Selected Topics on Nuclear Structure in Rare Electroweak Processes

O. Moreno, J. M. Boillos, and E. Moya de Guerra

Citation: AIP Conf. Proc. **1417**, 84 (2011); doi: 10.1063/1.3671042
View online: http://dx.doi.org/10.1063/1.3671042
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1417&Issue=1
Published by the American Institute of Physics.

Selected Topics on Nuclear Structure in Rare Electroweak Processes

O. Moreno, J.M. Boillos and E. Moya de Guerra

Dpto. Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid, Av. Complutense s/n, E-28040 Madrid, Spain

Abstract. We analyze the active-shell occupation probabilities of ^{76}Ge and their contribution to the single- and double-beta decay matrix elements of this isotope. Strength modifications of the nucleon spin-orbit interaction are introduced for a better comparison with experimental data. The relation of this spin-orbit interaction with the nucleon densities is explored, and the latter are analyzed for ^{76}Ge in the context of parity-violating electron scattering as a tool to extract accurate experimental information on neutron distributions.

Keywords: Microscopic nuclear models; Spectroscopic factors; Single and double beta decay; Neutron distribution; Elastic electron scattering
PACS: 21.60.Jz; 27.50.+e; 21.10.Pc; 23.40.Hc; 21.10.Gv; 25.30.Bf

The ground-state structure of the nuclei involved in the rare electroweak processes under study here is obtained within a selfconsistent mean field approach with pairing correlations. A Skyrme density-dependent force is used for the two-body nucleon-nucleon interactions, giving rise to the following form of the Schrödinger equation for the Hartree-Fock single-particle states [1]:

$$\left[-\vec{\nabla}\frac{\hbar^2}{2m^*(\vec{r})}\vec{\nabla}+U(\vec{r})-i\vec{S}(\vec{r})(\vec{\nabla}\times\vec{\sigma})\right]\phi_i(\vec{r})=e_i\,\phi_i(\vec{r})\,, \tag{1}$$

where m^* is an effective nucleon mass, U is a one-body central potential and $\vec{S}$ is a one-body spin-orbit potential. The Hartree-Fock (HF) equations together with the pairing BCS equations (HF+BCS) are solved iteratively to obtain after convergence the single-particle energies e_i, occupation probabilities v_i^2, and wave functions ϕ_i. The latter are expressed in an axially-deformed harmonic oscillator basis comprising 12 major shells.

The spin-isospin excitations related to beta decays can be computed as two-quasiparticle transitions where the connected levels have different isospin projections and differ at most in one unit of angular momentum. It is also possible to introduce spin-isospin correlations in the form of two-body separable interactions in the particle-hole (ph) and particle-particle (pp) channels, which will affect the two-quasiparticle beta transitions. The corresponding matrix elements, with different coupling constants χ_{ph} and κ_{pp}, are introduced in the QRPA equations. They account for the spin-isospin correlations between any two quasiparticles generated by the pairing correlation.

The active-shell occupation probabilities of ^{76}Ge and ^{76}Se have been recently measured [2], with some disagreements found with previous theoretical results. Using different Skyrme parametrizations, introducing nuclear deformation or modifying the pairing correlations do not avoid these discrepancies. To overcome them we have increased the

Workshop on Calculation of Double-Beta-Decay Matrix Elements
AIP Conf. Proc. 1417, 84-88 (2011); doi: 10.1063/1.3671042

strength of the spin-orbit interaction for neutrons in the Skyrme one-body potentials U and $\vec{S}$ (Eq. 1), reaching a good agreement with experimental results especially when axial deformation is considered [3]. The matrix element of the two-neutrino double-beta decay ^{76}Ge$\rightarrow$^{76}Se with the new active-shell occupation probabilities has also been obtained. We would like to focus now on the effect of these active-shell occupation probabilities on the Gamow-Teller (GT$^-$) transitions between the 0^+ spherical ground state of ^{76}Ge and the 1^+ excited states of ^{76}As. The corresponding strengths from a HF+BCS+QRPA calculation are shown in Fig. 1 as a function of the final-state excitation energies E^* (i.e. with respect to the ground state of the daughter nucleus). The latter are computed as $E_k^* = \omega_k - (\varepsilon_{\rm min}^p + \varepsilon_{\rm min}^n)$, where ω_k is the QRPA energy of the kth 1^+ excited state and $\varepsilon_{\rm min}^{p,n}$ are the minimum proton and neutron quasiparticle energies (not necessarily 1^+ excitations). This is an approximate procedure since, firstly, the daughter ground state is assumed to consist of a two-quasiparticle excitation of the parent even-even ground state; and secondly, because the QRPA energies ω_k are computed with respect to a correlated parent ground state, whereas the two-quasiparticle energies do not include ground-state correlations. A way to improve the transformation of QRPA energies to excitation energies is by shifting all the QRPA energies by the quantity $\mathscr{C} = E_1^{*\rm exp} - \omega_1$, which compares the experimental energy of the first 1^+ excited state in the daughter nucleus with the minimum QRPA energy.

