Coherence resonance in coupled excitable systems: dependence with system size

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\textbf{ABSTRACT}

We report on the existence of a coherence resonance effect as a function of the system size. We consider globally coupled FitzHugh–Nagumo systems under the effect of white noise and study the synchronization of the response as well as the regularity of the emitted pulses. By numerical integration of the stochastic equations, we find that the regularity, as measured both by the jitter and the time correlation, is optimal for an intermediate value of the number of coupled elements. A simple theory, based on the Gaussian approximation, is able to explain qualitatively the observed phenomenon.

\textbf{Keywords:} Noise induced effects, coherence resonance, excitable systems

1. INTRODUCTION

In most cases noise and “disorder” have been considered as synonymous concepts. This identification appears as natural when one looks at the erratic trajectories of the Brownian particle originated by the noise terms\textsuperscript{1}. However, the last decades are witnessing a change in our understanding of the noise terms. A paradoxical “ordering” effect of the fluctuating terms has been found in many situations and a non-exhaustive list for purely temporal dynamical systems includes phenomena such as noise–induced transitions\textsuperscript{2}, stochastic resonance\textsuperscript{3,4} and noise–induced transport\textsuperscript{5}; noise–induced phenomena in spatially–extended systems (a recent review of some of these aspects can be found in Ref.\textsuperscript{6}) include noise–induced patterns\textsuperscript{7,8}, noise–induced phase transitions\textsuperscript{9–11}, spatiotemporal stochastic resonance\textsuperscript{12}, noise–induced fronts\textsuperscript{13}, noise-supported traveling structures in excitable media\textsuperscript{14} and noise sustained convective structures\textsuperscript{15,16}.

Of all these examples, the stochastic resonance effect, besides being the first one in being discovered, has attracted considerable interest due, among other aspects, to its potential technological applications for optimizing the output signal-to-noise ratio in nonlinear dynamical systems, as well as to its connection with some biological mechanisms. The phenomenon shows unambiguously that the response of some systems to an external perturbation can be enhanced by the presence of noise. There is nowadays a wealth of papers, conference proceedings and reviews on this subject\textsuperscript{3,4} which show the large number of applications in science and technology, ranging from paleoclimatology to electronic circuits, as well as lasers, chemical systems, and the connection with some situations of biological interest (noise-induced information flow in sensory neurons in living systems, influence in ion-channel gating or in visual perception). A tendency shown in recent papers, and determined by the possible technological applications, points towards achieving an enhancement of the system response (that is: obtaining a larger output signal-to-noise ratio) by means of the coupling of several stochastic resonance units in what conforms an extended medium. In this respect, a very interesting recent paper by Pikovsky et al.\textsuperscript{17} shows that when one considers an ensemble of coupled bistable systems subjected to an external periodic forcing (and in the presence of a constant amount of noise), it turns out that an optimal response is obtained for an appropriate value of the number $N$ of coupled systems. In other words, that there is a resonance with respect to the number of coupled elements, rather than the usual one that involves the noise level. The authors speculate that

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this system size resonance might be relevant to neuronal dynamics, in which the neuronal connections or the coupling strengths between neurons can be tuned in order to achieve maximum sensitivity to external signals.

In this paper we consider the analogous role of the system size in a related, yet different, effect, namely the coherence resonance\textsuperscript{18}. This is a mechanism by which an excitable system shows a maximum degree of regularity in the emitted signal in the presence of the right amount of fluctuations. It is related to the stochastic resonance without the need of an external forcing discussed in Refs.\textsuperscript{19,20}, and it has been found to be relevant in dynamical systems close to the onset of a bifurcation\textsuperscript{21}, as well as in other bistable and oscillatory systems\textsuperscript{22,23}. It has also been analyzed in different neuronal models such as the FitzHugh–Nagumo\textsuperscript{24,25} and Hodgkin-Huxley\textsuperscript{26} models. It has been observed experimentally in electronic circuits, either excitable\textsuperscript{27,28} or chaotic\textsuperscript{29,30}, and in lasers operating in an excitable regime\textsuperscript{31}. We will show that a canonical model for excitability, the FitzHugh–Nagumo model, displays a coherence resonance effect which is optimal for a given number of coupled elements. We will analyze theoretically the effect as well as presenting the results of numerical simulation confirming the theoretical findings.

