Stakeholder Bargaining Games*

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Abstract

We study bilateral bargaining problems with an interested third party, the stakeholder, that enjoys benefits upon a bilateral agreement. To address the strategic implications of stakeholders over negotiations, we consider a model where two bargainers interact in the presence of a third party that (a) can transfer a share of her benefits to the bargainers but cannot receive a share of the bilateral surplus, and (b) while she may not participate in all periods of the negotiation, she cannot remain entirely inhibited. Our main findings are (1) that the stakeholder's (reverse) liquidity constraint implies the existence of a multiplicity of stationary subgame perfect equilibria that include outcomes with very asymmetric bilateral agreements, and (2) that the partial participation of the stakeholder may be the source of severe inefficiency.

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1 Introduction

Bargaining between two parties often affects *stakeholders*, third parties who care for the resolution of the conflict, and yet are unable to impose an agreement upon the contending bargainers. We model such bargaining problems as non-cooperative games with three players: two bargainers, who must reach an agreement on how to split one unit of bilateral surplus, and a third player, the stakeholder, who enjoys a positive externality upon the bilateral agreement. Two specific features distinguish stakeholder bargaining games from regular trilateral negotiations. The first is the (reverse) liquidity constraint that affects the stakeholder who can transfer a share of her benefits to the bargainers but cannot receive a share of the bilateral surplus. The second is that while the stakeholder cannot commit to remain inhibited, her participation is not assured in every period of the negotiation either.¹

The stakeholder can only be at best a facilitator who is not allowed however to extract any of the surplus from the bargaining parties. Scenarios of this type abound in industrial relations. In sectors of public interest the disruption of essential services like public transport, air traffic control, hospitals or the supply of utilities has a substantial impact on the population at large. Correspondingly, the government's stake in bilateral conflicts which are of public concern is of great consequence. In these situations, while the government might be willing to provide a handout to foster agreement, claiming a share of the bilateral surplus under negotiation is not an option. Our model also applies to international relations. For example, in an armed conflict that threatens the political stability of a region extending beyond the geographically disputed borders, the international community is a stakeholder. Third party intervention often takes the form of positive transfers (aid) to facilitate agreements; transfers from the contender to the international organizations are out of the question.

The effects of the liquidity constraint are addressed in the first part of the paper. We show that in a tripartite bargaining game - where the stakeholder is active throughout the negotiations - when the stake is small, this constraint causes a multiplicity of stationary subgame perfect equilibria. This multiplicity of equilibria supports outcomes in which the stakeholder does not transfer any portion of her stake and yet the bargainers share the bilateral surplus very differently from the usual Rubinstein-Stahl split. This occurs because the lack of transferability creates a slack in the equilibrium conditions that relate

¹Since the only action available to the stakeholder is to transfer her stake, our model does not apply to those settings whereby the interested third party has the ability to inflict damage onto the two contenders; nor to situations where the two original negotiators might be prepared to transfer resources to the stakeholder in order to keep him out of negotiations; in other words, we do not allow for the possibility of bribes.

the current offer to the expected payoff of rejecting and this slack can be taken up in various ways. This result requires that the stakeholder has some bargaining power in the sense that she can put forward proposals.

In the second part of the paper we focus on the effects of partial participation/abstention by the stakeholder, and we show that the sheer possibility that the stakeholder may intervene in negotiations creates the potential for delays. The reason is simple: if the institutional arrangements are such that the stakeholder is not a full participant in all periods, the bargainers may have an incentive to delay bilateral agreement in the hope to pressurize her into conceding extra resources. Hence, the potential participation of a stakeholder may have detrimental effects, introducing inefficiencies - in stationary strategies and under complete information - where there would not be any in its absence. The expectation of an increased surplus to share causes delays that, from the point of view of the bargainers, are efficient. This is a particular instance of the more general phenomenon discussed in [11]. In the present context, however, when all affected parties are taken into account, delays are inefficient.