The discrete spectrum and its gaussian folding (solid line) in Fig. 1 correspond to two-quasiparticle GT excitations, whereas the dashed line corresponds to the folded spectrum of QRPA GT excitations with residual interactions in the particle-hole and particle-particle channels. In the upper panel we show the results without any modification of the spin-orbit interaction strength (Sk3 force). The transitions between parent-neutron (ν) and daughter-proton (π) spherical states are clearly identifiable (no axial deformation is included in this analysis). The transitions identified in the figure, namely $\nu(2p_{3/2}) \rightarrow \pi(2p_{1/2})$, $\nu(1f_{7/2}) \rightarrow \pi(1f_{5/2})$ and $\nu(1g_{9/2}) \rightarrow \pi(1g_{7/2})$, together with the ones below 4 MeV, connect the active-shell states whose calculated occupations, as stated above, differ from the experimental ones. It is thus shown the relevance of these valence states for the computation of the single (and double) beta decay matrix elements of ^{76}Ge. The residual interactions mix the two-quasiparticle 1^+ states by means of the QRPA equations, yielding a new strength distribution (dashed line) that shows an accumulation of strength at a higher excitation energy, the Gamow-Teller giant resonance (GTGR).

In the lower panel of the same figure we present the same results but with a larger spin-orbit interaction strength for neutrons in the Skyrme-HF potentials. As a consequence, the energy splitting between neutron spin-orbit partners ($p_{3/2} - p_{1/2}$, $f_{7/2} - f_{5/2}$, $g_{9/2} - g_{7/2}$) increases, which translates into a change of the Gamow-Teller (proton-neutron) transition energies: the ones below 6 MeV decrease whereas the ones above increase. The latter transitions contribute most to the GTGR generated by the residual interactions, which as a result gets shifted to higher excitation energies. Thus, increasing the neutron spin-orbit strength improves the active-shell occupation probabilities and double-beta results, but worsens the GTGR energy position.

The spin-orbit interaction in the one-body Schrödinger equation for the non-relativistic HF single-particle wave functions can be thought of as coming from two sources. One is the non-local spin-orbit part of the two-body nucleon-nucleon force.

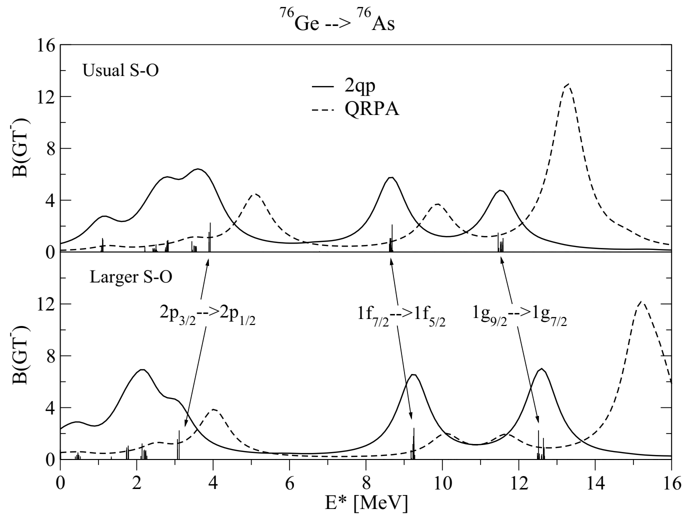

FIGURE 1. Gamow-Teller strength distribution of ^{76}Ge going to ^{76}As with HF+BCS ground state structures, with spin-isospin residual interactions treated in QRPA (dashed smooth line) and without residual correlations (two-quasiparticle excitations, solid smooth line and discrete spectrum).