The paper is organized as follows: in section 2 we review briefly the coherence resonance effect and introduce our model of coupled FitzHugh-Nagumo oscillators. An approximate analytical calculation shows that it is possible that the collective variable describing the average behavior shows a maximum regularity as a function of the system size. In section 3 we study the synchronization properties of the model. In section 4 we compute numerically some indicators of coherence resonance and show that they become optimal for a given number of coupled elements. Finally, section 5 draws some conclusions.

2. MODEL AND THEORETICAL ANALYSIS

We consider the FitzHugh–Nagumo model which provides the simplest representation of firing dynamics and has been widely used as a model for spiking neurons as well as for cardiac cells\textsuperscript{32,33}. The model is defined in terms of activation $x$ and inhibition $y$ variables, as follows:

\begin{align}
\epsilon \dot{x} &= x - \frac{1}{3} x^3 - y \\
\dot{y} &= x + a + D \xi(t)
\end{align}

where, following Ref.\textsuperscript{18}, a Gaussian white noise $\xi(t)$ of zero mean and correlations $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$ has been added to the slow variable $y$. $D$ will be called the noise intensity. The difference in the time scales of $x$ and $y$ is measured by $\epsilon$, a small number. Although we believe that our results are very general, our motivation for choosing this particular example is to consider it as a prototypical model, without making reference to any specific application, in order to clarify the minimal elements that an excitable model has to have in order to display the phenomenon of system size coherence resonance. This is equivalent to the approach used in Ref.\textsuperscript{17}, dealing with system size stochastic resonance, with focus on generic ($\varphi^4$, Ising) models for phase transitions, rather than on a specific application.

The deterministic system has $x = y = 0$ as the only stable fixed point. In the absence of any external perturbation, this fixed point is reached independently of the initial condition. When random perturbations act on the system, and in the so-called excitable regime, characterized by $a > 1$, the trajectories of the FitzHugh–Nagumo variables $x(t)$ and $y(t)$ eventually exit the basin of attraction of the stable fixed point and return to it after making a large excursion in phase space, i.e. a pulse. The regularity of the pulses can be quantified by the relative fluctuations $R$ of the time between two consecutive pulses. This indicator (also called jitter) is defined as the ratio of the root mean square, $\sigma[t_p]$, to the mean value, $\langle t_p \rangle$, of the time between pulses $t_p$. i.e. $R = \sigma[t_p]/\langle t_p \rangle$. $R$ can be estimated by noticing that $t_p$ is the sum of the pulse duration $t_e$ and the activation time of the pulse, $t_a$. The latter is a random variable whose mean value behaves as $\langle t_a \rangle \sim \exp(A/D^2)$ where $A$ is a constant and $D$ is the noise intensity (Kramers formula) and its variance is $\sigma^2[t_a] \approx \langle t_a \rangle^2$. Since the pulse duration $t_e$ depends only weakly on $D$, its mean value $\langle t_e \rangle$ can be considered constant and its variance can be estimated as $\sigma^2[t_e] \approx D^2 \langle t_e \rangle$. The detailed analysis carried out by Pikovsky and Kurths\textsuperscript{18} shows that for small noise $R \approx \sigma[t_a]/\langle t_a \rangle \approx 1$, whereas for large noise $R \sim D(t_e)^{-1/2}$. Therefore, it is expected the presence of a minimum for $R$ at intermediate values of the noise intensity $D$. This is the signature of coherence resonance.
According to the general discussion above, the next step is to consider an ensemble of \( N \) globally coupled such FitzHugh–Nagumo systems:

\[
\begin{align*}
\dot{x}_i & = x_i - \frac{1}{3}x_i^3 - y_i + \frac{K}{N} \sum_{j=1}^{N} (x_j - x_i) \\
\dot{y}_i & = x_i + a + D\xi_i(t), \\
\end{align*}
\]

with independent noises, \( \langle \xi_i(t)\xi_j(t') \rangle = \delta_{ij}\delta(t - t') \). The systems are globally coupled by a gap-junctional form, as indicated by the last term of Eq.(3), where \( K \) is the coupling strength. The assumption of global coupling is not strictly necessary, but it will allow us to justify a theoretical approach based on a mean-field type approximation. Similar globally coupled models have been used previously to study array enhanced stochastic resonance in the coupled FitzHugh–Nagumo equation\(^{34}\).

