On-line Quality Control of Non-linear Batch Systems: Application to the Thermal Processing of Canned Foods

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ABSTRACT

The recently developed Q-PI approach for quality control of linear continuous systems is extended to non-linear batch systems. The new control law contains proportional and integral terms with which the operators are intimately familiar. It is shown that control quality may be improved or costs reduced, consistent with the process specifications, by adjusting the tuning constants of the controller. The performance of the control law is demonstrated on a simulated batch retort, and the results indicate the excellent capability of the proposed approach for quality control of (nonlinear) batch processes.

NOTATION

α_1	0.000124
α_2	- 0.00139
A	Antoine's-law parameter = 24.633
A_b	Cross-section in bleeder, m ²
A_{rt}	Retort area, m ²
B	Antoine's-law parameter = 4893
C_{PI}	Specific heat capacity of water = 4180 J kg ⁻¹ K ⁻¹
C_{prt}	Specific heat capacity of retort = 500 J kg ⁻¹ K ⁻¹
C_{PS}	Heat capacity of steam = 1695.22 + 0.5713 T(K) J kg ⁻¹ K ⁻¹
F	Flow, kg/s
G_p	Laplace domain-transfer function
h	Enthalpy, J/kg
h_c	Convection heat-transfer coefficient = 5.475 W m ⁻² K ⁻¹

h_{rt}	Enthalpy of steam in retort
K_P	Equivalent-linear-system gain
M_s	Mass of steam accumulated, kg
M_s	Molecular weight of water, kg/mol
M_{rt}	Mass of retort, kg
P_{atm}	Atmospheric pressure = $100211.4 \text{ N m}^{-1}$
P_{rt}	Pressure of saturated steam in retort, N m^{-2}
Q	Rate of heat absorbed, J s^{-1}
R	Universal gas constant, $\text{J mol}^{-1} \text{ K}^{-1}$
s	Laplace transform operator
T_{rt}	Temperature of steam in retort, K
T_{ref}	Reference temperature = 373 K
T_{Rset}	Set-point temperature, K
U	Controller output, dimensionless: $U_{min} = 12$, $U_{max} = 254$
U_s	Internal energy of accumulated steam, J/kg
V	Pseudo-manipulated variable
V_{rt}	Volume of retort, m^3

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Greek

ξ	Damping coefficient (Q-PI tuning parameter)
τ	Characteristic time constant (Q-PI tuning parameter)
τ_P	Time constant of equivalent linear system
β_k	GLC design parameters
γ	$C_{ps}/(C_{ps} - R)$
λ	Latent heat of water, J/kg
θ	Stefan-Boltzmann constant = $5.676 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
ε	Emissivity of retort shell, dimensionless = 0.94

Subscripts

b	Outlet flow from bleeder
C	Condensate flow

con	Convection
i	Inlet steam flow

INTRODUCTION

Batch systems are widely used in a variety of applications, including food-processing, polymerization, biotechnology processing, and in the production of specialty chemicals. Automatic control of batch systems poses difficulties due to the non-linearities present. The two relevant problems in control of batch systems are 1) how to shift the nonlinear process from one operating level to another, and 2) how to regulate the process at the second level for the remainder of the batch cycle so that the production objectives are satisfied.

Research aimed at developing better methods for controlling nonlinear systems is being vigorously pursued and is reported in the literature (Boye & Brogan, 1986; Kravaris & Chung, 1987; Kulkarni et al., 1991). A review of non-linear control methods is also available (Bequette, 1991). One of these methods, of relevance to this work (Kravaris & Chung, 1987), exploits the concepts from differential geometry resulting in a linear transformation between the controlled variable and a new pseudo-manipulated variable. The linear transformation allows the user to design a linear-control law with which the operators are familiar but still achieve vastly superior performance of the non-linear system.

Recently, a new approach was presented (Deshpande, 1992; Deshpande et al., 1992) for quality control of continuous linear systems. The control laws, called Q-PID for regulatory control and Q-PI for servo control, contain PID, lead-lag, and dead-time compensation terms with which the operators are intimately familiar. It was shown that, by adjusting the tuning constants of these algorithms, one could improve quality or reduce costs, in terms of manipulated variable movements, consistent with the product-quality specifications.

Automatic control of batch retorts used in the thermal processing of canned foods poses difficulties due to the non-linear nature of the process. Previous work on this subject, reported in the literature (Holdsworth, 1983; Mulvaney et al., 1990), is basically limited

to the use of PID type of controllers. In this paper, we combine the linearizing-transformation ideas from differential geometry and the quality-control ideas from Q-PID/Q-PI approaches to develop a control law that can be used to achieve excellent control of batch retorts. The performance of the control law is demonstrated by a simulation example.