Another contribution is relativistic in origin and can be made explicit by writing the Dirac equation for the upper component in Schrödinger form as [4]:

$$\left[-\frac{\nabla^2}{2m} - W_C - W_{SO}\,\vec{\sigma}\cdot\vec{l}\right]\phi = E'\phi \quad \text{with} \quad W_{SO} = \frac{1}{2m}\frac{1}{rA}\frac{\partial A}{\partial r} \tag{2}$$

The central potential term W_C is a rather involved combination of scalar V_S, vector V_V and Coulomb V_C central potentials as well as of the energy E and mass m of the particle ($E' = (E^2 - m^2)/2m$). The spin-orbit interaction term, which depends on $A = E + m + V_S - V_V - V_C$, is obviously very sensitive to the details of the nucleon distribution in nuclei, especially where the derivatives are larger, i.e. in the nuclear surface. This behaviour can be also reabsorbed into the spin-orbit term of the Schrödinger equation for Skyrme interactions Eq. 1 ([5]): $\vec{S}\cdot(\vec{\nabla}\times\vec{\sigma}) \sim \frac{1}{r}\frac{\partial\rho}{\partial r}\,\vec{\sigma}\cdot\vec{l}$.

The relation between the spin-orbit strength and the nucleon distribution leads us to comment on the experimental extraction of the latter. Whereas the proton distribution can be accurately extracted using charged lepton probes, the one for neutrons is more uncertain. Hadronic probes have been used in the latter case in the form of hadron scatterings, nuclear resonances excitations or antiprotonic atoms, but they suffer from a complex entanglement between the nuclear structure and the reaction mechanism. There is a rare weak process that has been proposed to solve the experimental puzzle of neutron distributions in nuclei. It is the weak interaction between a nucleus and a scattered electron, which is made apparent through parity-violation (PV) observables [6]. One of these observables is the PV asymmetry in elastic scattering of polarized electrons by nuclei, which is proportional to the difference between the cross-sections of

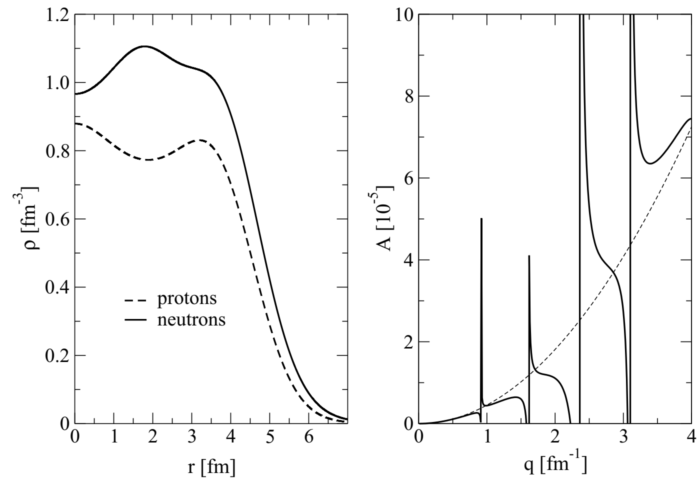

FIGURE 2. Left: Neutron (solid line) and proton (dashed line) spherical densities of ^{76}Ge from a HF+BCS calculation. Right: PV asymmetry (solid line) in PWBA for elastic electron scattering by ^{76}Ge in its ground state, using the nucleon densities appearing on the left (see text).

incoming electrons longitudinally polarized parallel and antiparallel to their momentum:

$$\mathscr{A} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} \approx 3.5 \cdot 10^{-6}\ \mathrm{GeV}^{-2}\ Q^2 \left[\frac{NF_n^0}{ZF_p^0} + \frac{\beta^p}{\beta^n} \right] , \tag{3}$$

which has been written in terms of the nucleon form factors $F_{p,n}^0$ of the target nucleus [7]. The latter expression is valid in plane-wave Born approximation (PWBA), for even-even nuclei and neglecting the neutron internal form factor. Since the nucleon weak charges ratio is small, $\beta^p/\beta^n = -0.08$, the experimental asymmetry is approximately proportional to the neutron to proton form factors ratio, what may cause a Q^2 dependence slightly different from the pure linear Q^2 (4-momentum transfer squared).