The simplest variables that can give us information about the collective behavior of the system are the average values of the activator and inhibitor variables:

\[
X(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t) \quad \quad Y(t) = \frac{1}{N} \sum_{i=1}^{N} y_i(t)
\]

In order to find a set of closed evolution equations for the \( X \) and \( Y \) variables we follow the approach by Desai and Zwanzig\(^{35}\) (see also reference\(^{17}\)): we write \( x_i = X + \delta_i \) and make the Gaussian approximation consisting in considering that the \( \delta_i \)'s are independent Gaussian variables with zero mean and variance \( M \). After replacing this decomposition in the evolution equations we average over the variables \( \delta_i \) and sum on \( i \). Since only even powers of these variables have a different form zero average we get:

\[
\begin{align*}
\dot{X} & = X - \frac{1}{3}X^3 - XM - Y \\
\dot{Y} & = X + a + \frac{D}{\sqrt{N}}\xi(t)
\end{align*}
\]

where \( \xi(t) = N^{-1/2} \sum_i \xi_i(t) \). The important fact is that \( \xi(t) \) is a Gaussian white noise of correlations independent of the number of coupled elements \( N \), i.e. \( \langle \xi(t)\xi(t') \rangle = \delta(t - t') \). We note that the equation for \( \dot{Y} \) is exact independently of the Gaussian approximation considered. In order to proceed, we need to find the variance \( M \) of the \( X \) variable. This is again obtained by writing \( y_i = Y + \Delta_i \) and assuming that the \( \Delta_i \)'s are Gaussian variables whose variance \( Q \) can be adiabatically eliminated. This leads to the value \( M(X) = \frac{1}{2}(1 - K - X^2 + \sqrt{(K - 1 + X^2)^2 + 4D}) \). The resulting equations adopt the final form

\[
\begin{align*}
\dot{X} & = F(X, K) - Y \\
\dot{Y} & = X + a + \frac{D}{\sqrt{N}}\xi(t)
\end{align*}
\]

with the function \( F(X, K) = X - \frac{1}{3}X^3 - \frac{1}{2}X \left(1 - K - X^2 + \sqrt{(K - 1 + X^2)^2 + 4D}\right) \). This function behaves approximately equal to the \( X - \frac{X^3}{3} \) for a wide range of the parameters \( D \) and \( K \), and therefore the general qualitative excitatory behavior is also recovered for the averaged system \( (X, Y) \). It is important to remark here that in the (exact) equation for \( \dot{Y}(t) \) the noise intensity appears rescaled as \( D/\sqrt{N} \). Therefore, this approximation suggests that the optimal effective noise intensity for the appearance of coherence resonance can be achieved by varying the number of coupled elements \( N \), as in the case of stochastic resonance for the bistable system considered in Ref.\(^{17}\). This is our main analytical result that will be checked in the next sections by extensive simulations of the (unsimplified) equations (3-4). Before that, in the next section, we study the synchronization properties of this model.
3. SYNCHRONIZATION PROPERTIES

In the coupled system given by Eqs. (3-4), and for some range of parameter values, it is observed that the different systems fire pulses at the same time. A general framework to study such synchronization phenomena is given by the work by Kuramoto. Kuramoto considers coupled phase-like variables $\phi_i(t)$ following a stochastic dynamics and discusses the existence of a synchronized regime in terms of the coupling strength and the noise intensity. It turns out that the model displays a genuine phase-transition in which synchronization disappears if the noise surpasses a given critical value. The same behavior can be observed in our model Eqs. (3-4). We are mostly interested here, however, in the effect that the number of oscillators has on the synchronization properties.

