

MDPI

Article

Family Restrictions at Work

Enriqueta Aragones

Institut d'Anàlisi Econòmica, CSIC, Campus UAB, 08193 Bellaterra, Spain; enriqueta.aragones@iae.csic.es

Abstract: This paper analyzes one of the causes of the current gender-unbalanced situation in the labor market: the discrimination that individuals face at work due to their commitment to unpaid care work. It aims at finding mechanisms that may induce a change from the current unbalanced situation to a world in which males and females are found in more equal shares in all professions and at all levels. I construct a formal model that includes the heterogeneity of individuals regarding their family commitments and I investigate how it affects the individual's optimal labor market participation. The welfare of individuals with commitment to family duties is reduced for two different reasons: for not being able to participate as much in the labor market and thus receive a lower labor income and for not being able to contribute as much to their family commitments. I compare the results for the female and male sections of the society and I illustrate the observed gender gaps in terms of labor market participation, income levels, and the overall utility obtained. I find that even though the gender wage gap may be alleviated with reductions in the cost associated to unpaid care work, the gender utility gap will persist.

Keywords: discrimination; labor market; unpaid care work

JEL Classification: J7; J31

1. Introduction

The under-representation of women with respect to men in many professions at all levels and in all professions at some levels is a fact. Given that women represent about 50 per cent of the population, it is reasonable to describe this situation as gender unbalanced. The demand for a balanced proportion of women in all professions and at all levels has been raised and its support has been increasing over time. This demand can be based on the claim that a world in which males and females are found in equal shares in all professions and at all levels would be optimal. But it also can be based on an equity claim: females and males should have the same professional opportunities.

This paper analyzes one of the causes of the current gender-unbalanced situation: the discrimination that individuals face at work due to their commitment to unpaid care work. It aims at finding mechanisms that may induce a change from the current unbalanced situation to a world in which males and females are found in more equal shares in all professions and at all levels.

The analysis proposed in this paper contributes to the theoretical literature on discrimination in the labor market. The existing literature is mostly devoted to explaining racial discrimination such us Peski and Szentes (2013) who analyze how racial stereotypes may determine the labor market outcomes. The theoretical models that focus on the gender gaps in the labor market assume specific strategies for the firms such as incentive contracts Albanesi and Olivetti (2009) or different types of jobs (Francois 1998; Dolado et al. 2013) and they display multiple self-fulfilling equilibria, some of which exhibit gender gaps. In addition, Cella and Manzoni (2023) analyze how discrimination in electoral contests produce fewer female candidates and fewer female elected politicians, even though in average they are more competent than their male counterparts. This paper proposes a theoretical analysis of the gender discrimination that originates in the supply side of the



Citation: Aragones, Enriqueta. 2024. Family Restrictions at Work. *Economies* 12: 101. https://doi.org/10.3390/economies12050101

Received: 23 February 2024 Revised: 22 April 2024 Accepted: 24 April 2024 Published: 26 April 2024



Copyright: © 2024 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

Economies **2024**, 12, 101 2 of 18

labor market with the aim of providing new insights that would complement the extensive empirical literature on the subject.

First, we briefly state some facts about the gender gaps observed in the labor market. Across the EU, the gender employment gap (the difference between the employment rates of men and women of working age: 20–64 years) was 10.8 percentage points in 2021, meaning that the proportion of men of working age in employment exceeded that of women by 10.8 percentage points. Women tend to work less hours and they are more likely to engage in low paid and informal work and in part-time jobs. Maternity leaves have a long-run negative effect on the participation of women in the labor force (Bertrand 2020; Isen et al. 2017).

The disproportionate representation of women in low paid and informal work also contributes to the observed gender earnings gap. The gender earnings gap measures the impact of the three combined factors (the average hourly earnings, the monthly average number of hours paid and the employment rate) on the average earnings of all women of working age compared with men. In 2018, the gender overall earnings gap was 36.2% in the EU. Across Member States, the gender overall earnings gap varied significantly (from 20.4% in Lithuania and Portugal to 44.2% in Austria).