PROCESS DESCRIPTION

A schematic diagram of the retort is shown in Fig. 1. In the first stage (venting), steam is used to eliminate air from the retort, and, as a result, the temperature goes up from room temperature to 100°C until the pressure is the equivalent to that of saturated steam at this temperature. This ensures the removal of air from the system. Once this stage is completed, the heating cycle begins. The goal in this stage is to drive the system to a pre-specified temperature and maintain it at this temperature in the presence of noise as long as required. Temperature and time are selected on the basis of specified lethality.

In the third stage (cooling stage), the control objective is to cool the canned food as soon as possible and avoid sudden pressure drops that could result in the bursting of cans with the subsequent waste of the product. This problem has been considered in the literature (Kelly & Richardson, 1987).

Although constant temperature set-point profiles (CRT) have been used in these operations, several authors indicate that a variable temperature set-point profile (VRT) may only marginally increase the retention of nutrients (Teixeira et al., 1975; Saguy & Karel, 1979; Nadkarni & Hatton, 1985), but others (Banga et al., 1991a, b) indicate that VRT may maximize surface retention of quality factors and minimize process time or energy consumption. Some examples of these profiles reported by Banga et al. (1991a) are shown in Fig. 2 along with a classical constant-temperature profile. In the case of variable-temperature profiles, the control objective is to make the retort temperature follow the set point profile as closely as possible.

From a control point of view, the heating stage represents a non-linear single-input single-output (SISO) operation. The cooling stage, on the other hand, is a MIMO (multi-input multi-output) operation.

The non-linearity of the system is a major contributing factor to control problems during the heating stage, whereas process interactions add to the complexities during the cooling stage. Bumpless switching from the first- to the second-stage operation is also a relevant issue. The focus in this paper is on quality control of the heating-stage operation.

MATHEMATICAL MODEL

The control technology we present is based on dynamic mathematical model of the batch process. Such a model may be developed from unsteady-state mass and energy balances. The detailed development is shown in the appendix. It is clear that the final expression relating the controlled variable (retort temperature) to the manipulated variable (steam flow) is a non-linear differential equation. Control of systems such as this with linear PID controllers poses problems due to the non-linearities present (Mulvaney et al., 1990). In the past, such problems in process industries have been overcome by incorporating adaptive features, such as gain-scheduling, self-tuning, model reference control, etc. (Mulvaney et al., 1990).

The approach we follow utilizes differential-geometry and Lie-algebra concepts (Kravaris & Kantor, 1990a, b), which enable the designer to represent the non-linear system relating an output Y to an input U as a new system, which linearly relates Y to a pseudo-manipulated variable V by means of a linearizing transformation. Once the system is linearized, a wide array of linear techniques available in the literature (Deshpande & Ash, 1988; Deshpande, 1989) can be brought to bear to solve the control problem for improved performance of the non-linear system. In particular, we demonstrate the application of the recently developed quality-control approach to the heating stage (Deshpande, 1992; Deshpande et al., 1992).

From the Appendix, the non-linear differential equation, eqn (A-8), relating the retort temperature to the steam flow may be represented as:

$$\frac{dT_r}{dt} = (T_r)F_i + g(T_r) \quad (1)$$

In accordance with the concepts from differential geometry, the linearizing transformation is:

$$\left(\sum_{k=0}^r \beta_k \frac{d^k}{dt} \right) T_r = V \quad (2)$$

Where r represent the relative order of the non-linear system, which is the smallest order of the derivatives of the controlled variable that depends explicitly on the manipulated variable (Kravaris & Kantor, 1990a,b). It can be easily shown that in this instance r is unity. Equation (2) then reduces to:

$$\beta_0 T_r + \beta \frac{dT_r}{dt} = V \quad (3)$$

Substituting for the derivative of T_r from eqn (1) into eqn (3) gives:

$$\beta_0 T_r + \beta_1 (T_r) F_i + \beta_1 g(T_r) = V \quad (4)$$

Solving eqn (4) for the manipulated variable, F_i , gives:

$$F_i = \frac{V - \beta_1 g(T_r) - \beta_0 T_r}{\beta_1 f(T_r)} \quad (5a)$$

In the present example, we use a linear relationship between the controller output and steam flow (Mulvaney et al., 1990) according to:

$$F_i = \alpha_1 U + \alpha_2 \quad (5b)$$

Combining eqns (5a) and (5b) gives:

$$U = \frac{V - \beta_1 g(T_r) - \beta_0 T_r + \beta_1 f(T_r) \alpha_2}{\beta_1 f(T_r) \alpha_1} \quad (5c)$$

In eqn (5c), β_0 and β_1 are parameters of the equivalent linear system that must be chosen by the designer for optimal performance. The linearized system may be controlled by a linear-control law for improved response of the non-linear plant. The control approach we have adopted is as follows.

