Polarization switching dynamics and bistability in mutually coupled vertical-cavity surface-emitting lasers

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ABSTRACT

We theoretically investigate the polarization dynamics of two vertical-cavity surface-emitting semiconductor lasers that are mutually coupled through coherent optical injection. We find a sequence of bistable polarization switchings that can be induced by either changing the coupling strength or the optical propagation phase. The successive polarization switchings are correlated to the creation of new compound-cavity modes when these parameters are continuously varied. The switching dynamics and the role of asymmetries are also discussed.

Keywords: VCSEL, Polarization, Optical bistability

1. INTRODUCTION

The understanding and control of the lightwave polarization in lasers is of fundamental importance in any polarization sensitive application. The heterostructure of a conventional edge-emitting laser induces a large anisotropy between TE and TM modes. Therefore, these devices generally emit in a single and well defined polarization state, unless induced strain or other band structure engineering techniques are applied. On the other hand, vertical-cavity surface-emitting lasers (VCSELs) preferentially emit linearly polarized (LP) light along one of two orthogonal preferred directions ($\hat{x}$ and $\hat{y}$) due to the weak material and cavity anisotropies. However, polarization switching between $\hat{x}$ and $\hat{y}$ is often observed when either varying the temperature and/or the injection current,¹ when feeding back part of the emitted light,² or when injecting external light.³

Mutually coupled semiconductor lasers have recently attracted a lot of attention. In mutually coupled edge-emitting lasers, achronal synchronization of chaotic coupling-induced instabilities, symmetry-breaking,⁴ and the role of asymmetries⁵ have been reported. Recent experimental studies⁶ have demonstrated that the mutual coupling of two similar VCSELs can also induce instabilities with high synchronization degree in both total intensity and polarization.

Here, we investigate the mutual coupling of two similar VCSELs in order to determine the role of light polarization dynamics in their mutual entrainment. In this configuration, we find a coupling-induced polarization switching (PS) scenario, where multiple PS occur when continuously varying either the coupling strength or the propagation phase between the two lasers. PS events are correlated to the creation of a new linearly polarized compound-cavity mode with higher gain. In addition, a bistable region appears around each PS which hysteresis width can be controlled by varying the coupling parameters. Controllable bistable PS in mutually-coupled VCSELs add new functionalities to those applications employing a bistable region for fast switching applications.⁷ Close to each polarization switching point, the VCSELs may exhibit a richer nonlinear dynamics including time-periodic, quasiperiodic or even chaotic behaviors. The nonlinear dynamics accompanying polarization switchings is analyzed as a function of the laser parameters, and in particular the spin-flip relaxation rate which is accounted for in our rate equation model. The coupling-induced PS scenario is however robust against modifications of the VCSEL parameters and against small mismatches, such as for example, the linear anisotropies or the frequencies of the free-running VCSELs.

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Figure 1. Time-averaged intensities in the $\hat{x}$ - and $\hat{y}$ -LP modes of one VCSEL 1 as a function of ξ and $\Omega\tau$. VCSEL 2 (not shown) has an identical behavior. Parameters are specified in the text.

2. MODEL AND PARAMETERS

Each solitary VCSEL is described according to the spin-flip model.⁸ For moderate coupling strength, the interaction between the lasers is taken into account by including delayed optical injection terms.⁹ The equations governing the dynamics of the electric field and carrier densities in each laser read

$$\dot{E}_{1\pm} = -i\Delta E_{1\pm} + \kappa(1 + i\alpha)[N_1 \pm n_1 - 1]E_{1\pm} - (\gamma_a + i\gamma_p)E_{1\mp} + \xi e^{(-i\Omega\tau \mp \theta)}E_{2\pm}(t - \tau) + F_{1\pm}(t), \quad (1)$$

$$\dot{N}_1 = -\gamma_e [N_1 - \mu + (N_1 + n_1)|E_{1+}|^2 + (N_1 - n_1)|E_{1-}|^2], \quad (2)$$