The current gender-unbalanced situation can be explained by causes that are related to specific gender conditions of the supply and demand in the labor market. Regarding the causes that produce a low demand for women in some professions, it is important to refer to different kinds of discrimination that originated in the decisions made by the employers that are biased in favor of men relative to women. Some kinds of discrimination are based on the fact that most employers are men and thus it is possible that the men are more likely to prefer men as employees or coworkers (taste-based discrimination). The taste-based discrimination model was first proposed by Becker (1957, 1971) and Schelling (1971). Given that in the past the female labor force has been disproportionately low with respect to the male one, employers have had more experience with male employees and thus have more information about the characteristics of male labor force (statistical discrimination). The theory of statistical discrimination was pioneered by Arrow (1973), Spence (1973) and Phelps (1972).

This paper focuses only on the factors that affect the female labor supply that originate in the individual's commitment to providing unpaid care work, that is "All unpaid services provided by individuals within a household or community for the benefit of its members, including care of persons and domestic work. Common examples include cooking, cleaning, collecting water and fuel and looking after children, older persons and persons with illness or disabilities. Voluntary community work that supports personal or household care, such as community kitchens or childcare, are also forms of unpaid care work" as defined by the United Nations (UN Women 2022). The United Nations report also claims that unpaid care work is basically provided by women: "Women and girls have disproportionate responsibility for unpaid care and domestic work; globally they spend three times as much time on this work as do men and boys. Unpaid care work is one of the main barriers preventing women from moving into paid employment and better quality jobs.".

Unpaid care work is absolutely necessary for a wellbeing of the society (Folbre 2001). The care, nurture and education of children is the basis for the growth of the economy and the evolution of the society. The care of elders is becoming more important over time. Maternity is unconditionally needed to the survival of our societies. Thus, it is of great importance to study the role of unpaid care work in the economy. Unpaid care work takes up a great amount of time and its responsibility falls disproportionately on women (Samman et al. 2016). The individuals, whether male or female, that assume this responsibility will experience a restriction in the quantity and quality of their labor supply. This restriction can be thought of in terms of the amount of time that they can offer to their professional activities. The decision of how much time one should devote to unpaid care work relative to professional work is complicated for most people since both activities are considered as very relevant or even necessary (Folbre 2001). This paper aims at analyzing the implications of such decisions. By making the economic costs and benefits of care

Economies **2024**, 12, 101 3 of 18

provision more visible, we might be able to find mechanisms that alleviate the gender gaps observed in the labor market.

This paper describes a formal model of individual choice in which the choice variable is the amount of time that an individual decides to devote to the labor market. This decision affects the total amount of income that an individual may obtain and also the total amount of cost that an individual has to bear depending on her or his previous commitments to unpaid care work. I assume that different individuals have different propensities to engage in family care; thus, the society exhibits a variety of costs associated to them. However, in the real world it is evident that in general women are very much committed to such activities while men are much less committed. Thus, women are expected to be more affected by the costs originating in family commitments.

The results offer a specific explanation of the lower labor market participation of women, of the salary gap that is observed in the labor market due to unpaid care work commitments and it also shows the reduction in overall welfare suffered by the female sector of the society due to both the lower income levels and the higher costs from family commitments. The results obtained also imply that the effects of unpaid care work commitments cannot be avoided. And since they are indispensable for the wellbeing of the society, the only possible way to diminish the discrimination against women and to attain a more gender-balanced labor market is to induce men to commit to take family responsibilities.

The next section introduces the formal model. Section 3 describes the optimal individual choices of labor participation. Section 4 analyzes the effects of discrimination in the society and compares the effects of discrimination between two sections of the society: female and male. Section 5 offers a discussion of the implications of the results which includes some policy implications derived from the analysis of the formal model. Finally, Section 6 contains some concluding remarks and possible extensions.

2. The Model

This model considers the choice of an individual about her or his participation in the labor market. Let $s \in [0,1]$ denote the corresponding choice variable and it is to be interpreted as follows: larger values of s denote higher levels of labor market participation, which could be thought of as time devoted to work but may include other considerations such as the quality of the time devoted to work, or the quality of the jobs that may be attained. And of course, higher values of the choice variable correspond to higher values of the individual's labor income.

