Limits for the vortex state spin torque oscillator in magnetic nanopillars: Micromagnetic simulations for a thin free layer

Gloria R. Aranda, Julian M. Gonzalez, Juan J. del Val, and Konstantin Y. Guslienko

Citation: *J. Appl. Phys.* **108**, 123914 (2010); doi: 10.1063/1.3524222

View online: <http://dx.doi.org/10.1063/1.3524222>

View Table of Contents: <http://jap.aip.org/resource/1/JAPIAU/v108/i12>

Published by the [American Institute of Physics](#).

Related Articles

Anomalous magnetic properties of 7nm single-crystal Co₃O₄ nanowires

J. Appl. Phys. **111**, 013910 (2012)

High field-gradient dysprosium tips for magnetic resonance force microscopy

Appl. Phys. Lett. **100**, 013102 (2012)

Dipolar interactions in magnetic nanowire aggregates

J. Appl. Phys. **110**, 123924 (2011)

Dielectric and spin relaxation behaviour in DyFeO₃ nanocrystals

J. Appl. Phys. **110**, 124301 (2011)

Finite size versus surface effects on magnetic properties of antiferromagnetic particles

Appl. Phys. Lett. **99**, 232507 (2011)

Additional information on J. Appl. Phys.

Journal Homepage: <http://jap.aip.org/>

Journal Information: http://jap.aip.org/about/about_the_journal

Top downloads: http://jap.aip.org/features/most_downloaded

Information for Authors: <http://jap.aip.org/authors>

ADVERTISEMENT

Submit Now

Explore AIP's new open-access journal

- Article-level metrics now available
- Join the conversation! Rate & comment on articles

Limits for the vortex state spin torque oscillator in magnetic nanopillars: Micromagnetic simulations for a thin free layer

Gloria R. Aranda,^{1,a)} Julian M. Gonzalez,² Juan J. del Val,^{1,2} and Konstantin Y. Guslienko^{2,3}

¹*Centro de Física de Materiales UPV/EHU-CSIC, San Sebastián, Spain*

²*Dpto. Física de Materiales, Universidad del País Vasco, 20018 San Sebastián, Spain*

³*IKERBASQUE, the Basque Foundation for Science, 48011 Bilbao, Spain*

(Received 23 June 2010; accepted 1 November 2010; published online 28 December 2010)

We report micromagnetic simulations of magnetization dynamics of a vortex state in the free layer of a circular nanopillar excited by the spin transfer torque effect of a perpendicular to the layer (dot) plane spin-polarized electrical current. The magnetization of the reference layer (polarizer) is assumed to be fixed. A new regime of the dynamic magnetization response to the current is reported: vortex expelling from the dot, subsequent in-plane magnetization oscillations in single domain state, and the vortex return with an opposite core polarization. We analyze conditions (limits of the vortex state as a nano-oscillator) to achieve steady magnetization oscillations corresponding to a gyrotropic motion of the vortex core in terms of the current intensity. These conditions are formulated via the critical currents and vary greatly with the magnetic damping parameter and the cell size used for micromagnetic simulations. The existing experiments on the current induced magnetization dynamics in nanopillars and nanocontacts are discussed. © 2010 American Institute of Physics.

[doi:[10.1063/1.3524222](https://doi.org/10.1063/1.3524222)]

I. INTRODUCTION

Since the prediction of the spin angular momentum transfer (ST) effect by Slonczewski¹ and Berger,² there were a lot of research on that subject. The possibility of changing the magnetic configuration of the sample without an applied magnetic field, only by the interaction between the spins of the conductivity electrons and the localized magnetic moments is promising for applications in information storage technology, and also for microwave power generation with a fixed frequency.^{3,4} One of the excitable magnetic configurations is a vortex state. Vortex state is a flux closure structure of the magnetization and presents the ground state in flat soft magnetic nanoparticles.⁵ This structure has been widely studied because of its characteristic features, such as vortex core polarization (orientation of the out-of-plane magnetization of the core) and chirality (sense of the curling of the in-plane magnetization). The vortex magnetization distribution stability as well as the vortex excitations were studied in Refs. 5–15.

The typical investigated systems revealing the ST induced magnetization dynamics are layered magnets with alternating ferromagnetic and nonmagnetic layers. One of the ferromagnetic layers is used for the polarization of the current (polarizer or reference layer) and magnetization dynamics are excited in other ferromagnetic layer (free layer, see Fig. 1). The nonmagnetic spacers can be metallic (nanopillars and nanocontacts) or thin insulators (tunnel junctions). Experimental information on the current induced magnetization dynamics can be extracted from measurements of the device resistance oscillations, by giant magnetic resistance (GMR) effect in the first case, or by tunneling magnetoresistance in the second case.

Using the knowledge of basic physics of the magnetic vortex dynamics, considerable efforts were done to measure, explain, and predict vortex core polarization switching, vortex motions, or oscillations in a free layer (dot) of nanopillar under the influence of the ST torque. The particular cases of an in-plane current,^{16–18} nanopillar magnetization dynamics under the influence of a perpendicular polarizer,^{19–28} a vortexlike polarizer^{29–31} or nanocontacts^{32–34} were considered.

The limits for the vortex state as a spin current driven oscillator were previously analytically and theoretically explored in Refs. 23, 27, and 28. By theoretical considerations, it was established that the ST induced magnetization dynamics of the free layer depends on its ground state, direction of the current, and the current spin-polarization (the magnetization distribution in the polarizer). It was established, as well, that the perpendicular to the plane spin polarized current I can excite permanent magnetic vortex motion in the free layer of nanopillar if the current intensity is within the interval $I_{c1} < I < I_{c2}$ (I_{c1} is the first critical current, necessary for the onset of vortex oscillations, and I_{c2} is a second critical current). In this current range, the moving vortex oscillates with a stationary orbit determined by the current value, I . If

FIG. 1. (Color online) Sketch of the magnetic nanopillar used for micromagnetic simulations.

^{a)}Electronic mail: gloria.r.aranda@csic.es.

the current exceeds I_{c2} ,^{21,22} the vortex steady motion state is not stable anymore, presumably because the vortex reaches a critical velocity⁹ and reverses its core. Vortex core precession and reversal effects are quite well-known now^{9,20–22,24–27} and have been numerically simulated. However, previous micromagnetic simulations of the spin torque excited vortex state by current perpendicular to plane (CPP) have always been performed for thick free layers. The authors of Ref. 25 presented analytical calculations using small free layer thickness of 4–5 nm, but they excluded this thickness from their micromagnetic simulations. Contrarily, the typical free layer thicknesses used in the experiments on nanopillars and nanocontacts^{32–39} are around 4–5 nm. In the case of vortex state in nanopillars, microwave measurements, and micromagnetic simulations of the vortex gyrotropic motion excited by CPP are reported in Ref. 19. This study is for thicker free layer (15 nm), with uniformly magnetized polarizer (whose magnetization was changed by an external applied field). Reduced thickness maximizes the ST torque strength, given that it is mainly an interfacial effect.⁴⁰ In order to minimize the current density needed to excite magnetization by this effect in the free layer, thickness should be reduced.

