

STUDIES IN FUZZINESS
AND SOFT COMPUTING

**Studies
in Fuzziness
and
Soft Computing**

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**Soft Computing
in Humanities
and Social Sciences**

 Springer

Soft Deontic Logic

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7.1 Deontic Logic

Deontic logic or logic of norms is a kind of special logics, so-called 'extended logics'.¹ This branch of logics tries to analyze the formal relations established among obligations, permissions, and prohibitions. To this end, the language of logic is supplemented with a new vocabulary that consists of three operators which refer to the above deontic qualifications: 'o' (obligation, duty), 'a' (allowed, permission) and 'v' (interdiction, forbidden). As Sánchez-Mazas says ([21], p. 25), the term 'Deontic Logic' is intended to cover today, in a general way, all studies on the peculiar logical structure of systems of norms of any kind or, if you will, on the sets of values, laws, and deduction rules that govern those systems. For example, if it is an obligation to vote for any political party, then is it allowed? If I have the right to move across the European Union, are prohibited to impede me? Is there a duty to provide the means to such places? If I am required to hold choice to do this or that, how will fulfil that duty, joining the first, or the second, or both requirements? If I have the obligation to attend the institutional acts of my university and I also have the duty not to lose any of my graduate lessons, am I subject to the joint obligation of both actions? Are the causal consequences of an allowed action lawful too? Etc. Many issues of this kind arise when we think about the framework of deontic qualifications or regulations that appear in all human relationships.

In sum, the terms that contain such qualifications are norms (moral, legal) whose structure and inferential relations analyzes deontic logic. Thus, we can say that deontic logic is the theory of valid inference rules, that is, the analysis of the conditions and rules in which reasoning including qualifications of prohibition, duty, or permission, is correct.

We assume, therefore, that there are structural relations among expressions that include qualifications as required, prohibited, permissible, right, duty, etc. That is, there is a principle of inference between the norms so that, from a structured set of norms, it is possible to establish deductive inferences (logical consequences). In this way, we treat norms as entities like propositions, which can be negated and

¹ The term 'deontic logic' is generalized from von Wright's work (1951)[25] in order to refer to the study of inferential relations among norms. The word 'deontik' has been previously used by Mally (1926)[16] related to his 'logic of will' and by Broad (1950)[6], speaking about 'deontic propositions'. Bentham also used the term 'deontology' referring to ethics. The source of that expression is greek word 'tò deón', translated as 'duty' or 'obligation'.

combined using logical connectives and quantifiers, clearly taking the view of the true possibility of the logic of norms.

7.2 Theoretical Assumptions of Standard Deontic Logic

There are three main assumptions in standard deontic logic that determine the usual logical analysis of norms: the modal conception of deontic logic (ideality), the gap between facts and norms (non-cognitivism/separatism), and the bivalence. We are going to analyze each of them.

7.2.1 The modal paradigm in deontic logic

All standard approaches consider deontic logic as a transcription of alethic modal logic, that is, the logic of necessity and possibility. Thus, deontic qualifications of obligation, prohibition and permission would be but the equivalent of alethic necessity, impossibility and possibility. This parallelism has come since the first studies about the logical inference of norms in the late scholastics and clearly in Leibniz: All the complications, transpositions and oppositions of the [Alethic] 'Modes' shown by Aristotle and others in their treaties about logics, can be transcribed to our 'Modes of Law'. ([15], p. 84). Leibniz defined the permission (lawful) as what a good man is possible to do; and the obligation (duty) as what a good man is necessary to do ([15], p. 83).

Such identification is inherited uncritically by modern deontic logic systems that arise simultaneously in the fifties of XX century ([14], [25], [4], [8] and [7]).

This isomorphism is not an arbitrary or trivial question: It seems understood that when we speak of obligations, duties, we are referring to what is "necessary", morally or legally speaking. Therefore, obligatoriness is interpreted as a kind of necessity, specifically, a normative necessity or requirement.

Expressed in terms of Kripkean semantics for modal logics, an action or behaviour is mandatory if it occurs in all alternative worlds relative to ours, i.e., what is compulsory is true in every possible normative world. Therefore, the duty is defined as the normative required, as a kind of necessity: For 'op' to be true in a world 'm', we should check if 'p' occurs in all the alternatives worlds of 'm'; that is, something is obligatory if it occurs in all the normative worlds relating to that used as reference -usually, the actual. This means that a duty has to be fulfilled in any normative world, context or situation -which is an idealized and maximalist view of the normative realm.

Furthermore, it appears very odd to try to say that not all obligations (norms) obtaining in the actual world obtain in its deontic alternatives. ([11], p. 71).

In this context, it is unquestionably the so-called 'Rule of Closure', whereby the logical consequences of duties are also mandatory. It means that obligatoriness is closed under the logical conditional: $(RC)p \supset q/op \supset oq$.