Individuals are characterized by their type. The type of an individual is a measure of her or his level of commitment to activities that are related to the labor market relative to the individual's commitment to unpaid care work. Let $t \in [0,1]$ denote the individual type and it is to be interpreted as the amount of time that an individual can devote to professional activities that is free of the cost derived from family commitments. This implies that lower values of t refer to individuals with strong family commitments, and higher values of t correspond to individuals with few family commitments. For an individual of type t, devoting an amount of time larger than t to the labor market implies a reduction in her or his commitment to family activities, and it represents a reduction in her or his welfare. In particular, if an individual of type t chooses to dedicate an amount of time t0 the labor market, this individual does not suffer any additional cost. However, if an individual of type t1 to the labor market, this individual will suffer a cost derived from her or his family restrictions.

This cost can be thought of as the amount of income that has to be devoted to pay someone else for the provision of care, or it can be thought as the loss produced due to the diminished care on other family members. It is reasonable to think that this cost is small for small deviations from the type because it produces a small distortion in the household organization. However, it is also reasonable to think that it becomes increasingly large for larger deviations from the type, because large distortions in the household organization may require drastic solutions. In order to formalize this argument I assume that this cost

Economies 2024, 12, 101 4 of 18

is represented by a convex function of the distance between the individual's type and the chosen level of dedication to labor market activities. Let $C(s) = \frac{\gamma}{2}(t-s)^2$ and $\gamma > 0$ denote the cost of labor market participation for an individual of type t.

Individuals obtain their income from their participation in the labor market. We assume that this income increases with the individual's level of participation in the labor market. The individual income is represented by $W(s) = \omega s$ where $\omega > 0$ denotes the income per unit of time devoted to the labor market that an individual may obtain. In particular, ω also refers to the total income that the individual obtains when he or she decides on full-time participation in the labor market, that is s=1.

The overall individual's welfare is measured by a utility function that combines the individual's income and the cost that he or she has to bear and it is represented by the following function:

 $U(s) = W(s) - C(s) = \omega s - \frac{\gamma}{2}(t - s)^2$

Larger values of the cost parameter relative to the income parameter imply larger reductions in welfare. Thus, $\frac{\gamma}{\omega}$ can be interpreted as a measure of the level of discrimination that an individual suffers due to her or his family commitments. Since the main trade-off is determined by the relationship between the parameters ω and γ , without loss of generality I normalize the wage to be $\omega=1$. This implies that the interpretation of the parameter γ is the weight of the cost relative to the maximal wage and therefore we can consider γ as the discrimination index derived from unpaid care work commitments.

I analyze the individual's optimal choice of labor market participation as a function of its type and of the discrimination index γ . In particular, I am interested in the effects that γ may have on the optimal individual choice and how it affects her or his labor income and total welfare. I also analyze the effects of the discrimination index on the overall society by considering the aggregate levels of labor market participation, income and welfare that the society may obtain, and also the aggregate level of cost that the society has to bear. Finally and most importantly, I analyze these effects on two segregated sections of the society: female and male. This distinction is important according to the empirical data since it is the female section of the society that bears most of costs derived from the family commitments or more generally from the unpaid care work. Thus, I compare the effects of discrimination between the female and male sections of the society.

3. The Optimal Individual Choice

In order to find the optimal level of labor market participation for an individual of type $t \in [0,1]$, I solve the maximization problem of her or his utility function with respect to the choice variable $s \in [0,1]$. This result is stated in the next proposition.

Proposition 1. The optimal level of labor market participation of an individual of type t is

$$s^*(t,\gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \le \frac{\gamma - 1}{\gamma} \\ 1 & if \quad t \ge \frac{\gamma - 1}{\gamma} \end{cases}$$

All proofs are relegated to Appendix A.