$$\dot{n}_1 = -\gamma_s n_1 - \gamma_e [(N_1 + n_1)|E_{1+}|^2 - (N_1 - n_1)|E_{1-}|^2], \quad (3)$$

$$\dot{E}_{2\pm} = i\Delta E_{2\pm} + \kappa(1 + i\alpha)[N_2 \pm n_2 - 1]E_{2\pm} - (\gamma_a + i\gamma_p)E_{2\mp} + \xi e^{(-i\Omega\tau \pm \theta)}E_{1\pm}(t - \tau) + F_{2\pm}(t), \quad (4)$$

$$\dot{N}_2 = -\gamma_e [N_2 - \mu + (N_2 + n_2)|E_{2+}|^2 + (N_2 - n_2)|E_{2-}|^2], \quad (5)$$

$$\dot{n}_2 = -\gamma_s n_2 - \gamma_e [(N_2 + n_2)|E_{2+}|^2 - (N_2 - n_2)|E_{2-}|^2], \quad (6)$$

where the subindices 1,2 label the lasers. $E_{\pm}$ are the circularly-polarized components of the electric field ($E_{\pm} = (E_x \pm iE_y)/\sqrt{2}$). N represents the total inversion population while n is the difference of population inversions between the up/down spin reservoirs associated to emission of opposite circularly-polarized photons. We consider the same internal parameters for both VCSELs. The only mismatches considered are a detuning between the free-running frequencies of both lasers $\Delta = \omega_2 - \omega_1$, and a misalignment between their two $\hat{x}$ and $\hat{y}$ eigenaxes, which is taking into account by the angle θ . The meaning and values of the parameters in Eqs. (1)-(6) are: linewidth enhancement factor $\alpha = 3$, cavity decay rate $\kappa = 300 \text{ ns}^{-1}$, total carrier number decay rate $\gamma_e = 1 \text{ ns}^{-1}$, amplitude anisotropy $\gamma_a = -0.1 \text{ ns}^{-1}$, and phase anisotropy $\gamma_p = 3 \text{ ns}^{-1}$. We fix the normalized pump at $\mu = 1.5$, where the solitary VCSELs emits in a stable $\hat{x}$ -polarization. It is worth noting that since $\gamma_a < 0$ and $\gamma_p > 0$, the x-LP mode exhibits a lower frequency and a larger gain than the y-LP mode. The spin-flip rate is $\gamma_s = 1000 \text{ ns}^{-1}$, as reported in experiments on PS in VCSELs.¹⁰ The distance between both VCSELs is only $L=6 \text{ cm}$ that provides a coupling time $\tau = 0.2 \text{ ns}$. We take $\Omega = (\omega_1 + \omega_2)/2$ as the reference optical frequency, and $\omega_{1,2}$ are the free-running frequencies of the lasers. The coupling strength ξ and propagation phase $\Omega\tau$ are our bifurcation parameters.

The last term in the field equations (1) and (4) are Langevin noise sources that account for spontaneous emission processes. Their expressions are $F_{\pm}(t) = \sqrt{\beta\gamma_e(N \pm n)}\chi_{\pm}(t)$, where $\chi_{\pm}(t)$ are independent complex random numbers with zero mean and δ -correlation. The spontaneous emission factor is $\beta = 10^{-5}$.

Figure 2. Panels (a)-(d) show the location of the symmetric fixed points in the N vs. ω phase space for $\xi = 13 \text{ ns}^{-1}$, $\xi = 13.25 \text{ ns}^{-1}$, $\xi = 13.6$, and $\xi = 13.8 \text{ ns}^{-1}$, respectively. Diamonds and triangles stand for the in-phase and anti-phase $\hat{x}$ -polarized solutions. Squares and circles stand for the in-phase and anti-phase $\hat{y}$ -polarized solutions. Arrows identify the attractor at which the system operates at every stage.

