Tax Evasion, Technology Shocks, and the Cyclicality of Government Revenues*

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Abstract

This paper analyzes the behavior of the tax revenue to output ratio over the business cycle. In order to replicate the empirical evidence, we develop a simple model combining the standard Ak growth model with the tax evasion phenomenon. When individuals conceal part of their true income from the tax authority, they face the risk of being audited and hence of paying the corresponding fine. Under the empirically plausible assumptions that the intertemporal elasticity of substitution exhibits a sufficiently small value and that productivity shocks are serially correlated, we show that the elasticity of government revenue with respect to output is larger than one, which agrees with the empirical evidence. This result holds even if the tax system displays flat tax rates. We extend the previous setup to generate larger fiscal deficits when the economy experiences a recession.

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1. Introduction

In this paper, we follow the approach introduced by Kydland and Prescott (1982) to study the role played by real technology shocks in driving business fluctuation. We will focus our analysis on the response of government revenue to technology shocks. The behavior of government revenue over the business cycle has received some attention in the empirical literature of recent years. It is well known that economic recessions tend to reduce the tax revenue and this makes difficult for governments to fund their existing spending programs. Moreover, during expansion periods tax revenue increases and this creates a new additional political pressure on the government to increase public spending. Therefore, the empirical analysis of this question focuses on obtaining estimates of the income elasticity of tax revenue in order to find out whether tax revenues exhibit a more than proportional response to output fluctuations.[1] It is important to distinguish between the long-run income elasticity of tax revenue, which shows how revenues will grow over time as permanent income grows, and the short-run income elasticity of tax revenue, which shows how much revenues will fluctuate over the business cycle. For instance, Holcombe and Sobel (1997) estimate both the short-run personal income elasticity of tax revenue and the short-run personal income elasticity of the tax base for U.S. states and find that on average they are equal to 1.392 and 1.192 respectively.[2] Hence, the average elasticity estimate suggests that a one percent increase in personal income should result in a 1.4 percent increase in the tax revenue. Recent studies by Dye and Merriman (2004) and Bruce et al. (2006) provide more accurate estimates that also support the idea that the short-run personal income elasticity of the tax base tends to be larger than one.

The main objective of this paper is to provide a theoretical setup that can be consistent with these empirical findings. The standard Ak growth model with flat tax rates predicts that the government revenue to output ratio remains constant when a technology (or total factor productivity) shock takes place. Under flat tax rates, a technology

[1] See Dye (2004) for a review of this literature.

[2] Researchers distinguish between two tax measures when estimating elasticities: the tax base or the tax revenue. Tax revenue data by the type of tax is easily available for several developed countries but they can embed tax rate changes and this leads to a bias in the short-run elasticity estimator.

shock affects symmetrically output and government revenue since government revenue is a constant proportion of output. Therefore, the standard Ak growth model with flat tax rates does not offer a plausible explanation for the empirical evidence as the value of the short-run income elasticity of tax revenue predicted by the model is equal to one.

There are several candidate explanations for the high empirical income elasticity of tax revenue. One obvious explanation consists of dispensing with the assumption of constant marginal tax rates and considering instead a progressive tax schedule on income. Clearly, as the average tax rates increase with income the government revenue will increase more than the aggregate income.

Another alternative explanation for the high income elasticity of government revenue relies on the behavioral responses to income shocks. When the economy deteriorates, individuals might increase their savings and reduce consumption, especially of items like durable goods. Then, after the economy starts to recover, they might make some of the purchases that had previously been put off during the recession. If the government collect taxes on consumption, then the previous behavior of consumption along the business results in a high elasticity of revenue.

In this paper we provide an alternative mechanism generating the desired pattern of cyclicality of government revenue. This mechanism complements the previous ones since relies exclusively on a different assumption, namely, the existence of tax evasion under a flat tax rate on income. We will show that even this simple tax structure is able to generate an income elasticity of tax revenue larger than one under serially correlated productivity shocks when the value of the inverse of the intertemporal elasticity of substitution (IES, henceforth) is larger than one. Of course, under a progressive tax system our mechanism based on tax evasion will reinforce the previous result and, thus, the government revenue will overshoot even more as a response to a productivity shock. The same can be said if taxes were imposed on other procyclical endogenous variables like consumption. Note that our model displays an income elasticity of government revenue larger than one even for economies having tax systems characterized by flat tax rates.[3]

[3] In this respect, it should be mentioned that during the last decade some countries made an important reform of their system of income taxation. They replaced their previous progressive tax structure by a pure flat tax rate. For instance, Russia, Serbia, Iraq, Slovakia and Ukraine set a flat tax rate of 13%, 14%, 15%, 19% and 13%, respectively.

In order to endow the standard Ak growth model with tax evasion, we assume that individuals have to choose in each period the amount of income they want to consume and the amount of income they want to evade. When individuals conceal part of their true income from the tax authority, they face the risk of being audited and hence of paying the corresponding fine. Both taxes and fines determine individual saving and the rate of capital accumulation. Thus, two types of shocks coexist in this model: the aggregate shock, which is given by changes in the total factor productivity of the economy and the idiosyncratic shock, which is introduced by means of the tax inspection policy. The main result of our analysis says that, when technology shocks are serially correlated, the value of the IES fully determines the behavior of the government revenue to GDP ratio. In particular, when the inverse of IES is larger than one, the government revenue increases more than output in the presence of a positive technology shock. In this case, the elasticity of tax revenue with respect to GDP is larger than one, which is consistent with the aforementioned empirical regularity. Moreover, when either the IES is equal to one or technology shocks are not serially correlated, the unitary elasticity of tax revenue is recovered. The intuition of this result lies in the fact that, when shocks are serially correlated, an increase in current total factor productivity means that the expected total productivity and, thus, the expected return of investment in the next period will be higher. Therefore, saving will increase or decrease depending on the value of the IES. Moreover, under tax evasion, underreporting the true income is also a mechanism that allows individuals to transfer present income to the future. This means that, if individuals decide to save more (less) as a response to a real business shock they will also decide to evade more (less) taxes and this will result in less (more) revenues raised by the government.