Notice that the optimal level of labor market participation for all types is positive; thus, everyone chooses to devote some time to work. Individuals with larger types decide to devote more time to the labor market and only the highest types decide to fully participate in the labor market, that is $s^*(t,\gamma)=1$.

Notice that $\frac{\gamma-1}{\gamma}$ increases with γ . Thus, larger values of γ imply smaller proportions of individuals that choose a full labor market participation. For $0<\gamma<1$, we have that all individuals decide to fully participate in the labor market and obtain the maximal wage, and thus they decide to bear all the cost that is required for it. Even if in this case all types obtain the same income, they do not obtain the same utility, because each individual has

Economies **2024**, 12, 101 5 of 18

to bear a different cost. Thus, we also have some discrimination: lower types bear higher costs and obtain lower utility levels.

However, full female participation in the labor market is not what we observe happening in the real world. Since we aim at explaining the existing gender-unbalanced features of the real labor market for most of the paper we consider that $1 < \gamma$. In this case, we have that only some individuals, those with higher types, decide to fully participate in the labor market and obtain the maximal wage. Instead individuals with lower types opt for partial labor market participation, which is exactly what the empirical results show. Therefore, for most of the paper we assume that $1 < \gamma$, which implies that the commitment to unpaid work is considered as very important to all individuals relative to the labor income. This assumption is relaxed towards the end of the paper, when we consider possible policies that may alleviate the existing gender gaps in the real labor market.

Assumption 1. $\gamma > 1$.

Proposition 1 highlights the twofold relevance of the discrimination in the individual's optimal decision about labor market participation: it determines which part of the society decides to participate full time in the labor market and it also explains the distribution of the part-time jobs derived from the individuals' optimal choice.

On the one hand, the ratio $\frac{\gamma-1}{\gamma}$ characterizes all the individual types whose decision on how much to participate in the labor market is negatively affected by the cost bearing of family commitments. All types with values below $\frac{\gamma-1}{\gamma}$ opt for a partial labor market participation and thus suffer from the effects of the discrimination because they are not able to obtain the full income. While all types with values above $\frac{\gamma-1}{\gamma}$ opt for a full labor market participation and obtain the full income from it. Notice that larger levels of discrimination imply larger sets of types that decide on a partial labor market participation.

On the other hand, the discrimination index also affects the level of participation in the labor market for those individuals that opt for partial labor market participation: their optimal choice of labor market participation decreases with γ . Thus, larger values of γ imply that most individuals decide to work less.

The gains and losses produced on the individuals' total welfare are represented by the individual's income, cost and utility computed at the optimal level of labor market participation given by proposition 1 and they are stated in the next proposition.

Proposition 2. The optimal income obtained by an individual of type t is

$$W^*(t,\gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \le \frac{\gamma - 1}{\gamma} \\ 1 & if \quad t \ge \frac{\gamma - 1}{\gamma} \end{cases}$$

The optimal cost supported by an individual of type t is

$$C^*(t,\gamma) = \begin{cases} \frac{1}{2\gamma} & \text{if } t \leq \frac{\gamma-1}{\gamma} \\ \frac{\gamma}{2}(1-t)^2 & \text{if } t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

The optimal utility obtained by an individual of type t is

$$U^*(t,\gamma) = \begin{cases} \frac{1}{2\gamma} + t & if \quad t \le \frac{\gamma - 1}{\gamma} \\ 1 - \frac{\gamma}{2} (1 - t)^2 & if \quad t \ge \frac{\gamma - 1}{\gamma} \end{cases}$$

Observe that the individual income and the individual utility received by all types are positive and they both increase with the type for those individuals that opt for a partial labor market participation, who in turn obtain only a fraction of the full income. Individuals with a larger type $\left(t \geq \frac{\gamma-1}{\gamma}\right)$ decide to fully participate in the labor market; they obtain

Economies **2024**, 12, 101 6 of 18

the maximal wage ω , and their individual utility increases with their type. However, this increase becomes smaller for larger values of t.