One could object that since with perfect information bilateral negotiations always ensure an efficient outcome², there is no reason why a third party, the stakeholder, should get involved. Yet, as it is often the case in services of public interest, the situation at hand may be one such that the stakeholder cannot get away from. It is thus important to emphasize that in the present analysis the stakeholder's does not choose to become involved; because she is unable to commit to inhibition, she sometimes finds herself drawn into the bargaining table. This immediately presents a potential for exploitation by the two bargainers: as long as the stakeholder suffers from disagreement, bargainers should succeed in extracting resources from the stakeholder, and this opens the door to inefficient delays.

In addition to the connection with [11] discussed above, our paper relates to the literature on bargaining with arbitration or mediation, and to the literature on coalition formation. The assumption that stakeholders enjoy a positive externality distinguishes our model from models of bargaining with arbitration or mediation, in which a third party (the mediator) derives no utility from an agreement (e.g. [3], [7] and [9]). Externalities are a main concern in the recent literature on coalition formation (see [1], [4], [6], [8], [12], [17]). The main focus of this literature is to characterize non-cooperative equilibrium outcomes for bargaining problems in partition function. Stakeholder bargaining situations can be viewed as pertaining to the class of three player bargaining games in partition function

²This is the case in the Rubinstein bargaining model. However it is well known that inefficient equilibria can obtain in alternating offers bargaining models even with complete information, once the original extensive form ([16]) is modified. See chapter 7 in [13].

analyzed in depth in [2] and [5]³. However, the crucial features of stakeholder bargaining are the liquidity and participation constraint of one of the players, and these have not been addressed in the preceding literature.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 characterizes equilibria in games where the stakeholder is present every period and points out at the difference with standard three player bargaining. The emergence of delays in equilibrium is addressed in section 4. Section 5 concludes.

2 Stakeholder bargaining games

The general setup for a Stakeholder Bargaining (SB) game is as follows. Two bargainers (indexed 1 and 2) have to agree between them over the division of some surplus, normalized to unity, which is available only in case of agreement. This agreement has an effect over player 3, the stakeholder, who obtains a positive payoff of s if the negotiators settle their dispute, and 0 otherwise. Thus the bargainers consensus on how to split their bilateral surplus of 1 is necessary and sufficient to generate the total surplus $1 + s$, where the stake s is an externality that the bilateral agreement generates over 3. The set of feasible agreements is a subset of what it would be if all parties involved had the same rights on $1 + s$ (which is the case in standard three player bargaining). Figure 1 depicts the bargaining set in the stakeholder bargaining game and clarifies this point. While the set of agreements in a standard three player bargaining problem consists of all points in three dimensional simplex of size $1 + s$, the feasible set of agreements in the present environment corresponds only to the shaded trapeze.

Bargaining proceeds over a (potentially infinite) number of rounds, $t = 0, \Delta, 2\Delta \dots$. At each period, the state of the game determines whether player 3 is active or not, specifies the set of admissible proposals and appoints the proposer. The initial state of the game is given (possibly at random). In rounds where 3 is not active the game is in a bipartite state, denoted $s_t = s_i^B$, tripartite states are denoted s_i^T and the subindex indicates the proposer. Proposals at bipartite states consist of a division of the bilateral surplus $x = (x_1, x_2)$ with $x_1, x_2 \geq 0$ and $x_1 + x_2 = 1$, and an agreement at such states yields a division of the total surplus (x_1, x_2, s) . At tripartite states, bargainers may receive resources from 3 in addition to their share of the bilateral surplus, but 3 cannot extract resources from the contending bargainers beyond the value of the stake s , thus a proposal is a division of the total surplus $y = (y_1, y_2, y_3)$ $y_1, y_2 \geq 0$, $0 \leq y_3 \leq s$ and $y_1 + y_2 + y_3 = 1 + s$.