In the present paper, we report simulations of the strongly nonuniform (vortex and quasingle domain) magnetization dynamics in a circular permalloy nanodot (free layer of nanopillar) excited by a perpendicular to the dot plane spin polarized current. We consider a dot with thickness of 4 nm, close to one usually used in the spintronic devices being explored experimentally. To consider the vortex state as a ST oscillator, its motion must be detected on the long time scale (minutes and hours), i.e., it is necessary to have a stability of the magnetization dynamics out of the equilibrium with the dissipated energy equal to the energy supplied by the spin transfer over one oscillation period. Within the widely used macrospin model, the magnetization trajectories that satisfy this condition are called precessional states. In the case of the strongly nonuniform (vortex) dynamics, we call them as “steady” states. The existence of such steady dynamic states is a key point in understanding the ST induced magnetization dynamics in spintronic devices.⁴⁰

The goal of the present work is to find a range of the currents where the steady state vortex oscillations exist (spin torque nano-oscillators). We show that the first critical current is well defined, whereas the definition of the second critical current is not so simple for the free layers with small radii R (≤ 100 nm), where the vortex core can be easily and quickly expelled from the dot.

II. MICROMAGNETIC SIMULATIONS

The simulations concern a permalloy dot of radius $R = 100$ nm, representing a free-layer in a spin valve nanopillar. This layer is the upper one shown in Fig. 1, where we also present the excitation scheme and the system of coordinates used. We use the Landau–Lifshitz–Gilbert (LLG) equation of the reduced magnetization $\mathbf{m} = \mathbf{M}/M_s$ motion with an extra ST term (M_s is the saturation magnetization, see Appendix). The magnetization dynamics is excited by the

FIG. 2. (Color online) Magnetic vortex shifted from the dot center by the bias in-plane field of 50 Oe, before applying the perpendicular spin polarized current.

spin-polarized dc current perpendicular to plane. This electrical current is polarized (by the polarizer layer magnetization, m_p) and traveling perpendicularly to the dot plane (Ox direction). Dimensions and excitation scheme are similar to the free layer in a nanopillar simulated in Refs. 24–27. The free layer thickness ($L = 4$ nm) is smaller in our case. Simulation details can be found in Appendix. We underline here the two important points in our treatment of the problem as follows:

- (1) the thin free layer (4 nm) was simulated. This is useful if we account that the ST effect is mainly interfacial;⁴⁰
- (2) the long time of simulations (up to 500 ns) in comparison to previous works,^{24–27} which is crucial for study of the vortex as a stable nano-oscillator with well defined frequency. In previous works, the typical time for micromagnetic simulations was 100 ns,³⁰ or at most 250 ns.²⁶ In Ref. 27, the authors simulated 100 ns from the reaching a steady oscillation state, but they did not supply any guideline to prove if the “steady state” was indeed reached. It could be only a change in the slope of the maxima in the volume averaged $\langle m \rangle$ -components dependence versus time. This is discussed in Appendix.

A. The magnetic vortex stability and core radius

The starting point of the simulations was to check that, actually, the vortex state is stable (i.e., there is not evolution to any other magnetization state) for our dot dimensions and magnetic parameters. This was theoretically predicted in Refs. 5 and 25 and confirmed by our simulations.

Next, we checked the shape of the vortex core for the static and moving vortex. For an external in-plane field $H_y = 50$ Oe applied to a centered vortex, we have a displaced vortex located at 25 nm away from the dot center (Fig. 2). The vortex core radius is approximately of 30 nm in this static configuration. Analytically estimated vortex radius is half of this value. This is a result of the small radius of our dot, which enhance the importance of the exchange interaction energy.

A spin-polarized electrical current is applied to this off-centered vortex equilibrium state (reason for this can be found in Appendix). When the vortex is moving, the vortex

FIG. 3. Time evolution of the in-plane magnetization averaged component $\langle m_y \rangle$ after the bias field of $H_y = 50$ Oe was removed at $t = 0$ ns. The applied current is $I = -0.4$ mA, with $P = 0.2$ and $m_p = (1, 0, 0)$, the cell size is $4 \times 4 \text{ nm}^2$.

core gets asymmetrically deformed. The smallest radius (27–30 nm) of the vortex core is in the core region away from the dot border. This vortex core region is limited by a negative out-of-plane magnetization zone (the start of the core reversal). The biggest radius (~ 34 nm) of the vortex core corresponds to the core zone nearer to the dot border. This distortion, when the vortex core is moving, of its circular shape and the core profile corresponds to a first stage of the vortex core magnetization reversal as it was already described in the review on magnetic vortices.⁴¹

B. Critical currents of the vortex steady oscillations

In order to find a range of the currents, I , for which the vortex steady oscillations are possible ($I_{c1} < I < I_{c2}$), we ap-

ply the CPP current of several different intensities I (from 0.1 to 5 mA, that corresponds to the current densities $J = I / \pi R^2$ from 0.3×10^6 to $15.9 \times 10^6 \text{ A/cm}^2$) to the magnetic layer with a off-centered vortex. The Oersted field of the current is accounted. It is equal to ~ 20 Oe at the edges of the layer when $I = 1$ mA.

For small current densities the vortex is approaching to the centered position performing a spiral motion around it. However, as we increase the current intensity, for $I = -0.4$ mA (Fig. 3), we find an oscillation regime which is the beginning of a steady state. The oscillations are almost steady, only small magnetization amplitude decay is observed. We present the vortex magnetization dynamic images during its almost steady oscillations around the dot center in Fig. 4. The oscillation period of this movement is 4.3 ns, some typical value for the vortex gyrotropic motion. So, we can consider that $I = -0.4$ mA ($1.3 \times 10^6 \text{ A/cm}^2$) is a little bit over the first critical current (I_{c1}) for existence of the steady state vortex motion and can serve as a good estimation of I_{c1} .

The current density range for the existence of this steady motion found in our simulations is narrow. When we increase/decrease the current by only ± 0.1 mA ($0.32 \times 10^6 \text{ A/cm}^2$) the steady state disappears. Instead, two different scenarios appear: the vortex movement gets damped (for decreased current) or the vortex motion is enhanced until the core is expelled from the dot and the free layer magnetization reaches an in-plane single domain state (for in-

FIG. 4. (Color online) Snapshots of the vortex dynamic magnetization distributions extracted from the simulations presented in Fig. 3 for time interval from 461.9 to 465.1 ns (each image corresponds to a quarter of the oscillation period).

FIG. 5. Reduced magnetization in-plane Y-component vs time, starting simulations from an initially displaced vortex by $H_y = 50$ Oe. Applied current is $I = -0.3$ mA. $P = 0.2$, $m_p = (1, 0, 0)$, and the cell size is 4×4 nm³.

creased current). An example of the first scenario is shown in Fig. 5 ($I < I_{c1}$), while the second is shown in Fig. 6. The average magnetization evolution in Fig. 6 (the vortex is expelled) inset corresponds to the vortex motion with increasing value of the orbit radius till reaching the dot border at $t \approx 400$ ns. The following large amplitude steady state magnetization dynamics corresponds to rotation of the almost uniform magnetization state ($\langle m_{y,z} \rangle \approx 0.89-0.97$), with no vortex. The frequencies of the magnetization oscillations after the expelling of the vortex core are shown in Fig. 6. They consist of a frequency close to the vortex gyrotropic frequency (a quasigyrotropic frequency), which is slightly increased from 254 to 303 MHz and its odd harmonics. Taking into account that the quasigyrotropic frequency is still present in the excitation spectra when the vortex core is not inside the dot, the situation regarding the second critical current definition is not clear. Considering I_{c2} as an upper current limit for the vortex gyrotropic steady oscillations frequency, this current intensity would be below I_{c2} , although the vortex core is not inside the dot anymore.

