic (sides)
Average time step	1.4085×10^{-2} s
Run time per simulation	~ 50 s

$$\delta^* = \frac{1}{2c_{33}^2} [2(c_{13} + c_{55})^2 - (c_{33} - c_{55})(c_{11} + c_{33} - 2c_{55})]. \quad (24)$$

The coefficients γ , ε , and δ^* are dimensionless anisotropy parameters that fully characterize the strength of the anisotropy. Thus, to quantify the magnitude of qP rotations in anisotropic elastic media, in this section we investigate the dependence of the peak rotation rate values of qP-waves on these Thomsen parameters.

As mentioned by Tsvankin and Thomsen, 1994 and Tsvankin, 1995b, the parameter γ does not affect the propagation of the qP- or the qSV-waves in TI media. Only the SH mode depends on this parameter. This can also be recognized from equation 13. For the coupled waves, neither the phase velocities nor the Kelvin–Christoffel equations depend on c_{66} . As a consequence, the rotation rates induced by qP-waves are also independent of c_{66} (see equation 18). This means that, in a certain TI medium with a constant value of β_0 (or equivalently c_{55}), the qP rotation rates are independent of γ (see equations 21 and 23). Hence, we consequently skip the parameter γ in the following. We quantify the peak rotation rates of the qP-waves generated in TI materials with different levels of anisotropy only as a function of the two parameters ε and δ^* .

We define a set of elastic modulus matrices $\mathbf{C}$ by assuming fixed values of α_0 , β_0 , ρ , and γ and letting ε and δ^* take different values. For each (ε, δ^*) couple, we aim to obtain the peak rotation rates produced by a qP-wave traveling in any direction. To this goal, for each elastic modulus matrix $\mathbf{C}$ and for each propagation direction, we first extract the phase velocity and the polarization of the considered qP-wave by solving equation 10. The corresponding rotation rates are calculated using equation 3. The corresponding peak rotation rate can be then obtained trivially from the full rotation rate vector. In this way, we obtain the peak qP rotation rate that can be generated by a certain plane wave in the considered materials, each characterized by its ε and δ^* values, in all propagation directions $\mathbf{l}$. We consider values of ε ranging from 0 to 0.35 and those of δ^* from -0.3 to 1 . These values cover most TI cases observed (Thomsen, 1986). Four case studies are investigated: (1) a plane wave of peak ground displacement (PGD) 355.78 nm and period $T = 1$ s (caused by a distant earthquake) propagating through bedrock; (2) a plane wave of PGD = 48×10^{-4} m and period $T = 0.8$ s (can be caused by a local earthquake M7.0, epicenter distance 100 km) propagating through bedrock; (3) a plane wave of PGD = 10^{-6} m and frequency $f = 150$ Hz (can originate from a microseismic event of $M_w = 0$ at hypocenter distance of approximately 1 km) propagating through reservoir rock; and (4) a plane wave of PGD = 180.35×10^{-6} m and $f = 5$ Hz (can be caused by an air-gun experiment) propagating through reservoir rock (see Chen et al., 2008). For each case study,

reasonable values of α_0 , β_0 , and mass density ρ are used and are detailed in Table 3. Because of the independence of the qP rotation rates on γ , any arbitrary value of this Thomsen parameter can be taken. Here we note that the first case study is based on measurements of the 25 September 2003 M8.1 Tokachi-oki earthquake at the Wettzell station in southeast Germany (see Pham et al., 2009a) except for the anisotropy, which has been solely added for the purpose of the research presented here.

We compute for each of the four study cases the peak rotation rates of qP-waves generated in different TI materials for a set of ε and δ^* values. The results are given in Figures 3–6. Every point in the plot depicts the maximum qP rotation rate among all propagation directions for a given couple of values of (ε, δ^*) . A general trend that can be observed is that peak qP rotation rates are higher with increasing values of ε and δ^* . In particular, if we look at the results obtained for the case of the distant earthquake (Figure 3), we can safely conclude that, if we consider weakly anisotropic bedrock under Wettzell (Lee et al., 2009b), we should measure qP rotation rates of at most 5×10^{-11} rad/s. This value is much smaller than the amplitude (6.3×10^{-10} rad/s) observed in P coda rotations, as reported by Pham et al. (2009a). However, for the other three study cases, significant amplitudes of qP rotation rates are expected (Figures 4–6). With peak qP rotation rates in the order of $\mu\text{rad/s}$, even in weakly anisotropic materials the signals can be recorded with current instrument technology (Nigbor et al., 2009; Wassermann et al., 2009).

