Polarization Instabilities in a Multi-Transverse-Mode
Vertical-Cavity Surface-Emitting Laser with Polarized Optical Feedback

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Abstract

We have experimentally studied polarization instabilities in the multi-transverse-mode regime of a vertical-cavity surface-emitting laser subject to polarized optical feedback. A dynamical regime that is similar to the transition from the so-called low frequency fluctuation (LFF) to coherence collapse (CC) in the single-transverse-mode VCSEL is found and investigated. The role of higher order transverse modes is to increase the irregularity of the dynamics as shown experimentally and numerically.

Key words: polarization dynamics, vertical-cavity surface-emitting lasers, optical feedback

I. Introduction

It is well known that a broad variety of dynamical behaviors can occur in edge-emitting semiconductor lasers subject to optical feedback that is usually unavoidable in applications; for example, feedback can result from reflections off the ends of fibers or surfaces of other elements. In recent years, studies have shown that a vertical-cavity surface-emitting laser (VCSEL) with optical feedback demonstrates dynamical regimes similar to those observed in edge-emitting lasers [1]. The interests on behaviors of VCSELs with optical feedback arise from two perspectives. For engineering purposes, it is necessary to explore different dynamical regimes in order to avoid occurrence of instabilities or to manipulate them. From the point of view of fundamental research, a VCSEL with optical feedback is an ideal vehicle for investigation of rich nonlinear dynamical phenomena and their underlying mechanisms. The behavior of single-transverse-mode VCSEL with optical feedback has been extensively studied both theoretically and experimentally [1-19]. Typical feedback-induced phenomena include destabilization of single-frequency operation, enhancement of intensity noise, linewidth narrowing and broadening, and coherence collapse (CC) [1-4]. Besides, polarization instabilities can be easily triggered by optical feedback. In a single-transverse mode VCSEL, the fundamental mode often consists of two orthogonally polarized components that usually have a 10-20 GHz frequency difference. Polarization switching, polarization modulation, and irregular fluctuations can be induced by optical feedback [7-19]. The orthogonally polarized components can manifest anticorrelated irregular fluctuations [9, 10]. Low frequency fluctuations (LFFs) have been observed near threshold in both orthogonal linearly polarized components with isotropic [9-11] and polarized [12] optical

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4 feedback. The polarization dynamics in the LFF regime depends on the intrinsic
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6 dichroism [13] and the relaxation rate of the magnetization [8]. Dependence of power
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8 dropouts in LFFs on control parameters is investigated experimentally [13, 15, 18].
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10 Correlation properties of polarization dynamics in LFFs are analyzed for both long [16]
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12 and short [17] external cavity. Dynamics derived from Lang-Kobayashi model are
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14 compared with experimental results [18]. Different polarizations of optical feedback
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16 affect the magnitude of anticorrelation between two polarized components [19].
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21 Because of its large aperture, a VCSEL can operate with several transverse modes
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23 when the injection current is high enough. The polarization of higher order transverse
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25 modes may be orthogonal or parallel to that of the fundamental mode. Multi-transverse
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27 mode VCSELs have much lower modal noise comparing to edge-emitting lasers and
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29 therefore are especially useful in multimode links [20, 21]. Simultaneously, the study of
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31 feedback-induced dynamics in the multi-transverse-mode VCSEL is more challenging
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33 due to the introduction of spatial complexity. Several theoretical efforts have been made
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35 on transverse dynamics when a VCSEL is subject to optical feedback [6, 22, 23]. Very
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37 recently, we studied polarization dynamics in the multi-transverse-mode regime of a
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39 VCSEL with isotropic feedback [24, 25]. In our experiment, simultaneous measurements
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41 of orthogonally polarized states were conducted for both total output and several spatial
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43 positions on the beam profile and anticorrelated irregular fluctuations were observed in
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45 the orthogonal polarizations in slow and fast time scales [24]. The theoretical analysis
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47 based on a spatially dependent dynamical model found good agreement with the
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49 experimental results [25].
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4 As a continuation of our study, we investigate, for the first time to our knowledge,
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6 the polarization dynamics when the multi-transverse-mode VCSEL is subject to polarized
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8 optical feedback. When the VCSEL enters multi-transverse-mode operation, we have
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10 found a dynamical regime in which fluctuations share some features of the LFF that are
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12 extensively studied near threshold in edge-emitters [26-34] and single-transverse-mode
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14 VCSELs [9-18]. In Section II, we describe our experimental setup and characteristics of
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16 the VCSEL. The polarization dynamics sharing some features of the LFF is reported in
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18 Section III. Numerical results are given in Section IV. Section V is for conclusion.
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26 II. Experimental setup and characteristics of the VCSEL