In the next section we develop the basic dynamic model of tax evasion. In Section 3, we will discuss the implications of a technology shock on the government revenue to GDP ratio. In section 4, we extend our model to cope with the implications for the budget deficits run by the government. Some final remarks conclude the paper.

2. The Model

Let us consider a competitive economy in discrete time with a continuum of ex-ante identical individuals who are uniformly distributed on the interval $[0, 1]$. Each individual i has access to a common technology represented by the production function $y_{i,t} = A_t k_{i,t}$ where $A_t > 0$ is the random total factor productivity (TFP), $y_{i,t}$ is the output per capita of individual i and $k_{i,t}$ is the capital per capita of individual i in period t.[4] We assume that capital fully depreciates after one period.

We assume that the stochastic process of strictly positive TFP shocks $\{A_t\}$ follow a logarithmic autoregressive process,

$$\ln A_{t+1} = \rho \ln A_t + u_{t+1}, \tag{2.1}$$

where $\rho \in [0,1]$ and u_{t+1} is i.i.d. and normally distributed with zero mean and variance σ^2. Note that the realization of TFP shocks are the same for all individuals. Therefore, production is exposed to macroeconomic (or non-idiosyncratic) TFP shocks.

Output can be devoted to either consumption or investment. After production has taken place, each individual i decides both his consumption $c_{i,t}$ and the amount $x_{i,t}$ of declared income, and then pays the corresponding income tax at the rate $\tau \in (0,1)$. If he is inspected by the tax enforcement agency, the total amount of unreported income is discovered and the taxpayer has to pay a penalty at the flat rate $\pi > 1$, which is imposed on the amount of evaded taxes (as in Yitzhaki, 1974).[5] Inspection of a particular individual is an event that occurs with probability $p \in (0,1)$. We also assume that $p\pi < 1$ in order to ensure positive tax evasion.

The amount of output remaining after consumption has taken place and taxes and (potential) penalties have been paid constitutes the capital stock $k_{i,t+1}$ that is used for production in the next period. Therefore, the budget constraint of an audited individual is

$$A_t k_{i,t} - \tau x_{i,t} - \pi\tau \left(A_t k_{i,t} - x_{i,t}\right) = c_{i,t} + k_{i,t+1},$$

whereas the budget constraint of a non-audited individual is

$$A_t k_{i,t} - \tau x_{i,t} = c_{i,t} + k_{i,t+1}.$$

We assume that the amount of taxes collected by the tax agency is devoted to financing government spending that enters into the instantaneous utility of individuals

[4] See Rebelo (1991) for a model where the Ak production function arises endogenously when physical and human capital are perfect substitutes. In this case the capital stock k embodies both types of capital.

[5] If the penalty rate π were smaller than one, tax evasion would be encouraged by the tax authority.

in an additive way. Therefore, the marginal rate of substitution of private consumption between two arbitrary periods is not affected by the level of government spending. Since consumers take as given the path of government spending, the utility accruing from this spending can be suppressed from the consumers' objective function. Individuals are assumed to maximize the following expected discounted sum of instantaneous utilities:

$$\sum_{s=0}^{\infty} \beta^t \mathbb{E}_t \left[U\left(c_{i,t+s}\right)\right], \tag{2.2}$$

where $\beta \in (0,1)$ is the discount factor and $\mathbb{E}_t[\cdot]$ is the conditional expectation given the information available at period t. We assume that the instantaneous utility function is isoelastic,

$$U\left(c_{i,t}\right) = \frac{\left(c_{i,t}\right)^{1-\gamma}}{1-\gamma},$$

where the parameter value γ plays the usual double role as the value of the (constant) relative risk aversion index and as the value of the inverse of the IES.

The amount of unreported income in period t for each individual i is $\epsilon_{i,t} = A_t k_{i,t} - x_{i,t}$. Hence, we can use the previous budget constraints to write the stochastic law of motion of capital per capita as

$$k_{i,t+1} = \begin{cases} \left(1-\tau\right) A_t k_{i,t} - c_{i,t} - \tau(\pi - 1)\epsilon_{i,t}, & \text{with probability } p, \\ \\ \left(1-\tau\right) A_t k_{i,t} - c_{i,t} + \tau\epsilon_{i,t}, & \text{with probability } (1-p), \end{cases}$$

or, equivalently,

$$k_{i,t+1} = \left(1-\tau\right) A_t k_{i,t} - c_{i,t} + \tau\epsilon_{i,t} h_i, \tag{2.3}$$

where h_i is a random variable with the following probability function:

$$f\left(h_i\right) = \begin{cases} p & \text{for} \quad h = 1-\pi, \\ \\ 1-p & \text{for} \quad h = 1, \end{cases} \tag{2.4}$$

for all $i \in [0,1]$. Moreover, the variables h_i are independently distributed across individuals. Note that $\mathbb{E}\left(h_i\right) = 1 - p\pi > 0$ as we have assumed that $p\pi < 1$. We define the net true income per capita as

$$n_{i,t} = \left(1-\tau\right) A_t k_{i,t}. \tag{2.5}$$

Then, using (2.3) we can write $n_{i,t+1}$ as

$$n_{i,t+1} = (1-\tau) A_{t+1} (n_{i,t} - c_{i,t} + \tau \epsilon_{i,t} h_i). \tag{2.6}$$

