Type-0 *T* duality and the tachyon coupling

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We consider the *T*-duality relation between type-0A and -0B theories, and show that this constrains the possible couplings of the tachyon to the Ramond-Ramond (RR) fields. Because of the "doubling" of the RR sector in type-0 theories, we are able to introduce a democratic formulation for the type-0 effective actions, in which there is no Chern-Simons term in the effective action. Finally, we discuss how to embed type-II solutions into type-0 theories.

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I. INTRODUCTION

Most ten-dimensional nonsupersymmetric superstring theories are plagued with tachyons, possibly endangering the consistency of the theory. This notwithstanding, they have become an active field of research mainly due to lessons learned from duality relations in the supersymmetric theories. One of the most studied examples, and the subject of this paper, is the so-called type-0 theories.

Type-0 theories can be obtained by a diagonal Ghozzi-Scherk-Olive (GSO) projection on the superstring spectrum or by orbifolding the corresponding type-II theories by $(-)^{F_s}$, the total target space fermion number [1]. Note that there are two type-IIB theories, denoted IIB_+ and IIB_-,[1] which are related by spacetime parity and lead to the same type-0B theory. From the supergravity point of view, IIB_+ differs from IIB_- in that IIB_+ has a self-dual five-form field strength whereas IIB_- has an anti-self-dual five-form field strength. Similar statements can be made for the type-$\mathrm{IIA}_\pm$ theories and their relation to the unique type-0A theory.

In the notation of [2], the spectra of the type-0 theories are represented as

$$(NS_-,NS_-)\oplus(NS_+,NS_+)\oplus(R_+,R_+)\oplus(R_-,R_-),\quad 0B,$$

$$(NS_-,NS_-)\oplus(NS_+,NS_+)\oplus(R_+,R_-)\oplus(R_-,R_+),\quad 0A, \tag{1}$$

which then consist of a tachyon, the string common sector, and a doubling, with respect to the analogous type-II theory of the Ramond-Ramond (RR) fields. Since there is a doubling of the RR fields, there is also a doubling of the D-brane content in the type-0 theories.

In Ref. [3] the lowest order field contributions to the type-0B string effective action were calculated from the appropriate string scattering amplitudes and, in our convention [4], and using the diagonal basis of RR fields $\{\hat{C}^+_{(p+1)},\hat{C}^+_{(4)},\hat{C}^-_{(p+1)}\}_{p=-1,1}$, the action reads

$$\hat{S}=\int d^{10}\hat{x}\sqrt{|\hat{g}|}\Bigg\{e^{-2\hat{\varphi}}\Bigg[\hat{R}-4(\partial\hat{\varphi})^2+\frac{1}{2\times 3!}\hat{\mathcal{H}}^2+\frac{1}{2}(\partial\hat{\mathcal{T}})^2-V(\hat{\mathcal{T}})\Bigg]+f_+(\hat{\mathcal{T}})\Bigg[\frac{1}{2}(\hat{G}^+_{(1)})^2+\frac{1}{2\times 3!}(\hat{G}^+_{(3)})^2+\frac{1}{2\times 5!}(\hat{G}^+_{(5)})^2\Bigg]+f_-(\hat{\mathcal{T}})\Bigg[\frac{1}{2}(\hat{G}^-_{(1)})^2+\frac{1}{2\times 3!}(\hat{G}^-_{(3)})^2\Bigg]\Bigg\}, \tag{2}$$

where the field strengths are defined by

$$\hat{\mathcal{H}}=d\hat{\mathcal{B}},\quad \hat{G}^\pm_{(n)}=d\hat{C}^\pm_{(n-1)}, \tag{3}$$

and the tachyon potential is

$$V(\hat{\mathcal{T}})=\frac{1}{2}m^2\hat{\mathcal{T}}^2-4c_1\hat{\mathcal{T}}^4,\quad m^2=-2/l_s^2, \tag{4}$$

where $l_s=\sqrt{\alpha'}$ is the string length. It was later argued in Ref. [5] that there should be no such potential in type-0 superstring effective actions. However, since the tachyon is inert under *T* duality, the value of the tachyon potential *V* will be immaterial in most of our discussion.

Finally, the functions $f_\pm(\hat{\mathcal{T}})$ are given by

$$f_\pm(\hat{\mathcal{T}})=1\pm\sqrt{2}\hat{\mathcal{T}}+\hat{\mathcal{T}}^2+O(\hat{\mathcal{T}}^4). \tag{5}$$

The RR fields are combinations of the (R_+,R_+) and (R_-,R_-) fields, denoted by C and respectively, $\overline{C}$, that diagonalize the kinetic terms:

$$\sqrt{2}\hat{C}^\pm_{(2n)}=\hat{C}_{(2n)}\pm\hat{\overline{C}}_{(2n)}, \tag{6}$$

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[1]These are denoted IIB and IIB′ respectively, in [2].

and we will denote a brane charged with respect to $C^{+}_{(p+1)}$ ($C^{-}_{(p+1)}$) as a Dp_{+}-brane (Dp_{-}-brane).

The fields $\hat{C}_{(4)}$ and $\overleftarrow{C}_{(4)}$ have self- and anti-self-dual field strengths and deserve further discussion. In principle, as in the type-IIB$_{\pm}$ cases, it is not possible to write a kinetic term for either of them separately without the help of auxiliary fields or without breaking covariance. Combining them, however, it is possible to write a kinetic term of the form $\hat{G}_{(5)}\overleftarrow{G}_{(5)}$. From this term one recovers a standard-looking equation of motion, but not self- or anti-self-duality which still has to be imposed by hand. It is easy to convince oneself that this cannot be done consistently in the presence of coupling to the tachyon: the equation of motion

$$d(f^{*}_{+}\hat{G}_{(5)})=0, \tag{7}$$

should give the Bianchi identity under the duality transformation, which has to be

$$f^{*}_{+}\hat{G}_{(5)}=\hat{G}_{(5)}, \tag{8}$$

which is clearly inconsistent for nonconstant f_{+}.

Another possibility is to combine both of them into a completely unconstrained five-form field strength $\hat{G}^{+}_{(5)}$ with standard kinetic term, as was done here in the action Eq. (2). All we require is that it leads to the right equations of motion associated with the propagators that can be calculated from string amplitudes. Defining

$$\hat{G}^{-}_{(5)}\equiv f^{*}_{+}\hat{G}^{+}_{(5)}, \tag{9}$$

we can immediately find an alternative to the action Eq. (2) in which all the kinetic terms of the fields with a minus superscript have a factor $f_{-}(\hat{\mathcal{T}})$ except for $\hat{G}^{-}_{(5)}$ which carries a factor $f^{-1}_{+}(\hat{\mathcal{T}})$.