By taking the Laplace transform of the terms in eqn (3), the transfer function of the linearized system may be obtained as:

$$G_p(s) = \frac{K_p}{1 + \tau_p s} \quad (6)$$

where:

$$K_p = \frac{1}{\beta_0} \quad \tau_p = \frac{\beta_1}{\beta_0}$$

Now, the closed-loop transfer function of the linear SISO system relating T_{rt} to T_{Rset} is:

$$\frac{T_{rt}}{T_{Rset}} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \quad (7)$$

Solving eqn (7) for $G_c(s)$ gives:

$$G_c(s) = \frac{\frac{T_{rt}}{T_{Rset}}}{G_p(s) \left(1 - \frac{T_{rt}}{T_{Rset}} \right)} \quad (8)$$

Equation (8) may be solved for G , provided that a suitable expression is adopted for (T_{rt}/T_{Rset}) . Utilizing a second-order trajectory gives:

$$\frac{T_{rt}}{T_{Rset}} = \frac{2\xi\tau s + 1}{\tau^2 s^2 + 2\xi\tau s + 1} \quad (9)$$

A variety of responses can be derived from eqn (9) by changing the values of α and τ as shown in Fig. 3. Note that perfect control is theoretically feasible. Also note that eqn (9) says nothing about the cost of achieving the specified performance in terms of the manipulated variable movements.

Substituting eqns (6) and (9) into eqn (8) gives:

$$G_c(s) = \frac{2\xi\tau_p}{k_p\tau} + \left(\frac{2\xi\tau s + \tau_p}{k_p\tau^2} \right) \frac{1}{s} + \frac{1}{k_p\tau^2} \cdot \frac{1}{s^2} \quad (10)$$

Equation (10) involves the familiar proportional and integral terms. The terms ξ and τ are the tuning constants of the algorithm, ξ determines damping and τ is a function of the settling time. This controller has been called Q-PI by its developers (Deshpande et al., 1992). By adjusting ξ and τ , the designer may improve servo quality or reduce costs in accordance with the process-specification requirements.

RESULTS AND DISCUSSION

A block diagram of the linearly transformed system is shown in Fig. 4. The block diagram of the equivalent linear system with the Q-PI linear controller is shown in Fig. 5. All simulation programs were written in FORTRAN and implemented on the VAX cluster at the University of Louisville. The system consists of a VAX 8650 mainframe linked to several 11/780 machines. The differential equation representing the plant, eqn (1), was solved by the RKG (Runge-Kutta-Gill) subroutine that is part of DEC's IMSL Library package. The integration was made over 200 seconds with a step size of 1 second.

As has been pointed out earlier, β_0 , β_1 , ξ , and τ are design parameters. The parameter values are taken from the work of Mulvaney et al. (1990). For the illustrative case where the canned food is to be heated from 100°C to 121°C, the optimized values of these parameters, based on the ISE (integral-of-the squared-error) criterion, were determined by a random-search procedure (Banga & Casares, 1987) giving $\beta_0 = 1.146$, $\beta_1 = 183.51$, $\xi = 9.89$, and $\tau = 89.69$ seconds. The closed-loop-response curves in the absence of and in the presence of noise, along with the manipulated-variable movements, are shown in Figs 6(a)-(c). For this test, a white-noise sequence having a standard deviation of 0.0004 was employed in accordance with the work of Mulvaney et al. (1990).

For comparison, the closed-loop responses with a traditional PI controller ($K_c = 150$; $\tau_i = 100$) employed by Mulvaney et al. (1990) are shown in Figs 7(a) and 7(b). These responses are similar to those depicted in Figs 6(a)-(c) but the manipulated-variable movements with the Q-PI controller are smoother. Furthermore, the Q-PI approach permits the designer to specify a variety of desired set-point trajectories by selecting appropriate values of ξ and τ . These parameters are also robustness tools; in the presence of modeling errors, they may be adjusted to maintain stability.

This example involves a weakly non-linear system and consequently the performance of the linear PI controller may be deemed adequate. As the process non-linearities increase, the Q-PI control strategy will yield significantly better performance relative to linear-PID-type control. For example, the cooling stage of the batch-retort operation involves a MIMO system that is more non-linear than the system under study here. It is expected that Q-PI controller will deliver superior results; further studies are in

progress, and the results will be communicated in due course. Finally, it may be noted that the Q-PI controller involves computing elements that are readily available on microprocessor-based programmable controllers, and additional expense on hardware is therefore not needed.