3. POLARIZATION SWITCHING AND HYSTERESIS

In this section, we first consider the case of two VCSELs with identical laser parameters, zero frequency detuning, and aligned polarization eigenaxes. We demonstrate multiple bistable polarization switchings in otherwise polarization stable VCSELs, upon continuously modifying the mutual injection parameters.

Figure 1 shows maps of the LP mode intensities of laser 1 upon the variation of the coupling strength (ξ) and propagation phase ($\Omega\tau \bmod 2\pi$). Intensities are plotted after removing transients and averaging over 50 ns. The alternation between dark and light regions demonstrates successive PS between orthogonal LP states. Moreover, PS events appear with a defined periodicity in the coupling parameters. The maps of laser 2 are not shown in the figure because PS occur at the same coupling conditions in both VCSELs.

The coupling-induced PS scenario described in Fig. 1 remains valid even for small modifications of the laser parameters including: the linear anisotropies, injection current, and spin-flip relaxation rate. In particular, we still observe the multiple bistable PS when the two modes operate on the lower side of the gain curve ($\gamma_a > 0$), i.e., with the lower frequency mode having a smaller gain. Numerical simulations show that the coupling-induced PS scenario is robust against small mismatch of the two laser parameters: linear anisotropies, detuning and misalignment of the polarization axes.

In order to gain insight into the origin of the coupling-induced PS, we study the bifurcations of LP solutions of Eqs. (1)-(6) when the coupling strength and phase are changed. Two kinds of monochromatic LP solutions appear. Symmetric (asymmetric) fixed points corresponds to identical (different) output power and inversion of both lasers. In our case, numerical simulations indicate that only symmetric LP solutions are stable. Therefore, we focus on the symmetric fixed points. They are obtained by imposing the steady-state conditions $E_{1+} = E_0 e^{i\omega t}$, $E_{1-} = E_0 e^{i(\omega t + \varphi)}$, $E_{2+} = E_0 e^{i(\omega t + \phi)}$, $E_{2-} = E_0 e^{i(\omega t + \phi + \varphi)}$, $N_{1,2} = N_0$, and $n_{1,2} = 0$, where φ controls the polarization direction and ϕ takes into account the relative phase between the electric fields of both lasers. These conditions are only satisfied for a relative phase $\phi = 0$ ($\phi = \pi$) leading to in-phase (anti-phase) solutions. After a little algebra, the frequency shift and inversion of the symmetric monochromatic solutions read

$$\omega = \pm(\alpha\gamma_a - \gamma_p) - \xi\sqrt{1 + \alpha^2} \sin(\phi - \Omega\tau - \omega\tau - \arctan \alpha), \quad (7)$$

$$N = \frac{1}{\kappa} [\kappa \pm \gamma_a - \xi \cos(\phi - \Omega\tau - \omega\tau)] , \quad (8)$$

where $\pm$ stands for $\hat{x}$ and $\hat{y}$ states. As shown in Fig. 2, the corresponding LP steady-states along the $\hat{x}$ and $\hat{y}$ polarization directions are located on two different ellipses in the frequency (ω) versus inversion (N) phase space. The steady-states are plotted for increasing values of the coupling strength near a PS event ($13 \leq \xi \leq 13.8$

Figure 3. Panels (a) and (b) show the evolution of the $\hat{x}$ -LP mode intensity as we increase (thin line) and then decrease (thick line) ξ (a) and $\Omega\tau$ (b). In (c) are shown the two hysteresis widths H_1 and H_2 [labelled in (a)] as a function of ξ .