In Fig. 7, we present snapshots of the magnetization configurations corresponding to the vortex core expelling and

FIG. 6. Amplitude of the FFT of the reduced magnetization in-plane Y-component shown in the inset, calculated for the time interval from $t = 399.4$ ns to $t = 500$ ns. Inset: reduced magnetization in-plane Y-component vs time simulated from an initially displaced vortex by $H_y = 50$ Oe. Applied current is $I = -0.5$ mA. $P = 0.2$, $m_p = (1, 0, 0)$, and the cell size is $4 \times 4 \times 4$ nm³.

subsequent in-plane magnetization oscillations. After the vortex core expelling, the magnetization distribution presents so-called S-state, and afterwards it turns into a dynamical C-state.^{7,29}

From this point, we compare our results on the critical currents with other published works containing experimental data, analytical calculations, and simulations, and explain the differences in results. For the magnitude of the critical currents for the vortex steady state oscillations for our simulated dot dimensions, we only have values coming from analytical approaches. There are no published experiments for the ST induced vortex dynamics by the perpendicularly magnetized polarizer in nanopillars for our free layer dimensions. The reported experimental intensities of the current where the vortex steady motion exists are generally bigger than ours. The differences with our results could be either assigned to the polarizer magnetization not exactly perpendicular to the plane and absence of lateral confinement in the free layer, as it happens in nanocontacts (see Refs. 32–34), or to an inhomogeneously magnetized polarizer.^{30,31}

The current intensities larger in 50–100 times were detected experimentally to excite the vortex gyrotropic motion in nanocontacts. This is because the polarization is not entirely perpendicular to the layer plane, but has an in-plane component incapable of exciting the vortex movements. As an illustration for this effect, we made calculations for $I = -0.5$ mA (when we get the vortex going out for the current polarization $m_p = (1, 0, 0)$ as shown in Fig. 6), but with $m_p = (\sqrt{2}, \sqrt{2}, 0)$, and found a damping in the vortex movement presented in Fig. 8. Contrary to the case shown in Fig. 6, the vortex is damped and returns to the dot center.

Besides, regarding lateral confinement in the nanocontact case, the orbit of the vortex was reported to be inside and outside the point contact area. Limited space for the vortex oscillations or distorted shape of the vortex core profile due to boundary conditions is not present in nanocontacts. For nanocontacts, the electrical current should cause a vortex nucleation that could increase I_{c1} existing previously in our system because of the shape effect.

In the case of inhomogeneously magnetized polarizer for nanopillars, a behavior similar to the one shown in Figs. 6 and 7, was found experimentally in Ref. 30. Two coexisting frequency modes were measured in a range of the applied current, and interpreted as vortex and single domain oscillations of the thinner layer in the nanopillar. In that experiment, the polarizer, that it is not fixed, results to be in a vortex state. However, this behavior did not appear in their simulations performed only during 100 ns (whereas our simulations cover 500 ns interval). The authors of Ref. 30 checked only by the micromagnetic simulations that both kinds of the oscillations were possible for a range of current densities, depending on the direction of the current ramp: increasing or decreasing.

Regarding analytical calculations of the critical currents in Ref. 25 for a free layer of the same material and dimensions as ours, the interval between the critical currents densities reduces to only single value of the current density close to our result for the steady state. The single value of the current density for the steady vortex motion is because of the

FIG. 7. (Color online) Snapshots of the dot magnetization distributions for $I = -0.5$ mA when the vortex leaves the dot (the snapshots are taken from 396.5 to 400.3 ns of the long time magnetization dynamics shown in the inset of Fig. 6).

authors of Ref. 25 did not take into account nonlinear terms in the Thiele's equation of motion with spin torque term^{10,23,27,28} which they used for calculations

$$\mathbf{G} \times \dot{\mathbf{X}} = -\frac{\partial W}{\partial \mathbf{X}} + \hat{D}\dot{\mathbf{X}} + \mathbf{F}_{\text{ST}}. \quad (1)$$

In this equation, $\mathbf{X} = (Y, Z)$ is the coordinate of the vortex core and the dot symbol is time derivative, $\mathbf{G} = \hat{x} 2\pi M_s L p / \gamma$ is the gyrovector describing the vortex core, p is the core polarization, γ is the gyromagnetic ratio, W is the energy of the moving vortex, $\hat{D}$ is the damping tensor, and $\mathbf{F}_{\text{ST}} = \pi L M_s \beta \varepsilon (\hat{x} \times \mathbf{X})$ is the force resulting from the ST torque²⁸ (β and ε are defined in Appendix and $\hat{x}$ is the unit

vector perpendicular to the dot plane). The nonlinear terms in the vortex core displacement appear in this equation because of the high values of the vortex core orbit radius $|\mathbf{X}|$ comparable with R , and are calculated in Ref. 28. The value of J_{c1} calculated on the basis of papers^{27,28} for our particle dimensions is -1.04 A/cm², $I_{c1} = -0.33$ mA, which fits very well with our simulations. The same occurs for the analytical calculations presented in Ref. 25. The critical current, I_{c2} , necessary for the vortex expelling has been theoretically considered and estimated only in Ref. 28, as a current needed for the vortex core to reach the border of the dot by subsequent increase in its orbit radius. The estimated value of I_{c2} is bigger than the one obtained by micromagnetic simulations.

Our simulations can be compared with those performed by Khvalkovskiy *et al.*²⁷ for thicker layers. In the micromagnetic simulations,²⁷ a wider current intensity range of stability for the steady movement was found in comparison to our result. The cell size $1.5 \times 1.5 \times 10$ nm³ was used in simulations in Ref. 27 and the authors only simulated the magnetization movement during 100 ns after reaching the steady state. If we take into account the different thickness of their free layer (10 nm), the different radius (150 nm), and the high slope of the curve representing the orbit radius versus current density $R_s(J)$, as explained in Ref. 31, their results are consequent with ours. Regarding the second critical current, we simulated a big slope curve $R_s(J)$, which results in early expelling of the vortex increasing the current density for the current relatively close to the first critical value, I_{c1} .

FIG. 8. Time evolution of the in-plane magnetization $\langle m_y \rangle$ component for $I = -0.5$ mA ($\alpha = 0.01$ and $P = 0.2$, $H_y = 10$ Oe), but with $\mathbf{m}_p = (\sqrt{2}, \sqrt{2}, 0)$. The cell size is $4 \times 4 \times 4$ nm³.

FIG. 9. (Color online) Reduced magnetization in-plane Y-component vs time. The parameters are $I=-2$ mA, $P=0.2$, $m_p=(1,0,0)$, $H_y=100$ Oe. $\alpha=0.01$, the cell size of $4 \times 4 \times 4$ nm³. Vortex leaves the dot, an oscillating high-frequency mode appears, and then vortex comes back with opposite core polarization and it is damped. In the down part of the figure, snapshots of the magnetization configuration when the vortex is out of the particle are shown.

C. Vortex core expulsion, reversal, and damping

If we further increase the applied current intensity to $I=-2$ mA (6.4×10^6 A/cm²) and $I=-5$ mA (15.9×10^6 A/cm²), a new feature appears. The vortex core, after expelling from the dot, returns to the dot with opposite polarization. Finally, this new entering vortex core is damped to the dot center. The in-plane reduced magnetization in the Y direction for $I=-2$ mA is shown in Fig. 9, along with different snapshots of the magnetization configurations. In this case, after the vortex leaves the particle, the vortex gyrotropic frequency disappears from the spectrum and we find in-plane oscillations of the magnetization with different frequency, in the gigahertz range.