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28 Our experimental setup is shown in Fig. 1. A proton-implanted VCSEL operating
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30 at 847 nm (Honeywell HFE-4083) is used in our experiments. The temperature of the
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32 VCSEL is stabilized within 0.01 °C using a temperature controller (Thorlabs TEC 2000).
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34 The smallest increment of the current driver (Thorlabs LD500) is 0.1 mA. A total
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36 reflector M is used to form an external cavity and provide optical feedback to the
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38 VCSEL. In the external cavity, we insert a half-wave plate (HWP_1) and a polarizing beam
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40 splitter (PBS) to achieve polarized feedback. The polarization perpendicular to the optical
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42 table is termed as Y polarization and the polarization parallel to the optical table is
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44 termed as X polarization. Another half-wave plate (HWP_2) rotates the polarized states
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46 back to their original orientations for detection purposes. The effective reflectivity of the
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48 external cavity is adjusted by placing a neutral density filter (ND) in the external cavity.
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50 The optical spectrum and beam pattern of the VCSEL are obtained with a Fabry-Perot (F-
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52 P) scanning spectrum analyzer and a charge-coupled-device (CCD) camera, separately. A
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4 neutral density filter is placed in front of the F-P spectrum analyzer to minimize
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6 reflection from the instrument and ensure that the F-P spectrum analyzer does not provide
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8 any unwanted feedback. Two photodetectors, PD_1 and PD_2 , are used for polarization
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10 resolving measurement, where PD_1 measures the Y-polarized state and PD_2 the X-
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12 polarized state. The dynamics of the total power is measured with a third detector PD_3 .
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14 The bandwidth of the photodetectors is 1 GHz (New Focus 1601). The ac outputs of the
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16 detectors are sent to a digital oscilloscope (Tektronix DPO 7254, bandwidth 2.5 GHz) to
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18 observe the time series. The sampling rate is 20 GS/s. The power spectrum of each
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20 polarized state and the total power is monitored with a rf spectrum analyzer (Tektronix
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22 2712, bandwidth 1.8 GHz). In our experiments, both X- and Y-polarized feedback
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24 induces similar dynamics, but the phenomenon obtained with Y-polarized feedback
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26 dynamics is more obvious and exists in a wider current range. In this paper we report the
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28 results obtained with Y-polarized feedback.
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36 The polarization-resolved L - I curve of the solitary VCSEL is shown in Fig. 2(a).
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38 The threshold current of the laser is ~ 2.65 mA. The VCSEL operates in the fundamental
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40 mode when the injection current I is less than 3.8 mA. The fundamental mode has both
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42 X- and Y-polarized components. The frequency of the X-polarized component is ~ 10
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44 GHz less than that of the Y-polarized component. The Y-polarized component is the
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46 dominant near the threshold. Polarization switching occurs at around 3.0 mA. From 3.8
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48 mA, a higher-order transverse mode is on. The frequency difference between the
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50 fundamental mode and the higher-order mode is 60-70 GHz depending on the injection
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52 current. In the two-transverse-mode regime, the laser is nearly linearly polarized in the X
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54 polarization. The third transverse mode, orthogonally polarized with respect to the first
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two transverse modes, starts oscillation at 4.1 mA. The mode spacing between the third transverse mode and the second transverse mode is 25- 30 GHz. We focus on the current region where the VCSEL operates with two or three transverse modes.