Taking $n_{i,t}$ as the state variable for individual i in period t, and $c_{i,t}$ and $\epsilon_{i,t}$ as the control variables, the Bellman equation for the stochastic dynamic problem faced by this individual in period t before knowing if he is going to be audited or not is

$$V(n_{i,t}) = \underset{\{c_{i,t},\, \epsilon_{i,t}\}}{Max} \left\{ \frac{(c_{i,t})^{1-\gamma}}{1-\gamma} + \beta E_t [V(n_{i,t+1})] \right\}, \tag{2.7}$$

where $n_{i,t+1}$ satisfies (2.6). It is well known that the value function for this problem is the isoelastic function, $V(n_{i,t}) = \frac{D}{1-\gamma} (n_{i,t})^{1-\gamma}$ with $D > 0$ (see Hakansson, 1970). Therefore, using (2.6) and computing the conditional expectation $\mathbb{E}_t [V(n_{i,t+1})]$, the optimization problem faced by a taxpayer with initial after-tax true income $n_{i,t}$ becomes

$$\underset{\{c_{i,t},\, \epsilon_{i,t}\}}{Max} \left\{ \frac{(c_{i,t})^{1-\gamma}}{1-\gamma} + \beta E_t \left[\frac{D}{1-\gamma} [(1-\tau) A_{t+1} (n_{i,t} - c_{i,t} + \tau \epsilon_{i,t} h_i)]^{1-\gamma} \right] \right\}, \tag{2.8}$$

Differentiating with respect to the control variables $c_{i,t}$ and $\epsilon_{i,t}$, we obtain the following first order conditions for the previous problem:

$$(c_{i,t})^{-\gamma} = \beta D E_t \left[((1-\tau) A_{t+1})^{1-\gamma} (n_{i,t} - c_{i,t} + \tau \epsilon_{i,t} h_i)^{-\gamma} \right], \tag{2.9}$$

and

$$\mathbb{E}_t \left[[(1-\tau) A_{t+1} (n_{i,t} - c_{i,t} + \tau \epsilon_{i,t} h_i)]^{-\gamma} h_i \right] = 0. \tag{2.10}$$

Using the independency between A_{t+1} and h_i and the distribution of the random variable h_i given in (2.4), condition (2.9) becomes

$$(c_{i,t})^{-\gamma} = \beta D (1-\tau)^{1-\gamma} \alpha_t \left[(1-p) (n_{i,t} - c_{i,t} + \tau \epsilon_{i,t})^{-\gamma} + p (n_{i,t} - c_{i,t} + \tau (1-\pi) \epsilon_{i,t}) \right], \tag{2.11}$$

with

$$\alpha_t \equiv \mathbb{E}_t \left[(A_{t+1})^{1-\gamma} \right],$$

while condition (2.10) becomes

$$(1-p) (n_{i,t} - c_{i,t} + \tau \epsilon_{i,t})^{-\gamma} = p (\pi - 1) (n_{i,t} - c_{i,t} + \tau (1-\pi) \epsilon_{i,t})^{-\gamma}. \tag{2.12}$$

Solving for $c_{i,t}$ and $\epsilon_{i,t}$ in the system composed of equations (2.11) and (2.12), we obtain

$$c_{i,t} = \theta_t n_{i,t}, \tag{2.13}$$

and

$$\epsilon_{i,t} = \frac{\lambda}{\tau}(n_{i,t} - c_{i,t}), \tag{2.14}$$

where

$$\theta_t = \frac{1}{1 + \left(\beta D(1-\tau)^{1-\gamma}\alpha_t \left[(1-p)(1+\lambda)^{-\gamma} + p\left(1-(\pi-1)\lambda\right)^{-\gamma}\right]\right)^{1/\gamma}}, \tag{2.15}$$

and

$$\lambda = \frac{\left(\frac{1-p}{p(\pi-1)}\right)^{1/\gamma} - 1}{1 + (\pi - 1)\left(\frac{1-p}{p(\pi-1)}\right)^{1/\gamma}} > 0. \tag{2.16}$$

Applying the envelope theorem, that is, $U'(c_{i,t}) = V'(n_{i,t})$, it must hold that

$$c_{i,t}^{-\gamma} = D n_{i,t}^{-\gamma}. \tag{2.17}$$

Substituting (2.13) in (2.17) and using (2.15) we obtain

$$D = \frac{1}{1 + \left(\beta D(1-\tau)^{1-\gamma}\alpha_t \left[(1-p)(1+\lambda)^{-\gamma} + p\left(1-(\pi-1)\lambda\right)^{-\gamma}\right]\right)^{1/\gamma}}.$$

Therefore, solving for D in the previous equation we get

$$D = \left[\frac{1}{1 - \left(\beta(1-\tau)^{1-\gamma}\alpha_t H\right)^{1/\gamma}}\right]^{\gamma}, \tag{2.18}$$

where

$$H = (1-p)(1+\lambda)^{-\gamma} + p\left(1-(\pi-1)\lambda\right)^{-\gamma}.$$

Substituting (2.18) into (2.15), and using (2.13), and (2.14), we get the following consumption and evasion policies:

$$c_{i,t} = \left[1 - \left(\beta(1-\tau)^{1-\gamma}H\alpha_t\right)^{1/\gamma}\right] n_{i,t}, \tag{2.19}$$

and

$$\epsilon_{i,t} = \frac{\lambda}{\tau}\left(\beta(1-\tau)^{1-\gamma}H\alpha_t\right)^{1/\gamma} n_{i,t}. \tag{2.20}$$