Yet another possibility, which we will use later on, is to write an almost standard kinetic terms for $\hat{C}^{+}_{(4)}$ and $\hat{C}^{-}_{(4)}$ with the understanding that self- and anti-self-duality have to be imposed on the subsequent equations of motion. In this case, the kinetic term would be

$$\int d^{10}\hat{x}\sqrt{|\hat{j}|}\left\{f_{+}\frac{1}{4\times 5!}(\hat{G}^{+}_{(5)})^{2}+f^{-1}_{+}\frac{1}{4\times 5!}(\hat{G}^{-}_{(5)})^{2}\right\}, \tag{10}$$

and we would impose Eq. (9) as a constraint. This *non-self-dual* (NSD) action would be a generalization of the type-IIB one [6,4]. Eliminating the $\hat{G}^{-}_{(5)}$ combination with the above constraint would take us to Eq. (2). Eliminating $\hat{G}^{+}_{(5)}$ would give us the alternative action in terms of $\hat{G}^{-}_{(5)}$.

A further remark must be made: the type-0 theories have to be invariant under a $\mathbb{Z}_2$ group associated with $(-)^{f_L}$, the world sheet fermion number. This implies that the effective action should be invariant under the transformation $\hat{\mathcal{T}}\rightarrow-\hat{\mathcal{T}}$ combined with an interchange of the + and − fields. A quick look at Eq. (2) then reveals that this can be true only if

$$f_{+}(\hat{\mathcal{T}})=f_{-}(-\hat{\mathcal{T}}). \tag{11}$$

Furthermore, since in the interchange of the + and − fields $\hat{G}^{+}_{(5)}$ is transformed into $\hat{G}^{-}_{(5)}$, it is clear that the action is not strictly invariant under this transformation. Actually, it takes us to the alternative action[2] but with the five-form kinetic term carrying a tachyon factor $f_{+}(-\hat{\mathcal{T}})$ instead of $f^{-1}_{+}(\hat{\mathcal{T}})$. This implies that

$$f_{+}(-\hat{\mathcal{T}})=f^{-1}_{+}(\hat{\mathcal{T}})=f_{-}(\hat{\mathcal{T}}). \tag{12}$$

These constraints, which will also coincide with the constraints coming from T duality, determine to some extent the form of the functions f, as we will discuss later.

Because of similarity of type 0B to type IIB we expect more terms in the RR field strengths and a Chern-Simons term in the action. These terms could in principle be determined from more complicated string amplitudes, but we are going to try to determine as many as we can of these additional terms by imposing T duality between the type-0B and type-0A string effective actions using dimensional reduction as in Refs. [7,8,4]. This is our main goal. The type-0A string effective action has not been calculated from first principles as yet. However, it is clear that the tachyon-independent part of the Neveu-Schwarz–Neveu-Schwarz (NSNS) sector effective action (the so-called common sector) is identical to the type-0B one. Furthermore, it is also clear that T duality acts on this sector according to the usual Buscher rules[3] [9] which also implies that the tachyon is invariant under T duality. We are going to show that these facts, plus our knowledge of the field content of the type-0A theory and its T duality relation to the type-0B theory, are enough to deter-

[2]We could also say that the transformation has to be supplemented by a dualization of $\hat{G}^{-}_{(5)}$ to be a symmetry of the action.

[3]In closed string theory, T duality is always a symmetry that interchanges momentum and winding modes associated with a given compact direction whose radius is simultaneously inverted. It is worth remarking that the string effective action does not contain any field associated with these modes (they are massive). One could take into account massive Kaluza-Klein (momentum) modes arising in the compactification of the massless fields contained in the effective action, but there is no known way to take into account winding modes, which are stringy (not field theoretical) objects. Given this fact, one may wonder how, if at all, the string effective action can give a description of T duality. The answer lies in the observation that all Kaluza-Klein modes are charged with respect to the massless Kaluza-Klein vector coming from the metric, while all winding modes are charged with respect to the *winding* vector coming from the Kalb-Ramond two-form. The interchange of momentum and winding modes implies the interchange of the Kaluza-Klein and winding vectors of the string effective action. Furthermore, the inversion of the radius is expressed in the effective action as the inversion of the Kaluza-Klein scalar that measures that radius. The transformation of the remaining massless fields follows from these and from covariance. This is the content of the Buscher rules and this is why they have to take the same form in the NSNS sector of any closed string theory effective action.

mine the effective action of both the type-0A and type-0B theories to identical orders in the fields.

In the next section we are first going to reduce the type-0B action Eq. (2) to nine dimensions. From the form of the nine-dimensional action plus the T duality invariance of the tachyon field, we will immediately be able to derive an effective action for the type-0A theory, including tachyon couplings about which we will obtain more information. Next, we will notice that we need additional terms in the RR field strengths to establish T duality with the type-0A effective action. This will follow from our knowledge of Buscher's rules in the NSNS sector. The introduction of the new terms in the RR field strengths will also force us to introduce a Chern-Simons term.

II. THE TYPE-0A ACTION AND T DUALITY

As was said above, the type-0A action has not been calculated from first principles, although such an action was proposed in Ref. [10]. In this section we will use T duality as a guideline for the construction of the type-0A effective action. We will leave the construction of the massive theory for Sec. III.

A. Reduction to $d=9$ of the type-0B effective action

Our Kaluza-Klein ansatz to reduce the the type-0B action Eq. (2) in the direction of the coordinate $y=x^9$ will be similar to the one used in establishing type-IIA–type-IIB T duality in Ref. [4] (identical in the NSNS sector, actually). The relation between the ten-dimensional fields

$$\{\hat{J},\hat{\mathcal{B}},\hat{\varphi},\hat{\mathcal{T}},\hat{C}^{+}_{(0)},\hat{C}^{-}_{(0)},\hat{C}^{+}_{(2)},\hat{C}^{-}_{(2)},\hat{C}^{+}_{(4)}\} \tag{13}$$

and the nine-dimensional fields

$$\{g,B,A^{(1)},A^{(2)},k,\phi,T,C^{+}_{(0)},C^{+}_{(1)},C^{+}_{(2)},C^{+}_{(3)},\\ C^{+}_{(4)},C^{-}_{(0)},C^{-}_{(1)},C^{-}_{(2)},\} \tag{14}$$

is, in the NSNS sector,

$$\hat{J}_{\mu y}=g_{\mu\nu}-k^{-2}A^{(2)}_{\mu}A^{(2)}_{\nu},\quad \hat{\mathcal{B}}_{\mu\nu}=B_{\mu\nu}+A^{(1)}_{[\mu}A^{(2)}_{\nu]},$$
$$\hat{J}_{\mu\underline{y}}=-k^{-2}A^{(2)}_{\mu},\quad \hat{\mathcal{B}}_{\mu\underline{y}}=A^{(1)}_{\mu},$$
$$\hat{J}_{\underline{yy}}=-k^{-2},\quad \hat{\varphi}=\phi-\frac{1}{2}\log k,$$
$$\hat{\mathcal{T}}=T, \tag{15}$$

and, in the RR sector,

$$\hat{C}^{\pm}_{(2n)\mu_1\cdots\mu_{2n}}=C^{\pm}_{(2n)\mu_1\cdots\mu_{2n}}-2nA^{(2)}_{[\mu_1}C^{\pm}_{(2n-1)\mu_2\cdots\mu_{2n}]},$$
$$\hat{C}^{\pm}_{(2n)\mu_1\cdots\mu_{2n-1}\underline{y}}=-C^{\pm}_{(2n-1)\mu_1\cdots\mu_{2n-1}}. \tag{16}$$

