Analysis of Garofalo equation parameters for an ultrahigh carbon steel

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ABSTRACT

Isothermal stress-strain curves data from torsion tests conducted at high temperature (950-1200 °C) and strain rates ($2\text{-}26\text{ s}^{-1}$) were analyzed in a ultrahigh carbon steel (UHCS) containing 1.3%C. The sine hyperbolic Garofalo equation was selected as an adequate constitutive equation for the entire range of the forming variables considered. The Garofalo parameters were assumed strain dependent allowing the prediction of stress-strain curves under transient and steady-state conditions. The average relative errors obtained were below 3% in stress. In addition, the creep deformation mechanisms in the UHCS were analyzed from the Garofalo equation parameters. For this aim, the stress exponent of the Garofalo equation was, for the first time, related to that of the power law equation. The results show that the controlled deformation mechanism at steady-state is lattice diffusion-controlled slip creep.

Keywords: Torsion tests; Modeling; Garofalo equation; Deformation mechanisms; Ultrahigh carbon steel

1. Introduction

Torsion tests at various temperatures and strain rates are usually carried out in order to evaluate the hot deformation behavior of materials. These tests are especially adequate for this aim since they allow high deformations minimizing plastic instability [1].

Torsion tests provide a set of data of stress, σ , strain rate, $\dot{\epsilon}$, and temperature, T . These variables are often related, at steady state, by means of a power law creep equation [2]:

$$\dot{\epsilon} = A \exp\left(-\frac{Q}{RT}\right) \sigma^{n_p} \quad (1)$$

where A is a constant, Q is the activation energy for creep, R is the gas constant and n_p is the stress exponent. A large number of materials have been investigated using this equation with a value of five for the stress exponent in order to describe the creep behavior at steady-state deformation [3].

In addition, the sine hyperbolic equation, or Garofalo equation, is also employed for describing the creep behavior of materials in a wider range of temperatures and strain rates [4, 5]:

$$\dot{\epsilon} = A \exp\left(-\frac{Q}{RT}\right) \sinh^{n_g}(\alpha \sigma) \quad (2)$$

where A , Q , α and n_g are material parameters. Usually, this equation is fitted at the strain corresponding to the peak stress or at the strain where the steady state is reached. In contrast, recent works fit the Garofalo equation at various levels of strain predicting the creep behavior under transient and steady state deformation [6-8].

As mentioned, the advantage of the Garofalo equation lies in its capability to predict the creep behavior of materials under very wide range of temperatures and strain rates.

For this reason, the Garofalo equation is nowadays more and more employed to describe the flow behavior of materials. However, the association of the Garofalo equation parameters with usual deformation mechanisms at high temperature is different from that involving the power law equation. Apart from α , which is not included in the power law equation, only the activation energy has been shown to be the same for both equations [3,9]. In contrast, the difference between n_p and n_g is usually large [4]. It should be mentioned that the relation between the parameters from both equations, (1) and (2), are poor and are only based on statistical correlations [4].

The objective of this work is threefold: a) to establish a physical based relation between n_p and n_g , b) to determine the creep deformation mechanism operating in an ultrahigh carbon steel containing 1.3%C (UHCS-1.3%C) using the relation for the evolution of the Garofalo equations parameters with strain and the relation established in a), and c) to predict the stress-strain behavior under transient and steady state conditions for this steel.

2. Material and experimental details

The UHCS studied in this investigation has the following composition: 1.3%C, 0.5% Mn, 0.6% Si, 0.18% Cr and balance Fe. The manganese was added to neutralize the deleterious effects of sulphur and phosphorus. The steel was produced at Sidenor Company as a cast of 8 liters by means of an induction furnace. The as-cast ingot was initially soaked at 1050°C and forged into a bar of 60 mm x 55 mm cross section.

Torsion tests were conducted in a SETARAM torsion machine at CENIM in the temperature range 950 to 1200 °C at strain rates in the range 2 to 26 s⁻¹. An induction furnace heats the test sample and the temperature is continuously measured by means of a two-color pyrometer. A silica tube with argon atmosphere ensures protection against

oxidation. A helium atmosphere is used to obtain, after testing, a cooling rate of 325 K/s. The torsion samples have an effective gage length of 17 mm and a radius of 3 mm.

3. Results and discussion

3.1. Stress-strain behavior

Torsion tests at various temperatures and strain rates were conducted in a UHC-1.3%C steel. For each torsion test, data of torque and number of turns were converted to shear stress and shear strain [10]. The von Mises method was selected to obtain the values of stress, strain and strain rate. Moreover, adiabatic heating usually occurring during high strain rates torsion tests were also considered in order to obtain stress-strain curves at constant temperature and strain rate. It is worth noting that adiabatic heating strongly affects the shape of the stress-strain curves as well as the absolute values of the Garofalo equation parameters. The correction of adiabatic heating was conducted by a new algorithm described elsewhere [11].

Fig. 1 shows the isothermal stress-strain behavior of the UHCS at all temperatures and strain rates. The curves are characterized by a continuous increase of the stress to a peak value following by a decrease to a steady-state previous to rupture. The beginning of a steady-state regime can be identified at strains of about 2. This behavior is usually attributed to the presence of dynamic recrystallization being the main softening phenomena [12].