³Their messages are mixed: The model of [2] yields non-unique stationary subgame perfect equilibria that may be inefficient and where externalities are irrelevant for equilibrium payoffs. In contrast, [5] obtains a unique equilibrium were the agents payoffs are monotone and (piece-wise) linearly dependent on the externalities they impose on others.

Figure 1: Set of feasible agreements in the stakeholder bargaining game; (y_1, y_2, y_3) is one such point.

Each period, the proposer chooses a proposal and the responder(s) accept or reject; in tripartite states replies follow the natural order.⁴ When a proposal is accepted (by all active responders), the game ends and the agreement is implemented. Upon rejection, play moves to the next time period and the state at $t + 1$ is determined as follows: if $s_t = s_i^B$ then with probability p the state becomes $s_{t+1} = s_j^B$, $j \neq i$, and with probability $1 - p$ the state becomes $s_{t+1} = s_3^T$; if $s_t = s_i^T$ then $s_{t+1} = s_{i+1}^T$ with probability 1.⁵ Agents have a common discount factor $\delta = e^{-r\Delta}$, where r is the discount rate. Consequently an agreement reached at time t allocating the total surplus according to some division $z = (z_1, z_2, z_3)$ yields utility $u_i(z, t) = \delta^t z_i$ to player i .

This general description includes as extreme cases pure bilateral bargaining without 3's intervention ($s_0 = s_1^B$ and $p = 1$); and tripartite stakeholder bargaining (TSB) where bipartite rounds are ruled out ($s_0 = s_i^T$). The extensive form of the SB game with $s_0 = s_i^B$ and the TSB game is sketched in Figure 2.

Strategies and Subgame Perfect Equilibria (SPE) are defined in the standard way. Stationary strategy profiles are fully characterized by a vector specifying proposals and acceptance thresholds (whichever applicable) for each player at each state of the game

⁴We follow the convention that 1 follows 3. More generality on the timing of replies is inconsequential. Also, we are formally assuming that at tripartite states the stakeholder has veto power over bilateral agreements, even if $x_3 = s$. This is only for expositional simplicity and has no consequence, since exercising this veto power is a dominated action.

⁵The analysis extends easily to other specifications of state transitions, including time and state dependent transition probabilities.

a) The SB subgame

where she is active. A *Stationary Subgame Perfect Equilibrium* (SSPE) is a profile of strategies that constitutes an SPE. A SSPE is an *Immediate Agreement Equilibrium* (IAE) if it yields agreement at $t = 0$; and a *Delayed Agreement Equilibrium* (DAE) is an SSPE where disagreement occurs at least in one state of the game.

We start by pointing out at the fundamental difference between an SB game and a trilateral negotiation over $1 + s$.

3 The stakeholder's liquidity constraint

Assume that the initial state of the game is tripartite and negotiations begin with the stakeholder proposing at time $t = 0$, $s_0 = s_3^T$.

Because she cannot access any of the bilateral bargaining surplus, the stakeholder's payoff can be at most s (if an agreement prevails and she makes no contributions to bargainers) - this produces quite dramatic effects. If s is not too great, the unique stationary equilibrium of the corresponding 'equal-rights' trilateral bargaining problem is no longer achievable; furthermore, the presence of a stakeholder introduces multiplicity of equilibria even in stationary strategies.