Its importance lies in expressing an important normative principle, namely, that one is committed to the logical consequences of her obligations.

This [RC] seems right and indeed useful in moral philosophy, for by means of this axiom we may persuade moral agents that they are committed to the logical consequences of their moral principles. ([22], p. 151).

7.2.2 The gap between facts and norms

One of the most controversial issues about arguments involving deontic matters (ethical, legal) is whether the statements of duty or right can be inferred from statements of fact, and conversely ([3]). Most philosophers, at least since Hume, have opted to give a negative answer to this question, arguing that obligations or permissions are not concerned with facts and that no combination of facts can imply a duty or a right, and no combination of duties or rights implies a fact. It is called the radical separation thesis between facts and norms (separatism), such that deontic expressions are a special kind of assertion whose content has nothing to do with the factual assertions. According to this thesis, factual assertions refer to states of affairs that exist or not in this world while deontic assertions refer to a particular type of entity - a right, an obligation-, whose existence or obtaining would be independent of the existence or obtaining of facts or states of affairs. This position is linked to so-called non-cognitive ethics (emotivism, precriptivism): 'good' is not a property of X , 'X is good' is not a descriptive term; the deontic qualifications do not bring new "knowledge" about X .

This approach to deontic logic is a consequence of the above mentioned isomorphism that exists between alethic modal logic and standard deontic logic in terms of possible-world semantics. We have said that, from this perspective, A is obligatory if and only if every normative possible world contains A (correspondingly, B is allowed if and only if at least one normative possible world contains B). But there would be no inferential link between the content of those normative possible worlds (ideal) and the content of the real or actual world or of any bunch of designated worlds. Consequently, a duty exists whether or not the facts and conditions are so and so. What is obligatory and what is licit would not change with any change in the facts of the world ([3], p. 45).

7.2.3 Bivalence

The standard deontic analysis is, as we said, a sort of supplementary logic, extended to the realm of norms, by introducing deontic operators which relate to the qualifications of permission, obligation and prohibition ([9]). However, it is not an alternative or divergent system to classical logic, to the extent that it does not question any of axioms or theorems of classical logic. Specifically, as we are concerned, standard deontic logic assumes bivalent conception of classical logic inherited from Parmenides and Aristotle, according to that may be envisaged only two truth values: (entirely) falsehood and (entirely) truth. So, it is impossible that something be and not be at

the same time; that is, stated as the basic principle the so-called 'principle of non-contradiction': it is not true that A and $\text{not}A : \sim (A \& \sim A)$.

The principle of non-contradiction has been, from Greek antiquity to the present, the most obvious of all the principles of logic. Medieval logicians bestowed supreme importance because they were convinced that reason could not accept contradictions; hence, a contradictory theory was invalid ipso facto. The contradiction would have such disastrous effects, when given, reason loses control over herself and, in particular on deductive processes, so anything could happen. This possibility was strongly expressed by the maxim *Ex Contradictoriis Quodlibet* ([20], p. 649, [5], p. 319 ff.). Thus, a system or theory that accepts as valid conflicting claims would not in any way differentiate between true and false. Therefore, such a system would be trivial as long as from a contradiction we can deduce anything. This is called the principle of Pseudo-Scotus (Cornubia) (*ex contradictione sequitur quodlibet*), which may be formulated in different ways: $A \& \sim A \supset B$; $A \supset (\sim A \supset B)$; $\sim A \supset (A \supset B)$.

Beyond logical, ontological and epistemological considerations, this principle of bivalence has favoured a dichotomous view of reality and a similar tendency in any discussion, debate or argumentation. That is, it has involved an approach to normative reasoning in terms of "all-or-nothing", so that the normative definitions and qualifications have been treated in sharp and abrupt terms. Remember that 'dichotomy' means to divide into two, separately or exclusively, as wholly possessing or not a particular property.

7.3 Shortcomings of Standard Deontic Logic

There are also three main challenges for a deontic logic that aspires to cope with normative reasoning in a significant way: the case of conditional and ensuant obligations (paradox of lesser evil), the gradualism of reality (factual and normative), and the normative conflicts.