Note that, when $p\pi = 1$, we have that $\lambda = 0$ and, hence, $H = 1$. Therefore, when $p\pi = 1$, individuals do not evade taxes, $\epsilon_{i,t} = 0$ for all $i \in [0,1]$. Moreover, under this full enforcement policy conducted by the tax agency, the optimal consumption policy is the one appearing in absence of tax evasion,

$$c_{i,t} = \left[1 - \left(\beta(1-\tau)^{1-\gamma}\alpha_t\right)^{1/\gamma}\right](1-\tau) A_t k_{i,t}.$$

In order to obtain the value of the aggregate after-tax true income n_{t+1} in equilibrium, which is given by (2.6), we compute

$$\begin{aligned} n_{t+1} &= \int_{[0,1]} n_{i,t+1} di = (1-\tau) A_{t+1} \left[\int_{[0,1]} n_{i,t} di - \int_{[0,1]} c_{i,t} di + \tau \int_{[0,1]} \epsilon_{i,t} h_i di \right] \\ &= (1-\tau) A_{t+1} \left[\int_{[0,1]} n_{i,t} di - \int_{[0,1]} c_{i,t} di + \tau \left(\int_{[0,1]} \epsilon_{i,t} di \right) \left(\int_{[0,1]} h_i di \right) \right] \\ &= (1-\tau) A_{t+1} \left[n_t - c_t + \tau (1 - p\pi) \epsilon_t \right], \end{aligned}$$

where the third equality follows from the independence between the variables h_i and $\epsilon_{i,t}$ at the beginning of period t, whereas the last equality comes from the law of large numbers for a continuum of i.i.d. random variables, according to which $\int_{[0,1]} h_i di = \mathbb{E}(h_i) = 1 - p\pi$, and from the definitions of aggregate consumption $c_t \equiv \int_{[0,1]} c_{i,t} di$, aggregate evasion $\epsilon_t \equiv \int_{[0,1]} \epsilon_{i,t} di$, and aggregate after-tax true income $n_t \equiv \int_{[0,1]} n_{i,t} di$. In consequence, as follows from (2.19) and (2.20), the aggregate values of consumption and evaded income are

$$c_t = \underbrace{\left(1 - \left(\beta (1-\tau)^{1-\gamma} H \alpha_t\right)^{1/\gamma}\right)}_{\theta_t} n_t, \tag{2.21}$$

and

$$\epsilon_t = \frac{\lambda}{\tau} \left(\beta (1-\tau)^{1-\gamma} H \alpha_t\right)^{1/\gamma} n_t = \frac{\lambda}{\tau} (1 - \theta_t) n_t. \tag{2.22}$$

In order to analyze the effect of a TFP shock on evaded income and on consumption, we must compute the value of α_t. Given that the random variable u_{t+1} is normal and thus the technology shock A_{t+1} is log-normal, the conditional expectation $\alpha_t \equiv \mathbb{E}_t \left[(A_{t+1})^{1-\gamma} \right]$ is equal to

$$\alpha_t = A_t^{\rho(1-\gamma)} \exp\left(\frac{(1-\gamma)^2 \sigma^2}{2} \right). \tag{2.23}$$

The next section discusses the effect of a TFP shock on both the amount of evaded income and the government revenue to GDP ratio.

3. Effects of TFP shocks

In order to analyze the effect of a technology shock on government revenue to output ratio, we should first compute the effect of an increase of the TFP value A_t on the evasion to income ratio ϵ_t / y_t. Since aggregate output satisfies $y_t = A_t k_t$ and the the after-tax aggregate income is $n_t = (1-\tau) A_t k_t = (1-\tau) y_t$, we substitute (2.23) into (2.22) to obtain the fraction of evaded income,

$$\frac{\epsilon_t}{y_t} = \frac{\lambda}{\tau} \left[\beta H (1-\tau) A_t^{\rho(1-\gamma)} \exp\left(\frac{(1-\gamma)^2 \sigma^2}{2} \right) \right]^{1/\gamma}. \tag{3.1}$$

The next preposition summarizes the impact of a variation of A_t on the amount of evaded income to GDP ratio ϵ_t / y_t :

Proposition 3.1. *(a) Assume that $\rho > 0$. Then,*

$$\frac{\partial (\epsilon_t / y_t)}{\partial A_t} \gtreqless 0 \quad \text{if } \gamma \lesseqgtr 1.$$

(b) Assume that $\rho = 0$. Then,

$$\frac{\partial (\epsilon_t / y_t)}{\partial A_t} = 0.$$

Proof: (a) It follows directly from (3.1) since $\lambda > 0$ and $H > 0$.

(b) When $\rho = 0$, A_{t+1} does not depend on A_t and, hence, $\alpha_t = \exp\left((1-\gamma)^2 \sigma^2 / 2\right)$. Therefore, the ratio ϵ_t / y_t is not affected by TFP shocks.■

The intuition of this result comes directly from the assumption made about the utility function. Recall that the parameter γ measures the inverse of IES. When $\gamma > 1$ individuals have a strong preference for consumption smoothing so that, if the expected return from saving increases, then they decide to decrease the fraction of income saved so as to preserve a low discrepancy between present and future consumption. Under positive autocorrelation, $\rho > 0$, a high value A_t of the total factor productivity in period t, implies a large expected value A_{t+1} of the shock in the period $t+1$. This results in a higher expected return from saving and then individuals decide optimally to reduce the saving to income ratio. Note that in our setup with tax evasion individuals decrease the fraction of income transferred to the next period through the two channels at their disposal. On the one hand, they reduce the direct fraction of their income devoted to capital accumulation and, on the other hand, they reduce the fraction ϵ_t / y_t of evaded income. Note in this respect that in our model tax evasion is a risky investment with positive expected return as $\mathbb{E}(h_i) = 1 - p\pi > 0$. The opposite argument applies when $\gamma < 1$. In this latter case, a rise of the expected return A_{t+1} from saving triggers a larger capital accumulation and larger tax evasion since individuals are not as concerned about consumption smoothing. As the substitution effect between present and future consumption is the dominating force in explaining the saving decision in this case, a large value of A_t results in a larger expected value of A_{t+1} and, hence, the fraction of saving out of present income should increase.