ns^{-1}). For $\xi = 13 \text{ ns}^{-1}$ in panel (a), the system operates in the lowest inversion fixed point, which in this case corresponds to an in-phase $\hat{y}$ -LP solution. An increase of the coupling strength in panel (b) creates a new pair of $\hat{x}$ -LP modes through a saddle-node bifurcation. One of them is a stable node, hence accessible as a stable attractor for the system. However, at this stage the system continues operating in the most stable maximum gain mode (MGM) of the $\hat{y}$ -polarized ellipse. For larger coupling strengths [panel (c)], the $\hat{y}$ -LP MGM destabilizes to a limit cycle through a Hopf bifurcation at the relaxation oscillation frequency. Further increasing ξ , the oscillatory dynamics is interrupted and the laser finally switches to the $\hat{x}$ -LP in-phase fixed point, which has become the new MGM [panel (d)]. If the coupling rate is continuously increased this process repeats and new PS are periodically induced following the same mechanism. When the procedure is repeated decreasing the coupling from large values, bistability is observed since the saddle-node bifurcation, that creates the stable x-LP mode, is located at a smaller ξ than the PS point. Numerical simulations show that the coupling-induced PS scenario is qualitatively preserved for different γ_s values ranging from 50 ns^{-1} to 10^4 ns^{-1} . However, the range of coupling strengths around PS where total intensity instability appears increases for small γ_s . Moreover, the amount of spontaneous emission noise slightly modifies the PS positions. Since two stable orthogonal LP attractors coexist around each PS, noise may favor the jump to an orthogonal LP.

From the simulations, we find that the periodicity of PS when ξ is varied approximately equals the periodicity in the creation of a new saddle-node pair. For a fixed polarization and relative phase, the number of symmetric steady-states of Eqs. (7)-(8) is proportional to $1 + \xi\tau(1 + \alpha^2)^{1/2}/\pi$. Consequently, taking into account that the creation of new steady-states alternates between in-phase and anti-phase modes, the periodicity in the PS events when ξ is changed in a definite direction is approximately $\Delta\xi_{PS} = \pi/\tau(1 + \alpha^2)^{1/2}$. This value corresponds to the increase in ξ necessary to create a new pair of modes with a given polarization. For our set of parameters this quantity corresponds to 4.96 ns^{-1} , which agrees very well with the numerical results shown in Fig. 1 where different PS, with the same transition ($\hat{x} \mapsto \hat{y}$, for example), are separated by $\sim 5 \text{ ns}^{-1}$.

The PS events induced by changing the propagation phase can also be understood in terms of the bifurcation of the LP solutions. When the phase is continuously decreased from 2π to 0 there is a pulling of the steady-states around the ellipses from the low to high inversion regions. At the same time, a new pair of saddle-node modes is created at the lowest vertex of each of the ellipses while they are annihilated by an inverse saddle-node

Figure 4. (Color online) Bifurcation diagram of the polarized-resolved optical power as a function of the coupling strength. The coupling rate is adiabatically varied upwards (upper panel) and downwards (downer panel) for both VCSELs. The spin-flip rate is $\gamma_s = 1000 \text{ ns}^{-1}$.

bifurcation at the highest vertex. Since the process of creation of new pairs of modes at the lowest corner of the ellipse occurs in alternation for the $\hat{x}$ and $\hat{y}$ polarization modes, this results into PS events when varying the propagation phase. The transformation $\Omega\tau \mapsto \Omega\tau + \pi$ interchanges the in-phase and anti-phase modes and defines the periodicity of the PS induced by phase changes. Similar bifurcation mechanism has been reported in mutually coupled edge-emitting lasers.¹¹

In our VCSEL system the selection of stable compound-cavity modes may be accompanied by new features such as polarization switching with hysteresis. Figure 3 shows the multiple PS events when varying ξ or $\Omega\tau$, clarifying the bistability that occurs when increasing and, then decreasing, a control parameter. When sweeping the coupling strength both the switch-off and switch-on events of the $\hat{x}$ -LP mode are accompanied by hysteresis, whose widths are labelled as H_1 and H_2 respectively in Fig. 3(a). The orthogonal polarization component (not shown) displays a complementary behavior. Interestingly, as shown in Fig. 3(c), H_1 and H_2 grow while increasing ξ , hence showing the hysteresis width can be tuned with the coupling parameters. A scan of the propagation phase also leads to multiple PS Fig. 3(b) but, in contrast to the previous case, i) only the switch-on events of the x-LP mode are accompanied by bistability, and ii) the hysteresis width keeps constant when changing $\Omega\tau$ as a consequence of the symmetry of Eqs. (1)-(6) with respect to a change of $\Omega\tau$ in π .