To illustrate the spatial profile of each transverse mode, we use an example in the three-transverse-mode regime, when the injection current is 4.6 mA. The polarization-resolved optical spectra of the solitary laser (Fig. 2c) show that the first two transverse modes are X-polarized and the third transverse mode is Y-polarized. The inset to the left is the X-polarized beam pattern, and the one to the right is the Y-polarized beam pattern. The X-polarized pattern is very similar to that of the fundamental mode because the fundamental mode is much stronger than the second transverse mode. The Y-polarized pattern indicates that the third transverse mode can be described as a LP_{11}^s mode [35]. The X-polarized spatial pattern is attenuated by a factor of 10 by placing a neutral density filter in front of the CCD camera to prevent the camera from saturation.

For Y-polarized optical feedback, we carefully align the feedback mirror M to achieve the greatest reduction in the threshold current. With the highest effective reflectivity of the external cavity (that is, no neutral density filter is inserted in the cavity), the reduction in the threshold current is $(6\pm1)\%$ for most trials in our experiment, where the uncertainty is attributed to environmental factors (e. g., variation in humidity and room temperature) that affects the coupling between the external cavity and the VCSEL. The typical polarization-resolved $L-I$ curve is shown in Fig. 2(b). Different from the solitary laser, polarization switching does not occur when the polarized feedback is applied. The Y-polarized state is the dominant in the whole current range we have studied. This is similar to what were reported in refs. 36 and 37, in which polarization

switching is suppressed by polarization-selective optical feedback. In general, however, polarization switching with polarized feedback can be more complex when the injection current is varied [38, 39]. The thresholds of the second and third transverse mode are approximately 0.1 mA lower than that of the solitary VCSEL.

For the three-transverse-mode example at 4.6 mA, the Y-polarized state becomes dominant when Y-polarized feedback is applied, as shown in Fig. 2(d). Its spatial pattern (the inset to the left), attenuated with the same neutral density filter as the X-polarized pattern shown in Fig. 2(c), is bigger than that of the dominant polarization of the solitary laser. This is because it is the superposition of three transverse modes, with the fundamental mode being the strongest. On the other hand, the X-polarized state is significantly weakened. It includes components of the first two transverse modes, both of which are similar in intensity. The corresponding spatial profile (the inset to the right) has a weaker lobe to the left. This suggests that the higher-order mode is an LP_{11}^c mode [35], with the central minimum of its spatial profile shifted slightly to the left instead of overlapping the central peak of the fundamental mode. Note that the spatial profiles of the two higher-order transverse modes are not perpendicular to each other. The optical spectra in Figs. 2(c) and 2(d) also reveal that the effect of optical feedback is most obvious on the fundamental mode. This indicates that the external cavity used in our experiment provides the highest coupling efficiency for the fundamental mode.

III. Polarization dynamics resembling the LFF-CC dynamics

As reported in the literature, the LFF has the following characteristics: (a) the output power manifests sudden large dropouts followed by gradual recovery; (b) the time

between dropouts is much larger than the round-trip time in the external cavity; (c) the power spectrum shows a broad low frequency peak that corresponds to the averaged time elapsing between dropouts. As the injection current increases, the dropouts gradually becomes less deep, less regular, and more frequent. When fluctuations become very irregular and all characteristics of LFF disappear, the dynamics is termed coherence collapse (CC). In our VCSEL, the polarization dynamics in the multi-transverse-mode regime demonstrates some features similar to the LFF. The major similarity is that the power spectra of orthogonally polarized states manifest a peak in the low frequency part (< 0.10 GHz). A typical example is shown in Fig. 3, when the VCSEL with Y-polarized feedback operates in the three-transverse-mode regime. Fig. 3(a) gives the polarization-resolved optical spectra when the injection current is 4.1 mA. The Y polarized state is much stronger than the X-polarized state. The polarization-resolved time series are shown in Fig. 3(b), in which the signals are offset (with the Y polarization to a greater degree) for visual convenience. The power of the dominant polarization (Y polarization) manifests irregular fluctuations at different time scales. One type of fluctuations is at the scale of round-trip time ($\tau=3.3$ ns) in the external cavity; another type consists of erratic power dropouts. The time interval between most two adjacent dropouts is much greater than the round-trip time τ ; however, the amplitudes of the dropouts have considerable differences. When the dominant polarization has a drop, the weaker polarization (X polarization) shows a burst. This indicates anticorrelation between dynamics of the two orthogonally polarized states. Fig. 3(c) is the power spectrum of the Y-polarized states. The peak at $\sim f_1=0.33$ GHz and its harmonics are related to the round-trip time in the external cavity. Between 0 and 0.1 GHz, there is a broad peak at $\sim f_2=50$ MHz, indicated