As it is well known (see e.g. [14]), the standard trilateral alternating bargaining game over $1+s$ yields a unique stationary equilibrium partition, i.e. $y^* = \left(\frac{1+s}{1+\delta+\delta^2}, \frac{\delta(1+s)}{1+\delta+\delta^2}, \frac{\delta^2(1+s)}{1+\delta+\delta^2} \right)$, which in the limit as the discount factor approaches unity converges to $y^* = \left(\frac{1+s}{3}, \frac{1+s}{3}, \frac{1+s}{3} \right)$, where the first, second and third entry refer to the first, second and third mover payoffs, respectively. Because in the TSB game the stakeholder's share is bounded above by s , when s is not 'too large' there is a continuum of partitions of the bilateral surplus that can be supported in a stationary equilibrium and moreover the splits of the bilateral surplus that are reached along such equilibria can be very different from the usual bilateral monopoly one. In the present set up if s is relatively small, when the stakeholder proposes the equilibrium payoff of the (responding) bargainers exceeds the equilibrium continuation payoff, and because of this slack, bargainers cannot take anything away from the stakeholder (whose equilibrium payoff is at any rate less than what she would get in trilateral bargaining over $1 + s$). Then we can state the following:

Proposition 1 *Assume the initial state is $s_0 = s_3^T$ with probability 1. In the limit as $\Delta \rightarrow 0$:*

i) When s is sufficiently small (i.e. $s < \frac{1}{2}$), any proposal $(x, 1 - x, s)$ where the stakeholder makes no contribution, with $x \in \left[\frac{1}{3}(1 + s), \frac{2-s}{3} \right]$ can be supported in a SSPE, and there are no other SSPE.

ii) Otherwise, when $s \geq \frac{1}{2}$, the unique SSPE outcome coincides with that of a trilateral bargaining game over $1 + s$.

iii) *There can be no DAE.*

Proof. See the Appendix.

To gain the intuition for this result, fix a discount factor $\delta \in (0, 1)$ and consider a node off the equilibrium path where, following a series of rejections, the stakeholder puts forward the equilibrium proposal $(x, 1 - x, s)$. Backtracking to the previous period, an offer by bargainer 2 would be accepted only if it yielded each respondent a payoff at least equal to the present discounted value of his continuation payoff following a rejection. This means that bargainer 2 would have to concede δs and δx to players 3 and 1, respectively, and retain for himself the residual payoff $1 + (1 - \delta)s - \delta x$. Similarly, backtracking one more period, the proposal by bargainer 1 would be accepted only as long as it yielded $\delta^2 s$ and $\delta(1 + (1 - \delta)s - \delta x)$ to players 3 and 2, respectively, leaving player 1 with the residual payoff $1 + s - \delta(1 + (1 - \delta)s - \delta x) - \delta^2 s = (1 + s)(1 - \delta) + \delta^2 x$. The latter implies that any proposal $(x, 1 - x, s)$ by the stakeholder should yield at least $\delta(1 + s)(1 - \delta) + \delta^2 x$ to player 1 and $\delta(1 + (1 - \delta)s - \delta x)$ to player 2, that is $x \geq \delta(1 + s)(1 - \delta) + \delta^2 x$ and $1 - x \geq \delta(1 + (1 - \delta)s - \delta x)$. Provided s is not too large, there is a whole range of possible proposals $(x, 1 - x, s)$ which are acceptable to both bargainers, and in all of them at least one bargainer obtains an equilibrium payoff in excess of his continuation payoff! This would never be possible in a stationary equilibrium of a standard trilateral bargaining game: in that case it cannot be optimal for a proposer to settle for an agreement that rewards opponents more than their continuation payoff, as she could improve on her own payoff by claiming a bit more, until she makes her opponents indifferent between accepting and rejecting. This is not always possible in the SB game, though, as the stakeholder can at most retain all of her stake s in equilibrium. Because of this reverse liquidity constraint, if the stake is ‘small’, this upper bound on the stakeholder’s payoff has a bite, and it is this which causes a multiplicity of stationary equilibria to arise. Note that this result does not depend on δ , and in the limit as the period between successive offers, Δ , becomes negligibly small (so that $\delta \rightarrow 1$) the range of values of x which can be supported in an SSPE lies in the closed interval $\left[\frac{1}{3}(1 + s), \frac{2-s}{3}\right]$.