3.2. Modeling of stress-strain behavior

The Garofalo equation, Eq. (2), was used to model the stress-strain curves of the UHCS-1.3%C. The equation is fitted to $\{ \dot{\epsilon}, \sigma, T \}$ data at various levels of strain in

order to predict the transient and steady-state regimes of deformation [6]. Concretely, these fittings are based on 23 torsion tests and the Garofalo parameters are determined at strains of 0.4, 0.5, peak (about 0.54), 0.7, 0.8, 1, 1.5, 2, 2.5, 3 and 3.5. This large selection of strains allowed us to accurately compare results from both equations, (1) and (2), with very different strain ranges of application. The determination of the Garofalo parameters is carried out by means of the Rieiro-Carsí-Ruano, RCR, method described elsewhere [13]. In contrast to the traditional methods, the RCR method is an improvement without the use of initial values providing, additionally, statistical parameters to quantify the goodness of the fit.

Table 1 shows the Garofalo equation parameters as a function of strain. The last two columns show the R^2 and F-Snedecor statistical parameters. The high F and R values are an indication of high goodness of the fit. Each parameter characterizes different aspects of the goodness of the fit of the Garofalo equation to the experimental data set [13]. For strains lower than 0.4 the quality of the fits are low and are not given in the table. This is attributed to spurious elastoplastic effects occurring at these high strain rates.

Once the Garofalo equation parameters are determined the stress at the strains previously selected can be predicted. For this aim, the Garofalo equation can be written as:

$$\sigma = \frac{1}{\alpha^\varepsilon} \sinh^{-1}(Z_A^\varepsilon) \quad (3)$$

where the superscript ε indicates a dependence with strain and

$$Z_A^\varepsilon = \left(\frac{Z^\varepsilon}{A^\varepsilon} \right)^{\frac{1}{n_g^\varepsilon}} \quad (4)$$

being Z^ε the Zener-Hollomon parameter, $Z^\varepsilon = \dot{\varepsilon} \exp(-Q^\varepsilon/RT)$.

Fig 2 shows the comparison between the experimental and predicted stress from Eq. (3). Except for the strain 0.4, and as expected from the high values of the statistical parameters given in the last two columns of Table 1, the average relative errors between the experimental and predicted stress are lower than 3.5 %. These errors are similar or even lower than other recently presented in the literature [14-16]. Fig. 2 confirms that Eq. (3), in average terms, may predict the stress under transient and steady state conditions. These results allow us to conclude that the Garofalo equation can be considered as a suitable constitutive equation for the UHCS-1.3%C under transient and steady state creep regime [6].

3.3. Relation between stress exponents and analysis of deformation mechanism

Fig. 3 shows the Garofalo equation parameters as a function of strain in the interval 0.54 (average value of peak strain) to 3.5. Except for the α parameter, a clear steady state regime is revealed for strains higher than 2 which is in agreement with the results obtained for the stress-strain curves. It is also observed that while n continuously decreases with strain, α continuously increases with strain. This behavior was previously reported by McQueen and coworkers as a general behavior in austenitic stainless steels [4].

According to Fig. 3, n_g is about 2.5 at the higher strains, a value that is quite lower than 5 which is the expected one for a large number of alloys when the power law is employed as constitutive equation [3]. An interpretation of this value of 2.5 as associated to a governing mechanism other than slip creep would be erroneous. However, this difference can be explained by understanding the physical meaning of both stress exponents. A new relation between the stress exponents of the power law

and the Garofalo equations, n_p and n_g has been developed for this aim. The stress exponent corresponding to the power law equation can be expressed as:

$$n_p = \left. \frac{\partial \ln(\dot{\epsilon})}{\partial \ln(\sigma)} \right|_T \quad (5)$$

and according to the Garofalo equation:

$$n_g = \left. \frac{\partial \ln(\dot{\epsilon})}{\partial \ln(\sinh(\alpha\sigma))} \right|_T \quad (6)$$

Applying the chain rule to Eq. (6) leads to:

$$n_g = \left. \frac{\tanh(\alpha\sigma)}{\alpha\sigma} \frac{\partial \ln(\dot{\epsilon})}{\partial \ln(\sigma)} \right|_T \quad (7)$$

The following relation between the n_p and n_g parameters is obtained from Eqs. (5) and (7):

$$n_p = \left. \frac{\alpha\sigma}{\tanh(\alpha\sigma)} n_g \right|_T \quad (8)$$

This relation can be rewritten as:

$$n_p^\varepsilon = \left. \frac{\alpha^\varepsilon \sigma^\varepsilon}{\tanh(\alpha^\varepsilon \sigma^\varepsilon)} n_g^\varepsilon \right|_T \quad (9)$$

This relation allows converting, for a given strain, stress exponents obtained by the fit of the Garofalo equation into stress exponents from the power law equation.

