Neighborhood Models of Minority Opinion Spreading

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Models of Consensus vs. Polarization, or Segregation:

Voter model, Minority opinion models, Axelrod, Sznajd, Schelling, Bounded confidence and Relative agreement, Threshold models, Social impact theory, …

Two general questions:

1) **Effects of “Who interacts with whom?”**: Comparison with average, Fully connected population, Random pairing, Neighboring interaction in regular lattices, Complex networks.

   *Here: Neighborhood models*

2) **Mechanisms for reaching or stopping consensus with convergent interactions**: Barriers, topology, size, noise, updating rules,…

   *Here: Mechanisms for consensus on initial minority*
NEIGHBORHOOD MODELS?

T. Schelling (Micromotives and Macrobehavior, 1978):

1) Spatial Proximity Model:
   - Interactions weighted by spatial location
   - Agents interact with neighbors in lattices

2) Bounded Neighborhood Model:
   - Common definition of neighborhood (system) and its boundaries
   - Interactions in a completely connected population (mean field)
   - Recurrence relations for the fraction of population with one option
   - Threshold models of collective action (M. Granovetter, Am. J. Soc. (1978))

PROPOSAL OF A THIRD WAY: Neighborhood Models

- Consider locally defined neighborhood cells within a system
- Complete connectedness within the neighborhood cell
- Neighborhood cells change shape and size during evolution
Mechanisms for consensus on initial minority?

**Question:** How an initially minority opinion can become majority?

Serge Galam’s model discusses a mechanism of *social inertia* resulting in democratic rejection of social reforms initially favored by a majority:

**A TIE IN A CELL VOTING MEANS NO FOR THE REFORM**

a) Irish vote on the EU Nice treaty

b) September 11 French rumor: no plane crashed the Pentagon

Madrid March 11 terrorist attack: ETA or AL-QAEDA
Galam's Model

• Models how an initially minority opinion can become majority

• Examples:
  • The September 11 no-plane Pentagon hoax
  • The spread of such rumor in France and UK
  • Minoritary opinion against an structural change in society finally becomes a majority
  • Maastritch related Ireland & France voting
  • Authorship of recent terrorists attacks in Madrid
  • Eta versus Al-Qaeda
Minority opinion blocks social reforms

EU Nice treaty: Ireland referendum yielded results against reforms, although initially a majority supported it. Result came as a surprise to the Irish people themselves.

Social inertia: Conservative response to the risk of a change
Maintain social status quo
Minority Opinion Spreading:
September 11th no-plane Pentagon hoax

Social inertia: Feelings of French society about US politics
Government representatives swiftly put the blame for bombs on ETA yesterday morning. However, throughout the day doubts circulated regarding whether or not the basque group was in fact responsible.

Binary opinion:
Each agent has one opinion: blue (+) or yellow (-)

Initially there is a (blue) minority against social reform
Social life: Agents gather and discuss in *meeting cells* (offices, houses, bars, clubs, etc.)

Cells are defined only by their size $k$

$k=16$

$M=\text{Maximum size of a cell}$
Interaction rules in meeting cells:

*Pre-campaign political party meeting*

Majority of *yellow* opinion
Interaction rules in meeting cells:

*Pre-campaign political party meeting*

Majority of *yellow* opinion

ALL the agents in the cell adopt the *yellow* opinion
Majority of blue opinion
Majority of **blue** opinion

ALL the agents in the cell adopt the **blue** opinion

*Individuals leave the cell as supporters of the majority*
Social inertia:

A tie in the voting is a NO for social reform

There is a bias toward one opinion in the society
Social inertia:

A tie in the voting is a NO for social reform

ALL the agents in the cell adopt blue opinion → minority against reform is favored
Dynamical evolution

Agents join a meeting cell randomly selected
Dynamical evolution

Decision rule is applied in all the cells
Dynamical evolution

Following cycle: Agents randomly redistributed in the meeting cells carrying their adopted opinion.
Mean-field analysis

$P_+(t)$ : density of agents with blue (+) opinion

$P_-(t)$ : density of agents with yellow (-) opinion

$P_-(t) + P_+(t) = 1$

$M$ : maximum cell size

$a_k$ : probability for an agent to be in a cell of size $k$

Recursion relations:

\[
P_+(t + 1) = \sum_{k=1}^{M} a_k \sum_{j=\left[\frac{M}{2}+1\right]}^{M} \binom{k}{j} P_+(t)^j \left[1 - P_+(t)\right]^{k-j}
\]

\[
P_-(t + 1) = \sum_{k=1}^{M} a_k \sum_{j=\left[\frac{M+1}{2}\right]}^{M} \binom{k}{j} P_-(t)^j \left[1 - P_-(t)\right]^{k-j}
\]
Asymmetric unstable fixed point: \textit{FAITH POINT}