The ‘disturbance’ introduced by the presence of the stakeholder makes itself felt even when the stake is negligibly small: as $s \rightarrow 0$, and $\delta \rightarrow 1$ the admissible range for x approaches $\left[\frac{1}{3}, \frac{2}{3}\right]$, whereas the division $(\frac{1}{2}, \frac{1}{2})$ would prevail uniquely in the absence of the stakeholder. The reason for this is that while in a standard bilateral bargaining game negotiators’ offers would make the respondent just indifferent between accepting and making a counterproposal, this threat to ‘punish’ a greedy proposer by rejecting an offer and getting the upper hand in the next round as a proposer is watered down by the presence of the stakeholder, who could come in and insist on any division of the bargaining surplus.

The multiplicity of equilibria is a fundamental feature that does not depend on our specific formulation of the protocol. It is not difficult to check that a proposition in the same vein as proposition 1 holds for a modification with random proposers.

The set of equilibria of proposition 1 is depicted in Figure 3 as the thick black line, where one possible equilibrium agreement $(x, 1-x, s)$ is highlighted. The delimiting sloping border on the left side is the set of feasible agreements all yielding a payoff of $\frac{1}{3}(1+s)$ for player 1. Similarly, the delimiting sloping border on the right hand side corresponds to those feasible agreements yielding a payoff $\frac{2-s}{3}$ to player 2.

Figure 3: The equilibrium agreements of proposition 1.

The proof of point **iii)** in proposition 1 implies that as long as all three agents are active, agreement is *not* delayed:

Corollary 2 *There can be no DAE due to disagreement in a tripartite negotiation round.*

Details are in the Appendix, but the intuition is straightforward. Since we are dealing with stationary strategies, the only way to obtain delayed agreement is that for some tripartite state, in equilibrium all proposal are rejected. The consequence is that the proposer in that state loses all the bargaining power deriving from her role as the proposer. On the other hand, all of the agents ‘pay’ the cost of a rejection by moving negotiations to the next round. This is however what destroys the possibility of delay in equilibrium, as the agent whose offer is always rejected can ‘bank’ on these rejection costs and make an offer which makes her better off and that responders have an incentive to accept. To see this, take for instance a candidate equilibrium where the stakeholder’s proposal

is always rejected. In this case the bargainers can coordinate and extract the entire stakeholder's surplus, by insisting on a complete handout each time they make a proposal. The stakeholder would have no incentive to refuse such a proposal, as a rejection would trigger either a subgame where the stakeholder is a proposer, in which case no agreement follows; or one in which one of the bargainers makes the same offer the stakeholder has just rejected. However, precisely for this reason, this cannot be an equilibrium, for the stakeholder has an incentive to make an offer that would be accepted: for instance, she could offer immediately the same surplus division as a bargainer would do in the next round, and give out all of her stake but a small amount. Similar arguments apply to the candidate equilibrium where a bargainer's proposal is always rejected. It is then immediate that TSB games, since all agents are active at all rounds, cannot have DAE.

A remark on delay and non-stationary equilibria: In multilateral bargaining games of alternating proposals, stationary equilibria do not exhaust the set of SPE, other non-stationary equilibria may exist (See eg. section 3.13 in [14]). This is the case in the present model, too. Non-stationarity allows us to construct equilibria with delayed agreement. At any rate, in general the punishments needed to support delayed equilibria of this type are rather extreme, while seldom observed in practice.⁶

4 Limited participation and delays

We now turn our attention to games where the stakeholder's participation is not always assured and we show that this translates in the possibility that in a stationary equilibria agreements are delayed. As discussed above, if a DAE exist, it must be that the stakeholder is not always active, and some rounds of play take place in a bipartite state where bargainers disagree. Thus the question is whether the stakeholder's limited participation is in itself sufficient for a DAE to exist, and if and under what conditions efficient equilibria obtain.