Assuming n_p as a constant for a given temperature and strain, Eq. (9) shows that n_g decreases with increasing α as supported experimentally [4]. The equation also reveals that the usual identification of the Garofalo equation stress exponent with the inverse of

the strain rate sensitivity, m , is only valid in the range where $\alpha\sigma < 1$ since $\frac{\alpha\sigma}{\tanh(\alpha\sigma)}$ approach to 1. The relation $m=1/n$ is only fulfilled at low stresses where $n_g \cong n_p$.

Eq. (9) can be applied at every temperature and strain in order to check the dependence of n_p with strain and temperature. Fig. 4 shows the evolution of the calculated n_p , obtained by Eq. (9), with temperature at strains of 2, 2.5 and 3. The figure shows that small differences in n_p values are present at the highest temperatures, which is an evidence of steady state. In addition, Fig. 4 shows that the n_p parameter, for a given strain, is temperature dependent revealing a variation from values of about 7.4 at 950 °C to 4 at 1200 °C. This result shows that an explicit temperature dependence of the n_p parameter should have been incorporated in the power law equation in order to describe the creep data of the UHCS-1.3%C. This was previously theoretically advanced in [17].

Finally, in order to compare the values of the n_p parameter with those of the stress exponent of the Garofalo equation, n_g , an average value of n_p , n_p^{av} , can be computed at each strain. These values are represented as a function of strain in Fig. 5. It is observed that the n_g parameter continuously decreases with strain. In contrast n_p is almost constant for most strains with a value of about 5 at steady state (strains from 2 to 3) and for the strain at peak (~ 0.54). In other words, the n_p^{av} values given in the figure is the result of two components, n_g and α , from the Garofalo equation. The evolution of these components with strain results in a value of n_p of about 5 for large strain ranges where the power law is valid. These results seem to indicate that, in average terms, the creep behavior of the UHCS-1.3%C responds to a five-power-law equation. It is worth noting that the high correlation obtained for n_g values at strains between 2 and 3 allows

concluding that the five power law in the steady state corresponds to a stress exponent of about 2.5 for the Garofalo equation.

On the other hand, a high activation energy is computed at peak, 270 kJ/mol. This value is much higher than that corresponding to the self-diffusivity of iron in the austenite for the UHCS, 225 kJ mol⁻¹ [18]. This high value is attributed to the microstructure that is strongly changing since it is affected by dynamic recrystallization starting before the peak stress is reached. In contrast, at steady state regime, i.e. at strains higher than 2, the activation energy is almost constant with an average value of 224 kJ/mol that agrees well with the self-diffusivity of iron in the austenite. This confirms that the controlling deformation mechanism at steady state is lattice diffusion-controlled slip creep. In addition, it confirms that the activation energy obtained at the peak strain varies from that obtained at the steady state.

4. Conclusions

1. The Garofalo equation can be considered as a suitable constitutive equation for the UHCS-1.3%C at steady state and at the various regimes that characterize the stress-strain curve. The average relative errors between the predicted stress and the experimental data are lower than 3.5%.
2. A relation between the stress exponents of the Garofalo, n_g , and power-law, n_p , equations has been established. This relation predicts an increase of n_g with a decrease of α . The usual identification of the stress exponent of the Garofalo equation as the inverse of the strain rate sensitivity is only valid for $\alpha\sigma < 1$.
3. The value of n_g of about 2.5 for the UHCS-1.3%C is similar to a value of about 5 for the stress exponent of the power law.

4. The controlling deformation mechanism is consistent with lattice diffusion-controlled slip creep.

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Figure Captions

Fig. 1. Isothermal stress-strain behavior of the UHCS-1.3%C.

Fig. 2. Comparison between predicted and experimental stress for the UHCS-1.3%C.

The predicted stress is given by Eq. (3).

Fig. 3. Garofalo equation parameters as a function of strain for the UHCS-1.3%C.

Fig. 4. Variation of the stress exponent of the power law, n_p , with temperature for strains of 2, 2.5 and 3 (steady-state). Data are obtained from Eq. (9).

Fig. 5. Comparison between the Garofalo stress exponent, n_g , and the average stress exponent of the power law n_p^{av} as a function of strain.

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Figure 5

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Table 1

ε	n_g	Q , kJ/mol	α , MPa ⁻¹	ln(A), A in s ⁻¹	R ²	F
0.4	3.66	324.83	0.0163	27.61	0.96	237.2
0.5	3.96	273.01	0.0103	25.45	0.99	850.3
Max.(~0.54)	4.31	270.12	0.0073	27.00	0.97	369.7
0.7	3.78	231.20	0.0076	23.34	0.99	787.5
0.8	3.99	223.69	0.0053	24.42	0.98	652.1
1	3.77	212.16	0.0063	22.66	0.98	434.8
1.5	3.53	211.55	0.0090	21.22	0.97	384.2
2	2.85	218.43	0.0173	19.24	0.98	651.2
2.5	2.7	223.12	0.0210	18.96	0.98	429.2
3	2.51	225.35	0.0246	18.68	0.97	376.3
3.5	2.47	226.76	0.0267	18.45	0.96	223.5

Table 1. Garofalo equation and statistical parameters as a function of strain for the UHCS-1.3%C.