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A Systematic Study of Elastic Proton-Nucleus Scattering

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Abstract. The aim of this study is to develop a state of the art tool to disentangle proton and neutron densities in exotic nuclei from hadron (proton and neutron mainly) scattering observables, and the extension to charge-exchange reactions. We start by folding of the nuclear densities with effective (one-boson-exchange) Nucleon-Nucleon (*NN*) interaction to describe scattering observables, via an optical potential. This has been shown to be suitable for kinetic energies in the range of 20 MeV to 1 GeV per nucleon, but has not been tested for exotic nuclei, which may have non standard nuclear densities (halos, *etc.*). We examine uncertainties associated with the choice of the effective *NN* interaction and improve on the treatment of nuclear corrections, such as Pauli blocking. We would provide with reasonable models for the nuclear densities when predictions of scattering observables are needed and study how a combination with electron scattering (conventional and parity violating) can constrain neutron and proton densities derived from hadron-nucleus scattering.

1 Motivation and Introduction

The study of the nuclear reactions is a challenging subject of nuclear physics both in theory and laboratory. This is useful to explain the nuclear structure of stable as well as exotic nuclei. The Nucleon-Nucleus interaction provides a wide source of information to determine the nuclear structure including spin, isospin, momenta, densities and gives a clear path towards the formation of exotic nuclei in the laboratory. One of the theoretical methods to study such type of reactions (Nucleon-Nucleus) is the "Relativistic Impulse Approximation" (RIA). The RIA provides an excellent quantitative description of complete sets of elastic proton scattering observables from various spin-saturated spherical nuclei at incident

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energies ranging from 200 to 400 MeV [1, 2]. The latter represents the theoretical framework for generating complex microscopic optical potentials for solving the elastic scattering Dirac equation. In particular, we focus on the relativistic Horowitz-Love-Franey (HLF) model [1–4] which parameterizes the *NN* scattering amplitudes as a number of Yukawa-type meson-exchange terms. More specifically, the latter microscopic optical potentials are generated by folding the HLF t-matrix with the relevant relativistic mean field Lorentz densities via the so-called $t\rho$-approximation. An attractive feature of HLF model is the existence of a simple relationship – lacking in conventional nonrelativistic models – between the Lorentz invariant *NN* scattering amplitudes and mesons mediating the interaction. Moreover, this model has a physical basis in the one-boson-exchange (OBE) picture, since the values of the real meson-nucleon coupling constants (at energies higher than 200 MeV) are similar to those arising from more sophisticated OBE models [1]. The reliability of the RIA has also been demonstrated by the fact that, for the above energies, predictions of elastic proton scattering observables are very similar to the corresponding results based on the highly successful global Dirac phenomenological optical potentials, which have been calibrated to provide excellent quantitative predictions of elastic proton scattering observables from stable nuclei ranging from ^{4}He to ^{208}Pb and for incident energies between 20 and 1040 MeV [5].

One of the frontier areas of research in nuclear physics is to understand the properties of nuclei far from the beta-stability line, the so-called exotic or unstable nuclei. Indeed, a number of radioactive ion beam facilities are currently under construction to study short-lived rare isotopes. In particular, there are plans to study exclusive proton-induced proton-knockout reactions (using inverse kinematics) at facilities such as RIKEN and GSI for both neutron- and proton-rich nuclei. For the eventual interpretation of these data it is essential to have reliable optical potentials. One can readily extend the above-mentioned folding procedure to calculate scattering potentials for proton-induced reactions on exotic nuclei. Moreover, the successful application of the relativistic $t\rho$ approximation to describe elastic proton scattering from stable nuclei, gives one confidence in extending this approach to study proton scattering on exotic nuclei. Two basic ingredients underly the realization of these folding potentials, namely a suitable analytical representation for the *NN* interaction in the energy range of interest, as well as an appropriate method (model) to obtain relativistic mean field Lorentz densities of the nucleus. Our motivation for this study can be summarized:

- Event generator for elastic proton-nucleus cross-sections and p-n reactions with focus on exotic nuclei
- Aiming for the best possible quantitative accuracy
- Derive an state of the art tool to analyze proton and neutron densities in exotic nuclei from hadron scattering observables
- Id. for charge-exchange reactions, i.e., include isovector channels, and

also we are looking for possible extensions to (^{3}H,^{3}He) and $(p, 2p)$

- Review the assumptions made in the existing (or to be developed) simulation packages when applied to the region of nuclei far from stability
- Include Jacobian transformations for inverse and other kinematics
- Provide reasonable models of nuclear densities for when predictions of scattering observables are needed

2 The Nucleon-Nucleon Scattering Amplitude

The non-linear RIA involves mainly two steps [6–8] of calculation. Basically a particular set of Lorentz covariant function [9], which multiply with the so called Fermi invariant Dirac matrix represent the *NN*-scattering amplitudes. This functions are then folded with the target densities of proton and neutron from the relativistic Langragian to produce a first order complex optical potential. The invariant *NN*-scattering operator $\widehat{\mathcal{F}}$ can be written in terms of five complex functions (the five terms involves in the proton-proton (*pp*) and neutron-neutron (*nn*) scattering). The function $\widehat{\mathcal{F}}$ can be expressed as [6–8],

$$\widehat{\mathcal{F}} = F_S + F_V \gamma_1 \gamma_2 + F^T {\sigma_1}^{\mu\nu} {\sigma_2}^{\mu\nu} + F^P {\gamma_1}^5 {\gamma_2}^5 + F_A {\gamma_1}^5 {\gamma_1}^\mu {\gamma_2}^5 {\gamma_2}_\mu . \quad (1)$$

Subscripts 1 and 2 distinguish Dirac operators in the spinor space of the two scattering particles. The five complex amplitudes for scalar (S), vector (V), tensor (T), pseudoscalar (P), and axial vector (A) interactions depend on the momentum transfer q and laboratory energy E. They are determined directly from the *NN* phase shifts which parametrize the physical *NN* scattering data.

Essentially Horowitz-Love-Franey model (which is the relativistic version of Love-Franey model (RLF)) parameterizes the complex relativistic amplitudes $F^L(q, E)$ in terms of a set of $N = 10$ meson exchanges in first-order Born approximation, such that both direct and exchange *NN* (tree-level) diagrams are considered separately, that is:

$$F^L(q, E) = \frac{iM^2}{2E_c k_c} [F^L_D(q) + F^L_X(Q)] \,, \quad (2)$$

where

$$F^L_D(q) = \sum_{i=1}^{N} \delta_{L,L(i)} \langle \vec{\tau}_1 \cdot \vec{\tau}_2 \rangle^{T_i} f^i(q) \quad (3)$$

$$F^L_X(Q) = (-1)^{T_{NN}} \sum_{i=1}^{N} C_{L(i),L} \langle \vec{\tau}_1 \cdot \vec{\tau}_2 \rangle^{T_i} f^i(Q) \,. \quad (4)$$

Here $T_i = (0, 1)$ denotes the isospin of the i^{th} meson, T_{NN} refers to the total isospin of the two-nucleon system, $\langle \vec{\tau}_1 \cdot \vec{\tau}_2 \rangle^{T_i}$ is 1 or -3 for different T_{NN} and

T_i, $C_{L(i),L}$ is the Fierz matrix, and

$$f^i(x) = \frac{g_i^2}{x^2 + m_i^2}(1 + \frac{x^2}{\Lambda_i^2})^{-2} - i\frac{\bar{g}_i^2}{x^2 + \bar{m}_i^2}(1 + \frac{x^2}{\bar{\Lambda}_i^2})^{-2} \tag{5}$$

where x represents the magnitude of either the direct three-momentum transfer q or the exchange-momentum transfer Q, $(g_i^2, \bar{g}_i^2)$, $(m_i, \bar{m}_i)$, and $(\Lambda_i, \bar{\Lambda}_i)$ are the real and imaginary parts of the coupling constant, mass, and cutoff parameter for the i^{th} meson.

These parameters are obtained by fitting the theoretical amplitudes with the values extracted from the *NN* scattering data.