7.3.1 Conditional and ensuant obligations

Many factual situations give rise to prescriptions which would not arise at all unless those factual situations existed in first place. This is the case in conditional norms: allow, ban or compel to the extent that certain facts exist [13]. Thus, many duties and permissions are contingent on facts. This kind of conditional norms are really abundant in normative systems and demands a proper rule of deontic detachment: How can we take seriously a conditional obligation if it cannot, by way of detachment, lead to an unconditional one? ([23], p. 263). But, standard deontic logic does not give currency to deontic detachment when the condition or factual premise is fulfilled: It is not possible to infer ' oq ' from ' p ' and ' $o(p \supset q)$ '. – Here we adopt a representation of conditional obligation as a whole norm, *in toto* ([26]), by means of a deontic operator with a wide scope: It is mandatory that, if p , then q .²

² Representing conditional obligation as a hybrid formulation (an implication with a factual antecedent and a deontic consequent) is unsatisfactory. Representing as a whole formula-

Moreover, relating to conditional obligations, we detect an important issue: the ensuing obligations or duties of lesser evil; namely, duties that arise as a result of antecedent factual situation, one wherein another duty has been breached. They are “reparational” or “compensatory” duties in the sense that they arise when one has failed in fact a prior obligation. Thus, for instance, resorting to war is forbidden by current international law but, in case such a prohibition is transgressed, new obligations arise as regards how to conduct the war, in accordance with international human conventions such as the Red Cross agreements.

This has led us to define the paradox of lesser evil as the pivot around which all deontic paradoxes hinge:³ A general principle of morals and law lays down that, if we act wrongly, at least, we have to act so as to implement the lesser evil. However, to the extent that the lesser evil is realized, evil is indeed done; but then, by means of the inference rule of logical closure RC, evil -without conditions- must be done.

7.3.2 Gradualism

Virtually any term, whether properties, definitions, qualifications, or states of affairs, involved in the normative reasoning -as in everyday life and in most sciences – is likely to have fuzzy edges and as regards borderline cases, without precise lines of demarcation. From this perspective, a particular action can have a more or less degree of licitness and, to the extent that it is not completely licit, it will have some degree of illicitness.

A property or a set is fuzzy in the sense that there are degrees of possession thereof or of belonging to such a set – so that the relationship of a member belonging to a group varies in different degrees, in the same way as a property may possess it or lack it in various amounts.

This gradualism of reality (normative and factual) appears itself in two types of language expressions: Adverbs of intensity or decay: pretty, little, a lot, entirely, somewhat, . . . ; and, comparative constructions: more or less, so and so, likeness, similar, close, etc. – for example, used in analogical reasoning, a crucial resource in legal realm.

Thus, there is a profound disagreement between a continuous and gradual reality, riddled with nuances and transitions, a reality in gray, and a logic (an analysis and description of it) bivalent, between sheer truth and complete falsehood, between utter duty and pure ban, in “all-or-nothing” terms, black or white (principle of bivalence, above mentioned). Consequently, similar behaviours and situations cannot receive a similar normative (ethical and legal) treatment, transgressing the elementary principles of fairness and proportionality.

tion $'o(p \supset q)'$ allows a way to fulfil the norm by completely refrain from $'p'$ (the condition) – which is not accounted for the narrow-scope rendering of conditional obligation.

³ Chisholm’s paradox (contrary-to-duty imperatives paradox), Good Samaritan paradox, knower paradox, gentle murder paradox, bi-conditional paradox, praise paradox, second best plan paradox, penitent paradox, and Ross’ paradox ([1], p. 59 ff).

7.3.3 Normative conflicts

As a consequence of gradualism of reality (something or someone is so and so, to some extent, and, at the same time, is not) we find in normative reasoning true conflicts or collisions but the normative ordering is not a trivial system (it does not follow everything from a contradiction).

Moreover, apart from the gradualism of many terms and properties (coercion, fraud, damage, equality, ...), legal normative conflicts are unavoidable since they arise from a variety of reasons: the multiplicity of sources of law; the own dynamics of legal systems (derogations, amendments); the indeterminacy – syntactic and semantic – of legal language; the legal protection of mutually conflict interests in social complex contexts (rules and exceptions); and the presence of gaps. There are three criteria used in legal doctrine to avoid antinomies, but, in fact, they are not sufficient to avoid all kind of legal normative conflicts.

7.4 Elements for a Soft Deontic Logic

In view of that challenges and shortages of standard deontic logic, we espouse soft deontic logic with the following five main characteristics: abandonment of modal paradigm, paraconsistency, fuzzy approach, non-relevantism, and quantification.

7.4.1 Abandonment of modal paradigm

We reject, as Hansson (1988)[10] and Weinberger (1991)[24], the idealized viewpoint of normative realm that presupposes alethic modal paradigm in deontic logic and, thus, the rule of closure (both in the core of deontic paradoxes). Obligatoriness has nothing to do with realization in ideal or optimal worlds. First, a state of affairs may be primitively obligatory while being quite undesirable. Second, many obligations exist only because the world is in fact thus or so. Many an obligation arises only when some factual circumstances are met. This is the main reason why all ideal-world approaches to deontic logic are doomed.

Then, an alternative is necessary to cope with the commitments of our rights, duties and prohibitions. A right – a licit course of action – is such that its owner may not be compelled or constrained to give it up. Rights imply an obligation for everybody else to respect those rights, and so a duty not to disturb the right's owners' enjoyment thereof. So, what forcibly prevents the exercise of a right is forbidden (principle of ensuant obligation). Here the key issue is causal, and no merely inferential, consequences that prevent the fulfilment of a right. What makes a right an entitlement for somebody to do some action is nothing else but: you are not entitled to force anyone not to enjoy one of her rights.