DISCUSSION

The rotational motion of qP-waves studied in this paper is another expression of the polarization deviation of the P-waves in anisotropic media studied in earlier papers (e.g., Pšenčík and Gajewski, 1998; Zheng and Pšenčík, 2002). However, at the beginning of this study it was not clear how anisotropy affects rotational ground motions and if rotational motions can help putting constraints on anisotropic properties. The results obtained in this paper under the assumption of plane wave propagation show that (1) anisotropic behavior in elastic media can be recognized from rotational motion records, especially by the appearance of qP rotations; (2) for typical reservoir situations and strong local earthquakes qP rotation signals are significant, recordable, and can be used for inverse problems.

It should be noted that our original motivation to study the relation between anisotropy and rotational motions comes from efforts to explain the observed P coda rotations reported by Igel et al. (2007) and Pham et al. (2009b). At the beginning of this study, it was unclear whether rotational motions induced by qP-waves in anisotropic media contribute to the observed rotational signals in the P coda. Relat-

Table 3. Parameters used for quantifying peak rotation rates of qP-waves in terms of the Thomsen parameters for four case studies.

Case study	Peak ground displacement (m)	Considered period (s)	α_0 (m/s)	β_0 (m/s)	ρ (kg/m ³)
1. Bedrock/distant earthquake	355.78×10^{-9}	1	6600	3700	2900
2. Bedrock/local earthquake	48×10^{-4}	0.8	6600	3700	2900
3. Reservoir/microseismic event	10^{-6}	1/150	3928	2055	2590
4. Reservoir/air-gun experiment	180.35×10^{-6}	0.2	3928	2055	2590

Figure 3. The maximum peak rotation rates (in 10^{-11} rad/s) caused by plane P-waves in a full space TI medium as a function of the Thomsen parameters ϵ and δ^* , as expected for a distant earthquake with PGD of 355.78 nm and period $T = 1$ s. A vertical P velocity $\alpha_0 = 6600$ m/s, vertical S velocity $\beta_0 = 3700$ m/s, and $\rho = 2900$ kg/m³ were assumed. Every point in the plot depicts the maximum qP rotation rate among all propagation directions for a given couple of values of (ϵ, δ^*) .

Figure 4. The maximum peak rotation rate (in 10^{-7} rad/s) caused by qP-waves in a full space TI medium as a function of the Thomsen parameters ϵ and δ^* , as expected for a local earthquake with PGD 48×10^{-4} m and period $T = 0.8$ s. A vertical P velocity $\alpha_0 = 6600$ m/s, vertical S velocity $\beta_0 = 3700$ m/s, and $\rho = 2900$ kg/m³ were assumed. Every point in the plot depicts the maximum qP rotation rate among all propagation directions for a given couple of values of (ϵ, δ^*) .

ed to this, several results obtained in the study presented here need to be mentioned. First, although anisotropy does generate qP rotations, for teleseismic wavefields the rotational energy induced by anisotropic effects is expected to be much smaller than the observed one in the P coda. Second, it is evident from equation 3 that translational acceleration and the resulting qP rotation rates would have the same waveforms coinciding in time. However, as reported by Pham et al. (2009b), the P coda rotations observed at the Wettzell station come later than the appearance of direct P-waves. These results demonstrate that anisotropic qP rotational signals cannot be the main cause of the observed P coda rotations. The difference in the arrival time of anisotropic qP rotational motions and scattering P coda rotations can be used to separate these two cases for specific purposes when a heterogeneous medium is present.

Figure 5. The maximum peak rotation rate (in 10^{-6} rad/s) caused by qP-waves in a full space TI medium as a function of the Thomsen parameters ϵ and δ^* as expected for a reservoir microseismic event with PGD of 10^{-6} m and frequency $f = 150$ Hz. A vertical P velocity $\alpha_0 = 3928$ m/s, vertical S velocity $\beta_0 = 2055$ m/s, and $\rho = 2590$ kg/m³ were assumed. Every point in the plot depicts the maximum qP rotation rate among all propagation directions for a given couple of values of (ϵ, δ^*) .

Figure 6. The maximum peak rotation rate (in 10^{-7} rad/s) caused by qP-waves in a full space TI medium as a function of the Thomsen parameters ϵ and δ^* as expected for an air-gun experiment in a reservoir area with PGD 180.35×10^{-6} m and frequency $f = 5$ Hz. A vertical P velocity $\alpha_0 = 3928$ m/s, vertical S velocity $\beta_0 = 2055$ m/s, and $\rho = 2590$ kg/m³ were assumed. Every point in the plot depicts the maximum qP rotation rate among all propagation directions for a given couple of values of (ϵ, δ^*) .

CONCLUSIONS

In summary, we conclude that rotational motions contain additional information about the material's anisotropic properties and that joint measurements of translational and rotational motions of qP-waves in anisotropic media might allow one to constrain anisotropic parameters. Our results not only demonstrate the potential benefit of measurements of rotational ground motions but also pose several issues that need to be addressed in the future.