The individual cost supported by all types is also positive. The cost supported by the individuals that opt for a full labor market participation decreases with her or his type, and this reduction becomes smaller for larger types. And all those types that opt for a partial labor market participation end up supporting the exact same total amount of individual cost (see Figure 1).

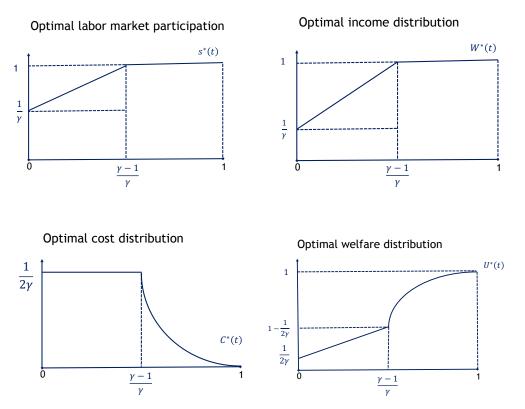


Figure 1. Optimal individual decisions.

Increases in the discrimination index γ produce decreases in the individual income, in the individual cost and in the individual utility for those types that choose a partial labor market participation. Notice that on the one hand, they decide to work less when the discrimination index increases, and that is why they obtain a lower income. However, their individual cost also increases and overall it produces a large reduction in individual utility. The individual income for the larger types, those that choose a full labor market participation, is not affected by increases in the cost parameter. However, their individual cost increases and thus their individual utility also decreases. Therefore, increases in the value of γ imply lower utility for all types. The formal description of the comparative statics is included in the proof of the proposition contained in Appendix A.

4. Discrimination in Society

In this section, we present and discuss the main results. First, we focus on the aggregated economic variables for the society and then we compare these variables for the two sections of the society: female and male types. Suppose that the individual types in the society are distributed according to a uniform probability distribution function over the support [0,1]. We compute the labor market participation of the society $T^*(\gamma)$, total income of the society $TW^*(\gamma)$, the total cost derived from labor participation $TC^*(\gamma)$ and the total utility obtained $TU^*(\gamma)$ as stated in the next proposition.

Economies **2024**, 12, 101 7 of 18

Proposition 3. The labor market participation, total income, total cost and total utility for the society are:

$$T^*(\gamma) = TW^*(\gamma) = \frac{\gamma^2 + 2\gamma - 1}{2\gamma^2}$$
$$TC^*(\gamma) = \frac{3\gamma - 2}{6\gamma^2}$$
$$TU^*(\gamma) = \frac{3\gamma^2 + 3\gamma - 1}{6\gamma^2}$$

Notice that since we have normalized the wage to be 1 we now have that the measure of the labor market participation is equal to the measure of the total income. We find that the labor market participation and total income decrease with γ because lower types choose to dedicate less time to work when its associated cost becomes more expensive. Total utility decreases with γ because an increase in the cost parameter reduces the individual utility for all types: lower types work less and higher types pay a higher cost.

The total cost may increase or decrease with γ depending on the value of the discrimination. Recall that the individual cost for lower types decreases with the cost parameter while the individual cost for higher types increases with the cost parameter. Overall, we have that the total cost increases with γ if the discrimination is very low $\left(1 < \gamma < \frac{4}{3}\right)$ because in this case most individuals decide to fully participate in the labor market and thus their individual cost increases with γ . However, the total cost decreases with γ if the discrimination is more severe $\left(\gamma > \frac{4}{3}\right)$ because in this case more individuals opt for a part-time participation in the labor market and they reduce their participation when the cost parameter increases. This implies that the reduction in cost due to the reduction in part-time labor market participation compensates for the increase in cost that the larger types suffer. The reason is twofold: the cost that higher types have to pay is relatively small compared to that of lower types, and for large values of the discrimination index the proportion of types that decide to work full time is smaller.