In previous works (except Refs. 24 and 28), mechanism of the vortex core polarization reversal was supposed to be vortex-antivortex pair formation.^{9,25–27,31} The vortex core expulsion excited by the spin polarized CPP has not been reported. Only in simulations concerned about the vortex core polarization reversal by CPP in Ref. 24, the authors found a mechanism similar to ours for small dot radius ($2R=80$ nm and 40 nm thickness), with appearance of an intermediate

magnetization C-state when the vortex leaves the dot. The appearance of this mechanism is due to the lack of space for the vortex-antivortex pair formation for thin dots and small R . The authors of Ref. 24 need bigger current densities, above 50×10^6 A/cm² to invert the vortex core polarization, while we can do it with 1.27×10^6 A/cm². For the high currents used in Ref. 24, the intermediate quasisingle-domain oscillations between the vortex up and vortex down states cannot be observed. The high Oersted field corresponding to their current intensities causes a quick appearance of the vortex with reversed core polarization.

We have similar situation as in Ref. 24 for the current densities of 15.9×10^6 A/cm² ($I=-5$ mA), with no intermediate in-plane magnetization oscillations. The vortex expelling is much faster in this case, and the transient state existing for $I=-2$ mA (Fig. 9) is absent. This results in the vortex expelling before a single oscillation is performed and its return with opposite vortex core polarization before 6 ns from the onset of the current. Afterwards, the new vortex core motion is damped to the center of the dot.

We observed that incrementing the applied current density provokes faster expelling of the vortex core and reduction in in-plane magnetization oscillations time after this expelling. When considering the oscillating behavior after the vortex core expulsion, we compare Fig. 6 ($I=-0.5$ mA) to the simulations showed in Fig. 9 ($I=-2$ mA). We found some discrepancies, due to the increase in the Oersted field and the spin torque interaction accompanying the current increment. The first discrepancy concerns the average out-of-plane magnetization component, which is much bigger for $I=2$ mA (changing from $-0.3M_s$ to $-M_s$) than for $I=-0.5$ mA (from $-0.05M_s$ to $-0.15M_s$). The magnetic configuration at $I=2$ mA is much like a vortex oscillating on the circumference of the dot, with a localized region of the perpendicular to plane magnetization (like a vortex core). Differently, for $I=-0.5$ mA, we have an almost homogeneous out-of-plane component of the magnetization and a wide opened C-state as the magnetic configuration. The second discrepancy arises from the oscillations' coherence. If we compare the oscillations of the $\langle m_y \rangle$ magnetization component for both the currents, after the vortex leaves the dot (from approximately 10 ns to 80 ns) for $I=-2$ mA and the ones corresponding to $I=-0.5$ mA (from 400 ns in Fig. 6), we find that they are quite incoherent for $I=-2$ mA. Finally, the third discrepancy involves the values of the oscillation frequencies: for $I=0.5$ mA (Fig. 6), a quasigyrotropic vortex frequency is still present in the simulated spectra when the vortex leaves the dot, while the magnetic configuration is a much more opened C-state with less out-of-plane component than in the case of $I=2$ mA. For the current $I=2$ mA: the quasivortex gyrotropic frequency disappears from $t \approx 10$ ns to ≈ 80 ns, and the main oscillation frequencies are a kind of the double wide peak around 1.25 GHz and 1.36 GHz.

For all our simulations, the change in the core polarization implies a change in sense of the vortex core gyrotropic motion. The change in sign of the core velocity when the vortex core polarization is reversed was experimentally observed in the x-ray imaging experiments.¹³ The simulations performed in Refs. 24 and 27 show a change in the sense of

vortex core rotation when the polarization of the vortex is changed. This change in the vortex sense of rotation after each switching of the vortex core polarization was also found in Ref. 31 for a circular polarizer. This change sense was calculated in the paper,¹⁰ where it was shown that the sense of the vortex core rotation (described by the vortex gyrovectector $\mathbf{G}$) depends on the product of the vortex core polarization ($p = \pm 1$) and the topological charge (vorticity, q), i.e., the gyrovectector in the Thiele's Eq. (1) is proportional to the product of pq .

Theoretical considerations predict, as well, the damping of the vortex motion when it returns to the particle with opposite core polarization, as we found in our simulations. According to theory, if the condition $pIm_{px} > 0$ (Ref. 28) for a perpendicular polarizer is not fulfilled, there is no vortex excitation (the ST term acts as an extra damping term), and the vortex gets damped. In this expression I is the sign of the current (direction of movement of the positive electric charges) and m_{px} is the component of the polarizer magnetization along the vortex core polarization direction. This is the case when the vortex is expelled and then comes back into the dot again, with opposite polarization (parallel to the one corresponding to the incident polarized electrons), as happens for higher current intensities (shown in Fig. 9). The vortex with opposite core polarization cannot be excited for the given current sign I and the magnetization oscillations stop. The regime of absence of the vortex oscillations due to change in the current sense or vortex core polarization also was observed in the simulations^{24–27} and experimentally.¹⁹

The vortex core polarization reversal is typically explained by the vortex-antivortex pair formation when the vortex velocity reaches some critical value. The vortex core position X and velocity result to be crucial parameters when considering either the vortex core expulsion or reversal. We presented our simulation results in these variables. In Fig. 10, we show evolution of the vortex orbit radius when the vortex going out the dot (Fig. 6) along with the vortex core velocities in this case and in the case presented in Fig. 3, when the core approaches a stationary orbit. In Fig. 10 we can observe the vortex core position $\mathbf{X}$ (in the Y direction) and module of its velocity [Fig. 10(b)] as functions of time for two cases: $I = -0.4$ mA (only velocity) and $I = -0.5$ mA. In both the cases, the in-plane bias field is $H_y = 50$ Oe. We can check that in none of the cases, the vortex reaches the critical velocity of 330 m/s (for permalloy) necessary for the vortex-antivortex pair formation and subsequent vortex core polarization reversal. The core velocity is only 30 m/s (steady) for $I = -0.4$ mA and ≈ 120 m/s for $I = -0.5$ mA. This is consistent with our observations of the vortex disappearance by expelling from the dot. We can estimate the velocity of a vortex leaving the dot, supposing its frequency of $\omega_0 \approx 250$ MHz corresponding to the vortex gyrotropic motion for these dimensions. We use estimation for the core velocity $|\dot{\mathbf{X}}| = \omega_0 R$, where ω_0 is the gyrotropic frequency. The velocity results to be 160 m/s for a vortex core on the dot border, which is not enough for the vortex-antivortex pair formation.

FIG. 10. (Color online) (a) The vortex core position Y coordinate vs time corresponding to the conditions of inset of Fig. 6, with the vortex finally leaving the free layer. (b) Absolute value of the vortex core velocity vs time corresponding to the motions shown in Fig. 3 (solid black line) and Fig. 6 (upper red line).

D. Critical currents dependence on the simulation cell size

We found that the simulation cell size is crucial for establishing the current density range of the vortex steady state motion. However, the reason for such an influence has a physical nature as we show below. We give an explanation for this fact and propose a way to overcome it.