There is a threshold value of initial minority supporters $p_c(M) < 1/2$ such that for $P_+(0) > p_c$ the minority opinion finally becomes majority:

$$P_+ \rightarrow 1 \quad P_- \rightarrow 0$$


For $P_+(0) > p_c$:

Social reform is rejected and status quo maintained

\textbf{Dynamics:} Time to reach consensus is fast and system–size independent
Beyond mean-field recursion relations:

• **Numerical simulations of the model with** $N$ **agents**

• We consider decision cells whose size is uniformly distributed in the interval $[1,M]$

• $p<1/2$: Initial proportion of agents with the minority *(blue)* opinion

• Consensus (unique opinion) is always reached for all values of $M$ and $p$

• **Order parameter**: $\rho$

  probability (over realizations) that the consensus coincides with the initial *minority blue* opinion (rejection of social reform).

  Mean field prediction:  
  
  \[ \rho \neq 1 \quad \text{if} \quad p>p_c \]
  
  \[ \rho = 0 \quad \text{if} \quad p<p_c \]
Finite Size Fluctuations:

Smoothed 1st-order transition

\[ M = 4 \]
\[ M = 5 \]
Finite Size Fluctuations:

Smoothed 1st-order transition

There is a region of width $N^{-1/2}$ in which the outcome of a run is not well known.
Consensus phase diagram

Small meeting cells required for minority opinion spreading
Dynamics:

Time to reach consensus, $T$, for different sizes $N$

$T$ diverges with $N$ for $p=p_c$
Time to reach consensus can be shown to scale as

\[ T \sim \ln N \]

Also, D. Stauffer, Int. J. Mod. Phys C 13, 975 (2002)
Neighborhood Models (NMs)

Meeting cells defined by their size do not incorporate local effects

“Primitive” society: Individuals interact predominantly with neighbors

NMs: Individuals are fixed at the sites of a regular lattice

Meeting cells locally defined by a tessellation of the lattice

Individuals in a meeting cell interact with original rules

Meeting cells redefined at each time step of dynamics

Consider 1D and 2D square lattices with synchronous and asynchronous updating schemes
**2D Neighborhood Model**: Synchronous update

Rectangular tessellation

\[ 1 \leq m_x \leq M \]

\[ 1 \leq m_y \leq M \]

- Interaction rules simultaneous in each cell.
- Time step defined by each iteration
2D Neighborhood Model:

Synchronous update

Rectangular tessellation

\[ 1 \leq m_x \leq M \]

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- Interaction rules simultaneous in each cell.
- Time step defined by each iteration
2D Neighborhood Model:

Asynchronous update

For each update, time increases as $t \rightarrow t + \frac{m_x \cdot m_y}{N}$

Consensus formation by Domain Growth
Neighborhood Models: Steady State

Consensus always reached in finite systems in a finite number of steps.

For $N \to \infty$, $p_c \to 0$

$p_c \sim N^{-\alpha}; \alpha(D,M)$

$\rho(p,N) = f(pN^\alpha)$

In an infinite system, the blue initial minority opinion wins regardless the amount of initial supporters: Social reform always rejected in a large system
T has a maximum for $p_c(N)$.

$T$ much larger than for nonlocal models.

The time to reach consensus, scales as two power laws of $N$, below and above the transition.
Neighborhood Models: Domain growth

Domains of a given opinion shrink or grow.

Characteristic size scales linearly with time: \( R(t) = \beta t \)

\[ \beta = \frac{2}{5} \]

\[ M=5, \ N=5000 \]
Why minority blue opinion always wins in Neighborhood Models for large systems?

A critical size for an initial local domain of minority supporters exists. Domains of larger size grow and occupy the system. A domain of overcritical size always exists in a large enough population.
Critical Size in NM and Nucleation theory

Nucleation:

- \( R^* \): competition between surface tension and different bulk energies
- \( R^* \) meaningless in \( D=1 \) (no surface tension)
- Overcritical droplet created by a dynamical fluctuation, and grows deterministically

NM:

- Overcritical droplet appears in random initial condition. Stochastic dynamics of growth.
- Critical size (probabilistic concept) appears in \( D=1 \)
- \( R^* \): competition between favoured opinion and stochastic interface dynamics
Conclusions

General interest of Neighborhood Models as a third way between strictly local interactions and completely connected populations

Importance of the population size in consensus models:
- Analytic recursion relations neglect finite size effects
- NM local effects: Social reform is always rejected by a large population in the model of Minority Opinion spreading

Neighborhood models of minority opinion spreading:
- Threshold value of initial concentration of minority supporters for minority overcome: $p_c \sim N^\alpha$ Time to reach consensus grows as a power law of $N$.
- Mechanism: Critical size for an initial local domain of minority supporters
- Neighborhood models describe a more efficient spreading of minority opinion, but spreading takes a much longer time.