The previous argument does not apply when the technology shocks are not serially correlated since then current shocks do not affect the expected future shocks. Therefore, when a technology shock takes place in this case, both the true net income n_t and the output y_t increase and, because of the assumption of isoelastic preferences, aggregate evasion and output rise in the same proportion so that the evasion to income ratio ϵ_t/y_t remains constant.

Finally, when $\gamma = 1$ (that is, when the instantaneous utility function is logarithmic), the consumption policy (2.21) is equal to

$$c_t = (1-\beta)\, n_t,$$

for all $\rho \in [0,1]$ since $\alpha_t = 1$ and $H = 1$ in this case (see (2.21)). Note that this consumption policy obtained under logarithmic preferences is independent of the parameters p and π characterizing the tax enforcement. Therefore, the impact of a variation in the tax enforcement policy is totally absorbed by the amount of unreported income for given values of both the tax rate τ and the capital stock k_t. Moreover, when $\gamma = 1$, the proportion of saving is independent of the expected return from capital and, thus, the fraction ϵ_t/y_t of evaded income is unaffected by contemporaneous TFP shocks.

The total government revenue R_t is given by the taxes that consumers voluntary pay plus the amount of evaded taxes and the corresponding penalty paid by inspected individuals,

$$R_t = \tau x_t + p\pi\tau(y_t - x_t),$$

which can be rewritten as

$$R_t = \tau y_t - \tau(1-p\pi)\epsilon_t.$$

Therefore, the government revenue to GDP ratio R_t/y_t equals to

$$\frac{R_t}{y_t} = \tau\left[1-(1-p\pi)\frac{\epsilon_t}{y_t}\right]. \tag{3.2}$$

The effect of a TFP shock on the government revenue to GDP ratio is given in the following proposition:

Proposition 3.2. *(a) Assume that* $\rho > 0$*. Then,*

$$\frac{\partial\left(R_t/y_t\right)}{\partial A_t} \gtreqless 0 \ \ \textit{if} \ \ \gamma \gtreqless 1.$$

(b) Assume that $\rho = 0$*. Then,*

$$\frac{\partial\left(R_t/y_t\right)}{\partial A_t} = 0.$$

Proof: It is immediate from Proposition 3.1 and equation (3.2).■

We have just shown that a technology shock not only affects the output per capita but also the government revenue to GDP ratio. The intuition of this result lies in the behavior of the evasion ratio ϵ_t/y_t when A_t changes. As Proposition 3.1 shows, the effect of a positive TFP shock on the fraction ϵ_t/y_t of evaded income depends on the value taken by the parameter γ. When $\gamma > 1$, the fraction ϵ_t/y_t of evaded income decreases and, therefore, the government revenue to output ratio increases.

In this framework, the capital k_t is given at the beginning of each period t. Then, the output at time t is only affected by the fluctuations of the technology shock A_t. Therefore, knowing the sign of the effect of a TFP shock on the government revenue to GDP ratio is the equivalent knowing whether the elasticity of tax revenue with respect to output is larger, equal or lower than one. Empirical evidence shows that the estimates of this elasticity are generally larger than one. This model can reproduce this fact when $\gamma > 1$ and technology shocks are serially correlated ($\rho > 0$). It is important to stress the crucial role played by tax evasion since, in the case where consumers are honest (that is, when $p\pi = 1$), a TFP shock will only affect the output per capita but not the ratio R_t/y_t, since this ratio is simply equal to τ (see (3.2).

4. Government budget and growth rates

In order to extend the scope of our results, we will discus some potential scenarios concerning the behavior of government spending that give raise in turn to some insights about the procyclical or countercyclical nature of government deficits. There is a large number of empirical studies that use data from developing countries to suggest that government spending tends to be procyclical.[6] The articles by Gavin and Perotti (1997),

[6] A procyclical fiscal policy is defined as the reduction in public spending (or an increase in taxes) during recessions and increases in public spending (or reductions in taxes) during expansions.

Stein et al. (1999), Braun (2001), Kamisky et al. (2004), Akitoby et al. (2004) and Strawczynski and Zeira (2007) are some examples of papers that find a positive relationship between public spending and GDP.

Let us first analyze the implications of TFP shocks on the stochastic rate of endogenous growth of our Ak-type economy. The net rate g_{t+1} of growth of GDP per capita from t to $t+1$ satisfies

$$1 + g_{t+1} = \frac{y_{t+1}}{y_t} = \frac{A_{t+1}k_{t+1}}{A_t k_t}. \tag{4.1}$$

We obtain evolution of the aggregate capital k_{t+1} as follows:

$$\begin{aligned}
k_{t+1} &= (1-\tau)\, A_t k_t - c_t + \tau(1-p\pi)\epsilon_t \\
&= (1-\tau)\, A_t k_t - c_t + \tau(1-p\pi)\frac{\lambda}{\tau}(n_t - c_t) \\
&= (1-\tau)\, A_t k_t - (1+(1-p\pi)\lambda)\, c_t + (1-p\pi)\lambda n_t \\
&= (1-\tau)\, A_t k_t - (1+(1-p\pi)\lambda)\, \theta_t\, (1-\tau)\, A_t k_t + (1-p\pi)\lambda\, (1-\tau)\, A_t k_t \\
&= (1-\tau)\, (1-\theta_t)\, (1+(1-p\pi)\lambda)\, A_t k_t.
\end{aligned} \tag{4.2}$$