We now define an order parameter that allows us to measure the degree of synchronization in the coupled system. First, in order to follow the Kuramoto scheme, we need to define a phase $\phi_i$ for oscillator $i$. Once this phase is defined, we introduce collective amplitude, $\rho(t)$, and phase, $\Psi(t)$, variables by:

$$\rho(t) e^{i\Psi(t)} = \frac{1}{N} \sum_{i=1}^{N} e^{i\phi_i(t)}.$$  

(10)

The variable $\Psi(t)$ is the global phase of the system and it will be useful to study the coherence of the collective signal. The order parameter $\rho$ is defined as the time average:

$$\rho = \langle \rho(t) \rangle_t.$$  

(11)

Two different approaches were taken in order to evaluate the phases $\phi_i$. The first, and most naive, approach is based upon the fact that the limit cycles in which the variables $(x_i, y_i)$ evolve are approximately centered around the origin. Then, the easiest choice is

$$\phi_i = \arctan \left( \frac{y_i}{x_i} \right).$$  

(12)

However, this choice is only valid for particular cases of the parameters of the FitzHugh-Nagumo model. For large noise intensities, for example, the pulses are not so clearly centered around the origin. A definition of more general validity uses the so-called Hilbert transform. Let’s consider, for instance, the real variable $x_i(t)$. From it we can construct the so-called “analytic signal”, $s_i(t)$, defined as

$$s_i(t) = x_i(t) + i\tilde{x}_i(t),$$  

(13)

where the notation $\tilde{g}(t)$ denotes the Hilbert transform of the function $g(t)$. Such a transform is defined as

$$\tilde{g}(t) = -\frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{g(\tau)}{t - \tau} d\tau$$  

(14)

where PV denotes the principal value of the integral. Once having a complex signal, it is easy to consider the argument of $s_i(t)$ as the phase, i.e. $\phi_i(t) = \arctan \left( \frac{\tilde{x}_i(t)}{x_i(t)} \right)$. However, to perform the convolution involved in the Hilbert transform is very costly from a computational point of view. We will show now that the same phase can be obtained by a much more efficient procedure. This is based upon the equality

$$g(t) + i\tilde{g}(t) = 2\mathcal{F}^{-1} \left[ \mathcal{F}[g(t)] \cdot \Theta(\omega) \right]$$  

(15)

involving the Fourier transform operator $\mathcal{F}$. Here $\Theta(\omega)$ is the Heaviside function: $\Theta(\omega) = 0$ for $\theta < 0$, $\Theta(\omega) = 1$ for $\theta \geq 0$ defined in the Fourier space $\omega$.

This relation can be proved by replacing

$$g(t) = \int_{-\infty}^{\infty} g(t_\omega) \delta(t - t_\omega) d\omega,$$  

(16)
in the right hand side of (15):

\[
2\mathcal{F}^{-1} \left[ \mathcal{F} \left[ \int_{-\infty}^{\infty} g(t_0) \delta(t - t_0) dt_0 \right] \cdot \Theta(\omega) \right] = 2 \int_{-\infty}^{\infty} \mathcal{F}^{-1} \left[ \mathcal{F} \left[ g(t_0) \delta(t - t_0) \right] \cdot \Theta(\omega) \right] dt_0
\]

Then one can achieve the calculation of the Hilbert transform by using two Fourier transforms. From the computational point of view, this is very efficient since the use of the fast Fourier transform involves a computer time of order \(O(T \log T)\) instead of \(O(T^2)\) which would be the case if one evaluates directly the convolution that defines the Hilbert transform (\(T\) is the length of the time series considered). All the results presented henceforth use the definition of the phase \(\phi(t)\) based upon the Hilbert transform of the \(x_i(t)\) variables. The numerical results come from a numerical integration of equations (3-4) using a stochastic Runge-Kutta method (known as the Heun method) with a time step \(h = 10^{-4}\).

In Fig 1 (left panel) we show the order parameter \(\rho\) as a function of the noise intensity \(D\) for different number of coupled systems. It turns out that the order parameter continuously decreases with increasing \(D\), thus showing that the quality of the synchronization worsens for large noise intensity. The dependence of the order parameter \(\rho\) on the system size for a fixed value of the noise intensity is shown in the right panel of the same figure. Note that the synchronization has only a very small dependence for relatively small values \(N > 100\).

![Figure 1](image-url)

**Figure 1.** Left panel: Order parameter \(\rho\) as a function of noise intensity for a system of globally coupled FitzHugh-Nagumo systems. Right panel: The same order parameter \(\rho\) as a function of the number of coupled systems \(N\). Values of the parameters: \(a = 1.1, \epsilon = 0.01, K = 2, D = 0.7\).