Next, suppose that the female types are represented by those t such that $0 \le t \le \frac{1}{2}$ and the male types are represented by those t such that $\frac{1}{2} \le t \le 1$. The reason is that it is a fact that women are much more committed to unpaid care work than men. I compute the labor market participation, the total income, the total cost and the total utility evaluated at the optimal individual choice for female types $(T^{F*}(\gamma), TW^{F*}(\gamma), TC^{F*}(\gamma), TU^{F*}(\gamma))$ and for male types $(T^{M*}(\gamma), TW^{M*}(\gamma), TC^{M*}(\gamma), TU^{M*}(\gamma))$ separately. I analyze how these variables are affected by changes in the discrimination index γ . Then, I compare the results obtained for each section of society in order to evaluate the extent of the effect of the gender discrimination in the labor market over participation, income, cost and utility. The next proposition shows the results obtained for the aggregated economic variables corresponding to the female types. Notice that for this analysis I have to consider two cases depending on the value of the parameter γ . For $\gamma > 2$, I have that all female types decide to work part time while for $\gamma < 2$ I have that some female types decide to work part time and some of them decide to work full time.

Proposition 4. The labor participation, total income, total cost and total utility for female types are:

$$T^{F*}(\gamma) = TW^{F*}(\gamma) = \begin{cases} \frac{2\gamma - 1}{2\gamma^2} & \text{if} \quad \gamma \leq 2\\ \frac{4+\gamma}{8\gamma} & \text{if} \quad \gamma \geq 2 \end{cases}$$
$$TC^{F*}(\gamma) = \begin{cases} \frac{24\gamma - 16 - \gamma^3}{48\gamma^2} & \text{if} \quad \gamma \leq 2\\ \frac{1}{4\gamma} & \text{if} \quad \gamma \geq 2 \end{cases}$$

Economies **2024**, 12, 101 8 of 18

$$TU^{F*}(\gamma) = \left\{ egin{array}{ll} rac{\gamma^3 + 24\gamma - 8}{48\gamma^2} & if \quad \gamma \leq 2 \\ rac{2+\gamma}{8\gamma} & if \quad \gamma \geq 2 \end{array}
ight.$$

Recall that for $\gamma \geq 2$ all female types work part time, while for $\gamma \leq 2$ some of them decide to work full time. In all cases, the total income and the total utility for female types decrease with the discrimination index, as we found in the overall society. For $\gamma \geq 2$, we have that the total cost for female types decreases with γ , because as we have seen before part-time workers decrease their labor market participation when the cost parameter increases; thus, in this case the total cost is reduced. For $\gamma \leq 2$, we have that some female types work full time. In this case, the, comparative statics for the female types resembles very much that of the society overall: the total cost for female types decreases with γ for relatively small values of the discrimination index $(\gamma < \overline{\gamma} < \frac{4}{3})$ and it increases with γ for larger values of the discrimination index. The only difference is that for values of γ such that $\overline{\gamma} < \gamma < \frac{4}{3}$ the total cost of the female types increases with γ while the total cost of the society decreases with γ . The reason is that for values of γ such that $\overline{\gamma} < \gamma < \frac{4}{3}$ the proportion of female types that decide to work full time relative to the female population is not large enough and thus the total cost for the female types decreases with the cost of the majority because of part-time workers that decide to work less. While for the same parameter values, the proportion of full-time workers in the society is large enough relative to the total population and thus the total cost of the society increases.

Now I replicate the previous analysis for the male types. Again, for this analysis I have to consider two cases depending on the value of the parameter γ . For $\gamma < 2$, I have all male types deciding to work full time while for $\gamma > 2$ I have that some male types decide to work part time.

Proposition 5. The labor participation, total income, total cost and total utility for male types are:

$$T^{M*}(\gamma) = TW^{M*}(\gamma) = \begin{cases} \frac{1}{2} & \text{if} \quad \gamma \leq 2\\ \frac{3\gamma^2 + 4\gamma - 4}{8\gamma^2} & \text{if} \quad \gamma \geq 2 \end{cases}$$
$$TC^{M*}(\gamma) = \begin{cases} \frac{\gamma}{48} & \text{if} \quad \gamma \leq 2\\ \frac{3\gamma - 4}{12\gamma^2} & \text{if} \quad \gamma \geq 2 \end{cases}$$
$$TU^{M*}(\gamma) = \begin{cases} \frac{24 - \gamma}{48} & \text{if} \quad \gamma \leq 2\\ \frac{9\gamma^2 + 6\gamma - 4}{24\gamma^2} & \text{if} \quad \gamma \geq 2 \end{cases}$$

In this case, we also have to consider two different situations. When discrimination is low ($\gamma \leq 2$), all male types decide to work full time and the total income is not affected by changes in γ . Total cost increases with γ because all male types are working full time, and therefore their total utility decreases with γ . Of course, increases of γ beyond 2 imply that some male types decide to reduce their labor market participation.