The field strengths are related, in flat indices, by

$$\hat{\mathcal{H}}_{abc}=H_{abc},$$
$$\hat{\mathcal{H}}_{aby}=kF^{(1)}_{ab}, \tag{17}$$

where

$$H=dB-\frac{1}{2}A^{(1)}F^{(2)}-\frac{1}{2}A^{(2)}F^{(1)},$$
$$F^{(1,2)}=dA^{(1,2)}, \tag{18}$$

in the NSNS sector and by

$$\hat{G}^{\pm}_{(2n+1)a_1\cdots a_{2n+1}}=G^{\pm}_{(2n+1)a_1\cdots a_{2n+1}},$$
$$\hat{G}^{\pm}_{(2n+1)a_1\cdots a_{2n}y}=-kG^{\pm}_{(2n)a_1\cdots a_{2n}}, \tag{19}$$

where

$$G^{\pm}_{(2n+1)}=dC^{\pm}_{(2n)}+F^{(2)}C^{\pm}_{(2n-1)},$$
$$G^{\pm}_{(2n)}=dC^{\pm}_{(2n-1)}, \tag{20}$$

in the RR sector. The reduced action is

$$S=\int d^9x\sqrt{|g|}\Bigg\{e^{-2\phi}\Bigg[R-4(\partial\phi)^2+\frac{1}{2\times 3!}H^2+(\partial\log k)^2 -\frac{1}{4}k^2(F^{(1)})^2-\frac{1}{4}k^{-2}(F^{(2)})^2+\frac{1}{2}(\partial T)^2-V(T)\Bigg]+f_{+}(T)\\ \times\Bigg[\frac{1}{2}k^{-1}(G^{+}_{(1)})^2-\frac{1}{4}k(G^{+}_{(2)})^2+\frac{1}{2\times 3!}k^{-1}(G^{+}_{(3)})^2 -\frac{1}{2\times 4!}k(G^{+}_{(4)})^2+\frac{1}{2\times 5!}k^{-1}(G^{+}_{(5)})^2\Bigg]+f_{-}(T)\\ \times\Bigg[\frac{1}{2}k^{-1}(G^{-}_{(1)})^2-\frac{1}{4}k(G^{-}_{(2)})^2+\frac{1}{2\times 3!}k^{-1}(G^{-}_{(3)})^2\Bigg]\Bigg\}. \tag{21}$$

B. The type-0A effective action and its reduction to $d=9$

We should compare the above action with the dimensionally reduced type-0A effective action which we do not know in detail. Let us summarize our knowledge of this action: first of all, it contains the same ten-dimensional NSNS fields as the type-0B action and all of them (except, possibly, the tachyon) appear in it in identical fashion. This implies that the T duality rules in this sector will be Buscher's and also implies that the tachyon will appear also in the same form and will be invariant under T duality (its reduction is trivial).

As for the RR fields, the type-0A string effective action contains two one-forms and two three-forms[4] $\hat{C}_{(1)},\hat{C}_{(3)},\overleftarrow{\hat{C}}_{(1)}$, and $\overleftarrow{\hat{C}}_{(3)}$ that may couple to the tachyon as in the type-0B case. Whatever the couplings to the tachyon are, we can always diagonalize the kinetic terms. We denote

[4]There must be a massive extension of the type-0A theory, with two constant field strengths $\hat{G}_{(0)}$ and $\overleftarrow{\hat{G}}_{(0)}$. We will consider it later.

the potentials in the diagonal basis by $\hat{C}^{+}_{(1)},\hat{C}^{+}_{(3)},\hat{C}^{-}_{(1)}$, and $\hat{C}^{-}_{(3)}$ but we will not make any assumption about the relation to the original potentials. It is now evident that the fields with index $+$ $(-)$ will couple to the tachyon through $f_{+}(\hat{T})$ $[f_{-}(\hat{T})]$, since otherwise it would be impossible to get the reduced action Eq. (21).

Thus, to the order considered, the type-0A string effective action must be of the form

$$\hat{S}_{0\mathrm{A}}=\int d^{10}\hat{x}\sqrt{|\hat{g}|}\Bigg\{e^{-2\hat{\phi}}\Bigg[\hat{R}-4(\partial\hat{\phi})^2+\frac{1}{2\times 3!}\hat{H}^2+\frac{1}{2}(\partial\hat{T})^2-V(\hat{T})\Bigg]+\sum_{\alpha=+,-}f_{\alpha}(\hat{T})\Bigg[-\frac{1}{4}(\hat{G}^{\alpha}_{(2)})^2-\frac{1}{2\times 4!}(\hat{G}^{\alpha}_{(4)})^2\Bigg]\Bigg\}, \tag{22}$$

where the field strengths are defined as in Eq. (3) and the tachyon potential and coupling functions are identical to those of the type-0B theory.

Let us now reduce this action to nine-dimensions in the direction of the coordinate x. The relation between the ten-dimensional fields

$$\{\hat{g},\hat{B},\hat{\phi},\hat{T},\hat{C}^{+}_{(1)},\hat{C}^{+}_{(3)},\hat{C}^{-}_{(1)},\hat{C}^{-}_{(3)}\} \tag{23}$$

and the nine-dimensional fields

$$\{g,B,A^{(1)},A^{(2)},k,\phi,T,C^{+}_{(0)},C^{+}_{(1)},C^{+}_{(2)},C^{+}_{(3)},C^{-}_{(0)},C^{-}_{(1)},C^{-}_{(2)},C^{-}_{(3)}\} \tag{24}$$

is, in the NSNS sector[5]

$$\hat{g}_{\mu\nu}=g_{\mu\nu}-k^2A^{(1)}_{\mu}A^{(1)}_{\nu},\quad \hat{B}_{\mu\nu}=B_{\mu\nu}-A^{(1)}_{[\mu}A^{(2)}_{\nu]},$$
$$\hat{g}_{\mu\underline{x}}=-k^2A^{(1)}_{\mu},\quad \hat{B}_{\mu\underline{x}}=A^{(2)}_{\mu},$$
$$\hat{g}_{\underline{xx}}=-k^2,\quad \hat{\phi}=\phi+\frac{1}{2}\log k,$$
$$\hat{T}=T. \tag{25}$$

This will give a nine-dimensional NSNS sector identical to that of the action Eq. (21). The only possible relation between the ten- and nine-dimensional RR fields is

$$\hat{C}^{\pm}_{(2n-1)\mu_1\cdots\mu_{2n-1}}=C^{\pm}_{(2n-1)\mu_1\cdots\mu_{2n-1}}+(2n-1)\times A^{(1)}_{[\mu_1}C^{\pm}_{(2n-2)\mu_2\cdots\mu_{2n-1}]},$$
$$\hat{C}^{\pm}_{(2n-1)\mu_1\cdots\mu_{2n-2}\underline{x}}=C^{\pm}_{(2n-2)\mu_1\cdots\mu_{2n-2}}, \tag{26}$$

and we remark that it involves $A^{(1)}$ and not $A^{(2)}$, as in the type-0B case. We cannot change this without spoiling T duality in the NSNS sector.