4. NUMERICAL INDICATORS OF COHERENCE RESONANCE

In this section we study numerically the existence of a coherence resonance with respect to the number of coupled systems. The qualitative behavior can already be observed by looking at the time traces. We have plotted in the right panel of figure 2 the time trace for the averaged variable \(X(t)\) while the left panel of the same figure shows the time trace for the variable \(x_1(t)\) of the first element (all the elements behave similarly), for three different
values of the number of coupled elements (see the caption of the figure for details of the parameters). Notice that for \( N = 128 \) the regularity of the amplitude of the emitted pulses is better than that corresponding to larger or smaller values of \( N \). This is a clear signature of coherence resonance. Moreover, it can be seen that the regularity in the averaged variable \( X(t) \) is better than in one of the individual elements, showing that the coupling allows for a smoothness of the trace. It is worth noting that the peaks in the collective variable \( X(t) \) and in \( x_1(t) \) are very well synchronized in time indicating that the individual systems are pulsing synchronously in time. In Figure 3 (right panel) we plot the time trace for the angle variable \( \Psi(t) \), as well as a time trace for a single \( \theta_1(t) \) (left panel). The same qualitative results that when looking at the \( x_1 \) and \( X \) variables are observed.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{Time series for the averaged variable \( X(t) \) (right panel), and for the individual variable \( x_1(t) \) (left panel) of the set of coupled FitzHugh-Nagumo systems, as obtained from a numerical integration of Eqs. (3-4), for different values of the number of coupled elements: \( N = 1 \) (top), \( N = 128 \) (middle) and \( N = 1024 \) (bottom). Observe that the largest regularity is obtained for the intermediate value of \( N \). Same parameters as in figure 1.}
\end{figure}

We now study two indicators commonly used to quantify this effect. The first indicator to quantify the regularity of the emitted pulses can be obtained by the \textit{jitter} of the time between pulses\(^8\). A pulse in the \( X(t) \) variable is defined when \( X(t) \) exceeds a certain threshold value \( X_0 \) (taken arbitrarily as \( X_0 = 0.3 \), although other values yield similar results). To define a pulse for the collective angle variable \( \Psi(t) \), we first set it to the interval \([-\pi, \pi]\) and then, for technical reasons, we take the modulus \(|\Psi|\). A pulse is defined whenever \(|\Psi|\) crosses the value \( 1.5 \). The jitters \( R_X \) and \( R_\Psi \) of both variables are defined as the ratio of the root mean square of the time between two consecutive pulses normalized to its mean value:

\[
R_X = \frac{\sigma[T_X]}{\langle T_X \rangle} \quad R_\Psi = \frac{\sigma[T_\Psi]}{\langle T_\Psi \rangle}
\]

(17)
Figure 3. Time series for the phase variable $\Psi(t)$ (left panel), and the individual phase variables $\phi_i(t)$ (right panel) of the set of FitzHugh–Nagumo systems, Eqs. (3-4). Similarly as in Figure 2, observe that again the largest regularity for the averaged $\Psi$ variable is obtained for the intermediate value of $N$. Note again that the largest regularity is obtained for the intermediate value of $N$. Same parameters as in figure 1.

The smaller the value of $R_{X,\Psi}$, the larger the regularity of the pulses (a value of $R_{X,\Psi} = 0$ indicates a perfectly periodic signal). In left panel of figure 4 we show the jitter for a single FitzHugh–Nagumo system, $N = 1$, as a function of the noise intensity. This displays a clear minimum at around $D \approx 0.02$. According to our theoretical treatment, we have considered a set of $N$ globally coupled oscillators at a higher value for the noise intensity, namely $D = 0.7$. We expect that the collective variables are subjected to an effective noise intensity $D/\sqrt{N}$ and, hence, the optimal value will be reached for a particular value of $N > 1$. It is shown in figure 5 that indeed the jitter $R_X$ and $R_{\Psi}$ in the collective variables $X$ and $\Psi$ have a well defined minimum at a value of $N \approx 100$, again showing the existence of the system size resonance.