For higher values of the discrimination index $(\gamma > 2)$, some male types decide to work part time and in this case we have that total income and total utility for male types decrease with γ while total cost only decreases with γ for large values of the discrimination index $(\gamma > \frac{8}{3})$. The only difference with respect to the comparative statics of the society is found for values of γ such that $\frac{4}{3} < \gamma < \frac{8}{3}$. For these values, the total cost of the male types increases with γ while the total cost of the society decreases with γ . The reason is that for values of γ such that $\frac{4}{3} < \gamma < \frac{8}{3}$ the proportion of male types that decide to work full time relative to the male population is large enough and thus the total cost for the male types increases with the cost of the majority because full-time workers have to pay a larger cost. While for the same parameter values, the proportion of full-time workers in the society is not large enough relative to the total population and thus the total cost of the society decreases.

Economies **2024**, 12, 101 9 of 18

Now we compare the aggregated economic variables found before for female and male types. The next proposition illustrates the extent of the discrimination suffered by female types overall in the labor market by stating the shares of total income, total cost and total utility that correspond to the female types relative to the male types.

Proposition 6. The shares of labor market participation, total income, total cost and total utility for female types relative to male types decrease with γ and they are bound by:

$$\begin{split} \frac{T^{F*}(\gamma)}{T^{M*}(\gamma)} &= \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} \in \left[\frac{1}{3}, 1\right] \\ &\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} \in \left[1, 7\right] \\ &\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} \in \left[\frac{1}{3}, \frac{17}{23}\right] \end{split}$$

Total income and total utility for the female types are always smaller than those of the male types because the ratios are always smaller than 1. Labor market participation and total income for the female types approaches those of the male types when discrimination becomes very low ($\gamma \to 1$). This implies that a reduction in the discrimination index produces a reduction in the gender wage gap and more gender-balanced labor market participation (see Figure 2). The gender gap may even vanish completely if the discrimination index is low enough. However, with respect to the gender utility gap the implications are not as optimistic. Even though this gap is reduced with decreases in the discrimination index, it is not possible to eliminate it completely. That is, as long as there are costs associated with the unpaid care jobs, even if they are very small, there will be a gap in the utility obtained by the female and male sections of society. In fact, the share of total utility for female types is always smaller than $\frac{3}{4}$ of the total utility for males types. This is due to the fact that the total cost associated with the unpaid care jobs that is mostly borne by the female types represents a magnitude of many times that of the total cost borne by the male types for all values of the discrimination index (see Figure 3).

Higher discrimination implies lower ratios of total income and total utility for the female types, but they are always above 1/3. At the same time, the ratio of total cost borne by the female types decreases with the discrimination index, because lower types decide to work less when γ increases. The total cost borne by females and males are equal (and equal to zero) when the discrimination index reaches its maximum. However, at this point the inequality in terms of total utility is maximal. This is due to the fact that maximal discrimination implies maximal inequality in terms of income because when the salary is so low relative to the cost everybody has an incentive to offer exactly its own type and pay no cost. This implies, of course, that all male types obtain a higher income than female types.

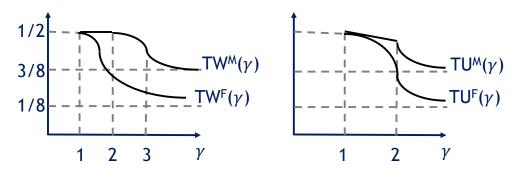


Figure 2. Comparing total income and total utility for female and male types.