In our calculations we use the following parametrization of these parameters to find Dirac optical potential:

RLF: D.P. Murdock and C.J. Horowitz, *Phys. Rev. C* **35** (1987) 1442.
This parametrization is appropriate at energies: 200 MeV $\leq E_{\text{lab}} \leq$ 400 MeV

Maxwell: O.V. Maxwell, E.D. Cooper, *Nucl. Phys.* **A656** (1999) 231;
O.V. Maxwell, *Nucl. Phys.* **A600** (1996) 509.
This parametrization is appropriate at energies: 200 MeV $\leq E_{\text{lab}} \leq$ 800 MeV

Hillhouse: Z.P. Li, G.C. Hillhouse, and J. Meng, *Phys. Rev. C* **77** (2008) 014001.
This parametrization is appropriate at energies: 50 MeV $\leq E_{\text{lab}} \leq$ 200 MeV

global: E.D. Cooper, S. Hama, B.C. Clark, *Phys. Rev. C* **80** (2009) 034605.
Global Dirac phenomenological optical potential from ^{4}He to ^{208}Pb;
This potential is appropriate at energies: 20 MeV $\leq E_{\text{lab}} \leq$ 1040 MeV

3 Nucleon-Nucleus Optical Potential

The Dirac optical potential is given by:

$$U_{\text{opt}} = -\frac{4\pi i p_{\text{lab}}}{M}\langle\overline{\psi}_2|\widehat{\mathcal{F}}|\psi_2\rangle, \tag{6}$$

where p_{lab} is the laboratory momentum of the incident proton and ψ_2 is the ground state wave function of the target nucleus, typically a (Hartree) product of single-particle four-component wave functions ϕ_α, with the states labeled by α. For spherical nuclei, parity implies only the S, V, and T parts of $\widehat{\mathcal{F}}$ contribute:

$$U_{\text{opt}}(q) = -\frac{4\pi i p_{\text{lab}}}{M}\left[F^S(q)\rho_S(q) + \gamma^0 F^V(q)\rho_V(q) - 2i\boldsymbol{\alpha}\cdot\widehat{\mathbf{r}}F^T(q)\rho_T(q)\right], \tag{7}$$

where the target densities are the Fourier transforms of the r-space scalar, vector (baryon), and tensor densities.

$$\rho_S(r) = \sum_\alpha^{\text{occ}}\overline{\phi}_\alpha\phi_\alpha;\ \rho_V(r) = \sum_\alpha^{\text{occ}}\phi_\alpha^\dagger\phi_\alpha;\ [\widehat{\mathbf{r}}\rho_T(r)]^i = \sum_\alpha^{\text{occ}}\overline{\phi}_\alpha\sigma^{0i}\phi_\alpha \tag{8}$$

In Ref. [2], Horowitz and Murdock have shown that the tensor potential has a negligible effect on all the observables and nuclei of interest. In our calculation we also neglect the small tensor term.

These optical potentials serve as input to solve the Dirac equation for elastic proton scattering so as to generate the relevant partial-wave scattering phase shifts for computing the scattering observables, namely the differential cross section $d\sigma/d\Omega$, analyzing power A_y and spin rotation function Q from the scattering amplitude.

4 Results

Our aims in this study can be summarized in the following points:

- Fold nuclear density with effective (OBE) *NN* interaction to describe scattering observables, via an optical potential
- Examine uncertainties associated with the choice of the effective *NN* interaction
- Improve the treatment of nuclear corrections, such as Pauli blocking (PB)
- Benchmark against existing data and phenomenological optical potentials. In Figure 1 we showed the results for the cross sections and analyzing power at different energies and nuclei using RLF parametrization and relativistic mean-field (RMF) theory with NLSH parameters [10, 11] to find proton and neutron densities of nuclei.

In this paper (due to limited number of pages), we present only the research of the scalar density on the basis of known vector density. Next we describe the treatment of the vector and scalar densities. The point proton density is fixed by using the results from electron scattering as follows,

$$\rho_V^p(r) = \frac{1}{(2\pi)^3} \int d^3q e^{-i\vec{q}\cdot\vec{r}} \tilde{\rho}_V^p(q) \tag{9}$$

where,

$$\tilde{\rho}_V^p(q) = \frac{\int d^3q e^{i\vec{q}\cdot\vec{r}} \rho_c(r)}{G(q)}. \tag{10}$$

Here $\rho_c(r)$ is the experimental charge density, $G(q)$ is the proton form factor. The point proton density, $\rho_V^p(r)$ is normalized to Z and the neutron density $\rho_V^n(r)$ is normalized to N. In our study we want to investigate different methods to find scalar density in the case of spherical nuclei, so as a first approximation we can assume that $\rho_V^n(r) \approx \rho_V^p(r)$. Firstly, we are concentrated on ^{12}C nucleus, the experimental charge density is taken from Ref. [12].