- 1) Because we have shown that the amplitudes of the qP rotational motions generated in anisotropic materials are high enough to be observed and recorded, one of the key questions is: Can we extract additional information about the material's properties using qP rotations?

- 2) In the study presented here, we focus mainly on qP-waves, the rotational motions of which are completely independent of the Thomsen parameter γ . Thus, the consideration of this measure of S anisotropy (i.e., γ) is out of the framework of this paper. Theoretically, qS motions also contain additional information. Can we constrain the Thomsen parameter γ using measurements of rotational motions of qS-waves? What additional information can be extracted if these motions are used?
- 3) The well-known result for isotropic media that the rotation vector is orthogonal to the propagation direction (i.e., the component of the rotation rate vector in the propagation direction is zero) is theoretically and numerically demonstrated to also hold for anisotropic media. This opens an opportunity to investigate scattering properties of heterogeneous media on the basis of variations of rotational motions around the wave propagation direction.

Finally, the applicability of the presented theory to real data will rest on the ability to develop reliable sensors (borehole- and surface-based) for rotational motions with a broad enough frequency bandwidth and dynamic range.

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APPENDIX A

ORTHOGONAL CARTESIAN COORDINATE SYSTEM $\mathbf{l}, \boldsymbol{\theta}, \boldsymbol{\phi}$

For the purpose of visualization of the influence that anisotropy has in the behavior of the rotational wavefield, an orthogonal Cartesian coordinate system $\mathbf{l}, \boldsymbol{\theta}, \boldsymbol{\phi}$ is used in the paper (see Figure A-1). The system is defined as follows. We first build the unit vector $\mathbf{l} = (\ell_1, \ell_2, \ell_3)$ so that its axis coincides with the wave propagation direction. Furthermore, we generate a set of three auxiliary vectors $\mathbf{p}$, $\mathbf{q}$, and $\mathbf{r}$ that will help us define the two remaining axes of our Cartesian coordinate system. These are defined as

$$\mathbf{p} = (\ell_1, -\ell_2, 0), \quad (\text{A-1})$$

$$\mathbf{q} = (0, -\ell_3, \ell_2), \quad (\text{A-2})$$

$$\mathbf{r} = (\ell_3, 0, -\ell_1). \quad (\text{A-3})$$

The definition of the two remaining Cartesian axes will depend on whether the auxiliary vectors are collinear or not and whether one of

them is the vector $\mathbf{0}$. The three distinct cases are described as follows. If $|\mathbf{l} \times \mathbf{p}| > 0$, then $\boldsymbol{\theta}$ is defined as

$$\boldsymbol{\theta} = \mathbf{l} \times \mathbf{p} = (\ell_2 \ell_3, \ell_1 \ell_3, -2\ell_1 \ell_2). \quad (\text{A-4})$$

If $|\mathbf{l} \times \mathbf{p}| = 0$ and $|\mathbf{l} \times \mathbf{q}| > 0$, then $\boldsymbol{\theta}$ is defined as

$$\boldsymbol{\theta} = \mathbf{l} \times \mathbf{q} = (\ell_2^2 + \ell_3^2, -\ell_1 \ell_2, -\ell_1 \ell_3). \quad (\text{A-5})$$

If $|\mathbf{l} \times \mathbf{p}| = 0$ and $|\mathbf{l} \times \mathbf{q}| = 0$, then $\boldsymbol{\theta}$ is defined as

$$\boldsymbol{\theta} = \mathbf{l} \times \mathbf{r} = (-\ell_1 \ell_2, \ell_3^2 + \ell_1^2, -\ell_2 \ell_3). \quad (\text{A-6})$$

The vector $\boldsymbol{\phi}$ is always obtained by cross product of $\mathbf{l}$ and $\boldsymbol{\theta}$,

$$\boldsymbol{\phi} = \mathbf{l} \times \boldsymbol{\theta}. \quad (\text{A-7})$$

APPENDIX B

AXIS TRANSFORMATIONS OF ROTATIONS

To set up the relationship between components of a rotational motion in two orthogonal Cartesian coordinate systems (x, y, z) and (x^*, y^*, z^*) , we start by recalling the definition of the rotation rate

$$\begin{pmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \end{pmatrix} = \frac{1}{2} \nabla \times \mathbf{V} = \frac{1}{2} \begin{pmatrix} \partial_y V_z - \partial_z V_y \\ \partial_z V_x - \partial_x V_z \\ \partial_x V_y - \partial_y V_x \end{pmatrix}, \quad (\text{B-1})$$

where $\mathbf{V}$ is the ground translational velocity and $\dot{\Omega}$ is the rotation rate.