by an arrow. In this paper, we name this peak as the *low frequency peak* (LFP). Combining this information with the irregular dropouts in the time series of the Y polarization, we conclude that the broad LFP reflects occurrence of the power dropouts of the Y polarization. The same low frequency peak is also observed in the power spectrum of the total output. Similarly to the results in ref. 24, the intensity of the peak is much weaker, which reflects the anticorrelation between the orthogonal polarized states.

Between the low frequency peak and the frequency peaked at external cavity resonance, there is a weaker peak at $\sim f_3=0.23$ GHz. The frequency of this peak, f_3 , approximately equals f_1-2f_2 and may be caused by nonlinear mixing of fluctuations. The power spectrum of the X-polarized state (Fig. 3(d)) resembles that of the Y-polarized state, but the peak corresponding to f_1 and its harmonics is relatively weaker. This is understandable because the feedback is Y polarized so that the X polarization does not form a strong resonance in the external cavity.

To compare the LFP dynamics with the LFF in single-transverse-mode VCSELs, we give the polarization resolved time traces and power spectrum of the Y polarization (the dominant polarization) for 2.9 mA in Fig. 4. The time trace of the Y-polarized component is the same as the LFF reported in literatures [e. g., 9, 18, 30-34]. With the same feedback strength, the time series near the threshold manifests fewer dropouts than in Fig. 3. The time intervals between dropouts are similar to each other and the dropouts have approximately the same amplitude. Correspondingly, the low frequency peak in the power spectrum is obviously sharper and has a lower frequency value (~ 20 MHz). The sharp peak represents a strong periodicity of the LFF. To evaluate the depth of dropout, we consider the ratio of the amplitude of the dropouts to the average power of the

polarized state, which is termed the normalized amplitude of the dropout. In Fig. 4a, the normalized amplitude of the Y-polarization is 40%, whereas in Fig. 3b it is only 20%. For the normalized amplitude in the time trace of the total output, it is $\sim 26\%$ at 2.9 mA, less than that of the Y-polarization because of the anticorrelated fluctuations in orthogonal polarizations; however, it is only $\sim 3\%$ at 4.1 mA. Hence the depth of modulation is significantly less in the LFP regime.

As the injection current is increased, the frequency of the LFF increases and the corresponding peak in the power spectrum becomes broader. When the VCSEL is near and enters operation of more than one transverse mode, the peak in the power spectrum evolves to the LFP as shown in Figs. 3c and 3d. The broad LFP in the power spectrum indicates that though the dynamics still carries some features of LFF, irregularity in fluctuations is enhanced when the higher order modes are starting oscillation.

It has been well known that feedback-induced dynamics of a single mode semiconductor laser can evolve from LFF regime to CC regime when the injection current is increased from threshold [e. g., 32, 34]. As the laser transits from LFF dynamic to CC dynamics, the variation in power spectrum and time trace is quite similar to the LFP dynamics. Naturally, one would wonder whether the observed LFP dynamics is simply the transition of fundamental mode from LFF to CC regime (the so called LFF-CC in ref. 34), and this transition occurs coincidentally with onset of higher order transverse modes. In other words, do higher order modes play any role in the LFP dynamics?

To find out the role of each transverse mode in the observed dynamics, we have conducted spatially resolved measurements by using a pinhole aperture with a 0.50 mm

diameter. The aperture is installed on a holder that can be adjusted horizontally and vertically with micrometers. The approach is the same as that in ref. 24. Here we give an example in the three-transverse-mode regime, with $I=4.4$ mA, $l=45$ cm, and the strongest feedback. We choose two spatial positions— P_1 and P_2 —on the beam pattern. The position P_1 is at the peak of the fundamental transverse mode; P_2 is 0.5 mm to the left of P_1 , on the left lobe of the LP_{11}^c mode. We choose the center-to-center separation to be 0.5 mm so that there is no overlap between any positions.