As we show below, in a SB game that combines bipartite states and some tripartite states (where the stakeholder acts as responder), when the stake is sufficiently sizeable, the unique SSPE is a DAE. The intuition for this result is that whenever the stakeholder is involved in negotiations as a responder, the possibility arises for the bargainers to extract a sizable part of her stake. Thus, because bargainers can extract additional surplus from the stakeholder, if the stake is sufficiently large they have the incentive to 'sit it out', that is to disagree in bipartite states and wait for the stakeholder to become active. On the other hand, when the stake is sufficiently small, the unique equilibrium is an IAE:

⁶Complexity arguments can be used to justify stationary strategies in bargaining games. See for instance [18].

Proposition 3 *For each SB game with $s_0 = s_i^B$ and $0 < p < 1$, in the limit as $\Delta \rightarrow 0$:*

- i) if $s \geq \frac{1}{2}$ the unique equilibrium is a DAE in which disagreement prevails at all $s_t = s_i^B$ and immediate agreement prevails at $s_t = s_i^T$;*
- ii) if $s < \frac{1}{2}$ the unique equilibrium is an IAE.*

Proof. See the Appendix.

To see why Proposition 3 holds, recall that Corollary 2 establishes that there can be no disagreement in tripartite states. Consequently, the only way for a DAE to occur is because of disagreement in bipartite states. This can be supported in equilibrium only if the expected discounted continuation joint payoff exceeds the joint payoff from a bilateral agreement. A necessary condition for this to happen is that some contribution from the stakeholder is forthcoming, which is possible as we saw above only when the stake is sufficiently large. In short, games where at any round the involvement of the stakeholder is not assured generate disagreement whenever the stake is sufficiently attractive.

5 Conclusions

In this paper we have introduced stakeholder bargaining games as a novel frame of reference for bilateral negotiations when the interests of third parties - stakeholders - can be affected, so that - and this is the crucial feature - the stakeholder may have an interest to inject additional resources in the dispute in order to encourage agreement. Our analysis makes two main points: i) When the stakes are small and the participation of the stakeholder is assured a continuum of equilibria exist and very asymmetric bilateral agreements may prevail. ii) When the participation of the stakeholder is uncertain and the stake is high inefficiencies abound, in the sense that delayed agreements obtain in equilibrium with positive probability.

Besides being suited to analyze negotiations which are originally configured as stakeholder bargaining games, our model lends itself to a number of possible extension. For instance it could be used to investigate the incentives that exists in bilateral bargaining to relinquish irrevocably one's bargaining rights and thereby opt for a decentralized bargaining protocol. For concreteness, a public authority with the ability to relocate property rights may want to separate into two (a stakeholder and a bargainer) when negotiating with another party. Our analysis suggests that in the context of public sector negotiations, this implies that reducing the size of government (i.e. the stake) can be advantageous from the perspective of the public sector, in the sense that governments may have an incentive to decentralize - and become stakeholders rather than directly involved negotiating partners - as they may gain as stakeholders an advantage which would be

lost in direct negotiations (i.e. retaining their stake). An issue which is closely related is the extent to which a stakeholder might be able to *choose* whether or not to intervene in negotiations.⁷ Finally, a more realistic approach would be to consider the effects of uncertainty on the part of the bargainers over the exact amount of resources available to the stakeholder. We leave these issues to be addressed properly in future research.

6 Appendix

Proof of proposition 1

i) As a preliminary, let $x^i = (x_1^i, x_2^i, x_3^i)$ denote the surplus division of $1 + s$ proposed by agent i , where $x_1^i + x_2^i + x_3^i = 1 + s$ and $x_3^i \leq s$. Assume that $x^3 = (x, 1 - x, s)$. With $x \in \left[\frac{\delta(1+s)}{1+\delta+\delta^2}, \frac{1+\delta-\delta^2s}{1+\delta+\delta^2} \right]$ and $\delta \in (0, 1)$, equilibrium strategies are for bargainer 1 to propose x^1 with $x_1^1 = \delta^2 x + (1 - \delta)(1 + s)$ and $x_3^1 = (1 - \delta^2)s$; for bargainer 2 to propose x^2 with $x_2^2 = 1 - \delta x + (1 - \delta)s$ and $x_3^2 = (1 - \delta)s$; and for the stakeholder to propose $(x, 1 - x, s)$. Regarding responses, each agent accepts an offer yielding at least the present discounted value of the equilibrium continuation payoff and rejects otherwise. Equilibrium proposals are derived based on the following considerations.