7.4.2 Paraconsistency

Contradictions, normative collisions and conflicts, do not trivialize normative orderings. Thus, we refuse the principle of Pseudo-Scotus which, if admitted, will become Law and normative sets something absurd and useless since, from a conflict, would collapse all normative distinctions.

7.4.3 Fuzzy approach

Deontic descriptions of licitness, prohibition and duty should be treated as gradual notions. From this perspective, a particular action can have a more or less degree of licitness and, to the extent that it is not completely licit, it will have some degree of illicitness. The core of this type of analysis is the principle of graduation, according to which, when two facts are similar, their deontic treatment must also be similar.⁴

As an alternative to the 'principle of bivalence' that permeates the standard approach to deontic logic, we maintain the 'principle of graduation', which says that facts and deontic qualifications are a matter of degree and therefore a fuzzy-logic approach is an appropriate theoretical method in deontic logic.

Suggesting the principle of graduation in relation to norms leads to the rejection of the idea that matters involving rights, duties and prohibitions, i.e., moral or juridical matters, are "all-or-nothing" questions. Consequently, graduation leaves room for flexibility and adaptability when dealing with particular and contingent circumstances, and it gives an important role to jurisprudence -understood as reasonableness in the normative domain. From a juridical point of view, this gradualist approach brings in an ingredient of malleability and flexibility which rehabilitates, in some ways, a certain spirit closer to the Anglo-Saxon *common law*. In addition to that, this approach also opposes the eagerness for legislative fixation which characterizes a codifier strand, in the line of Justinian's, with the perverse effects it carries.

So, the fuzzy approach to deontic logic entitles us to soften the sharp dichotomies usually stated in normative reasoning. The main consequence of the fuzzy approach to deontics is that it allows us to cope with thousands of dilemmas and conflicts that arise in a way less wrenching, traumatic, and arbitrary than the "all-or-nothing" approach.

7.4.4 Non-relevantism

Our approach recognizes deontic postulates that follow from any set of normative rules, by means of the axiological basis of normative systems. That is, soft deontic logic is not purely formal but, as any kind of deontic logic, involves assumptions about the theory of duty, the normative cognitivism, etc. ([12], pp. 393-410).

⁴ This idea is clearly related to the Leibnizian principle of transition or continuity: *lex iustitiae* [2].

7.4.5 Quantification

Most deontic systems developed have omitted the quantifiers in their formulations. In addition, the few times that have been introduced, it was in a mechanical way, tracing the relevant modal principles without elaborate on their normative significance or not.

In our view, it is absolutely essential the use of quantifiers in deontic logic. The introduction into the language of deontic logic of expressions like 'all', 'some', 'none', etc. will provide a inestimable clarification of the content and meaning of rights, duties and prohibitions, and the relationships and commitments among them, especially when involving the existential quantifier as in the case of positive rights [19].

7.5 The Underlying Quantificational Calculus

Our present treatment is based on *transitive logic*, a fuzzy-paraconsistent nonconservative extension of relevant logic **E**, which has been developed in the literature [18]. Transitive logic is a logic of truth-coming-in-degrees, and of truth being mixed up with falseness (the truer a proposition, the less false it is, and conversely). Thus it is mainly conceived of as a logic applicable to comparisons involving 'more', 'less' and 'as as'. The implicational functor, ' \rightarrow ', is here construed as a functor of alethic comparison: $[p \rightarrow q]$ is read as "To the extent [at least] that p , q ". Since our main idea is that an action can be both (to some extent) licit and (up to a point) also illicit, and that one out of two actions, both licit (both illicit), can be more licit (less licit) than the other, our logic's intended use is to implement valid inferences involving 'more', 'less' and 'as as' in deontic and juridical contexts.

Transitive logic (system **P10**) is built up by strengthening the logic of entailment **E**. (Our notational conventions are *à la* Church: no hierarchy among connectives; a dot stands for a left parenthesis with its mate as far to the right as possible; remaining ambiguities are dispelled by associating leftwards.)