Although the permalloy exchange length $l_{ex} = \sqrt{A/2\pi M_s^2}$ is around 5 nm (A is the exchange stiffness) (Ref. 42) and our cell size, $4 \times 4 \times 4$ nm³, should be sufficient to properly mimic the time evolution of any magnetization distribution in this material, nevertheless we reduced the cell size to $2 \times 2 \times 2$ nm³, and run the same simulation sequence. Results are shown in Fig. 11 and we can notice that the behavior with smaller cell size is essentially different regarding stability of the vortex core motion (conditions are equivalent to ones used in Fig. 3, but with smaller initial $\langle m_y \rangle$ magnetization). In the case of the cell size $2 \times 2 \times 2$ nm³, the magnetization oscillation amplitude increases rapidly with time, the vortex core approaches the dot border for the time ~ 200 ns and no vortex oscillation steady state exists, i.e., the vortex motion is steady for the cell size $4 \times 4 \times 4$ nm³, but it is not for $2 \times 2 \times 2$ nm³ for the same current $I = -0.4$ mA and other parameters. This means reducing the critical currents I_{c1} , I_{c2} ($I > I_{c2}$ now, whereas there was $I_{c1} < I < I_{c2}$ before). In the frequency domain, contrarily, the power spectral density reveals a peak at the same main frequency, as shown in Fig. 11(b).

FIG. 11. (Color online) (a) Time evolution of the in-plane magnetization component (along the Y direction) under excitation by the applied current $I = -0.4$ mA, $P = 0.2$, $m_p = (1, 0, 0)$, $H_y = 10$ Oe, $\alpha = 0.01$, for two different cell sizes. Red dashed line corresponds to $2 \times 2 \times 2$ nm³ cell size and black solid line to $4 \times 4 \times 4$ nm³. (b) Amplitude of the FFTs of both the time dependences presented in Fig. 11(a). Red dashed line corresponds to $2 \times 2 \times 2$ nm³ cell size and black solid line to $4 \times 4 \times 4$ nm³.

We performed a new trial, with the cell sizes $4 \times 1 \times 1$ nm³ and $4 \times 2 \times 2$ nm³, to study if we could obtain some asymptotic behavior and could make some extrapolation to smaller cell sizes. We checked that making the cell size smaller in Ox direction does not modify the results, as it was expected, because of the negligible variation in magnetization along the thickness of the free layer. We observed that, as the cell side in the YZ plane was reduced to half the value presented in Fig. 3, the current necessary for the steady state was reduced in the same way, by a factor of 0.5. If we reduce the cell size without reducing CPP, this leads to $I > I_{c2}$ regime, out of the vortex steady state range existing for bigger cell size. The current value to get the vortex steady state motion is proportional to cell size, for the cell sizes smaller than the exchange length value.

The simulation results for the same conditions as used in Fig. 11(a), but with the cell size $4 \times 2 \times 2$ nm³ and reduced current intensity, are shown in Fig. 12. The same steady state as for $4 \times 4 \times 4$ nm³ cell size in Fig. 11(a) is reached with decreased applied current. We also performed simulations with $4 \times 5 \times 5$ nm³ cell size, and obtained damping of the vortex gyrotropic motion for the currents, which correspond to a steady state motion simulated for $4 \times 4 \times 4$ nm³. Increasing the cell size without increasing current intensity means that we get the regime $I < I_{c1}$, outside of the vortex steady motion range existing for smaller cell sizes. This confirms the conclusion that increase in the in-plane cell size leads to increase in the critical currents (I_{c1}, I_{c2}). This depen-

FIG. 12. Time evolution of the in-plane magnetization component (along the Y direction) for the $4 \times 2 \times 2$ cell size, $I = 0.2$ mA, $m_p = (-1, 0, 0)$. Initial bias field is $H_y = 10$ Oe. $P = 0.2$ and $\alpha = 0.01$.

dence is unexpectedly strong. We checked that reduction in the cell size implies then a reduction in effective damping. Simulations for $I = 0.4$ mA and $4 \times 2 \times 2$ nm³ cell size, but for the damping constant $\alpha = 0.02$ (see Eq. (A1) in Appendix) provided magnetization oscillations almost identical to presented in Fig. 11(a) for $4 \times 4 \times 4$ nm³. Only the high-frequency harmonics of Fig. 11(b) for the $4 \times 4 \times 4$ nm³ cell size were not present in the $4 \times 2 \times 2$ nm³ case with increased damping constant. In our simulations, the smaller cell sizes mean smaller effective damping and enhanced amplitude of the vortex movement, so we would need smaller currents to get the same vortex orbit amplitude.

Looking for an explanation for the observation of the critical current variation with the cell size change, we conducted studies on the vortex core of magnetic configurations at the beginning of the vortex movement (first minimum in $\langle m_y(t) \rangle$ function after starting its movement). We observe by simple examination of the different magnetic configurations that when the cell size is reduced, the vortex core presents a wider area with almost perpendicular to plane magnetization. However, this is not enough to explain the strong dependence of I_{c1} on the cell size.

Variation in the results with the simulation cell size has been found in some previous works. In Ref. 43, the authors used the cell size of 3.6 nm for Cobalt nanopillar and observed considerable changes, when they varied the cell size. When the cell size was reduced by a factor of two, they found the same excitation frequencies, but difficulties in reproducing trajectories because of ‘the nonlinear character’ of the dynamics. The problem of variation in simulation results with the cell size was also found for simulations with finite temperature included.^{44–49} A summary of the attempts to overcome this problem is presented in Ref. 44. It was concluded in Ref. 45 that the LLG damping increases with the cell size increasing. There was a proposal for using a different equation to calculate magnetization dynamics (the Landau–Lifshitz–Bloch equation) in Ref. 46. The authors of Ref. 47 noticed the cutoff in the partition function for bigger cell sizes, others discussed the difference of disorderly oriented moments depending on the cell size and varied magnetization and exchange constants.^{48,49} Some of our previous results (not published) for excitation of a vortex by an in-plane alternating field demonstrated the same behavior with the reduction in cell size: the vortex was expelled for a

smaller field when we reduced the cell size. This allows us to conclude that this change in behavior does not depend on the excitation scheme. What happens with cell size decreasing is a decrease in the effective damping. We have performed simulations of the vortex getting back to the dot center from a displaced position, without any further excitation, for the cell sizes of $4 \times 4 \times 4$ and $4 \times 2 \times 2$ nm³. We fitted the $\langle m_y \rangle$ oscillations versus time to the function $\langle m_y(t) \rangle = m_0 \exp(-d\omega t) \cos(\omega t - \varphi)$ with m_0 , d , ω , φ as fitting parameters,⁵⁰ $d = -D/|\mathbf{G}|$ is the damping parameter and ω is the intrinsic vortex frequency. From these fittings, we conclude that the damping of the movement when the cell size was $4 \times 4 \times 4$ nm³ is twice the one for $4 \times 2 \times 2$ nm³ and the frequency, that depends on the recovering constant, also changes (from 215 MHz for $4 \times 4 \times 4$ nm³ to 232 MHz for $4 \times 2 \times 2$ nm³). We checked how other driving frequencies can excite the vortex gyrotropic motion in the calculations performed in Ref. 51 for excitation of the azimuthal spin waves in the vortex ground state. When the cell size is reduced, magnon modes with smaller wavelength can be activated, that were forbidden for bigger cell sizes because of the micromagnetic approximation that considers the magnetization constant inside each simulation cell. These modes add a new room for magnon-magnon interaction that must be accounted in simulations. The damping is the most suitable parameter to include these outcoming interactions. It should be increased in the LLG equation proportionally to the new spin wave modes appearing (as we reduce cell size) and capable to interact with vortex motion. The appearing magnon modes suitable for interaction with the vortex gyrotropic mode have the same symmetry (azimuthal spin waves⁵¹). To summarize, we need to increase damping parameter as we decrease cell size to reproduce the same critical current values.