Note that eqn (A-8) in the Appendix does not include the heat-conduction term in Q to account for the presence of cans. If desired, this term can be added, which along with the Fourier law, would permit the user to solve the control problem by following exactly the same procedure. In this instance, the Fourier equation would have to be solved by a numerical procedure, such as finite differences. In any event, our studies indicate that the dynamic performance of the control system is unaffected even when the heat absorbed by the cans is considered in the analysis.

CONCLUSIONS

A new approach to improved control of non-linear batch systems is presented. Its capabilities are demonstrated for the heating phase of a retort application with excellent results. It is shown that the new approach gives the user the ability to improve quality or to reduce costs consistently with the process-specification requirements. The approach can be extended to the cooling stage, as will be shown in an ensuing paper.

REFERENCES

- Banga, J. R. & Casares, J. J. (1987). ICRS: application to a wastewater treatment plant model. I. Chem. E. Symp. Ser., 100, 183-92.
- Banga, J. R., Perez-Martin, R. I., Gallardo, J. M. & Casares, J. J. (1991a). Optimization of thermal processing of conduction-heated canned foods: study of several objective functions. J. Food Engng., 14, 25-51.
- Banga, J. R., Simpson, R., Almonacid, S. & Torres, J. A. (1991b). MOPT: multipurpose optimization of thermal processing. Paper presented at the 52nd Annual Meeting of the Institute of Food Technologists (IF191), 1-5 June, Dallas, TX, USA.
- Bequette, B. W. (1991). Non-linear control of chemical processes - a review. Industr. Engng. Chem. Res., 30, 1391-413.

- Bird, R. B., Stewart, W. E. & Lightfoot, E. N. (1960). Transport Phenomena. John Wiley & Sons, New York, NY, USA.
- Boye, A. J. & Brogan, W. L. (1986). A non-linear system controller. *Int. J. Control*, 44, 1209-18.
- Deshpande, P. B. (Editor) (1989). Multivariable Process Control, Instrument Society of America.
- Deshpande, P. B. (1992). On-line quality control with PID-type controllers. *Chem. Engng. Prog.*, 88, 71-6.
- Deshpande, P. B. & Ash, R. H. (1988). Computer Process of Control with Advanced Applications, 2nd edition. Instrument Society of America.
- Deshpande, P. B. et al. (1992). On-line quality control of servo systems, *Hydrocarbon Processing*, 71, 75-6.
- Holdsworth, S. D. (1983). Developments in the control of sterilising retorts. *Process Biochem.*, 16 (5), 24-8.
- Kelly, P. T. & Richardson, P. S. (1987). Computer modelling for the control of sterilization processes. Campden Food Preserving Research Association Technical Memorandum No. 459, Chipping Campden, Glos., UK.
- Kravaris, C. & Chung, C. (1987). Nonlinear state feedback synthesis by global input-output linearization. *AIChE J.*, 33, 592-603.
- Kravaris, C. & Kantor, J. (1990a). Geometric methods for nonlinear process control: 1 - Background. *Industr. Engng. Chem. Res.*, 29, 2295-310.
- Kravaris, C. & Kantor, J. (1990a). Geometric methods for nonlinear process control: 2 - Controller synthesis. *Industr. Engng. Chem. Res.*, 29, 23 10-23.
- Kravaris, C., Wright, R. A. & Carrier, J. F. (1989). Nonlinear controllers for trajectory tracking in batch processes. *Computers Chem. Engng.*, 13, 73-82.
- Kulkarni, B. D., Tambe, S. S., Shukla, N. V. & Deshpande, P. B. (1991). Nonlinear pH control. *Chem. Engng. Sci.*, 46, 995-1003.
- Lee, P. L. & Sullivan, G. R. (1988). Generic model control. *Computers Chem. Engng.*, 12, 573-80.
- Mulvaney, S. J., Rizvi, S. S. H. & Johnson, C. R. (1990). Dynamic modelling and computer control of a retort for thermal processing. *J. Food Engng.*, 11, 273-89.
- Nadkarni, M. M. & Hatton, T. A. (1985). Optimal nutrient retention during the thermal processing of conduction-heated canned foods: application of the distributed minimum principle. *J. Food Sci.*, 50, 1312-21.

Saguy, I. & Karel, M. (1979). Optimal retort temperature profile in optimizing thiamine retention in conduction-type heating of canned foods. *J. Food Sci.*, 44, 1485-90.