Transitive logic introduces a distinction between strong (\neg) and simple (\sim) negation. Simple contradictions are nothing to be afraid of, whereas contradictions involving strong negation are completely to be rejected. Alternatively, transitive logic has a primitive functor of strong assertion, 'H', such that $[\neg p]$ abbreviates $[H \sim p]$ ("Hp" can be read as "It is completely (entirely, wholly, totally) the case that p"). Within transitive logic disjunctive syllogism holds for strong negation; thus we define a conditional ' \supset ', such that $[p \supset q]$ abbreviates $[\neg p \vee q]$. Notice that conjunction $\&$ is such that $[p \& q]$ is as true as $[q]$, provided $[p]$ is not utterly false; we read $[p \& q]$ as 'It being the case that p, q '. ' \wedge ' is simple conjunction, 'and' ($[p \& q]$ is defined as $\neg \neg p \wedge q$). We also define $[p \setminus q]$ as $[p \rightarrow q \wedge \neg(q \rightarrow p)]$ (" $p \setminus q$ " means that it is less true that p than that q). Our reading of ' \rightarrow ' is as follows: $[p \rightarrow q]$ is read: "To the extent [at least] that p , q ". We add a further definition: let $[\blacktriangle p]$ abbreviate $[\sim(p \rightarrow \sim p)]$ – $[\blacktriangle p]$ meaning that $[p]$ is true enough. ($[\blacktriangle p]$ could be read 'It is sufficiently (or amply or abundantly, or the like) true that p', or something

like that.) Last let ' α ' be a primitive sentential constant meaning the conjunction of all truths.

Here is an axiomatization of system **P10**. Primitives: $\wedge, \rightarrow, \sim, H, \alpha$. Definitions of ' $\vee, \neg, \blacktriangle$ ' are as above. $[p \vee q]$ abbreviates $[\sim(\sim p \wedge \sim q)]$.

7.5.1 Axioms

P10.01 α	P10.02 $\blacktriangle(p \rightarrow p) \rightarrow .p \rightarrow p$
P10.03 $\alpha \sim \alpha$	P10.04 $\alpha \rightarrow p \vee .p \rightarrow q$
P10.05 $Hp \rightarrow q \vee . \sim p \rightarrow r$	P10.06 $p \rightarrow q \rightarrow r \wedge (q \rightarrow p \rightarrow r) \rightarrow r$
P10.07 $p \rightarrow q \rightarrow .q \rightarrow r \rightarrow .p \rightarrow r$	P10.08 $p \rightarrow q \rightarrow .p \rightarrow r \rightarrow .p \rightarrow .q \wedge r$
P10.09 $p \rightarrow q \rightarrow . \sim Hq \rightarrow \neg Hp$	P10.10 $p \rightarrow (q \wedge \sim q) \rightarrow \sim p$
P10.11 $\sim p \rightarrow q \rightarrow . \sim q \rightarrow p$	P10.12 $p \rightarrow \sim \sim p$
P10.13 $p \wedge q \rightarrow p$	P10.14 $p \wedge q \rightarrow .q \wedge p$

Sole primitive *Rule of Inference*: **DMP** (i.e. disjunctive *modus ponens*):

for $n \geq 1$: $p_1 \rightarrow q \vee (p_2 \rightarrow q) \vee \dots \vee p_n \rightarrow q, p_1, \dots, p_n \vdash q$

MP [*Modus Ponens*] for implication ' \rightarrow ' is a particular case of the rule – the one wherein $n = 1$. *Adjunction* is a derived inference rule.

This axiomatic basis seems to us reasonably clear, elegant and functional. It may however contain some redundancy.

Modus ponens for the mere conditional, ' \supset ', is also a derived inference rule: from $[p \supset q]$ and $[p]$ to infer $[q]$. The fragment of **P10** containing only functors $\wedge, \vee, \supset, \neg$ is exactly classical logic (both as regards theorems and also as regards rules of inference). That's why **P10** is a conservative extension of classical logic.

7.5.2 Quantification

Our quantificational extension of system **P10** is obtained by adding further axiomatic schemata plus three inference rules. We introduce universal quantifier as primitive and define $[\exists x p]$ as $[\sim \forall x \sim p]$. By $[r[(x)]]$ we mean a formula $[r]$ with no free occurrence of variable ' x '. $[p[(x/z)]]$ expresses a formula resulting from substituting " z " to " x ".

Additional axiomatic schemata:

$$\begin{aligned} \exists x(p \wedge \forall x q) &\leftrightarrow \forall x(\exists x p \wedge q) \\ \forall x(p \wedge q) &\rightarrow .\forall x p \wedge q \\ \forall x(s \setminus r[(x)]) &\supset \exists x(s \setminus r) \\ \forall x p \wedge \exists x q &\rightarrow \exists x(p \wedge q) \\ H\exists x p &\rightarrow \exists x H p \end{aligned}$$

Quantifiational Inference Rules:

rinfq01: universal generalization

rinfq02: Free-variables change rule

rinfq03: Alphabetic Variation

Deontic Postulates

We introduce as primitives these symbols: $\lceil ap \rceil$ is read as ‘It is allowed that p’ or ‘It is licit that p’; $\lceil op \rceil$ abbr. $\lceil \sim a \sim p \rceil$; ‘o’ means obligation; $\lceil vp \rceil$ abbr. $\lceil \sim ap \rceil$; ‘v’ means interdiction (unlawfulness); ‘ \blacktriangleright ’ means a causal relation between facts, whereas ‘ \blacktriangleleft ’ means a relation of thwarting (obstructing, hindering) — not necessarily by brute force or violence, but in any case by material actions which make it practically impossible or very hard for the thus coerced person to perform the action she is prevented from doing.