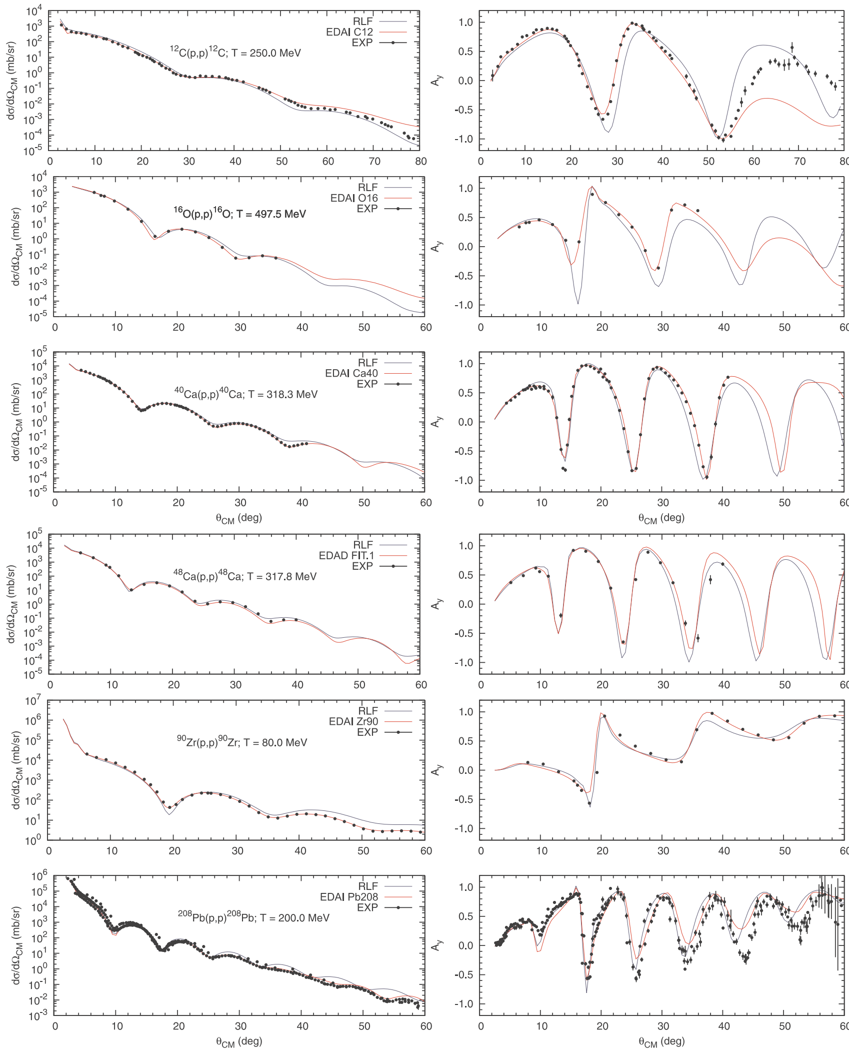

Figure 1. Benchmarking with proton data. The results for the cross sections $d\sigma/d\Omega$ and analyzing power A_y at different energies and nuclei using RLF parametrization (proton and neutron densities of nuclei are obtained within relativistic mean-field theory with NLSH parameters) and "global" Dirac phenomenological optical potential are compared with the experimental data [13].

We consider two prescriptions to find scalar densities:

method 1: based on the relativistic mean field theory and nuclear matter approximation [2]

The prescription is to approximate $\rho_S(r) = \langle(1 - v^2/c^2)^{1/2}\rangle\rho_V$ by

$$\rho_S(r) = \rho_V(r)\left[1 - \frac{3}{10}\frac{k_F^2(r)}{(M^*(r))^2}\right], \tag{11}$$

with $k_F(r)$ being the local Fermi momentum $k_F^3(r) = (3\pi^2/2)\rho_V(r)$. Since at the saturation vector density of $0.1934\ \text{fm}^{-3}$ the effective mass is $M^* = 0.56M_p$, the authors of Ref. [2] use a linear interpolation,

$$\frac{M^*(r)}{M_p} \approx 1 - 0.44\frac{\rho_V(r)}{0.1934\ \text{fm}^{-3}} = 1 - a\frac{\rho_V(r)}{0.1934\ \text{fm}^{-3}}. \tag{12}$$