The field strengths are related in flat indices by

$$\hat{H}_{abc}=H_{abc},$$
$$\hat{H}_{abx}=k^{-1}F^{(2)}_{ab} \tag{27}$$

in the NSNS sector where the nine-dimensional field strengths are also given by Eq. (18).

The RR field strengths are related by

$$\hat{G}^{\pm}_{(2n)a_1\cdots a_{2n}}=G^{\pm}_{(2n)a_1\cdots a_{2n}},$$
$$\hat{G}^{\pm}_{(2n)a_1\cdots a_{2n-1}x}=k^{-1}G^{\pm}_{(2n-1)a_1\cdots a_{2n-1}}, \tag{28}$$

where the nine-dimensional ones are defined as follows:

$$G^{\pm}_{(2n+1)}=dC^{\pm}_{(2n)},$$
$$G^{\pm}_{(2n)}=dC^{\pm}_{(2n-1)}+F^{(1)}C^{\pm}_{(2n-2)}. \tag{29}$$

The even ones involve $F^{(1)}$ while in type 0B the odd ones involve $F^{(2)}$.

Summarizing, we have obtained the action

$$S=\int d^9x\sqrt{|g|}\Bigg\{e^{-2\phi}\Bigg[R-4(\partial\phi)^2+\frac{1}{2\times 3!}H^2+(\partial\log k)^2-\frac{1}{4}k^2(F^{(1)})^2-\frac{1}{4}k^{-2}(F^{(2)})^2+\frac{1}{2}(\partial T)^2-V(T)\Bigg]+\sum_{\alpha=+,-}f_{\alpha}(T)\Bigg[\frac{1}{2}k^{-1}(G^{\alpha}_{(1)})^2-\frac{1}{4}k(G^{\alpha}_{(2)})^2+\frac{1}{2\times 3!}k^{-1}(G^{\alpha}_{(3)})^2-\frac{1}{2\times 4!}k(G^{\alpha}_{(4)})^2\Bigg]\Bigg\}. \tag{30}$$

This action is different from the one we obtained from the type-0B theory, Eq. (21), in two points: the definition of the nine-dimensional field strengths involves only one of the two nine-dimensional vectors, the Kaluza-Klein one. Since they are interchanged by T duality, we need both vectors to appear in the field strengths. On the other hand, in the type-0B case we have obtained one RR field strength which is not present in the reduced type-0A theory, $G^{+}_{(5)}$, and in the type-0A case we have obtained another RR field strength absent in the reduced type-0B action, $G^{-}_{(4)}$.

The first problem can be solved only by making the winding vector appear in the reduced RR field strengths, which implies that $\hat{B}$ must appear in the ten-dimensional RR field strengths. Up to possible field redefinitions, there is only one way of doing this: precisely defining the RR field strengths as in the type-II theories, i.e.,

$$\hat{G}^{\pm}_{(n)}=d\hat{C}^{\pm}_{(n-1)}-\hat{\mathcal{H}}\hat{C}^{\pm}_{(n-3)}, \tag{31}$$

[5]We use the T-dual Kaluza-Klein ansatz. This ensures that the resulting nine-dimensional actions are the same instead of being related by $k\to k^{-1}$ and $A^{(1)}\leftrightarrow A^{(2)}$.

in both the type-0B and -0A theories. This is consistent with the fact that the amplitudes involving two RR fields of the same sector and a NSNS field (different for the tachyon) are identical to those of the type-IIB$_\pm$ theories. The only difference might be the sign of the second term. We can set it to minus, as above, for the type-0B $\hat{G}^+_{(2n+1)}$ field strengths by fixing the relative sign between $\hat{\mathcal{B}}$ and the $\hat{C}^+_{(2n)}$ potentials. In principle, the sign of the second term in the type-0B field strengths $\hat{G}^-_{(2n+1)}$ could still be arbitrarily chosen by changing the sign of all the RR potentials $\hat{C}^-_{2n}$ because the $+$ and $-$ RR potentials are decoupled in the action Eq. (21). However, as we are going to argue next, we are going to have to introduce a Chern-Simons term that may couple them and we have to be open to the two possible signs.

The second problem can only be solved by Hodge-dualizing $G^+_{(5)}$ and identifying the dual field with $-G^-_{(4)}$. This is somewhat reminiscent of the procedure followed in type-II theories [4]. For this dualization to give the right form of $G^-_{(4)}$ it will be necessary to add to the ten-dimensional type-0B action a Chern-Simons term and this will force us to introduce another one in the ten-dimensional type-0A action. A subtle point arises here: when one dualizes a field strength whose kinetic term comes multiplied by a function, the kinetic term of the dual field comes multiplied by the *inverse* function. Thus, the $G^-_{(4)}$ kinetic term will carry an $f_+^{-1}(T)k$ factor. We expected the k factor, but we also expected an $f_-(T)$ factor. To establish T duality, then, we must have $f_+^{-1}=f_-$, which together with Eq. (11) implies that

$$f_\pm(\hat{\mathcal{T}})=\exp[\pm h(\hat{\mathcal{T}})], \tag{32}$$

where h is an odd function of $\hat{\mathcal{T}}$. Note that this result was anticipated in Ref. [11] by means of tadpole considerations; here it arises as a necessity for T duality to work.