where the first equality comes from (2.3), the second comes from (2.14), the third from collecting terms, and the fourth comes from the policy function (2.21) for consumption and from the definition of n_t in (2.5). Note that we have applied the law of large numbers in all these equalities. Therefore, the gross rate of growth becomes

$$\begin{aligned}
1 + g_{t+1} &= A_{t+1}\, (1-\tau)\, (1-\theta_t)\, (1+(1-p\pi)\lambda) \\
&= A_{t+1}\, (1-\tau)\, \left(\beta(1-\tau)^{1-\gamma} H\alpha_t\right)^{1/\gamma} (1+(1-p\pi)\lambda) \\
&= A_{t+1}\, (1-\tau)\left(\beta(1-\tau)^{1-\gamma} H A_t^{\rho(1-\gamma)} \exp\left(\frac{(1-\gamma)^2\sigma^2}{2}\right)\right)^{1/\gamma} (1+(1-p\pi)\lambda) \\
&= A_{t+1}\, [(1-\tau)\,\beta H]^{1/\gamma} A_t^{\rho(1-\gamma)/\gamma}\, (1+(1-p\pi)\lambda) \exp\left(\frac{(1-\gamma)^2\sigma^2}{2\gamma}\right),
\end{aligned} \tag{4.3}$$

where the first equality comes from (4.1) and (4.2), the second comes from the expression of θ_t in (2.21), the third from the expression of α_t in (2.23), and the fourth from some simplification. Note that the rate of growth from t to $t+1$ depends not only on the realization of the technology shock A_{t+1} at $t+1$ but also on the shock A_t at t. Taking into account in (4.3) that

$$\mathbb{E}_t\, (A_{t+1}) = A_t^{\rho} e^{\sigma^2/2},$$

we can easily compute the expected future rate of growth at t,

$$\mathbb{E}_t\left(1+g_{t+1}\right)=\left[(1-\tau)\,\beta H\right]^{1/\gamma}A_t^{\rho/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{\left(1-\gamma+\gamma^2\right)\sigma^2}{2\gamma}\right) \tag{4.4}$$

All the unexpected increases in the rate of growth from t to $t+1$ are fully explained by the realization of the technological shock A_{t+1}. To see this, we can simply use (4.3) and (4.4) to compute the following ratio between the actual and the expected gross rates of growth:

$$\frac{1+g_{t+1}}{\mathbb{E}_t\left(1+g_{t+1}\right)}=\frac{A_{t+1}}{A_t^{\rho}e^{\sigma^2/2}}.$$

We see that the the rate of economic growth is larger (smaller) than the expected rate if and only if $A_{t+1}>(<)A_t^{\rho}e^{\sigma^2/2}=\mathbb{E}_t\left(A_{t+1}\right)$.

After establishing the connection of TFP shocks and the rate of economic growth under tax evasion, we can analyze the potential implications of these shocks for government spending and fiscal deficits. Let us first assume that the government can adjust instantaneously its own spending and wants to keep the government spending to output ratio constant, which is equivalent to fixing a unitary elasticity of public spending with respect to GDP. In this case, the behavior of the tax revenue to output ratio fully characterizes the path of fiscal deficits or fiscal surpluses. We know that, under serially correlated TFP shocks ($\rho>0$), a positive technology shock has a larger effect on the tax revenue to GDP ratio than on the public spending to GDP ratio if $\gamma>1$. Therefore, the budget deficit decreases. However, when $\gamma<1$ a positive TFP shock results in a larger budget deficit. Note that when either $\gamma=1$ or $\rho=0$, the government budget does not vary as a response to a TFP shock. Nevertheless, when the elasticity of government spending with respect to output is larger than one and $\rho>0$, fiscal deficits can arise whatever value γ takes.[7] For $\gamma<1$ and $\gamma=1$ a positive technology shock gives rise to a fiscal deficit, while in the case of $\gamma>1$ deficits or surpluses may appear depending on the specific value of both the elasticity of public spending and the elasticity of tax revenue.

Another scenario where government spending is adjusted instantaneously appears when government is committed to devoting all tax revenue to financing public spending. Obviously, in this case government deficits are not affected by TFP shocks. This setup

[7] Akitoby et al. (2004) estimated the long and short term elasticity of government spending and output and found that this elasticity is on average greater than 1.

can be interpreted as the extreme case of the model developed by Talvi and Végh (2005) where the government comes under political pressure to increase public spending when tax revenue increases. In the setup of our paper the value of parameter γ will determine the magnitude of the elasticity of public spending with respect to output. In particular, when $\gamma > 1$ the elasticity of government revenue with respect to the output is larger than one. However, this scenario is not supported by data since the elasticity of government spending exhibits empirical values larger than one but smaller than those of the elasticity of government revenue (see Akitoby et al., 2004).

We next present an scenario where the amount of government spending is set one period in advance as it occurs, for instance, when this spending is mostly devoted to infrastructure building. In this case, there is an obvious lag from the period at which the spending decision is taken to the period where the infrastructure is fully operative. Alternatively, we can assume that the government (or the corresponding legislative body) has to approve the spending budget for the next period and, once the budget bill is passed, no further changes in the amount of spending are feasible during the next period. Due to this lag in the government spending decision, the spending to GDP ratio will fluctuate with the business cycle. Thus, the objective of the government will be to keep the expected government spending to GDP ratio as close as possible to its exogenous target value $\nu \in (0, 1)$. More precisely, we assume that the government sets in each period the government spending in the next period so as to minimize the expected square of the deviation of the government spending to GDP ratio from the exogenous constant ratio target ν. We maintain in this framework the assumption of constant tax rates, which in turn also gives raise to fluctuations in total government revenues.