Teixeira, A. A., Zinsmeister, G. E. & Zahradnik, J. W. (1975). Computer simulation of variable retort control and container geometry as a possible means of improving thiamine retention in thermally processed foods. *J. Food Sci.*, 40, 656-9.

APPENDIX

For the material balance, we have:

$$\frac{dm_s}{dt} = F_i - F_b - F_c \quad (\text{A-1})$$

For the energy balance, we have:

$$\frac{d}{dt}(m_s U_s) = F_i h_i - F_b h_c - Q \quad (\text{A-2})$$

We assume saturated steam to be an ideal gas. The mass of steam accumulated in the retort may be computed according to:

$$m_s = \frac{P_{rt} V_{rt} M_s}{RT_{rt}} \quad (\text{A-3})$$

where P_{rt} and T_{rt} are related by Antoine's law according to the equation:

$$P_{rt} = \exp\left(A - \frac{B}{T_{rt}}\right) \quad (\text{A-4})$$

The derivatives of m_s and U_s are:

$$\frac{dm_s}{dt} = \frac{BP_{rt}}{T_{rt}^2} \frac{dT_{rt}}{dt} \quad (\text{A-5})$$

and

$$\frac{dU_s}{dt} = \left(C_{ps} - \frac{R}{M_s}\right) \frac{dT_{rt}}{dt} \quad (\text{A-6})$$

Substituting eqn (A-5) into eqn (A-1) and solving for F_c gives:

$$F_c = F_i - F_{cb} - \left(\frac{M_s V_{rt} P_{rt} (B - T_{rt})}{RT_{rt}^3}\right) \frac{dT_{rt}}{dt} \quad (\text{A-8})$$

Substituting eqns (A-6) and (A-7) into equation (A-2) and solving for $d T_{rt}/dt$, we obtain the working equation:

$$\frac{dT_{rt}}{dt} = \left(\frac{\alpha_1 - \alpha_2}{\xi - \alpha_2 \beta} \right) F_i - \left(\frac{\alpha_3 - \alpha_2}{\xi - \alpha_2 \beta} \right) F_b - Q \quad (\text{A-8})$$

where:

$$\alpha_1 = \Delta h_i - \Delta h_{rt} - \frac{RT_{rt}}{M_s}$$

$$\alpha_2 = C_{pl}(T_{rt} - T_{ref}) - \lambda - \Delta h_{rt} + \frac{RT_{rt}}{M_s}$$

$$\alpha_3 = \frac{RT_{rt}}{M_s}$$

$$\beta = \frac{M_s V_{rt} M_s (B - T_{rt})}{RT_{rt}^3}$$

$$\xi = \frac{P_{rt} V_{rt} M_s}{RT_{rt}} \left(C_{ps} - \frac{R}{M_s} \right) + M_{rt} C_{prt}$$

The terms Δh_i and Δh_{rt} represent variations in enthalpy from the reference state (saturated steam at 373 K). The term F_b is the flow from the bleeder assuming isentropic flow (Bird et al., 1960). This is given by:

$$F_b = A_b \frac{P_{rt}}{\sqrt{RT_{rt}}} \sqrt{\frac{2\gamma}{\gamma-1}} \left(\frac{P_{atm}}{P_{rt}} \right)^{1/\gamma} \sqrt{\left(1 - \frac{P_{atm}}{P_{rt}} \right)^{\gamma-1/\gamma}} \quad (\text{A-9})$$

where

$$\gamma = \frac{C_{ps}}{C_{ps} - R}$$

The heat lost by radiation and convection is given by the term:

$$\dot{Q} = \dot{Q}_{rad} + \dot{Q}_{conv}$$

where

$$\dot{Q}_{rad} = \theta \varepsilon A_{rt} (T_{rt}^4 - T_a^4)$$

$$\dot{Q}_{conv} = h_c A_{rt} (T_{rt} - T_a)$$

Fig. 1. Schematic of a batch steam retort.

Fig. 2. Temperature set-point profiles: VRT1 maximizes overall retention of nutrients (e.g., thiamine); VRT2 maximizes surface retention of nutrients; VRT3 minimizes process time with a constraint on surface retention of a quality factor; CRT is typical constant-temperature set-point profile for maximum overall retention of nutrients.

Fig. 3. Second-order servo-response curves for several values of ξ and τ .

Fig. 4. Block diagram of linearized open-loop system.

Fig. 5. Block diagram of Q-PI-based control system.

Fig. 6. Performance of the control system with and without noise: (a) temperature responses; (b) manipulated variable profiles; (c) pseudo-manipulated variable profiles.

Fig. 7. Performance of the PI control system with and without noise: (a) temperature responses; (b) manipulated variable profiles.