The second indicator of coherence resonance uses the correlation function obtained from the time correlation function $C_X(t)$ of the averaged $X$ and phase $\Psi$ variables, defined as

$$C_X(t) = \frac{\langle \delta X(t') \delta X(t + t') \rangle}{\langle \delta X(t')^2 \rangle} \quad \delta X(t) = X(t) - \langle X(t') \rangle$$

$$C_{\Psi}(t) = \frac{\langle \delta \Psi(t') \delta \Psi(t + t') \rangle}{\langle \delta \Psi(t')^2 \rangle} \quad \delta \Psi(t) = \Psi(t) - \langle \Psi(t') \rangle$$

(18)

Here the averages $\langle \rangle$ are with respect to the time $t'$, after a small transient has been neglected. Figure 6 shows this correlation function for both the $X$ and $\Psi$ variables. It can be seen that the correlations extend further in time for an intermediate value, neither very large nor very small, of the number of coupled systems $N$. To obtain a quantitative indicator of this effect, we define the characteristic correlation times $\tau_X$ and $\tau_{\Psi}$ for each variable as

$$\tau_{X,\Psi} = \int_0^\infty |C_{X,\Psi}(t)| \, dt$$

(19)
Figure 4. We plot jitters $R_X$ and $R_\Psi$ of a single FitzHugh-Nagumo system, $N = 1$, as a function of noise intensity. The minimum at around $D \approx 0.02$ is the well known signature of coherence resonance for a single system\textsuperscript{18}. Same parameters as in figure 1.

![Diagram](image4)

Figure 5. We plot the jitters as a function of the number $N$ of coupled elements for a fixed value of the noise intensity $D$. Notice the presence of a minimum at around $N \approx 80$. Same parameters as in figure 1.

![Diagram](image5)

In practice, the upper limit of the integral is replaced by a value $t_{\text{max}}$ such that the correlation function can be considered as decayed to its asymptotic value $C_{X,\Psi} = 0$ ($t_{\text{max}} = 50$ for the data shown in figure 6). We have plotted these two correlation times in figure 7. Both times reach a maximum at approximately the same value $N \approx 100$, indicating that, for the set of parameters chosen, the maximum extent of the time correlation occurs for this number of coupled excitable systems.

5. CONCLUSIONS

In summary, we have shown that an ensemble of globally coupled FitzHugh-Nagumo excitable systems subjected to independent noises pulse on average with a regularity that is maximum for a given value of the number $N$ of coupled systems. An approximate calculation indicates that the collective variable $Y(t)$ is subjected to a noise of effective intensity $D/\sqrt{N}$. Therefore, even in the presence of a large amount of noise ($D$ large), it is possible to couple the right number of systems in order to optimize the periodicity of the emitted pulses $X(t)$. Notice that, since the individual variables $x_i(t)$ follow the same pattern than the collective one $X(t)$ (see Fig. 1), the periodicity is optimal in the individual trajectories $x_i(t)$ as well. The same optimal periodicity has been observed in the phase, defined by means of the Hilbert transform of the signal $X(t)$.
Figure 6. Correlation functions $C_X(t)$ and $C_\Psi(t)$ of the collective variables $X(t)$ and $\Psi(t)$, respectively, for the cases of $N = 1$ (dotted line), $N = 64$ (solid line) and $N = 1024$ (dashed line). Notice that, in agreement with the qualitative results derived from figures 1 and 2, the slower decay of the correlations corresponds to the intermediate values of the system size $N$. Same parameters as in figure 1.

An interesting connection of the stochastic and coherence resonance effects with some mechanisms of biological interest has been recently put forward by different authors who have analyzed the role that the dimension of the system (as measured by the number of components) may have in optimizing the periodicity of the output signal. For instance, references\textsuperscript{37–39} show that specific models of the Hodgkin-Huxley type predict that the pulses of $K^+$ and $Na^+$ concentration along biological cell membranes follow optimally an external periodic signal (stochastic resonance) for a given size of the number of ionic gates implied in the ionic transport. These latter results also show that, in the absence of external stimulus, the periodicity of the pulses is also optimal (coherence resonance) for an appropriate number of gates. Given that the FitzHugh–Nagumo system has been used previously to model some biological systems, we believe that our results, in the same lines than those of references\textsuperscript{37,38}, can be relevant when analyzing the collective response of such systems in a noisy environment, and can help to explain the observed size of some ensembles of excitable cells in living organisms.

Figure 7. Correlation times $\tau_X$ and $\tau_\Psi$ as obtained by integration of the absolute value of the respective correlation functions. Clear maxima (maximum extent of the correlations) can be observed around $N = 100$. 
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