Fig. 5a is the polarization-resolved optical spectra when the aperture is placed at P_1 , where the fundamental mode is much stronger than other transverse modes in both polarizations. The polarization-resolved time series (Fig. 5b) demonstrate anti-correlation between the two polarized states. Figs. 5c and 5d are the power spectra of the Y- and X-polarized states, respectively, in which the frequency of the LFP is at ~ 43 MHz. At position P_2 , the second transverse mode is not negligible. As shown in Fig. 6a, the intensity of the LP_{11}^c mode is comparable to that of the fundamental mode for the Y polarization; in the X-polarized state, the LP_{11}^c mode is stronger than the fundamental mode. The time traces shown in Fig. 6b reveal that anti-correlation between the two orthogonal polarizations is less obvious. This is qualitatively similar to what was observed with isotropic feedback [24]. The less definite anti-correlation is explained by the analysis on modal dynamics [25], which has shown that correlation of orthogonal components of higher-order transverse modes can be different from that of the fundamental mode. Figs. 6c and 6d display the power spectrum of Y- and X-polarized states, respectively. There is no distinguishable LFP in the power spectrum of either polarized state. Instead, the power spectra demonstrate a broadband feature in low

frequency region (<0.1 GHz). This implies that the fundamental mode plays a dominant role in carrying the LFF-like features, whereas the higher-order transverse modes contribute more to irregularity of the fluctuations. Our numerical results support this observation, which will be given in next section.

Fig. 7 shows the variation of the low frequency peak in the power spectrum with the injection current from near the threshold to the three-transverse-mode regime. The frequency, amplitude, and linewidth of the peak are illustrated in Figs. 7a, 7b, and 7c, respectively. Near the threshold of the solitary VCSEL, the VCSEL demonstrates LFF that is characterized by a clear, well-defined low frequency peak in the power spectrum (e. g., Fig. 4b). Both the frequency and the linewidth of the peak increase with increasing current. When the current reaches 3.0 mA, the amplitude of the peak has a significant decrease while the linewidth is at a local maximum, which makes the peak appear less sharp. For $3.0 \text{ mA} \leq I \leq 3.2 \text{ mA}$, the frequency decreases to a minimum while the amplitude increases to a maximum, which manifests a “valley” in the amplitude curve and a “peak” in the frequency curve. Since the intensity floor of the power spectrum also increases with the current, the difference between the amplitude of the peak and the floor, ΔA , is depicted in the inset in Fig. 7b. The variation of ΔA also shows a local maximum at 3.2 mA. We are not sure of the mechanism behind this variation; it may be related to the growth of the weaker polarization of the fundamental mode. For $I \geq 3.3 \text{ mA}$, the frequency of the peak increases while both the amplitude and ΔA decrease; the corresponding linewidth also has some decrease. The linewidth of the peak is less than 140 MHz for $I \leq 3.6 \text{ mA}$, when the VCSEL operates with the fundamental mode.

When the current reaches 3.7 mA, the linewidth of the peak is increased to above 160 MHz. This is where the second transverse mode starts operation. For $I \geq 4.0$ mA, the VCSEL operates with three transverse modes, and the linewidth is ~ 180 MHz. However, the amplitude of the peak does not increase in the same way. Though there is some increase in the amplitude, the effect actually comes from the increase of the intensity floor of the power spectrum. As shown in the inset, ΔA always stays at a low level in the whole regime of multi-transverse-mode operation. The low frequency peak in the power spectrum is always broad in this regime, looking like a bulge as shown in Figs. 3c and 3d. We name such a broad peak as LFP. For a LFP, the center of the peak is used as the frequency value of the peak. As for the linewidth measurement of the broad peak, it is often difficult to determine the full width at half maximum (FWHM) because the center frequency is too close to zero frequency. For such cases, we get the FWHM linewidth from the half width at half maximum by assuming the peak is symmetric. When the current is greater than 4.6 mA, the LFP in the power spectrum is indistinguishable. Instead, a broadband distribution in the low frequency part of the power spectrum is observed, as what is shown in Fig. 6. The data shows that onset of the higher order transverse modes significantly broaden the width of the peak while leaving the amplitude at a low level. When the higher order modes become strong enough, the LFP-like feature is suppressed.