For example, in the case when $M^*(0) = M^* = 0.56M_p$ and $\rho_V(0) = 0.1934\ \text{fm}^{-3}$ $\Rightarrow\ a = 1 - 0.56 = 0.44$.

method 2: following clear relation between scalar and vector densities

$$\frac{\rho_S^{p,n}(r)}{\rho_V^{p,n}(r)} \simeq \text{const.} \tag{13}$$

Thus scalar densities can be calculated using the relation between vector and scalar densities estimated by the relativistic Hartree which are introduced by Horowitz and Serot [14]. Following Ref. [14] and using RMF theory with NLSH parameters we can find the value of the constant of Eq. (13):

$$\text{const.} = \frac{4\pi\int_0^\infty \rho_S^{\text{nlsh}}(r)r^2dr}{4\pi\int_0^\infty \rho_V^{\text{nlsh}}(r)r^2dr} \quad\Rightarrow\quad \rho_S(r) \simeq \frac{\int_0^\infty \rho_S^{\text{nlsh}}(r)r^2dr}{\int_0^\infty \rho_V^{\text{nlsh}}(r)r^2dr}\rho_V(r) \tag{14}$$

The individual χ^2 for each data set are given by

$$\chi_\sigma^2 = \frac{1}{N_\sigma}\sum_{i=1}^{N_\sigma}\left[\frac{\sigma_{\text{exp}} - \sigma_{\text{theo}}}{\Delta\sigma_{\text{exp}}}\right]^2, \chi_{A_y}^2 = \frac{1}{N_{A_y}}\sum_{i=1}^{N_{A_y}}\left[\frac{A_{y_{\text{exp}}} - A_{y_{\text{theo}}}}{\Delta A_{y_{\text{exp}}}}\right]^2, \text{ and}$$

$$\chi_Q^2 = \frac{1}{N_Q}\sum_{i=1}^{N_Q}\left[\frac{Q_{\text{exp}} - Q_{\text{theo}}}{\Delta Q_{\text{exp}}}\right]^2, \tag{15}$$

where x_{exp}, Δx_{exp}, x_{theo}, and N are the experimental data, the error of the data, the calculated result, and the number of experimental points, respectively.