Increasing damping constant for fixed cell size leads to a different scenario. Higher values of the damping constant for permalloy are reported in Ref. 52. The higher values of damping are attributed to “grain structure or due to formation of magnetic domains at low field.” There is an effect of increase in the damping constant value when we consider inhomogeneous magnetization configurations as a whole. In Ref. 36, the high damping value is related to “nonequilibrium spin pumping effects and/or spin relaxation arising from defects.” As a summary, when we apply the LLG equation for the evolution of the magnetization to a strongly non-uniform state, the damping is enhanced. We performed another series of simulations using an increased damping parameter ($\alpha=0.03$) and the smaller polarization, $P=0.15$ [see Eq. (A1)], to study the changes with this parameter without cell size modification. For these simulations we chose to change the current sense and m_{px} sense to match a realistic experimental situation, as is shown in Fig. 1. Simulation results for $I=2$ mA are shown in Fig. 13. The steady state is present for bigger intensity (change in the critical currents) and the oscillations of the magnetization are enhanced (increase in microwave power). Effects of the polarization decrease and damping constant increase can be directly seen in Eq. (A1) in Appendix.

FIG. 13. Reduced magnetization in-plane Y-component vs time for different current polarization $P=0.15$ and damping parameter $\alpha=0.03$, $I=2$ mA. The initial bias field is $H_y=75$ Oe, $m_p=(-1,0,0)$. The cell size is 4×4 nm³.

There is another effect that we ascribe to micromagnetic discretization although it has been also found in experiments: the frequency harmonics present in both Figs. 11(b) and 6. These kinds of harmonics have been found in other simulations³² as well as experimentally³⁴ in the frequency spectra of nanocontacts. In these references, the harmonics appeared at every integer multiple of the vortex gyrotropic frequency. In our case, they are odd multiples of the vortex gyrotropic frequency. In Ref. 32, the frequency harmonics are related to the degree of circularity of the vortex core trajectory. In Fig. 6, when the core is out, this is not a suitable explanation. Our suggestion is to relate the harmonics with the influence of some periodical structure, such as cell border, in the magnetostatic potential. These distortions in the potential could be also present in experiments, but for different reasons.

III. SUMMARY

We have explored by the micromagnetic simulations the vortex motion and stability for a free layer of nanopillar excited by a perpendicularly to plane spin polarized current. Changing the applied current density, we found different behaviors of magnetization for the ferromagnetic thin layer which was initially in the vortex state. There is a lack of experiments for thin free layers of nanopillars in the vortex state and we only found published works for nanocontacts of the same thickness or nanopillars with thicker free layers or nonuniform polarizers. We obtained the vortex oscillation frequencies that match experimental results and analytical predictions for nanocontacts. However, two qualitative differences with previous works were found. The first one is that the reversal of the vortex core polarization in our simulations is always achieved by the vortex expelling and subsequent return to the dot with opposite polarization. The second one is the achievement of finite in-plane oscillations of the magnetization when the vortex core leaves the dot, with their stability and coherence and oscillation frequency depending on the current density magnitude and magnetic configuration.

Absence of a vortex-antivortex pair formation in our case is justified by the vortex core arriving to the border of the dot before it reaches the critical velocity of the vortex

core polarization reversal. The dot thickness to radius ratio determines a gyrotropic frequency that is not enough to reach the critical velocity for the core reversal inside the particle. In all the cases, the ultimate vortex core velocity was below 330 m/s [see Fig. 10(b)], the calculated in Ref. 9 critical velocity for starting a vortex-antivortex pair formation. So, according to this, another mechanism is responsible for reversal of the vortex core polarization. This mechanism is the vortex core expelling from the dot, as it was shown in our simulations.

Regarding the oscillation frequencies, we found two different regimes of the steady state oscillations: the vortex state and single domain state oscillations. For the vortex gyrotropic motion, we can see that our frequencies agree quite well with the extrapolation of the simulated in Ref. 12 results to our dot aspect ratio and with the ones supplied by the two vortex ansatz,¹⁰ assuming no side surface magnetic charges. There is also a very good agreement of our simulations with the experimental frequencies detected in Ref. 33, where the radius of nanocontact and thickness of the free layer have the same dimensions as our device.

The orbit radius of the vortex steady gyrotropic motion strongly depends on the ST term value (as one can conclude comparing Figs. 11 and 13). Its value is enhanced when we increase the magnitude of the damping factor that we use for micromagnetic simulations. However, the oscillation frequency does not depend so strongly on the ST term value (it is determined by the restoring force that changes with the current change just because of the contribution of the Oersted field²⁸).

It was found a strong dependence of the vortex core trajectory on the spin torque force and on simulation cell size. Theoretical calculations to obtain critical current^{27,28} used a damping constant α of 0.01. With this same damping constant, we can reproduce the predicted in Ref. 27 and 28 values for the first critical current for the cell size of 4 nm. This cell size is suitable to describe magnetization configuration dynamics in this system, for being below the exchange correlation length.⁴² If we decrease the cell size, we have to increment proportionally the damping parameter in the LLG equation used for simulations to obtain similar results in the first critical current value. Reason for this is of physical nature: when the cell size is reduced, we allow new magnon modes to enhance the vortex motion. In Ref. 54, describing the Bloch points responsible for the vortex polarization reversal, considerable changes for different cell sizes are found for the fields needed for reversal. However, big cell sizes (but always below exchange length) are proposed to be useful for resembling pinning effects similar to those existing in nature, and the changes found for different cell sizes are always smaller than the ones found experimentally. Anyway, the simulation results should converge as the cell size is reduced.

ACKNOWLEDGMENTS

G.R.A. is supported by a JAE-doc grant. K.G. acknowledges support by IKERBASQUE (the Basque Science Foundation). The work was partially supported by the SAIOTEK

Grant No. S-PC09UN03. The authors also thank the Donostia International Physics Center and UPV/EHU Scientific Computing Service, and their technicians, as well as the Center of Materials Physics CSIC-UPV, for their help and access to computation facilities.

APPENDIX

In order to perform micromagnetic simulations, we have discretized circular permalloy dot ($\text{Ni}_{80}\text{F}_{20}$ alloy), of 200 nm diameter ($2R$) and 4 nm thickness, into $4 \times 4 \times 4$ nm³ cubes, with size down the exchange length of this material.⁴² Such discretization should describe fairly well the magnetization configuration. This discretization size, or even higher, had been used previously in the micromagnetic simulations of vortex state dynamics.^{11,16,18,29-33,53}

The magnetic parameters used in simulations were: the saturation magnetization $M_s = 800$ emu/cc, the exchange constant $A = 13 \times 10^{-12}$ J/m, the gyromagnetic ratio $\gamma/2\pi = 2.95$ MHz/Oe.^{11,55} We use the code object oriented micromagnetic framework (OOMMF)⁵⁶ with the LLG equation of motion modified by a term that includes the interaction with a spin polarized current.¹ Using this code, magnetization dynamics can be explored as a function of current, applied magnetic field, and the sizes/shape of the free layer and polarizer. The equation governing the evolution of the free layer magnetization under the action of a spin polarized current is the LLG equation of magnetization motion plus two extra spin transfer torque terms⁵⁶ (SI units)

$$\begin{aligned} \frac{d\mathbf{m}}{dt} = & -|\gamma|\mathbf{m} \times \mathbf{H}_{eff} + \alpha \left(\mathbf{m} \times \frac{d\mathbf{m}}{dt} \right) + |\gamma|\beta\varepsilon(\mathbf{m} \times \mathbf{m}_p \\ & \times \mathbf{m}) - |\gamma|\beta\varepsilon'\mathbf{m} \times \mathbf{m}_p, \\ \beta \equiv & \left| \frac{\hbar}{\mu_0 e} \right| \frac{J}{LM_s}, \quad \varepsilon = \frac{P\Lambda^2}{(\Lambda^2 + 1) + (\Lambda^2 - 1)(\mathbf{m} \times \mathbf{m}_p)}, \end{aligned} \quad (\text{A1})$$