4. DYNAMICS ACCOMPANYING POLARIZATION SWITCHINGS

Figure 3(a) shows the evolution of the time-averaged $\hat{x}$ -LP mode intensity as the coupling strength is increased and then decreased. Any dynamics in the light intensity that occurs on time-scale faster than the averaging time is therefore removed from the time-series analysis. In Figure 4 we complement the analysis of Figure 3(a) by showing the corresponding bifurcation diagram. Here, the extrema of the $\hat{x}$ -LP ($\hat{y}$ -LP) mode intensity time-trace are plotted in black (red) for each value of the coupling strength ξ , and for either VCSEL 1 (left panels) or VCSEL 2 (right panels). In the upper panels (downer panels) of Fig. 4 the coupling strength ξ is adiabatically increased (decreased).

As the coupling strength is increased, the VCSEL displays a sequence of polarization switchings to steady-state solutions that are compound-cavity modes analyzed in the previous section. However, the VCSEL may

Figure 5. (Color online) Bifurcation diagram of the polarized-resolved optical power as a function of the coupling strength. The coupling rate is adiabatically varied upwards (upper panel) and downwards (downer panel) for both VCSELs. The spin-flip rate is $\gamma_s = 100 \text{ ns}^{-1}$.

also exhibit a pulsating dynamics when the coupling strength is slightly smaller than that corresponding to a polarization switching point. The pulsating dynamics may correspond to a regular time-periodic or even quasiperiodic or chaotic dynamics. An example of limit cycle attractor was shown in Fig. 2(b) in a projection of the infinite dimensional phase plane.

The value of the spin-flip relaxation rate γ_s considered so far is $\gamma_s = 1000 \text{ ns}^{-1}$ being comparable to measured values from experiments on several VCSEL devices. Other experiments have, on the other hand, concluded on smaller values of the spin-flip relaxation rate, of the order of $\gamma_s = 100 \text{ ns}^{-1}$ or even less.^{12,13} It is known from the literature that the value of γ_s may have a strong influence on the VCSEL light polarization dynamics, in particular for the dynamics that accompany the polarization switchings.^{14–17} In Fig. 5 we plot the same bifurcation diagrams than in Fig. 4 but for a smaller γ_s value ($\gamma_s = 100 \text{ ns}^{-1}$). Interestingly, each VCSEL still exhibits a sequence of polarization switchings to steady-state single LP mode solutions. However, between the polarization switching points the VCSELs may exhibit a richer dynamical behavior, including a two-mode steady-state solution that destabilizes to time-periodic, quasiperiodic or chaotic behaviors. Typical time-traces of the two LP modes of one of each VCSEL are shown in Fig. 6 for specific values of ξ . The corresponding optical spectra are shown in Fig. 7.

In particular, in Fig. 6(a), the VCSELs exhibit a synchronous chaotic dynamics. The $\hat{y}$ -LP mode is dominant and exhibits fast pulsations with large intensity modulation. When the $\hat{y}$ -LP power drops, the $\hat{x}$ -LP mode is suddenly excited and becomes the dominant mode. The LP modes, therefore, exhibit a mode hopping on a time-scale much larger than that corresponding to the fast intensity pulsations in each modes. For a larger coupling strength (b), the VCSELs bifurcate to a steady-state solution, with both LP modes emitting. As shown in the optical spectra the two LP modes are locked to the same frequency. Moreover, the optical spectra shown of the two LP modes of the second VCSEL (not shown in Fig. 7) unveil that the two VCSELs are locked. Four modes (LP modes of the two VCSELs) are therefore locked to the same frequency. The two-mode steady-state solution Fig. 6(b) bifurcates to a limit cycle dynamics Fig. 6(c) with a frequency close to 420 MHz. The limit cycle frequency is close to half the birefringence frequency splitting γ_p/π . The two-mode limit cycle dynamics then bifurcates for a larger coupling strength to a single polarization mode steady-state solution Fig. 6(d). When analyzing the bifurcation diagram of Fig. 5, we observe that this transition seems to occur at a defined coupling