The need to introduce a ten-dimensional Chern-Simons term can also be seen directly in ten-dimensions starting with the NSD action with the kinetic terms Eq. (10). It is instructive to derive the Chern-Simons term using an argument different from T duality. Let us for the moment set the tachyon field to zero, in order to simplify the calculations (in any case, it does not play any role in the determination of the Chern-Simons term). The kinetic terms are just

$$\int d^{10}\hat{x}\sqrt{|\hat{j}|}\left[\frac{1}{2\times 5!}(\hat{G}_{(5)})^2+\frac{1}{2\times 5!}(\hat{\tilde{G}}_{(5)})^2\right]. \tag{33}$$

The Chern-Simons term has to be such that the self-duality of $\hat{G}_{(5)}$ and the anti-self-duality of $\hat{\tilde{G}}_{(5)}$ can be consistently imposed, i.e., such that the equations of motion are identical to the Bianchi identities,

$$d(\hat{G}_{(5)}+\hat{H}\hat{C}_{(2)})=0,$$

$$d(\hat{\tilde{G}}_{(5)}+\hat{H}\hat{\tilde{C}}_{(2)})=0, \tag{34}$$

using the (anti-)self-duality constraints. The Chern-Simons term is, therefore, given by the addition of the type-IIB$_+$ and type-IIB$_-$ Chern-Simons terms, namely,

$$\int d^{10}\hat{x}\sqrt{|\hat{j}|}\left\{\frac{1}{2\times 5!}(\hat{G}_{(5)})^2+\frac{1}{2\times 5!}(\hat{\tilde{G}}_{(5)})^2 + \frac{10}{(5!)^2}\hat{\epsilon}[\hat{G}_{(5)}\hat{\mathcal{H}}\hat{C}_{(2)}-\hat{\tilde{G}}_{(5)}\hat{\mathcal{H}}\hat{\tilde{C}}_{(2)}]\right\}, \tag{35}$$

which, written in terms of the diagonal fields, is

$$\int d^{10}\hat{x}\sqrt{|\hat{j}|}\left\{\frac{1}{4\times 5!}(\hat{G}^+_{(5)})^2+\frac{1}{4\times 5!}(\hat{G}^-_{(5)})^2 + \frac{1}{4!\times 5!}\hat{\epsilon}[\hat{G}^+_{(5)}\hat{\mathcal{H}}\hat{C}^-_{(2)}+\hat{G}^-_{(5)}\hat{\mathcal{H}}\hat{C}^+_{(2)}]\right\}. \tag{36}$$

We can now Poincaré-dualize $\hat{G}^-_{(5)}$, adding to the above action a Lagrange-multiplier term to enforce its Bianchi identity:

$$\int d^{10}\hat{x}\sqrt{|\hat{j}|}\,\frac{1}{4!\times 5!}\hat{\epsilon}\partial\tilde{\hat{C}}^-_{(4)}(\hat{G}_{(5)}+10\hat{H}\hat{C}_{(2)}); \tag{37}$$

then, solving for $\hat{G}^-_{(5)}$,

$$\hat{G}^-_{(5)}={}^*\tilde{\hat{G}}^-_{(5)}, \quad \tilde{\hat{G}}^-_{(5)}=5\partial\tilde{\hat{C}}^-_{(4)}-10\hat{\mathcal{H}}\hat{C}^-_{(2)}, \tag{38}$$

and substituting this solution into the action and, finally, identifying $\tilde{\hat{G}}^-_{(5)}=\hat{G}^+_{(5)}$, we find

$$\int d^{10}\hat{x}\sqrt{|\hat{j}|}\left[\frac{1}{2\times 5!}(\hat{G}^+_{(5)})^2-\frac{1}{4\times 5!}\frac{\hat{\epsilon}}{\sqrt{|\hat{j}|}}\hat{G}^+_{(5)}\partial\hat{C}^-_{(2)}\mathcal{B}\right], \tag{39}$$

which contains the actual kinetic term that we have and the Chern-Simons term that we should expect.

C. Corrected ten-dimensional type-0A/B effective actions and T-duality rules

It is quite straightforward to carry on with the program. First, we consider again the action Eq. (2) but with RR field strengths given by Eq. (31) in both $+$ and $-$ sectors and repeat the dimensional reduction. The Kaluza-Klein ansatz is the same for all the fields, and the ten-dimensional field strengths decompose to nine-dimensional field strengths in the same form, and so we get an action of the form Eq. (21), but with lower dimensional RR field strengths defined by

$$\begin{aligned} G^\pm_{(2n+1)}&=dC^\pm_{(2n)}-HC^\pm_{(2n-2)}+F^{(2)}C^\pm_{(2n-1)},\\ G^\pm_{(2n)}&=dC^\pm_{(2n-1)}-HC^\pm_{(2n-3)}+F^{(1)}C^\pm_{(2n-2)}. \end{aligned} \tag{40}$$

Now, both the Kaluza-Klein and winding vector fields are present in the nine-dimensional RR field strengths.

The next step is to Poincaré-dualize $G^+_{(5)}$ into $G^-_{(4)}$: we add to the nine-dimensional action a Lagrange-multiplier term to enforce the Bianchi identity:

$$d[G^+_{(5)}+HC^+_{(2)}-F^{(2)}C^+_{(3)}]=0. \tag{41}$$

The Lagrange multiplier has to be a three-form that will become the dual potential $C^-_{(3)}$. Thus, the Lagrange multiplier term will take the form

$$\alpha\int d^9x\,\epsilon\partial C^-_{(3)}[G^+_{(5)}+10HC^+_{(2)}-10F^{(2)}C^+_{(3)}], \tag{42}$$

where α is a constant whose value has to be chosen so as to get the right normalization for the kinetic term of $C^-_{(3)}$. In the action Eq. (21) with the above Lagrange-multiplier term, $C^+_{(4)}$ appears only through $G^+_{(5)}$. We can consider it as a functional of $G^+_{(5)}$ since we can always recover the expression of $G^+_{(5)}$ in terms of $C^+_{(4)}$ through the equation of motion of the Lagrange multiplier. Now, the $G^+_{(5)}$ equation of motion is

$$G^+_{(5)}=-\alpha f_+^{-1}(T)k\frac{\epsilon}{\sqrt{|g|}}\partial C^-_{(3)}. \tag{43}$$

We expected

$$G^+_{(5)}=-\sqrt{f_-/f_+}k^*G^-_{(4)}, \tag{44}$$

with

$$G^-_{(4)}=dC^-_{(3)}-HC^-_{(1)}+F^{(1)}C^-_{(2)}. \tag{45}$$

This fixes the normalization constant $\alpha=+1/3!\times5!$, implies $f_+^{-1}(T)=f_-(T)$, and also tells us that there should be a nine-dimensional Chern-Simons term in the nine-dimensional action Eq. (21) of the form

$$-\frac{1}{2\times3!\times5!}\int d^9x\,\epsilon G^+_{(5)}[2HC^-_{(1)}-3F^{(1)}C^-_{(2)}], \tag{46}$$

to get Eq. (44). This term can come only from the ten-dimensional Chern-Simons term in Eq. (39) that we can also write, up to total derivatives, in the form

$$-\frac{1}{96}\int d^{10}\hat{x}\,\hat{\epsilon}\partial\hat{C}^+_{(4)}\partial\hat{C}^-_{(2)}\hat{\mathcal{B}}, \tag{47}$$

which gives rise to the term we wanted and another term not involving $C^+_{(4)}$ in any way:

$$-\frac{1}{2\times3!\times5!}\int d^9x\,\epsilon[G^+_{(5)}(2HC^-_{(1)}-3F^{(1)}C^-_{(2)})-5G^+_{(4)}HC^-_{(2)}]. \tag{48}$$

Observe that the Chern-Simons term Eqs. (39), (39) is very similar to the Chern-Simons term in the type-IIB NSD string effective action [6,4]. Here, however, it mixes nontrivially the two RR sectors.