Near the threshold, the LFF frequency demonstrates a monotonic increase with the injection current, which qualitatively agrees with the trend reported by many researchers [12, 13, 15, 18]. Different from the result in ref. 13, however, the frequency depicted in Fig. 7a does not show a linear dependence on the current. This difference may

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4 arise from the fact that our VCSEL has a large value of dichroism because it is essentially
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6 linearly polarized near threshold [15] whereas ref. 13 is for a VCSEL with low dichroism.
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10 our VCSEL. Besides, since the precision of our current driver is limited to 0.1 mA, we
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12 may miss the region of linear dependence if that region is narrow.
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16 The variation of the frequency of the LFP with the external cavity length in the
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18 two-transverse-mode regime is shown in Fig. 8. The injection current is $I=3.9$ mA and the
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20 external cavity provides the strongest feedback. For each cavity length l , we aligned the
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22 feedback mirror to obtain the maximum threshold reduction. When the cavity length is
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24 increased from $l=21$ cm, the trend of the frequency is decreasing, but not in a monotonic
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26 way. For $l > 55$ cm, the LFP peak in the power spectrum becomes indistinguishable at
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28 this current. A similar variation of the frequency of the LFP versus cavity length is also
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30 observed in the three-transverse-mode regime.
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34 The trend shown in Fig. 8 is qualitatively similar to the result in ref. 15, which
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36 studied the dependence of LFF near threshold on the length of external cavity. However,
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38 the longest cavity length for the LFP to be observed at 3.9 mA is only 55 cm, which is
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40 much shorter than many cavity lengths used for the study of LFF [e. g., 11,15]
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44 The occurrence of LFP dynamics depends on the strength of feedback. The
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46 feedback strength can be changed by varying effective power reflectivity, R , of the
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48 external cavity. We adjusted the feedback mirror to get optimal alignment, and used the
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50 neutral density filter to change the effective reflectivity of the external cavity. The highest
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52 value of R is determined to be 0.195 in the single-transverse-mode regime and 0.175 in
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54 the multi-transverses-mode regime. This difference is attributed to the fact that the power
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of X polarization becomes greater in the multi-transverse-mode domain and it is filtered out of the external cavity by the polarizing beam splitter. We checked the effect of R for $I=4.6$ mA and $l=45$ cm. When the effective reflectivity is very weak ($R < 0.03$), there is no distinguishable LFP in the power spectrum. Instead, the low frequency part (< 0.10 GHz) of the power spectrum has a broadband feature, similar to what is shown in Figs 5c and 5d. When $R \geq 0.053$, the LFP is observed; however, the value of its frequency does not show obvious change even for the highest value of R. This differs from the case of LFF, in which the frequency of LFF decreases significantly with increasing feedback strength [15].

IV. Numerical results

We consider in this section the theoretical model used in Ref. [25] with the appropriate modifications to take into account the polarized nature of the feedback considered in the experiment. All the equations, parameters and details of the model are detailed in that reference. The numerical values of the parameters are maintained with a just few exceptions: the round-trip delay time, τ , is changed to 3.3 ns, the dichroism, γ_a , is changed to -3 ns^{-1} , and the relative loss of the higher order transverse mode (LP_{11}) with respect to the fundamental mode (LP_{01}), κ_r is changed to 1.02. The polarized feedback in the y-polarization is taken into account by considering feedback strengths in the x-polarization, κ_{0x} and κ_{1x} for the LP_{01} and LP_{11} modes respectively, such that $\kappa_{0x} = \kappa_{1x} = 0$. The strength of the feedback in the y-polarized LP_{01} and LP_{11} modes, κ_{0y} and κ_{1y} respectively, is chosen such that $\kappa_{0y} = \kappa_{1y} = 25 \text{ ns}^{-1}$. In this way the theoretical reduction in the threshold current is 7%, a value similar to the experimental one. Also the frequency