Figure 6. Temporal traces for the optical power of VCSEL 1 for different coupling strengths. a) $\kappa = 3.5$, b) $\kappa = 4.5$, c) $\kappa = 5.2$, d) $\kappa = 7.0$, e) $\kappa = 8.5$, and f) $\kappa = 9.0 \text{ ns}^{-1}$. The rest of parameters are as in Fig. 5.

Figure 7. Optical spectra of VCSEL 1 for different coupling strengths. a) $\kappa = 3.5$, b) $\kappa = 4.5$, c) $\kappa = 5.2$, d) $\kappa = 7.0$, e) $\kappa = 8.5$, and f) $\kappa = 9.0 \text{ ns}^{-1}$. The rest of parameters are as in Fig. 5.

Figure 8. (Color-online) Bifurcation diagram of the polarized-resolved optical power as a function of the coupling strength in the presence of mismatch in the linear anisotropies. $\gamma_{a1} = -0.1 \text{ ns}^{-1}$, $\gamma_{a2} = -0.08 \text{ ns}^{-1}$, $\gamma_{p1} = 3 \text{ ns}^{-1}$, and $\gamma_{p2} = 5 \text{ ns}^{-1}$. The coupling rate is adiabatically varied upwards (upper panel) and downwards (downer panel) for both VCSELs. The spin-flip rate is $\gamma_s = 1000 \text{ ns}^{-1}$.

strength. The bifurcation to a single polarization mode is found at the coupling where the minimum of the intensity oscillations of the $\hat{x}$ -mode becomes equal to the maximum of the oscillations of the $\hat{y}$ -mode. This observation hints to the fact that a collision of limit cycles may have an influence on the polarization switching. As we further increase the coupling strength the single polarization mode steady-state Fig. 6(d) bifurcates to a single-mode time-periodic dynamics with frequency close to 3.6 GHz Fig. 6(e) and then to a two-mode time-periodic dynamics with frequency close to 2.2 GHz Fig. 6(f).

Therefore, the two-mode limit cycle dynamics exhibited in panels (c) and (f) of Fig. 6 occur at very different frequencies. Another interesting observation comes from the correlation analysis between both VCSELs intensities. In Fig. 6(c) the two polarization modes in VCSEL 1 are completely in-phase with those of VCSEL 2, while in Fig. 6(f) they are in a perfect anti-phase regime. Consequently, different correlation properties between the two VCSELs (from complete in-phase to complete anti-phase) are possible depending on the value of the coupling strength. In any case, the correlation between the polarization modes within the same VCSEL are phase locked in anti-phase.

5. PARAMETER MISMATCH AND DETUNING

In this section, we investigate the behavior of two mutually coupled VCSELs which exhibit slight mismatch in their parameters.

We begin by analyzing the influence of a mismatch between the linear anisotropies of the VCSEL cavities. VCSEL 1 has $\gamma_a = -0.1 \text{ ns}^{-1}$ and $\gamma_p = 3 \text{ rad/ns}$, while VCSEL 2 has $\gamma_a = -0.08 \text{ ns}^{-1}$ and $\gamma_p = 5 \text{ rad/ns}$. The other parameters are the same as in Fig. 1. Figure 8 shows the bifurcation diagrams of the two LP mode intensities of both VCSELs when either increasing (upper panels) or decreasing (downer panels) the coupling strength. The effect of parameter mismatch on the bifurcation diagram is quite small, when comparing Fig. 8 with Fig. 4. Interestingly, new laser instabilities occur for small coupling strength, which are not observed when the parameter mismatch is not taken into account.