In the (x^*, y^*, z^*) system, we can similarly find that

$$\dot{\Omega}_x^* = \frac{1}{2} (\partial_{y^*} V_{z^*} - \partial_{z^*} V_{y^*}). \quad (\text{B-2})$$

The relationship between the components of the translational motion in the two systems can be expressed as

$$\mathbf{V}^* = \mathbf{A} \mathbf{V}, \quad (\text{B-3})$$

which explicitly can be written as

Figure A-1. Illustration of the orthogonal Cartesian coordinate system $\mathbf{l}, \boldsymbol{\theta}, \boldsymbol{\phi}$ used in this study.

$$\begin{pmatrix} V_x^* \\ V_y^* \\ V_z^* \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}, \quad (\text{B-4})$$

where $a_{ij} = \mathbf{e}_i^* \cdot \mathbf{e}_j = \cos \alpha_{ij}$ ($i = 1, 2, 3; j = 1, 2, 3$) are the so-called direction cosines; α_{ij} are angles between two sets of axes; and $(\mathbf{e}_1^*, \mathbf{e}_2^*, \mathbf{e}_3^*)$ and $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ denote the unit basis vectors of the axes of the two systems.

Using B-4, equation B-2 can be rewritten as

$$\begin{aligned} \dot{\Omega}_x^* = & \frac{1}{2} [(a_{31}\partial_y^* V_x + a_{32}\partial_y^* V_y + a_{33}\partial_y^* V_z) - (a_{21}\partial_z^* V_x \\ & + a_{22}\partial_z^* V_y + a_{23}\partial_z^* V_z)]. \end{aligned} \quad (\text{B-5})$$

Now we use the Jacobian of the transformation to obtain the derivatives in the new coordinate system.

$$\partial_{x^*} = \frac{\partial x}{\partial x^*} \partial_x + \frac{\partial y}{\partial x^*} \partial_y + \frac{\partial z}{\partial x^*} \partial_z = a_{11}\partial_x + a_{12}\partial_y + a_{13}\partial_z, \quad (\text{B-6})$$

$$\partial_{y^*} = \frac{\partial x}{\partial y^*} \partial_x + \frac{\partial y}{\partial y^*} \partial_y + \frac{\partial z}{\partial y^*} \partial_z = a_{21}\partial_x + a_{22}\partial_y + a_{23}\partial_z, \quad (\text{B-7})$$

$$\partial_{z^*} = \frac{\partial x}{\partial z^*} \partial_x + \frac{\partial y}{\partial z^*} \partial_y + \frac{\partial z}{\partial z^*} \partial_z = a_{31}\partial_x + a_{32}\partial_y + a_{33}\partial_z. \quad (\text{B-8})$$

Substituting B-6–B-8 and B-5, we find that

$$\begin{aligned} \dot{\Omega}_x^* = & \frac{1}{2} \left[(a_{33}a_{22} - a_{32}a_{23}) \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right) + (a_{31}a_{23} \right. \\ & - a_{33}a_{21}) \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right) + (a_{32}a_{21} - a_{31}a_{22}) \left(\frac{\partial V_y}{\partial x} \right. \\ & \left. \left. - \frac{\partial V_x}{\partial y} \right) \right] \end{aligned} \quad (\text{B-9})$$

which leads to the final form

$$\begin{aligned} \dot{\Omega}_x^* = & (a_{33}a_{22} - a_{32}a_{23})\dot{\Omega}_x + (a_{31}a_{23} - a_{33}a_{21})\dot{\Omega}_y + (a_{32}a_{21} \\ & - a_{31}a_{22})\dot{\Omega}_z. \end{aligned} \quad (\text{B-10})$$

Similarly, we also obtain

$$\begin{aligned} \dot{\Omega}_y^* = & (a_{13}a_{32} - a_{33}a_{12})\dot{\Omega}_x + (a_{11}a_{33} - a_{31}a_{13})\dot{\Omega}_y + (a_{12}a_{31} \\ & - a_{32}a_{11})\dot{\Omega}_z, \end{aligned} \quad (\text{B-11})$$

$$\begin{aligned} \dot{\Omega}_z^* = & (a_{23}a_{12} - a_{13}a_{22})\dot{\Omega}_x + (a_{21}a_{13} - a_{11}a_{23})\dot{\Omega}_y + (a_{11}a_{22} \\ & - a_{12}a_{21})\dot{\Omega}_z. \end{aligned} \quad (\text{B-12})$$

The last three equations can be written in the compact form

$$\dot{\Omega}^* = (\mathbf{A}^{-1})^T \dot{\Omega}, \quad (\text{B-13})$$

which we can compare to B-3 to see how the rotation-rate vector can be transformed into a rotated Cartesian system once we know the expression of the unitary vectors $(\mathbf{e}_1^*, \mathbf{e}_2^*, \mathbf{e}_3^*)$ that form the basis of the new system.

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