differences between the polarized transverse modes and the current at which the LP_{11} mode appears are similar to the experimental ones. The power spectra of the total output, the x and y polarized states are plotted in the left column of Fig. 9 for different values of the VCSEL current. We also show in the right column of Fig. 9 the results obtained with the same model but with $\kappa_r=1.6$. In this case the losses of the LP_{11} mode are much higher than the corresponding to the fundamental mode. We will refer to those results as the single mode results because the LP_{11} mode is negligible for all the values of the VCSEL current. We have also plotted in Fig. 10 the time traces of the power of the polarized transverse modes corresponding to the situations depicted in Fig. 9. In the single-mode case (see Figs. 10(d)-(f)) the power of the polarized higher-order transverse modes ($LP_{11,x}$ and $LP_{11,y}$) are always negligible while in the multimode case the higher-order transverse mode ($LP_{11,y}$) becomes relevant as the current is increased (see Figs. 10(a)-(c)). The comparison between the multimode (left column) and single-mode (right column) results is useful to assess the influence of the excitation of the higher order mode on the dynamics. Figs. 9(a),(d) show that when the current is 1.2 times the threshold current, I_{th} , multimode and single mode results are very similar because the power of the higher order mode is very small ($5 \cdot 10^{-4} \%$ of the total power in Fig. 10(a)). A peak at low frequencies appear in both spectra at 12 MHz. When the current increases to $1.3 I_{th}$ (Figs. 9(b),(e)) the multimode and single mode results keep on being qualitatively similar but some differences appear in the location of the low frequency peak (30 and 24 MHz, for the multimode and single mode results, respectively). In this case the contribution of the higher order mode to the dynamics (the power of the LP_{11} mode is 2% of the total power in Fig. 10(b)) is small. The fundamental transverse mode is the main contributor to the

observed low frequency peak in the multimode results. Figs. 9(c),(f) show that the situation clearly changes when the contribution of the LP_{11} mode to the dynamics is significant (the power of the LP_{11} mode accounts for 34 % of the total power in Fig. 10(c)). The low frequency peaks disappear in all the power spectra obtained with the multimode results while in the single mode results are still clear. This behavior is similar to the experimental one reported in the previous section. The role of the higher order transverse modes is then to increase the irregularity of the dynamics as demonstrated by the disappearance of the low frequency peak and the subsequent appearance of a “shoulder” of lower peak amplitude. In our simulations the dynamical regime characterized by the appearance of the low frequency peak during the emission in multiple transverse modes goes from 1.3 to 1.5 I_{th} (at this value the peak has broadened and is barely visible).

V. Conclusion

We have experimentally studied polarization dynamics in the multi-transverse-mode regime of a VCSEL with polarized optical feedback. A dynamical regime is observed when the VCSEL operates with more than one transverse mode up to 1.7 times higher than the threshold of the solitary VCSEL. This regime, named as LFP regime, is featured by a low frequency (< 100 MHz) peak in the power spectrum. The dependence of the low frequency peak on injection current and external cavity length is measured.

The LFP dynamics resembles the dynamical transition from LFF to CC (the so called LFF-CC regime [34]) in a single-mode semiconductor laser. The resemblance includes a distinguishable peak in the power spectrum and irregular power dropouts in

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4 the dominant polarization with a normalized amplitude much smaller than that in LFF..
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6 However, the LFP regime is not simply the LFF-CC regime of the fundamental mode.
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8 The spatially resolved measurements and numerical simulations show that higher order
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10 transverse modes affect the dynamics. While the fundamental mode is the main
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12 contributor to the low frequency peak in the power spectrum, higher order transverse
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14 modes do not support it. Instead, their growth in strength increases irregularity in
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16 dynamics, which leads to the broadband distribution in the power spectrum. Therefore we
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18 think that the observed LFP is a dynamical regime between the single-transverse-mode
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20 dynamics and the multi-mode dynamics when higher order modes have powers
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22 comparable to that of the fundamental mode.
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Captions:

1. Experimental setup, where BS stands for nonpolarizing beam splitter, PBS for polarizing beam splitter, HWP for half-wave plate, A for pinhole aperture, L for collimating lens, PD for photodetector, and ND for neutral density filter.
2. The polarization-resolved L - I curves of (a) the solitary VCSEL and (b) the VCSEL subject to the strongest polarized optical feedback. (c) The polarization-resolved optical spectra of a solitary three-mode example for $I=4.6$ mA, where the inset to the left is the spatial profile for stronger polarization (X polarization) and the inset to the right is the spatial profile for weaker polarization (Y polarization). (d) The polarization-resolved optical spectra and spatial profiles for the VCSEL with Y-polarized feedback and the same injection current, where the stronger polarization becomes Y-polarized. The left inset is the spatial profile for the stronger polarization (Y polarization) and the inset to the right is the spatial profile for the weaker polarization (X polarization). From (a) to (d), the solid curve is for Y-polarized state and the dashed curve is for X-polarized state. For spatial profiles, a neutral density filter is used to attenuate the stronger polarized state by a factor of ten.
3. Polarization-resolved measurement of the total output for 4.1 mA and a 45 cm external cavity with the strongest feedback. (a) Optical spectra show the VCSEL is in the three-transverse-mode regime, with the Y polarization being much stronger than the X-polarized component. (b) Time series of the orthogonally

polarized components. In both (a) and (b), the solid curve is for the Y polarization and the dashed one is for the X polarization. (c) and (d) give power spectrum for the Y-polarization and the X-polarization, respectively, in which the low frequency peak is indicated by an arrow.

4. Polarization-resolved measurement of the total output for 2.9 mA and a 45 cm external cavity with the strongest feedback. (a) Time series of the orthogonally polarized components, in which the solid curve is for the Y polarization and the dashed one is for the X polarization. (b) Power spectrum for the Y-polarization.
5. Polarization resolved measurement at position P_1 for $I=4.4$ mA, $l=45$ cm, and strongest feedback. (a) Polarization resolved optical spectra of the VCSEL. (b) Polarization resolved time series. In both (a) and (b), the solid curve is for the Y polarization and the dashed one is for the X polarization. (c) Power spectrum of the Y polarization. (d) Power spectrum of the X polarization.
6. Polarization resolved measurement at position P_2 for $I=4.4$ mA, $l=45$ cm, and strongest feedback. (a) Polarization resolved optical spectra of the VCSEL. (b) Polarization resolved time series. In both (a) and (b), the solid curve is for the Y polarization and the dashed one is for the X polarization. (c) Power spectrum of the Y polarization. (d) Power spectrum of the X polarization.

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4 7. Variation of the (a) frequency, (b) amplitude, and (c) linewidth of the low
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6 frequency peak with the injection current, where the external cavity length is 45
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8 cm and the feedback is the strongest. The inset in Fig.7b manifests the difference
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10 between the peak amplitude and the intensity level of the floor of the power
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12 spectrum.
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19 8. Variation of the LFP frequency with the external cavity length in the two-
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21 transverse-mode regime, where the injection current is 3.9 mA and the feedback is
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23 the strongest.
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29 9. Power spectrum of the x (red dashed) and of the y (blue dotted) polarizations.
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31 Results corresponding to the total power are also plotted with a solid line. Parts (a)-
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33 (c) correspond to the results obtained with the multimode model while parts (d)-(f)
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35 are the single-mode results. Results for VCSEL currents equal to 1.2, 1.3 and 1.6 I_{th}
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37 are plotted in the first, second and third rows, respectively.
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43 10. Time traces of the power of the polarized transverse modes. Parts (a)-(c)
44
45 correspond to the results obtained with the multimode model while parts (d)-(f) are
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47 the single-mode results. Results for VCSEL currents equal to 1.2, 1.3 and 1.6 I_{th} are
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49 plotted in the first, second and third rows, respectively. Different colors are used for
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51 the time traces of different polarized transverse modes.
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Figure 1

Fig. 1

Figure 2a,b

(a)

(b)

Fig. 2

Figure 2c,d

(c)

(d)

Fig. 2

Figure 3a,b

(a)

(b)

Fig. 3

Figure 3c,d

(c)

(d)

Fig. 3

Figure 4

(a)

(b)

Fig. 4

Figure 5a,b

(a)

(b)

Fig. 5

Figure 5c,d

(c)

(d)

Fig. 5

Figure 6a,b

(a)

(b)

Fig. 6

Figure 6c,d

(c)

(d)

Fig. 6

Figure 7

(a)

(b)

(c)

Fig. 7

Figure 8

Fig. 8

FIG. 9

FIG. 10