Figure 9. (Color online) Temporal traces for the polarized-resolved optical power. Panel a) shows the two modes power in VCSEL 1. Panel b) shows the two modes power in VCSEL 2. Panel c) plots the cross-correlation between the different modes. The parameters are the same as in Fig. 8 and the coupling rate is fixed at $\xi = 6.1 \text{ ns}^{-1}$.

Figure 9 shows the temporal traces of the $\hat{x}$ -LP mode (black) and $\hat{y}$ -LP mode (red) intensities of VCSEL 1 (a) and VCSEL 2 (b), for a specific value of coupling strength $\xi = 6.1 \text{ ns}^{-1}$. Panel (c) shows the correlation analysis between the different polarization-resolved time-series. Specifically, we show the cross-correlation function of the $\hat{x}$ -modes of both VCSELs (solid line), of the $\hat{y}$ -modes of both VCSELs (dashed line), of the $\hat{x}$ - and $\hat{y}$ -LP modes of VCSEL 1 (dotted line), and of the $\hat{x}$ - and $\hat{y}$ -LP modes of VCSEL 2 (dashed-dotted line). The corresponding LP modes in the two VCSELs exhibit a very good correlation coefficient (close to 1), i.e., an almost perfect synchronization, but when one time-trace is shifted by the injection delay time τ with respect to the other. Moreover, the two orthogonal LP modes of each VCSEL are in an almost perfect antiphase dynamics, as unveiled by the cross-correlation coefficients close to -1 at zero time-lag. A more systematic study would be needed to analyze the physical origin of the synchronization with time-lag. Still, our preliminary investigations indicate that such a time-lag in the synchronization of LP mode time-traces is directly linked to the presence of parameter mismatch including mismatch in the linear cavity anisotropies of the two VCSELs.

Figures 10 and 11 analyze the effect of a non-zero frequency detuning on the bifurcation diagrams when sweeping the coupling strength. A small frequency detuning, like in Fig. 10, does not strongly modify the polarization switching points and the accompanying polarization dynamics. However, as shown in Fig. 11, the increase in frequency detuning leads to large regions of chaotic dynamics between the polarization switching points. The intervals of coupling strength in which the VCSELs exhibit steady-state solutions also significantly decrease when the detuning between the two VCSELs is increased.

5.1. Conclusions

In conclusion, we have shown that mutual coupling may induce multiple bistable polarization switchings in otherwise polarization stable VCSELs. A sequence of PS events appear when varying the coupling strength or the propagation phase, with a periodicity that is related to the creation of new compound-cavity modes with higher gain and orthogonal polarization. Each PS event is accompanied by a large hysteresis whose width can be tuned by the coupling parameters. Such controllable bistable PS system is interesting for fast optical switching applications. The coupling-induced PS scenario is robust against modifications of the corresponding

Figure 10. (Color online) Bifurcation diagram of the polarized-resolved optical power as a function of the coupling strength in the presence of detuning ($\Delta = 2$ rad/ns). The coupling rate is adiabatically varied upwards (upper panel) and downwards (downer panel) for both VCSELs. The spin-flip rate is $\gamma_s = 1000$ ns $^{-1}$.

Figure 11. (Color online) Bifurcation diagram of the polarized-resolved optical power as a function of the coupling strength in the presence of detuning ($\Delta = 5$ rad/ns). The coupling rate is adiabatically varied upwards (upper panel) and downwards (downer panel) for both VCSELs. The spin-flip rate is $\gamma_s = 1000$ ns $^{-1}$.

laser parameters, but also against small mismatches between the VCSEL linear anisotropies and small frequency detuning.

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