Hindering is of course a particular case of causation, but while an appeal can prompt someone to refrain from acting in a certain way, the link is not one of coercion. Preventing is causing an omission against the agent’s will. Thus, a state of affairs, that Peter’s house’s door is key-locked by John, blocks another state of affairs, namely for Peter to leave his house.

We are aware that it would be nice to have axiomatic treatments of the two relations of causation and prevention, but no interesting set of such axioms has occurred to us.

We also take for a common assumption, namely the equivalence between the obligatoriness of A and the illicitness of not-A.

Axioms

[DI] $a(p \rightarrow q) \rightarrow .op \rightarrow q$	(duty implication)
[RI] $o(p \rightarrow q) \rightarrow .ap \rightarrow q$	(right implication)
[Sa] $o(op \rightarrow ap)$	(deontic subalternation)
[RA] $ap \wedge aq \rightarrow a(p \wedge q)$	(right aggregation)
[RD] $a(p \vee q) \wedge \sim p \rightarrow aq$	(right disjunction)
[DD] $o(p \vee q) \wedge \sim p \rightarrow oq$	(duty disjunction)
[DS] $o(p \wedge q) \wedge p \rightarrow oq$	(conditional duty simplification)
[RS] $a(p \wedge q) \wedge p \rightarrow aq$	(conditional right simplification)
[DE] $oop \rightarrow op$	(duty enforcement)
[RE] $oap \rightarrow ap$	(right enforcement)
[LC] $p \blacktriangleright q \wedge ap \rightarrow aq$	(licit causation)
[NH] $p \blacktriangleleft q \wedge ap \rightarrow \sim vq$	(non-hindrance)

Inference rules

[RP] If $\lceil op \rceil$ is not a theorem, then $\vdash a \sim p$	(Rule of Permission)
[Eq] $\vdash p \leftrightarrow q \Rightarrow \vdash op \leftrightarrow oq$	(Rule of Equivalence)

7.6.3 Comments on Deontic Postulates

Most of those axioms are to be understood with an implicit clause: provided $[p]$ and $[q]$ are separately and jointly contingent – that is to say, each expresses a contingent state of affairs and neither necessarily implies the other or its negation. That restriction applies to **[DI]**, **[RI]**, **[RD]** and **[DD]**.

[RI] (or, equivalently, $[o(p \rightarrow q) \rightarrow .p \rightarrow oq]$) means that whenever you have a duty to do A -only-to-the-extent B is the case, then you only are entitled to A to the extent B happens. Suppose the consequent is false: you are entitled to A to a higher extent than B is the case. And suppose the antecedent is true: it is mandatory for you to realize A only to the extent B happens to be the case. Then you are put under two mutually incompatible deontic determinations: on the one hand you are allowed to do A to some high degree – higher than the one of fact B being realized; but on the other hand you are obliged to do A in at most as high a degree as that of B 's being realized. Thus supposing you choose to perform A to such a high degree, you are both enjoying your right and yet breaching your duty. In order to avoid such a deontic incongruity, **[RI]** ought to be accepted as a principle of deontic logic.

The case for **[DI]** (which is equivalent to $[a(p \rightarrow q) \rightarrow .p \rightarrow aq]$) is quite similar – the reasoning being exactly parallel.

A usual objection against **[RI]** and **[DI]** is that they allow us to draw factual inferences from deontic premises, which seems to be odd or inconvenient. Yet those inferences are legitimate. What duties and what rights exist depends not only on what enactments have been passed by the legislators but also on what factual situations exist. Under certain factual circumstances, A , once a right has been granted with an implicative content, $a(A \rightarrow B)$, B is automatically licit. If the legislator pronounces $A \rightarrow B$ licit and A obligatory while B is not realized at all, either of the two rules is null and void. Thus his enacting the obligation to A may be taken to imply repealing the right to $A \rightarrow B$.

The subalternation principle **[Sa]** is a deontic version of Bentham's well-known thesis that whatever is obligatory is also licit. Our particular version makes it into a deontic norm. It is an implicit rule of any deontic system that the system itself is bound to regard as rightful what it renders mandatory. In other words, any ruleer has to be aware that the system of norms as such contains a rule to the effect that, obligatorily, what is mandatory is also, to that extent, licit. Otherwise the ensuing set of rules would not count as a normative system at all, but would be an incongruous ensemble.