For a proposal to be accepted by the stakeholder in a subgame where bargainer i proposes it must be that

$$\begin{aligned} x_3^2 &= \delta s \\ x_3^1 &= \delta^2 s \end{aligned}$$

Bargainer i 's proposed share x_i^i for herself must satisfy

$$\begin{aligned} 1 + (1 - \delta^2)s - x_1^1 &= \delta x_2^2 \\ 1 + (1 - \delta)s - x_2^2 &= \delta x \end{aligned}$$

Given their strategies, the stakeholders' proposal $(x, 1 - x, s)$ is acceptable to the bargainers only insofar as

$$\begin{aligned} x &\geq \delta x_1^1 = \delta \left(\delta^2 x + (1 - \delta)(1 + s) \right) \Rightarrow x \geq \frac{\delta(1+s)}{1+\delta+\delta^2} \equiv \underline{x} \\ 1 - x &\geq \delta x_2^2 = \delta \left(1 + s - x_1^1 - \delta^2 s \right) = \delta^2 (1 - \delta x + (1 - \delta)s) = \delta^2 x_2^2 \Rightarrow x \leq \frac{1+\delta-\delta^2s}{1+\delta+\delta^2} \equiv \bar{x} \end{aligned}$$

whereby

$$\bar{x} > \underline{x} \Leftrightarrow s < \frac{1}{\delta(1+\delta)} = \underline{s}.$$

From the equilibrium conditions above it is clear that only proposals $(x, 1 - x, s)$ with $x \in \left[\frac{\delta(1+s)}{1+\delta+\delta^2}, \frac{1+\delta-\delta^2s}{1+\delta+\delta^2} \right]$ can be supported in an SSPE. Moreover, at least one bargainer

⁷In [15] and [10], we explore games where the stakeholder's decision to participate is strategic, modelling the environment as a concession game.

obtains a payoff in excess of his continuation payoff. If there were another SSPE, then it would have to be the case that the stakeholder makes a contribution $\gamma_i \geq 0$ to each bargainer, $0 < \gamma = \gamma_1 + \gamma_2 \leq s$, so that the outcome is $(x + \gamma_1, 1 - x + \gamma_2, s - \gamma)$, and at least one bargainer obtains a payoff in excess of his continuation payoff. It is clear that both players getting a payoff in excess of their continuation value cannot be an equilibrium, since the stakeholder could decrease her contribution a bit and still get acceptance. Say that only player 2 gets a payoff in excess of his continuation value, then the stakeholder could improve on this offer by a proposal $(x + \gamma'_1, 1 - x + \gamma'_2, s - \gamma')$ choosing $\gamma'_1 = \gamma_1 + \varepsilon$ and $\gamma'_2 = \gamma_2 - 2\varepsilon$: provided that ε is sufficiently small, both bargainers will still want to accept, and the stakeholder will retain a higher portion of her payoff. Taking the limit as $\Delta \rightarrow 0$ (so that $\delta \rightarrow 1$) yields the statement.

ii) The proof is standard thus omitted for the sake of brevity, see e.g. [14].