where $\mathbf{H}_{eff}$ is the effective magnetic field (which includes exchange, dipolar, and applied external field contributions, and also the Oersted field of the current), α is the Gilbert damping parameter, β is defined by the electron charge e , the current density J , and the free layer thickness L , along with the vacuum magnetic permeability μ_0 , as well as by the reduced Planck constant $\hbar$. It is assumed that the ST torque is distributed over the thickness L of the free layer of nanopillar because typically the free layers of spin-injection devices need to be ultrathin for an optimized sensitivity to the ST torque, which is essentially an interface effect. The definition of the parameter ε contains P , which is the degree of spin polarization of the current and Λ , the parameter that describes the angular dependence of the ST torque.⁵⁷ The parameter ε' denotes the secondary spin transfer term, and $\mathbf{m}_p$ is the polarization direction of the electron spin polarization [$\mathbf{m}_p = (-1, 0, 0)$ in Fig. 1, after being reflected by the polarizer-spacer interface]. In our simulations, $\varepsilon' = 0$ and $\Lambda = 1$ as in previous papers.^{17,19,21,24-27,31,43} Nonuniform circular Oersted field created by the current (I) flowing perpendicularly to the free layer plane is given by (SI units)

$$H_{\varphi} = \frac{1}{2\pi} I \frac{r}{R^2}, \quad (\text{A2})$$

where r is the radial coordinate within the circular free layer of radius R , and φ is the azimuthal angle.

We obtained no vortex excitation when starting with the centered vortex structure [$|\mathbf{X}|=0$, see Eq. (1)], as a consequence of not taking into account temperature effects in the micromagnetic approach. At finite temperature, the vortex is always moving around the centered (equilibrium) position because of the thermal energy leading to finite amplitude of this low-frequency excitation mode. We overcame this handicap by displacing the vortex from the centered position and applying CPP to the off-centered vortex. An in-plane bias field H_y was introduced to displace the vortex core and it was removed when the current was applied.

At a first stage, we performed simulations with the damping parameter $\alpha=0.01$ as it was extracted from ferromagnetic resonance measurements⁵² for permalloy and widely used for micromagnetic calculations of the vortex state dynamics. Regarding the value of the degree of polarization of the incident current, the values from 0.17 to 0.7 were used in previous works, depending on the polarizer material. We finally chose $P=0.2$,²⁷ corresponding (0.2–0.3) to a permalloy polarizer.

We use the volume average of an in-plane magnetization component to describe the vortex core position. The averaged magnetization and core displacement are proportional to each other.^{10,58} Using Fast Fourier Transform (FFT) of the averaged reduced magnetization component in the OY direction $\langle m_y \rangle$, we obtained the oscillation frequencies for the system. The magnetization configuration was saved each $\Delta t=0.1$ ns. This corresponds to a maximum frequency that can be studied by the Fourier transform of the magnetization components of $1/(2\Delta t)=5$ GHz. Our simulations were conducted for 500 ns after the ST term was on. We have precision in frequency (Δf) for the FFT depending in this simulation time, T . For $T=500$ ns: $\Delta f=1/T=2$ MHz.

Here we discuss a clear example of what should or should not be considered as steady state: in Figs. 11(a) and 3, the conditions for vortex excitation are the same, the only difference is H_y value, 50 Oe for Fig. 3 and 10 Oe for Fig. 11(a). We could be tempted to consider the dynamics shown in Fig. 3 as steady state, but we observe in Fig. 11(a) that the final amplitude of steady excitations $\langle m_y(t) \rangle_{\max}$ is much lower (0.15 for Fig. 3. and 0.04 for Fig. 11), if the initial bias field and corresponding core position are smaller.

¹J.C. Slonczewski, *J. Magn. Magn. Mater.* **159**, L1 (1996).

²L. Berger, *Phys. Rev. B* **54**, 9353 (1996).

³S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, R. J. Schoelkopf, R. A. Buhrman, and D. C. Ralph, *Nature (London)* **425**, 380 (2003).

⁴A. Ruotolo, V. Cros, B. Georges, A. Dussaux, J. Grollier, C. Deranlot, R. Guillemet, K. Bouzehouane, S. Fusil, and A. Fert, *Nat. Nanotechnol.* **4**, 528 (2009).

⁵K. L. Metlov and K. Yu. Guslienko, *J. Magn. Magn. Mater.* **242–245**, 1015 (2002).

⁶P. O. Jubert and R. Allenspach, *Phys. Rev. B* **70**, 144402 (2004).

⁷W. Scholz, K. Yu. Guslienko, V. Novosad, D. Suess, T. Schrefl, R. W. Chantrell, and J. Fidler, *J. Magn. Magn. Mater.* **266**, 155 (2003).

⁸B. Krüger, A. Drews, M. Bolte, U. Merkt, D. Pfannkuche, and G. Meier, *Phys. Rev. B* **76**, 224426 (2007).

⁹K. S. Lee, S. Kim, Y. S. Yu, Y. S. Choi, K. Y. Guslienko, H. Jung, and P. Fischer, *Phys. Rev. Lett.* **101**, 267206 (2008); K. Y. Guslienko, K. S. Lee, and S. K. Kim, *ibid.* **100**, 027203 (2008).

¹⁰K. Y. Guslienko, B. A. Ivanov, V. Novosad, Y. Otani, H. Shima, and K. Fukamichi, *J. Appl. Phys.* **91**, 8037 (2002).

¹¹F. G. Aliev, J. F. Sierra, A. A. Awad, G. N. Kakazei, D. S. Han, S. K. Kim, V. Metlushko, B. Ilic, and K. Yu. Guslienko, *Phys. Rev. B* **79**, 174433 (2009).

¹²K. Y. Guslienko, W. Scholz, R. W. Chantrell, and V. Novosad, *Phys. Rev. B* **71**, 144407 (2005).

¹³B. Van Waeyenberge, A. Puzic, H. Stoll, K. W. Chou, T. Tylliszczak, R. Hertel, M. Fähnle, H. Brück, K. Rott, G. Reiss, I. Neudecker, D. Weiss, C. H. Back, and G. Schütz, *Nature (London)* **444**, 461 (2006).

¹⁴G. de Loubens, A. Riegler, B. Pigeau, F. Lochner, F. Boust, K. Y. Guslienko, H. Hurdequint, L. W. Molenkamp, G. Schmidt, A. N. Slavin, V. S. Tiberkevich, N. Vukadinovic, and O. Klein, *Phys. Rev. Lett.* **102**, 177602 (2009).

¹⁵K. Y. Guslienko, X. F. Han, D. J. Keavney, R. Divan, and S. D. Bader, *Phys. Rev. Lett.* **96**, 067205 (2006).

¹⁶S. Kasai, Y. Nakatani, K. Kobayashi, H. Kohno, and T. Ono, *Phys. Rev. Lett.* **97**, 107204 (2006).

¹⁷M. Bolte, G. Meier, B. Krüger, A. Drews, R. Eiselt, L. Bocklage, S. Bohlens, T. Tylliszczak, A. Vansteenkiste, B. Van Waeyenberge, K. W. Chou, A. Puzic, and H. Stoll, *Phys. Rev. Lett.* **100**, 176601 (2008).

¹⁸J. Shibata, Y. Nakatani, G. Tatara, H. Kohno, and Y. Otani, *Phys. Rev. B* **73**, 020403(R) (2006).