[RA] is one of the most significant features of our approach. Standard systems of deontic logic tend to embrace duty-aggregation, which we reject, since for a system to make separately mandatory two courses of action – e.g. one by agent X and another by agent Y – does not necessarily entail that the system makes the (perhaps impossible) conjunction of those two courses of action mandatory. On the other hand, unless right-aggregation, **[RA]**, is embraced no right is unconditional. For a right to A to be unconditional the agent has to be entitled to A whether or not s/he chooses to do B or to refrain from doing B . If the agent is both unconditionally entitled to A and unconditionally entitled to B , he is unconditionally entitled to do

both, even if doing both is impossible. Being obliged to perform an impossible action is indeed quite irksome, but being entitled to do it is absolutely harmless. In fact once a course of action is commonly held to be impossible, any prohibition to perform it tends to be automatically dropped or otherwise voided.

Notice that **[RA]** escapes usual objections, such as that a man can be allowed to marry a woman while also being allowed to marry another woman without thereby being allowed to marry both. In fact his right to marry either is conditional – upon a number of assumptions, among others his refraining from marrying any other woman. This shows that **[RA]** applies to unconditional rights only and that many rights are implicitly or silently conditional.

[DS] is merely a weakened version of a principle of standard deontic logic, namely that whenever a joint action, *A-and-B*, is obligatory, each conjunct, (*A,B*) is obligatory, too. As such the principle is wrong, since for you to have an unconditional duty to jointly perform those two actions, *A* and *B*, does not imply for you to be unconditional allowed – let alone obliged – to realize *A* in case you fail to perform *B* (for whatever reason, whether by your own will or because you are prevented), since perhaps doing *B* without *A* may be a worse course of action falling afoul of the normative system’s purposes. Yet our version avoids such a difficulty, since deontic simplification is made conditional on one of the two jointly-obligatory actions being realized.

As for **[RS]** quite similar considerations apply, replacing “duty” by “right”. (In standard deontic logic **[RS]** is not needed as a primitive axiom, but our system needs both.)

[RD], or right disjunction, is also a principle of deontic congruence, which means that, whenever you are entitled to realize either *A* or *B*, and in fact *B* is not the case at all (whether in virtue of your own choice or for whatever reason), the only way you can enjoy your right to *A-or-B* is by realizing *A*; hence, under such assumptions, you are unconditionally entitled to *A*. In deontic or normative contexts, being entitled to *A-or-B* is usually confined to free choices. Your right to *A-or-B* entails that you are free to choose either. Hence, if you refrain from *A*, you are entitled to *B*. Notice, though, that you may be allowed to *A-or-B* without being permitted to *A-and-B*. (For instance, you may either pay a higher tax or else renounce a pay increase, not both.)

[DD] is exactly similar to **[RD]**, only replace “right” with “duty”. For *A-or-B* being mandatory implies that, in case *A* completely fails to be realized – for whatever reason, whether by your own choice or not – *B* is mandatory, since, under such assumptions, doing *B* is the only way for you to abide by your duty to *A-or-B*. Notice, though, that neither **[RD]** nor **[DD]** apply when one of the disjuncts, *A*, is partly realized. Then even if *A* is not completely realized and thus it is partly true that $\sim A$, the right or the duty to *B* cannot be derived. This is why we have chosen strong negation, \neg , for those two axioms. Thus our system of axioms fails to countenance your claiming a right to *B*, in case you are entitled to *A-or-B*, when you have already chosen *B*, even if only in a low degree. It is only by wholly refraining from the alternative course that your disjunctive right entitles you to embrace the remaining disjunct.

[DE], $[oop \rightarrow op]$ (or, equivalently, $[ap \rightarrow aap]$) is a principle of (dis)iteration. According to it, when a normative system makes an obligation mandatory, it implicitly makes the content of the obligation also mandatory; or, whenever the system makes an action rightful, it makes the right to perform that action also licit. This means that there are no unlawful rights. Notice, though, that for a legal situation, oA , to exist is not the same as for the law-giver to enact an act making A mandatory. The legislator's enactment may be a sufficient cause for the obligation to arise (under certain conditions), but the two states of affairs are different. Thus our axiom does not mean that, whenever the legislator is bound to enact a law, the law is already implicitly enacted – which is of course false in a number of cases. What alone is ruled out is that a situation should be made licit while the right to realize it is forbidden. To forbid the right implies to forbid its content.

Likewise **[RE]**, $[oap \rightarrow ap]$ – or, equivalently, $[op \rightarrow aop]$ – means that the normative system only contains lawful duties. To the extent the system, as such, makes an action mandatory, it makes the duty to perform that action licit. You cannot be illicitly obliged to do A . In other words, to the extent the obligation is unlawful, its content is not mandatory. Or: nothing is forbidden unless it is rightfully forbidden. Of course there can be sets of rules lacking such a principle, but they are not normative systems, but un-congruous amalgams, which cannot discipline society's set of activities.