iii) Consider state s_i^T and let V_j $j = 1, 2, 3$ denote players' ex-ante expected payoffs in the continuation after disagreement prevails in this state. In any DAE it must be that $V_1 + V_2 + V_3 \leq 1 + s$, so that $1 + s > \delta(1 + s) \geq \delta(V_1 + V_2 + V_3)$. The two extremes of this last set of inequalities guarantee that the proposer, i , is better off by having her proposal accepted, as a deviation to an acceptable proposal (yielding responders $j, k \neq i$ a total share $\delta(V_j + V_k)$) guarantees a higher payoff than any proposal that j and k reject (since $1 + s - \delta(V_j + V_k) > \delta V_i$). Note that when the proposer is the stakeholder, either $s = \max\{s, 1 + s - \delta(V_1, V_2)\}$, in which case the previous argument applies; or $s \neq \max\{s, 1 + s - \delta(V_1, V_2)\}$, in which case by deviating to an acceptable offer the stakeholder would achieve a payoff of s , which exceeds any discounted continuation payoff.

■

Proof of proposition 3

Recall from the proof of proposition 1 that $\underline{s} \equiv \frac{1}{\delta(1+\delta)}$. From corollary 2, a DAE requires that (equilibrium) proposals in bipartite states are rejected. Denote by $W_i^B = V_{i1}^B + V_{i2}^B$ the joint expected payoffs of bargainers upon disagreement at state s_i^B , and by $W^T = V_1^T + V_2^T$ the joint expected payoff in tripartite states s_3^T . Obviously W_i^B must solve $W_i^B = p\delta W_j^B + \delta(1-p)W^T$. The continuation payoff to the bargainers in tripartite states s_3^T depends on how large the stake s is. Thus there can be two cases:

(A) Suppose $s \geq \underline{s}$, so that the outcome in state s_3^T is immediate agreement at the standard trilateral partition $(\frac{\delta(1+s)}{1+\delta+\delta^2}, \frac{\delta^2(1+s)}{1+\delta+\delta^2}, \frac{1+s}{1+\delta+\delta^2})$. In this case W_i^B solves $W_i^B = p\delta W_j^B + \delta(1-p)\frac{(\delta+\delta^2)(1+s)}{1+\delta+\delta^2}$, so that $W_i^B = W_j^B = \delta^2 \frac{(1+s)(1+\delta)(1-p)}{(1+\delta+\delta^2)(1-p\delta)} \equiv W^B$. Then, in bipartite states disagreement must prevail when $\delta W^B > 1$, and this inequality holds provided that $\delta^3 \frac{(1+s)(1+\delta)(1-p)}{(1+\delta+\delta^2)(1-p\delta)} > 1$, or equivalently for $s > \hat{s}$ (case (i) in proposition 3). If instead $s \leq \hat{s}$, then $\delta W^B \leq 1$, so that the unique stationary equilibrium is with immediate

agreement at all $s_t = s_i^B$ (case **(ii)** in proposition 3). Note that $\hat{s} \geq \underline{s}$.⁸

(B) Suppose $s < \underline{s}$, so that the outcome in state s_3^T is an immediate agreement on a partition in which the stakeholder makes no contribution. Then the bargainers joint payoff in state s_3^T is 1, and there are no incentives for delay, thus agreement in the bipartite state is immediate. Supporting strategies are the obvious ones, thus omitted for the sake of brevity (i.e. in DAE bargainers make unacceptable offers and in tripartite states follow the strategies which support the equilibria of proposition 1, while in IAE proposers make the just acceptable offers, that is the expected present discounted value of the equilibrium continuation payoff in case of disagreement). Taking the limit as $\Delta \rightarrow 0$ (so that $\delta \rightarrow 1$) yields the statement of the proposition. ■

⁸In fact

$$\hat{s} - \underline{s} = \frac{\delta(1-\delta)(1+\delta)^2 + 1 - p\delta(1+\delta-\delta^3)}{\delta^3(1+\delta)(1-p)} - \frac{1}{\delta(1+\delta)} = (1-\delta)(1+\delta+\delta^2) \frac{1+\delta(1-p)}{\delta^3(1+\delta)(1-p)} > 0$$

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