In Fig. 2 we show to the left the neutron (solid line) and proton (dashed line) densities of ^{76}Ge from a spherical HF+BCS calculation, and to the right the PV asymmetry in elastic electron scattering (solid line) obtained from these densities in PWBA including nucleon internal electric form factors. The asymmetry is plotted against the momentum transfer q (1 fm^{-1} = 197.3 MeV). The reference value (dotted line with a linear Q^2 dependence) is exclusively due to the weak interaction and ignores the nuclear and nucleon structure effects (i.e. it assumes $\rho_n(r)/N = \rho_p(r)/Z$ for every r).

The proton form factor is, as mentioned above, usually well known, so the neutron distribution may be extracted from experimental PV asymmetries, as planned in the PREX experiment for ^{208}Pb [8]. This kind of experimental information will be useful not only in relation to the strength of the nucleonic spin-orbit interaction, but also to describe halos or neutron skins in exotic neutron-rich nuclei, to determine accurately the neutron equation of state (important for the structure of neutron stars), or to extract

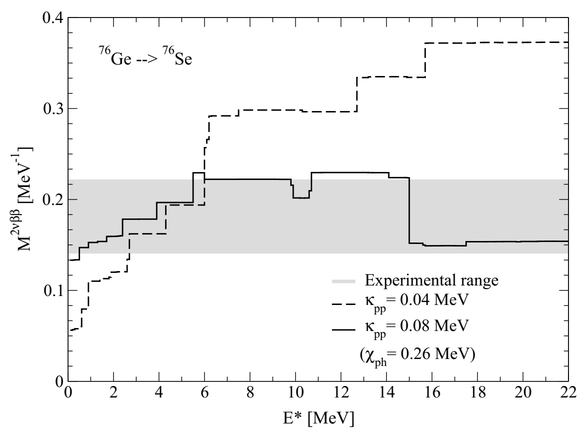

FIGURE 3. Nuclear matrix element running sums of the two-neutrino double-beta decay ^{76}Ge→^{76}Se from a HF+BCS+QRPA calculation using different *pp* residual interaction strengths.

precise values of fundamental constants from atomic PV experiments.

The last question we would like to address here is that of the strengths of the spin-isospin residual interaction in the *pp* channel. In Fig. 3 we show the running sum of the nuclear matrix element for the two-neutrino double-beta decay ^{76}Ge→^{76}Se from a HF+BCS+QRPA calculation including an increased spin-orbit strength for neutrons. Results for different values of the *pp* correlation strength are shown. It can be seen that increasing the value of κ_{pp} definitely modifies the shape of the matrix element running sum by introducing negative contributions at higher excitation energies that compensate the positive ones. This behaviour is similar to the one shown in [9] for larger values of the renormalization constant g_{pp} of the particle-particle residual interaction matrix element (whose exact relation to κ_{pp} is not simple).

This work was supported by Ministerio de Ciencia e Innovación (Spain) under Contract FIS2008-01301. J.M.B. thanks 'Beca de Colaboración' of Ministerio de Educación.

REFERENCES

1. D. Vautherin and D. M. Brink, *Phys. Rev. C* **5**, 626 (1972).
2. J.P. Schiffer *et al.*, *Phys. Rev. Lett.* **100**, 112501 (2008); B.P. Kay *et al.*, *Phys. Rev. C* **79**, 021301R (2009).
3. O. Moreno, E. Moya de Guerra, P. Sarriguren and A. Faessler, *Phys. Rev. C* **81**, 041303R (2010).
4. J.M. Udias, P. Sarriguren, E. Moya de Guerra, E. Garrido and J.A. Caballero, *Phys. Rev. C* **5**1, 3246 (1995).
5. O. Moreno, P. Sarriguren and E. Moya de Guerra, *J. Phys: Conf. Ser.*, accepted for publication (2011).
6. T.W. Donnelly, J. Dubach and I. Sick, *Nucl. Phys. A* **503**, 589 (1989); O. Moreno, P. Sarriguren, E. Moya de Guerra, J.M. Udias, T.W. Donnelly and I. Sick, *Nucl. Phys. A* **828**, 306 (2009).
7. O. Moreno, E. Moya de Guerra, P. Sarriguren and J.M. Udias, *J. Phys. G: Nucl. Part. Phys.* **37**, 064019 (2010).
8. http://hallaweb.jlab.org/parity/prex/
9. D. Fang, A. Faessler, V. Rodin, M.S. Yousef and F. Simkovic, *Phys. Rev. C* **79**, 014314 (2009).