¹⁹A. Dussaux, B. Georges, J. Grollier, V. Cros, A. V. Khvalkovskiy, A. Fukushima, M. Konoto, H. Kubota, K. Yakushiji, S. Yuasa, K. A. Zvezdin, K. Ando, and A. Fert, *Nat. Commun.* **1**, 8 (2010).

²⁰D. D. Sheka, Y. Gaididei, and F. G. Mertens, *Appl. Phys. Lett.* **91**, 082509 (2007).

²¹Y. Liu, H. He, and Z. Zhang, *Appl. Phys. Lett.* **91**, 242501 (2007).

²²J. G. Caputo, Y. Gaididei, F. G. Mertens, and D. D. Sheka, *Phys. Rev. Lett.* **98**, 056604 (2007).

²³B. A. Ivanov and C. E. Zaspel, *Phys. Rev. Lett.* **99**, 247208 (2007).

²⁴W. Jin, H. He, Y. Chen, and Y. Liu, *J. Appl. Phys.* **105**, 013906 (2009).

²⁵Y. S. Choi, K. S. Lee, and S. K. Kim, *Phys. Rev. B* **79**, 184424 (2009).

²⁶Y. S. Choi, M. W. Yoo, K. S. Lee, Y. S. Yu, H. Jung, and S. K. Kim, *Appl. Phys. Lett.* **96**, 072507 (2010).

²⁷A. V. Khvalkovskiy, J. Grollier, A. Dussaux, K. A. Zvezdin, and V. Cros, *Phys. Rev. B* **80**, 140401 (2009).

²⁸K. Y. Guslienko, G. R. Aranda, and J. Gonzalez, preprint at <http://arXiv.org/abs/0912.5521> (unpublished).

²⁹V. S. Pribiag, I. N. Krivorotov, G. D. Fuchs, P. M. Braganca, O. Ozatay, J. C. Sankey, D. C. Ralph, and R. A. Buhrman, *Nat. Phys.* **3**, 498 (2007).

³⁰V. S. Pribiag, G. Finocchio, B. J. Williams, D. C. Ralph, and R. A. Buhrman, *Phys. Rev. B* **80**, 180411(R) (2009).

³¹A. V. Khvalkovskiy, J. Grollier, N. Locatelli, Ya. V. Gorbunov, K. A. Zvezdin, and V. Cros, *Appl. Phys. Lett.* **96**, 212507 (2010).

³²M. van Kampen, L. Lagae, G. Hrkac, T. Schrefl, J. V. Kim, T. Devolder, and C. Chappert, *J. Phys. D: Appl. Phys.* **42**, 245001 (2009).

³³Q. Mistral, M. van Kampen, G. Hrkac, J. V. Kim, T. Devolder, P. Crozat, C. Chappert, L. Lagae, and T. Schrefl, *Phys. Rev. Lett.* **100**, 257201 (2008).

³⁴T. Devolder, J.-V. Kim, M. Manfrini, G. Hrkac, P. Crozat, L. Lagae, T. Schrefl, and C. Chappert, *Proc. SPIE* **7398**, 739808 (2009).

³⁵W. H. Rippard, M. R. Pufall, S. Kaka, S. E. Russek, and T. J. Silva, *Phys. Rev. Lett.* **92**, 027201 (2004).

³⁶P. M. Braganca, I. N. Krivorotov, O. Ozatay, A. G. F. Garcia, N. C. Emley, J. C. Sankey, D. C. Ralph, and R. A. Buhrman, *Appl. Phys. Lett.* **87**, 112507 (2005).

³⁷I. N. Krivorotov, N. C. Emley, J. C. Sankey, S. I. Kiselev, D. C. Ralph, and R. A. Buhrman, *Science* **307**, 228 (2005).

³⁸N. C. Emley, I. N. Krivorotov, O. Ozatay, A. G. F. Garcia, J. C. Sankey, D. C. Ralph, and R. A. Buhrman, *Phys. Rev. Lett.* **96**, 247204 (2006).

³⁹D. Houssameddine, U. Ebels, B. Delaët, B. Rodmacq, I. Firastrau, F. Ponthenier, M. Brunet, C. Thirion, J. P. Michel, L. Prejbeanu-Buda, M. C. Cyrille, O. Redon, and B. Dieny, *Nature Mater.* **6**, 447 (2007).

⁴⁰M. D. Stiles and J. Miltat, in *Spin Dynamics in Confined Magnetic Structures III*, Topics in Applied Physics Vol. 101, edited by B. Hillebrands and A. Thiaville (Springer-Verlag, Berlin, 2006), p. 225.

⁴¹K. Yu Guslienko, *J. Nanosci. Nanotechnol.* **8**, 2745 (2008).

⁴²Exchange energy representations in computational micromagnetics (2002),

- M. J. Donahue and R. D. McMichael, <http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.6.6611>
- ⁴³K. J. Lee, A. Deac, O. Redon, J. P. Nozières, and B. Dieny, *Nature Mater.* **3**, 877 (2004).
- ⁴⁴E. Martínez, L. López-Díaz, L. Torres, and C. J. García-Cervera, *J. Phys. D: Appl. Phys.* **40**, 942 (2007).
- ⁴⁵X. Feng and P. B. Visscher, *J. Appl. Phys.* **89**, 6988 (2001).
- ⁴⁶U. Atxitia, O. Chubykalo-Fesenko, N. Kazantseva, D. Hinzke, U. Nowak, and R. W. Chantrell, *Appl. Phys. Lett.* **91**, 232507 (2007).
- ⁴⁷G. Grinstein and R. H. Koch, *Phys. Rev. Lett.* **90**, 207201 (2003).
- ⁴⁸M. Kirschner, T. Schrefl, F. Dorfbauer, G. Hrkac, D. Suess, and J. Fidler, *J. Appl. Phys.* **97**, 10E301 (2005).
- ⁴⁹M. Kirschner, T. Schrefl, G. Hrkac, F. Dorfbauer, D. Suess, and J. Fidler, *Physica B* **372**, 277 (2006).
- ⁵⁰M. Paz, *Structural Dynamics: Theory and Computation*, 3rd ed. (Van Nostrand Reinhold, New York, 1991), p. 26.
- ⁵¹K. Y. Guslienko, G. R. Aranda, and J. M. Gonzalez, *Phys. Rev. B* **81**, 014414 (2010).
- ⁵²S. Serrano-Guisan, K. Rott, G. Reiss, and H. W. Schumacher, *J. Phys. D: Appl. Phys.* **41**, 164015 (2008).
- ⁵³G. Finocchio, V. S. Pribiag, L. Torres, R. A. Buhrman, and B. Azzèrboni, *Appl. Phys. Lett.* **96**, 102508 (2010).
- ⁵⁴A. Thiaville, J. M. García, R. Dittrich, J. Miltat, and T. Schrefl, *Phys. Rev. B* **67**, 094410 (2003).
- ⁵⁵L. C. C. M. Nagamine, J. Geshev, T. Menegotto, A. A. R. Fernandes, A. Biondo, and E. B. Saitovitch, *J. Magn. Magn. Mater.* **288**, 205 (2005).
- ⁵⁶<http://math.nist.gov/oommf/doc/>
- ⁵⁷J. Xiao, A. Zangwill, and M. D. Stiles, *Phys. Rev. B* **70**, 172405 (2004).
- ⁵⁸K. Y. Guslienko, K. S. Buchanan, S. D. Bader, and V. Novosad, *Appl. Phys. Lett.* **86**, 223112 (2005).