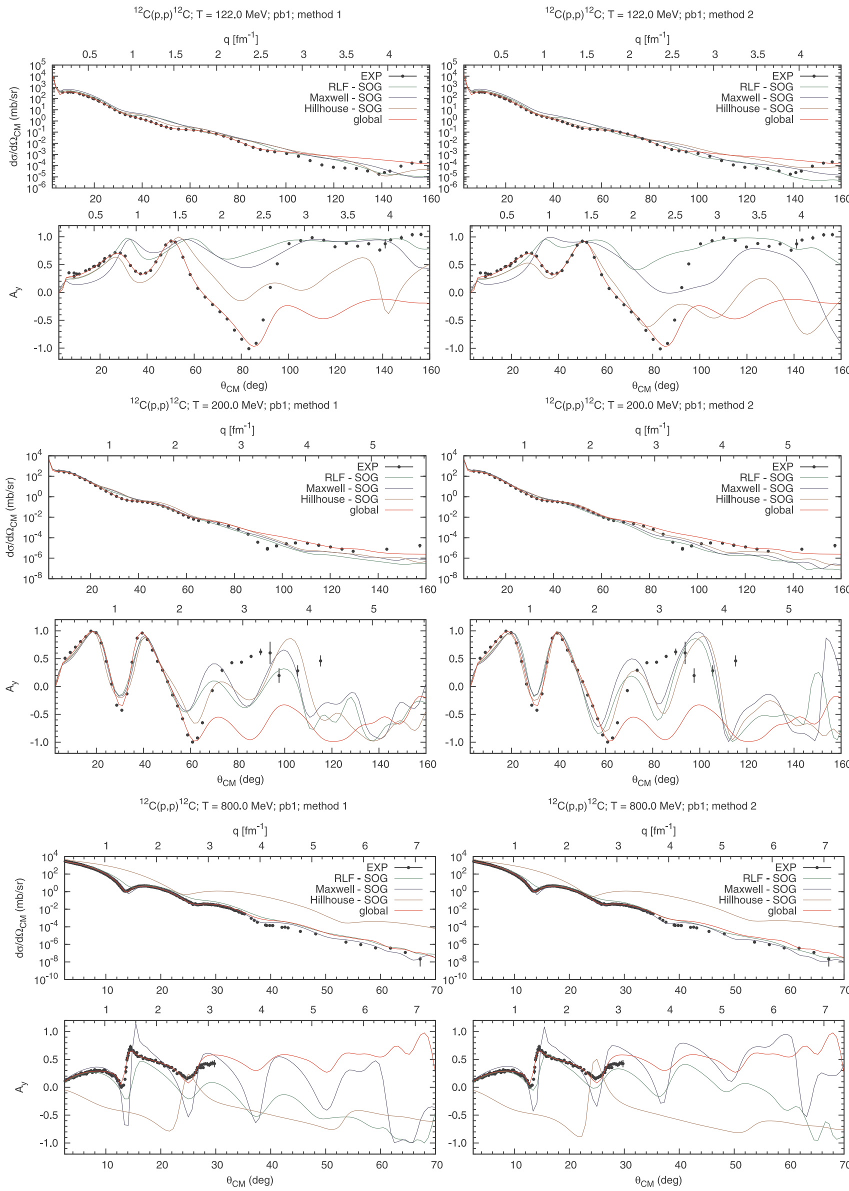

Figure 2. The results for the cross sections $d\sigma/d\Omega$ and analyzing power A_y at energies $E_{\text{lab}} = 122$, 200 and 800 MeV using RLF, Maxwell and Hillhose parameterizations and "global" Dirac phenomenological optical potential are compared with the experimental data [13].

Table 1. The individual χ^2_σ and $\chi^2_{A_y}$ for each data set of proton elastic scattering from ^{12}C nucleus, using different ways (**method 1** and **method 2**) to obtain scalar density and global OP [5]. Experimental data are taken from Ref. [13].

		^{12}C(p,p) – χ^2_σ			^{12}C(p,p) – $\chi^2_{A_y}$		
E_{lab}	paramet.	global	method 1	method 2	global	method 1	method 2
122 MeV	Hillhouse	3.95	674.16	**214.11**	3.31	430.97	**53.32**
160 MeV	Hillhouse	1.57	1052.78	**145.87**	9.02	259.41	**58.14**
200 MeV	RLF	10.40	446.33	**227.21**	14.23	**190.69**	223.13
250 MeV	RLF	5.21	90.67	**51.83**	14.21	**44.77**	68.86
494 MeV	Maxwell	18.91	3613.88	**1176.26**	7.57	374.23	**328.11**
800 MeV	Maxwell	38.47	329.49	**204.29**	3.30	283.25	**184.41**

In Table 1 are given results for individual χ^2_σ and $\chi^2_{A_y}$ calculated using Eq. (15) for different sets (122 MeV$\leq E_{\text{lab}} \leq 800$ MeV) of experimental data of proton elastic scattering from ^{12}C nucleus. The results for the cross sections and analyzing power are obtained with RLF, Maxwell and Hillhose parameterizations (above mentioned). In Table 1 are showed the name of parametrization for which (at considered energy E_{lab}) χ^2 has a minimum value and in red color is indicated the method which gives better values of χ^2. In almost of all the cases "method 2" gives better results for χ^2 than "method 1". This behavior can be seen also in Figure 2, where are presented results for the cross sections $d\sigma/d\Omega$ and analyzing power A_y at energies $E_{\text{lab}} = 122, 200$ and 800 MeV using RLF, Maxwell and Hillhose parameterizations and "global" Dirac phenomenological optical potential. As can be expected, the results for χ^2 in the case of "global" potential are the best. In our future calculations we will use **method 2** for the scalar density, improving the treatment of nuclear corrections (such as Pauli blocking) and fitting neutron density of nucleus we will try to find the results which give the χ^2 values better than those obtained from global potential.

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