Adding to this nine-dimensional Chern-Simons term the Lagrange-multiplier term Eq. (42) with the value of α that we have calculated, we find the equation of motion Eq. (44), and, using it to eliminate $G^+_{(5)}$, we finally get the nine-dimensional type-0 string effective action

$$\begin{aligned}S=\int d^9x\sqrt{|g|}\Bigg(&e^{-2\phi}\left[R-4(\partial\phi)^2+\frac{1}{2\times3!}H^2+(\partial\log k)^2-\frac{1}{4}k^2(F^{(1)})^2-\frac{1}{4}k^{-2}(F^{(2)})^2+\frac{1}{2}(\partial T)^2-V(T)\right]\\&+\sum_{\alpha=+,-}f_\alpha(T)\left[\frac{1}{2}k^{-1}(G^\alpha_{(1)})^2-\frac{1}{4}k(G^\alpha_{(2)})^2+\frac{1}{2\times3!}k^{-1}(G^\alpha_{(3)})^2-\frac{1}{2\times4!}k(G^\alpha_{(4)})^2\right]\\&-\frac{1}{36}\frac{\epsilon}{\sqrt{|g|}}\Bigg\{\partial C^+_{(3)}\partial C^-_{(3)}A^{(2)}-\frac{9}{2}C^+_{(2)}C^-_{(2)}\partial A^{(1)}(\partial B-A^{(1)}\partial A^{(2)}-A^{(2)}\partial A^{(1)})\\&+\frac{3}{2}[\partial C^+_{(3)}\partial C^-_{(2)}(B+A^{(1)}A^{(2)})+2\partial C^+_{(3)}C^-_{(2)}A^{(2)}\partial A^{(1)}A^{(2)}+\partial C^-_{(3)}\partial C^+_{(2)}(B+A^{(1)}A^{(2)})+2\partial C^-_{(3)}C^+_{(2)}A^{(2)}\partial A^{(1)}A^{(2)}]\Bigg\}\Bigg).\end{aligned} \tag{49}$$

In order to establish T duality then, we have to find a ten-dimensional Chern-Simons term to add to the type-0A string effective action Eq. (30), leading to the above nine-dimensional type-0 string effective action using the same ansatz as before. This is a very nontrivial check of our construction. It takes little time to see that the sought for Chern-Simons term is

$$-\frac{1}{72}\int d^{10}\hat{x}\,\hat{\epsilon}\partial\hat{C}^+_{(3)}\partial\hat{C}^-_{(3)}\hat{B}. \tag{50}$$

Again, this Chern-Simons term looks very similar to the one in the type-IIA string effective action. In fact, we could rewrite it in the form

$$-\frac{1}{144}\int d^{10}\hat{x}\,\hat{\epsilon}[\partial\hat{C}_{(3)}\partial\hat{C}_{(3)}\hat{B}-\partial\hat{\tilde{C}}_{(3)}\partial\hat{\tilde{C}}_{(3)}\hat{B}], \quad (51)$$

which would be the sum of the Chern-Simons terms of the type-IIA$_+$ and type-IIA$_-$ theories (which are related by target space parity).

The resulting type-0A string effective action [Eq. (22) plus the Chern-Simons term Eq. (50)] is left-right invariant (i.e., invariant under the interchange of the two RR sectors $\hat{C}^{\pm}\to\pm\hat{C}^{\pm}$ and sign reversal of the Kalb-Ramond form $\hat{B}\to-\hat{B}$), as it should be. In the same way, the complete type-0B action [Eq. (2) plus Eq. (47)] is invariant under the transformation that changes the sign of the tachyon and interchanges the + and − RR field strengths if we dualize $\hat{G}^{-}_{(5)}$ into $\hat{G}^{+}_{(5)}$.

Please note that the Chern-Simons terms (47) and (50) or (51) differ from the ones proposed in Ref. [10].

We have just established T duality between the type-0A and -0B string effective actions, as we intended to do. The T duality rules are identical to those of the type-II theories [4], but now working inside each of the + and − diagonal RR sectors.

III. DEMOCRATIC TYPE-0 ACTIONS AND MASSIVE 0A

In Ref. [12] a "democratic" pseudoaction for type-II theories was proposed in which all RR potentials appear on the same footing. The pseudoaction has to be supplemented by duality constraints relating "electric" and "magnetic" RR fields (hence the "pseudo") and one of its properties is that it has no Chern-Simons term and only the kinetic terms for all the field strengths appear in it. In the type-0 case, it is a simple exercise to get an action in which RR field strengths of all orders appear in the same footing: in the 0B action we can dualize $\hat{G}^{-}_{(3)}$ and $\hat{G}^{+}_{(1)}$ into $\hat{G}^{+}_{(7)}$ and $\hat{G}^{+}_{(9)}$, respectively, and in this order[6] by the standard Poincaré-dualization procedure. There is no need to impose any duality constraint as the resulting $\hat{G}^{+}_{(2n+1)}$, $n=0,1,2,3,4$, field strengths are independent. Actually, not all "electric" and "magnetic" field strengths appear, but only some electric and some magnetic. In any case, the action obtained in this way is really much simpler than the one we arrived at in the previous section given by Eq. (2) plus Eq. (39) or Eq. (47) with RR field strengths given by Eq. (31). In particular, there is no Chern-Simons term and only the kinetic terms of all field strengths $\hat{G}^{+}_{(2n+1)}$ $(n=0,1,2,3,4)$ appear:

$$\hat{S}_{0\mathrm{B}}=\int d^{10}\hat{x}\,\sqrt{|\hat{j}|}\left\{e^{-2\hat{\varphi}}\left[\hat{R}-4(\partial\hat{\varphi})^2+\frac{1}{2\times 3!}\hat{\mathcal{H}}^2+\frac{1}{2}(\partial\hat{\mathcal{T}})^2-V(\hat{\mathcal{T}})\right]+f_+(\hat{\mathcal{T}})\sum_{n=1}^{n=4}\frac{1}{2\times(2n+1)!}(\hat{G}^{+}_{(2n+1)})^2\right\}. \quad (52)$$

We could have dualized instead the $\hat{G}^{+}_{(5)}$, $\hat{G}^{+}_{(3)}$ and $\hat{G}^{+}_{(1)}$ field strengths, in this order, and we would have obtained the above action with the + indices replaced by − indices. The transformation that changes the sign of the tachyon and interchanges the two RR sectors would take us back to the above action which is thus invariant under a combination of that transformation and the Poincaré-dualization of all the field strengths.