[LC] $[p \blacktriangleright q \& ap \rightarrow aq]$ means that the causal consequences of licit actions are also licit. Again the legislator is not almighty. He can forbid a course of action, B . But if, at the same time, he allows A and, as a matter of fact, A causes B , that entails that either of his rulings is null – usually we must take the earlier one to be repealed by the later.

[NH] $[p \blacktriangleleft q \& ap \rightarrow vq]$ means that only to the extent an action is forbidden can be the case that a licit action hinders or prevents it. Suppose again that the legislator allows A while, as a matter of fact, A impedes B . Now suppose that the same legislator also allows B . Either of the two rulings is voided by the other (usually the later one will repeal the earlier). Notice that for an action to be unlawful is not the same as for it to fall afoul of, e.g., the criminal code or any other particular regulation. By proclaiming B legal, while A prevents B , the legislator is making A unlawful, even if A is not considered by the criminal code – a general rule for unlawful damage compensation will apply.

Principle **[NH]** has to be restrictively interpreted. The adequate notion of hindrance is not a purely factual one, but a concept which is partly defined and construed by the normative system. It is not empty or idle, though. The principle does not mean that unlawful hindrances of licit actions are forbidden. What it means is that all hindrances of legal actions are forbidden, yet the apposite concept of hindrance is not wholly determined by nature but partly by law, which sorts out allowed and forbidden ways of interfering with other people's behaviour.

The Rule of Permission is a peculiar inference rule, which makes our system non-recursively axiomatizable, since it entitles us to infer the licitness of any course of action when the negation thereof cannot be proved to be forbidden. This rule is in effect a presumption rule which imposes the burden of proof upon such as

claim that the considered course of action is forbidden. Unless and until it is proved to run against the law, it has to be assumed to be licit. This is one of the many differences between modal and deontic logics. Deontic logics have a practical job to perform. Practice cannot wait for logicians to carry their inferential work on and on. When legal operators are satisfied that the prohibition cannot be proved, they are automatically entitled to regard its content as in compliance with the law.

Finally, the Rule of Equivalence is pretty obvious: any two logically (or, more generally, necessarily) equivalent situations are liable to share their deontic determinations or qualifications. Of course that does not apply to other sorts of loose equivalences, such as practical equivalence or the like; only to strict equivalences which exist in virtue of logic, or at least are metaphysically necessary. Thus $[A]$ and $[\sim\sim A]$, $[A \wedge A]$, $[A \vee A]$ and so on, since they express equivalent states of affairs, are to be ascribed the same deontic attributes.

7.6.4 Quantificational axioms

Since existential quantification is similar to infinite disjunction and universal quantification is like an infinite conjunction, axioms for quantifiers are easily guessed:

[URA]	$\forall xap \rightarrow a\forall xp$	(universal right aggregation)
[UDS]	$o\forall xp \wedge \forall x(y \neq x \supset p[x/y]) \rightarrow op$	(universal duty simplification)
[URS]	$a\forall xp \wedge \forall x(y \neq x \supset p[x/y]) \rightarrow ap$	(universal right simplification)
[UDD]	$o\exists xp \wedge \forall x(y \neq x \supset p) \rightarrow op[x/y]$	(universal duty disjunction)

Other universal generalizations of our axioms are left as an exercise for the reader.

As particular examples of those axioms we shall mention the following ones. As for **[URA]**: to the extent everybody is entitled to do A it is also lawful for all to do A. (Otherwise, please notice, those separate courses of action are not unconditionally allowed, but only permitted to the extent other people refrain from following the same course of action. (Thus it is no the case that everybody is allowed to visit the museum on Monday.) As for **[UDS]**, to the extent it is obligatory for all children of Rosa to help their mother, to that extent Rosa's son, Jacob, is bound to help his mother if in fact all his brothers and sisters do help her. As for **[UDD]** take this example: to the extent it is mandatory for at least one of the co-debtors to pay the borrowed amount and John is one of them and in fact all other co-debts completely fail to pay, John is bound to pay. Of course the same restrictions apply to universal generalisations as do to its sentential-logic counterparts (the facts under consideration have to be contingent and logically independent of one another).

7.7 Conclusion

The **DV** system allows for degrees of licitness and obligatoriness, and also for degrees of compliance or realization. It countenances partial contradictions. But it puts limits on incongruity, beyond which the purported normative system does not

count as one. One of the practical purposes this logic serves it to provide us with a criterion to know when a fresh regulation or statute repeals earlier regulations. We have already mentioned several cases. Our system is fact-sensitive. Norms and facts are partly interdependent? despite Hume's and Moore's qualms or strictures. What norms are in operation is not an issue entirely independent of what facts happen in the world. And the other way round. There is a certain solidarity between facts and norms. Facts can abolish norms.

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