The objective of the government in period $t-1$ is thus to choose the amount G_t of spending in period t to minimize

$$\mathbb{E}_{t-1}\left(\frac{G_t}{y_t} - \nu\right)^2,$$

This objective is fully achieved when $\mathbb{E}_{t-1}\left(G_t/\, y_t\right) = \nu$, that is, when

$$G_t = \frac{\nu}{\mathbb{E}_{t-1}\left(y_t^{-1}\right)} = \frac{\nu}{\mathbb{E}_{t-1}\left(\left[(1+g_t)y_{t-1}\right]^{-1}\right)} = \frac{\nu y_{t-1}}{\mathbb{E}_{t-1}\left(\left[1+g_t)\right]^{-1}\right)} \tag{4.5}$$

To compute explicitly the denominator of the previous expression, we first get from

(4.3) that

$$(1+g_t)^{-1} = \frac{A_t^{-1}}{\left[(1-\tau)\,\beta H\right]^{1/\gamma} A_{t-1}^{\rho(1-\gamma)/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{(1-\gamma)^2\sigma^2}{2\gamma}\right)}$$

so that

$$\begin{aligned}
\mathbb{E}_{t-1}\left([1+g_t)]^{-1}\right) &= \frac{\mathbb{E}_{t-1}\left(A_t^{-1}\right)}{\left[(1-\tau)\,\beta H\right]^{1/\gamma} A_{t-1}^{\rho(1-\gamma)/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{(1-\gamma)^2\sigma^2}{2\gamma}\right)} \\
&= \frac{A_{t-1}^{-\rho}e^{\sigma^2/2}}{\left[(1-\tau)\,\beta H\right]^{1/\gamma} A_{t-1}^{\rho(1-\gamma)/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{(1-\gamma)^2\sigma^2}{2\gamma}\right)} \\
&= \frac{1}{\left[(1-\tau)\,\beta H\right]^{1/\gamma} A_{t-1}^{\rho/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{(1-3\gamma+\gamma^2)\sigma^2}{2\gamma}\right)}, \qquad (4.6)
\end{aligned}$$

where the second equality comes from the fact that

$$\mathbb{E}_{t-1}\left(A_t^{-1}\right) = A_{t-1}^{-\rho}e^{\sigma^2/2},$$

and the third comes from some straightforward simplification.

Therefore, using (4.5) and (4.6), the amount of government spending in date t is

$$G_t = \frac{\nu y_{t-1}}{\mathbb{E}_{t-1}\left([1+g_t)]^{-1}\right)} = \frac{\nu A_{t-1}k_{t-1}}{\mathbb{E}_{t-1}\left([1+g_t)]^{-1}\right)}$$

$$\begin{aligned}
&= \nu A_{t-1}k_{t-1}\left[(1-\tau)\,\beta H\right]^{1/\gamma} A_{t-1}^{\rho/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{(1-3\gamma+\gamma^2)\sigma^2}{2\gamma}\right) \\
&= \nu A_{t-1}^{(\gamma+\rho)/\gamma}k_{t-1}\left[(1-\tau)\,\beta H\right]^{1/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{(1-3\gamma+\gamma^2)\sigma^2}{2\gamma}\right).
\end{aligned}$$

Note that the government spending in t depends on the values of two variables known a $t-1$, namely, the capital k_{t-1} and the the TFP shock A_{t-1}.

Concerning the effective government spending to GDP ratio in period t, note that

$$\frac{G_t}{y_t} = \frac{G_t}{(1+g_t)y_{t-1}} = \frac{\nu}{(1+g_t)\mathbb{E}_{t-1}\left([1+g_t)]^{-1}\right)} =$$

$$\frac{\nu\left[(1-\tau)\,\beta H\right]^{1/\gamma} A_{t-1}^{\rho/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{(1-3\gamma+\gamma^2)\sigma^2}{2\gamma}\right)}{A_t\left[(1-\tau)\,\beta H\right]^{1/\gamma} A_{t-1}^{\rho(1-\gamma)/\gamma}\left(1+(1-p\pi)\lambda\right)\exp\left(\frac{(1-\gamma)^2\sigma^2}{2\gamma}\right)} = \frac{\nu A_{t-1}^{\rho}e^{-\sigma^2/2}}{A_t}, \qquad (4.7)$$

where the second equality comes from (4.5) and the third from (4.3) and (4.6).

As we have shown in the previous section, the government revenue to GDP ratio can fluctuate in each period t with the technological shock A_t in the presence of tax evasion (i.e., when $p\pi < 1$) even if the tax rate remains constant across periods (see (4.9)). Moreover, we have just seen in this section that the government spending to output ratio at t also fluctuates with the shock A_t as the amount of government spending was decided in period $t-1$.

Concerning the fiscal deficit to GDP ratio, we can compute $\frac{G_t - R_t}{y_t}$ from (4.7) and (4.9). Note from (4.7) that the government spending to GDP ratio strictly decreases with the innovation shock in A_t. However, the government revenue to output ratio increases (decreases) with the innovation shock in A_t if $\gamma > 1 (< 1)$ when $\rho > 0$, while it does not vary with A_t if either $\rho = 0$ or $\gamma = 1$. Therefore, we get the following result:

Proposition 4.1. *For a given value of A_{t-1}, the government deficit to output ratio $\frac{G_t - R_t}{y_t}$ is decreasing in the value A_t of TFP if $\gamma \geq 1$ and $\rho > 0$. Moreover, the same result holds for all $\gamma > 0$ when $\rho = 0$.*

Proof: Note that, if $\gamma \geq 1$ and $\rho > 0$, then the government revenue to output ratio weakly increases with A_t and, since the government spending to GDP ratio strictly decreases with A_t for a given value of A_{t-1}, the result immediately follows. When $\rho = 0$, the government deficit is decreasing since the government revenue to output ratio is not affected by changes in A_t, while the government spending to output ratio strictly decreases with A_t for a given value of A_{t-1}.■