Needless to say, one can also write down a democratic formulation of the type-0A action created in the foregoing section and it has the same features as the democratic 0B action, namely, only kinetic terms appear. T duality is then established by extending the decomposition rules (16), (26) to include the higher ("magnetic") RR forms.

In Ref. [8] it was shown that in order to establish T duality between type-IIB and massive type-IIA, Romans' theory for short [13], one has to apply *generalized dimensional reduction* (GDR) on the type-IIB side and standard dimensional reduction on Romans' side. The symmetry abused to perform the GDR is the invariance under the addition of constants to the type-IIB RR scalar, i.e., $\delta\hat{C}_{(0)}=m=cte$.

In the democratic formulation, the symmetry under constant shifts of the two RR zero-forms also acts on the higher RR forms and can be written as

$$\hat{C}^{\pm}=\hat{C}^{\pm}+a^{\pm}e^{\hat{B}}, \quad (53)$$

so we can apply GDR in much the same way as in Ref. [8] and oxidize the nine-dimensional theory to the massive 0A action. In the democratic 0B action there is, however, only one RR scalar present so that it might seem that we would end up with only one mass parameter, whereas type 0A can support two mass parameters associated with the two D8-branes present in its spectrum. This is, however, only an illusion: the nine-form field strength in type 0B will induce a nine-form field strength in $d=9$, which in its turn can only be related to a ten-form field strength in type 0A. It is this ten-form field strength that couples to the second D8-brane.

Generalized dimensional reduction, then, boils down to using the decomposition[7]

$$\hat{C}^{+}_{(2n)}=C^{+}_{(2n)}-C^{+}_{(2n-1)}(d\underline{y}+A^{(2)})+\underline{y}G^{+}_{(0)}\frac{1}{n!}\hat{B}^n \quad (54)$$

instead of Eq. (16), in the reduction carried out in Sec. II. The resulting nine-dimensional action can be obtained by standard dimensional reduction from the action

$$\hat{S}_{0A}=\int d^{10}\hat{x}\,\sqrt{|\hat{g}|}\left\{e^{-2\hat{\phi}}\left[\hat{R}-4(\partial\hat{\phi})^2+\frac{1}{2\times 3!}\hat{H}^2+\frac{1}{2}(\partial\hat{\mathcal{T}})^2-V(\hat{\mathcal{T}})\right]-f_+(\hat{\mathcal{T}})\sum_{n=0}^{n=5}\frac{1}{2\times(2n)!}(\hat{G}^{+}_{(2n)})^2\right\}, \quad (55)$$

where the RR field strengths read

[6]We cannot directly dualize $\hat{G}^{-}_{(1)}$ because there are explicit $\hat{C}^{-}_{(0)}$ potentials in $\hat{G}^{-}_{(3)}$. We could absorb them into a redefinition of $\hat{C}^{-}_{(2)}$, but this would introduce unnecessary complications.

[7]Please note the caret on the B.

$$\hat{G}^{\pm}=d\hat{C}^{\pm}-\hat{H}\hat{C}^{\pm}+G^{\pm}_{(0)}e^{B}. \tag{56}$$

The nondemocratic formulation of massive 0A can be obtained by dualizing the ten-, eight-, and six-form field strengths, resulting in

$$\begin{aligned}\hat{S}_{0A}=\int d^{10}\hat{x}\,\sqrt{|\hat{g}|}\Bigg\{ & e^{-2\hat{\phi}}\Bigg[\hat{R}-4(\partial\hat{\phi})^2+\frac{1}{2\times 3!}\hat{H}^2\\ &+\frac{1}{2}(\partial\hat{\mathcal{T}})^2-V(\hat{\mathcal{T}})\Bigg]-\sum_{\alpha=\pm}f_\alpha\sum_{n=0}^{2}\frac{1}{2\times(2n)!}G^{\alpha 2}_{(2n)}\\ &-\frac{1}{72\sqrt{\hat{g}}}\hat{\epsilon}\Bigg[\partial\hat{C}^{+}_{(3)}\partial\hat{C}^{-}_{(3)}\hat{B}+\frac{1}{4}\hat{G}^{-}_{(0)}\partial\hat{C}^{+}_{(3)}\hat{B}^3\\ &+\frac{1}{4}\hat{G}^{+}_{(0)}\partial\hat{C}^{-}_{(3)}\hat{B}^3+\frac{9}{80}\hat{G}^{+}_{(0)}\hat{G}^{-}_{(0)}\hat{B}^5\Bigg]\Bigg\},\end{aligned} \tag{57}$$

which is just what one would expect.

IV. TYPE-0 D-BRANE SOLUTIONS FROM TYPE-II

In this section we are going to see how to adapt type-II Bogomol'nyi-Prasad-Sommerfield (BPS) solutions to the type-0 setting. This will be done under the assumption of a constant tachyon field $\hat{\mathcal{T}}_0$, and we will absorb any tachyon dependence in the equations of motion into the RR fields, whose field strengths will be denoted by $F^{\pm}_{(n)}$. In order to do this consistently, however, we must investigate the tachyon equation of motion, i.e.,

$$\begin{aligned}&\nabla^{\mu}(e^{-2\phi}\partial_\mu\mathcal{T})-h'(\mathcal{T})\left[\sum_n\frac{(-)^n}{2\times n!}F^{+2}_{(n)}-\sum_n\frac{(-)^n}{2\times n!}F^{-2}_{(n)}\right]\\ &=0,\end{aligned} \tag{58}$$

where we made use of Eq. (32) and following Ref. [5] we have put the tachyon potential to zero. We will assume that $h'(\hat{\mathcal{T}}_0)$ is finite.[8] The tachyon equation of motion, then, leads to the constraint

$$\sum_n\frac{(-)^n}{2\times n!}F^{+2}_{(n)}-\sum_n\frac{(-)^n}{2\times n!}F^{-2}_{(n)}=0. \tag{59}$$

In terms of the rescaled RR field strengths, denoted by F, the equations of motions can be written as,[9]

$$0=d{*F^{\pm}}+H\wedge F^{\pm},$$

$$0=d(e^{-2\phi}{*H})+\frac{1}{2}\sum_\alpha {*F^{\alpha}}\wedge F^{\alpha},$$

$$0=R+4(\partial\phi)^2-4\nabla^2\phi+\frac{1}{2\times 3!}H^2,$$

$$\begin{aligned}R_{\mu\nu}=&\,2\nabla_\mu\nabla_\nu\phi-\frac{1}{4}H^{\rho\sigma}_{\mu}H_{\nu\rho\sigma}\\ &+\frac{1}{4}e^{2\phi}\sum_{n,\alpha}\frac{(-1)^n}{n!}T^{\alpha(n)}_{\mu\nu},\end{aligned} \tag{60}$$

where $T^{\pm(n)}_{\mu\nu}$ are the energy-momentum tensors of the RR field,

$$T^{\pm(n)}_{\mu\nu}=nF^{\pm\rho_1\cdots\rho_{n-1}}_{(n)\mu}F^{\pm}_{(n)\nu\rho_1\cdots\rho_{n-1}}-\frac{1}{2}g_{\mu\nu}F^{\pm 2}_{(n)}. \tag{61}$$

The type-II equations of motion can be obtained from these by taking, for example, all the $-$ RR fields to vanish.

Now, a typical type-II brane solution cannot, except for the D3-brane, be a solution of the constraint Eq. (59), and so the best thing we can do is to distribute each type-II D-brane charge evenly over the $+$ and $-$ $(p+2)$-form field strength.[10] The constraint is then automatically satisfied and the solution reads

$$\begin{aligned}&ds^2=H^{-1/2}(dt^2-d\vec{y}^2_{(p)})-H^{1/2}d\vec{x}^2_{(9-p)},\\ &e^{2\phi}=H^{(p-3)/4},\\ &C^{+}_{ty_1\ldots y_p}=\pm C^{-}_{ty_1\ldots y_p}=\frac{1}{\sqrt{2}}H^{-1},\end{aligned} \tag{62}$$

where H depends only on the transverse coordinates $\vec{x}_{(9-p)}$ and is harmonic. Since this solution bears $+$ and $-$ charge, but has the form of only one object, we are destined to interpret these solutions as the $Dp_{\pm}$-brane [16], a bound state of a Dp_{+}- and a Dp_{-}-brane.[11] Observe that these solutions are simpler in terms of the original (but rescaled) nondiagonal $C_{(p+1)},\overline{C}_{(p+1)}$ RR potentials because they are charged only with respect to one of them. They are also trivially related by T duality as type-II Dp-branes are.

The fact that this pairing occurs follows naturally from the type-IIB $D3$-brane solution: Since it is self-dual it automatically satisfies the condition (59), but as before the $D3$-brane charge must be divided by $\sqrt{2}$ in order to satisfy the equations of motion. Consider then T duality in a world volume direction; in the democratic formulation, the electric component of the five-form field strength gives rise to the electric component of $G^{+}_{(4)}$, whereas its magnetic part gives

[8] If $h'(\hat{\mathcal{T}}_0)=0$ we end up with no extra constraint and we can embed every type-II solution in type 0. Therefore, from now on we will consider the $h'\neq 0$ case only.

[9] We will adapt the same philosophy as in [14] meaning that in the equations of motion the dual fields are also given. This means, for instance, that not only $F^{+}_{(5)}$ but also $F^{-}_{(5)}$ will contribute.

[10] A similar idea was proposed in Ref. [15]. For solutions concerning type-0 branes on orbifolds and their behavior under T duality the reader is referred to [19].

[11] Please note that in this notation the system of a coincident electric and magnetic $D3$-brane [11] is denoted by $D3_{\pm}$.

rise to a magnetic $G^{+}_{(6)}$ and thus leads to an electric $G^{-}_{(4)}$. Needless to say, it works also in the other direction, implying that all the $Dp_{\pm}$-branes are connected by T duality.

In Ref. [16] it was shown that the potential between a $D(p+r)_{\pm}$-brane and a $D(p+s)_{\pm}$-brane vanishes if $r+s=4$. This means that we can expect adapting the notation of [18] to the case at hand, the $[p|D(p+r)_{\pm},D(p+4-r)_{\pm}]$ intersection to be described by the *harmonic superposition rule* [17]. In type II the $r+s=4$ class can be generated by T duality from the $(1|D3,D3)$, which in the type-0 setting has to be interpreted as a $(1|D3_{\pm},D3_{\pm})$ intersection. Since we embed a solution that is based on a self-dual five-form, Eq. (58) is identically satisfied, and as before will give rise to a solution once we divide the type-II RR field ansatz by $\sqrt{2}$. Applying T duality to this type-0 intersection, we can generate the whole class of $[p|D(p+r)_{\pm},D(p+4-r)_{\pm}]$ intersections.[12] Please observe that Eq. (58) is automatically obeyed: by using the $Dp_{\pm}$-branes instead of a Dp_{+}-brane, say, we are effectively identifying the $+$ and $-$ sectors, automatically satisfying Eq. (59).

It should be clear that we can embed every type-II solution into type 0: just distribute the type-II RR charge over the $+$ and $-$ sectors in the appropriate way. For type-II brane solutions this means changing the Dp-brane for a $Dp_{\pm}$-brane. In particular, this means that the type-II BPS intersections will give rise to type-0 intersections of various $Dp_{\pm}$-branes. However, as was pointed out in Refs. [20], once loop corrections are taken into account, a dilaton tadpole develops. Since no such effects are expected on the type-II side, we see that the above correspondence is a mere tree-level coincidence.

V. CONCLUSION

In this paper, starting from the type-0B effective action, we have constructed the type-0A effective action by means of T duality. Although there is a doubling of RR fields in the type-0 theories with respect to the type-II theories, T duality does not mix RR fields from the different sectors. Due to this doubling of RR fields, one can write down a democratic formulation of the action, in which we dualize the fields of one sector giving rise to an action with only kinetic terms for the RR fields, i.e., in the democratic formulation there is no Chern-Simons term in the action. Using this democratic formulation we applied generalized dimensional reduction based on the translational symmetry of the RR scalar(s), in order to find the type-0 analogue of Romans' theory, massive type-0A.

Type-0 inherits a $\mathbb{Z}_2$ symmetry from the left world sheet fermion number operator, which takes the tachyon to its negative and interchanges the electric $(+)$ and magnetic $(-)$ RR sectors. This discrete symmetry together with the consistency of T duality between the type-0 effective actions then constrains the possible couplings of the tachyon to the RR fields.

In Ref. [21] the type-0 string theories were conjectured to correspond to certain supersymmetry breaking orbifolds of M theory. From this identification it follows that type-0B string theory should be symmetric under S duality. Looking at the form of the tree-level action presented in Sec. II and remembering that the tachyon can only couple, at tree level, with even powers to the dilaton, the way this S duality comes about in the effective action seems puzzling.[13] Clearly this point deserves further investigation.

Finally, we have shown how to create type-0 solutions starting from type-II solutions, assuming a constant tachyon. In short, it all boils down to changing a type-II Dp-brane to a type-0 $Dp_{\pm}$-brane, which is nothing but a bound state of a Dp_{+}- and a Dp_{-}-brane. In particular, we can embed the type-II BPS intersections.

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[12]Note that this will not reproduce [16]'s $D5_{\pm}D1_{\pm}$ solution as we took the tachyon potential to be zero in contrast to [16].

[13]For more puzzles associated with the proposed S duality of type 0B, see [22].

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