The previous result agrees with the empirical evidence since tell us that, under the empirically relevant case with $\gamma \geq 1$ and $\rho > 0$, fiscal deficits increase when the current rate of growth is lower than the expected one. Note in this respect that, as we have shown at the beginning of this section, the deviation of the actual rate of growth in period t and its expectation at $t-1$ for a given value A_{t-1} in period $t-1$ is fully explained by the realization A_t of the TFP in period t. However, for the empirically most implausible case $\gamma < 1$, if TFP shocks are positively correlated, $\rho > 0$, the overall effect on the public deficit to GDP ratio is ambiguous. In this case the revenue to GDP ratio decreases when there is a positive shock on TFP, which coupled with the decrease in the government spending to GDP ratio, gives raise to an ambiguous effect on the government deficit to output ratio.

Let us finish this section with some comments about the selection of the tax rate when the amount of government spending is chosen a period in advance. Note that we have bee implicitly assuming in our previous analysis that the selection of tax rates is subject to less discretion than government spending, that is, that tax rates are set for longer periods than the amount of government spending. In fact, we were making the extreme assumption that the value of the tax rate was exogenously given. One way of rationalize this assumption and make it consistent with balanced budget in the long run consists of assuming that the government (or the legislative body) chooses at date 0, before observing any technological shock, the tax rate in order to minimization of the unconditional expected square of the government deficit to output ratio. Therefore, the objective of the government is to choose the tax rate τ in order to minimize

$$\mathbb{E}\left(\frac{G_t - R_t}{y_t}\right)^2$$

This target is fully achieved achieved when

$$\mathbb{E}\left(\frac{R_t}{y_t}\right) = \mathbb{E}\left(\frac{G_t}{y_t}\right),$$

which according to the government spending objective becomes

$$\mathbb{E}\left(\frac{R_t}{y_t}\right) = \nu. \tag{4.8}$$

as, from the law of iterated expectations, $\mathbb{E}\left(G_t/\,y_t\right) = \mathbb{E}\left(\mathbb{E}_{t-1}\left(G_t/\,y_t\right)\right) = \nu$. Combining (3.2) with (3.1) we obtain the government revenue to GDP ratio

$$\frac{R_t}{y_t} = \tau - (1-p\pi)\lambda\left[\beta H(1-\tau)\right]^{1/\gamma} A_t^{\rho(1-\gamma)/\gamma} \exp\left(\frac{(1-\gamma)^2\,\sigma^2}{2\gamma}\right). \tag{4.9}$$

The unconditional expectation (i.e., the expectation at the initial date 0 before observing any realization of the TFP shock) of the previous government revenue to GDP ratio can be easily computed by taking into account the following unconditional expectation:

$$\mathbb{E}\left(A_t^{\rho(1-\gamma)/\gamma}\right) = \exp\left(\frac{\rho^2\,(1-\gamma)^2\,\sigma^2}{2\gamma^2\,(1-\rho^2)}\right).$$

Plugging the previous expression in the unconditional expectation of the ratio (4.9), we get

$$\mathbb{E}\left(\frac{R_t}{y_t}\right) = \tau - (1-p\pi)\lambda\left[\beta H(1-\tau)\right]^{1/\gamma} \exp\left(\frac{\rho^2\,(1-\gamma)^2\,\sigma^2}{2\gamma^2\,(1-\rho^2)}\right) \exp\left(\frac{(1-\gamma)^2\,\sigma^2}{2\gamma}\right)$$

$$= \tau - (1 - p\pi)\lambda \left[\beta H(1-\tau)\right]^{1/\gamma} \exp\left(\frac{(1-\gamma)^2\sigma^2}{2\gamma}\left[1 + \frac{\rho^2}{(1-\rho^2)\gamma}\right]\right).$$

It is immediate to see that the previous expectation is strictly increasing in the tax rate and tends to 1 as τ converges to 1 and to a negative number when τ approaches 0. Thererefore, there exists a unique value τ^* of the tax rate solving the equation (4.8) for $\nu \in (0,1)$. This is the tax rate that balances the government budget in (unconditional) expected terms and that is kept constant for all periods in our analysis.

5. Final Remarks

In this paper, we have shown that, by introducing tax evasion in the standard Ak model growth with flat tax rates, it is possible to obtain an elasticity of tax revenue with respect to output larger than one, which agrees with the empirical evidence. Therefore, tax evasion offers by itself an explanation to the high income elasticity of government revenue that complements other explanations relying either on progressive income taxation or on taxes imposed on procyclical variables. we have extended the model to account for the cyclical behavior of fiscal deficits when government has a target concerning the value of its spending relative to GDP. we show that, under a plausible parameter restriction, fiscal deficits become larger in recessions.

We have used for our analysis a very simple model of capital accumulation where the static portfolio choice model of tax evasion presented by Allingham and Sandmo (1972) has been extended to a dynamic setup.[8] In this framework, consumers' decisions about how much income they want to report not only affect their present consumption but also their future consumption. Therefore, the response of consumers to positive TFP shocks affects both the tax evasion decision and government revenue. In this setup, we have shown how the effect of a positive technology shock on the government revenue to GDP ratio is fully characterized by the value of IES parameter when TFP shocks are serially correlated. In particular when the IES exhibits a sufficiently small value, a positive technology shock makes individuals to lower more than proportionally their amount of evaded income in order to maintain a smooth path of consumption over time. Therefore, the government revenue increases more than output and in consequence the income elasticity of tax revenue becomes larger than one.

[8] See Lin andYang (2001) for a similar context.

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