Economies 2024, 12, 101 10 of 18

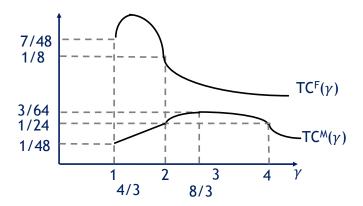


Figure 3. Comparing total cost for female and male types.

Until now, I have assumed that $\gamma > 1$; that is, the commitment to activities not related to the labor market is considered as very important to all individuals relative to the salary that they may obtain in the labor market. This assumption has allowed me to obtain significant results in terms of being able to explain the gender variations observed in the labor market. In particular, I have characterized the set of individuals that decide to work part time and those that decide to work full time. If instead we consider that the discrimination index is much lower, such that $0 < \gamma < 1$, we have that all individuals prefer to bear all the cost that allows them to obtain the maximal income. Therefore, they all decide on full participation in the labor market and we have that $s^*(t, \gamma) = 1$ for all types. This is not what we observe in the real world, and this is the reason we have assumed $\gamma > 1$ so far since our goal was a description of the real-world facts. However, if instead of trying to explain the observed gender variation in labor market participation in the real world now we want to see how much of that gender variation can be reduced through a reduction in the discrimination index, we have to consider values of γ that go below 1. The next proposition shows that if we allow for lower indices of discrimination we find that the inequality between female and male types does not disappear.

Proposition 7. If
$$0 < \gamma < 1$$
, we have that $\frac{T^{F*}(\gamma)}{T^{M*}(\gamma)} = \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = 1$, $\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = 7$, and $\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{24-7\gamma}{24-\gamma} < 1$ with $\frac{\partial \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)}}{\partial \gamma} < 0$ and $\frac{TU^{F*}(0)}{TU^{M*}(0)} = 1$.

This proposition states that for lower values of the discrimination index we have that the gender wage gap disappears and labor market participation becomes gender balanced. The gender utility gap decreases when γ becomes smaller; however, it only disappears when $\gamma=0$. Indeed, the burden of the personal cost derived from family commitments and other unpaid care jobs cannot be avoided and even though when we push down the discrimination index we manage to balance labor market participation and the salary gap, the utility difference between females and males remains significantly different. This is because our discrimination index relates the magnitudes of the cost derived from commitments to unpaid care jobs and the labor market salary. And as long as we have a positive cost we will have a positive discrimination.

5. Implications of the Results

This paper has analyzed an individual choice model about labor market participation that includes a specific cost that individuals have to bear if they are committed to unpaid care work or other types of non-professional activities. This analysis shows that the presence of this cost produces a great distortion in the supply of labor in society which in turn produces a wage gap and a much larger utility gap between the types that are committed to unpaid care work and those that are not.

Economies **2024**, 12, 101 11 of 18

The general conclusion is that since women devote more time to unpaid care work and thus it is more costly for them to devote time to professional activities; that implies that they obtain lower wages, more part-time jobs, less promotions and so on. Thus, the commitment to unpaid care work is one of the clear causes of the observed gender *wage gap* and *glass ceiling*.

Since it is a fact that unpaid care work is of vital importance for the proper development of a society, the discrimination of the individuals that have family committments has to be considered as a very relevant economic and social problem. This paper has made the economic costs and benefits of unpaid care provision more visible so that we might be able to find mechanisms that should allow us to change the currently gender-unbalanced labor market.

The present paper has shown that there does not exist a solution that solves this problem completely, but there are at least two ways to alleviate it. On the one hand, a reduction in the cost that individuals that are committed to family care have to bear implies a reduction in gender discrimination and at the same time an enhancement of efficiency for the overall society. On the other hand, a more gender-balanced distribution of the family commitments should imply a more equitable distribution of the costs derived from the unpaid care work and thus it would reduce the gender gaps observed in the labor market. In what follows, we describe the effects of these two mechanisms.

The reduction in the cost that individuals have to bear because of their family commitments can be achieved, for instance, with the provision of care by the public sector. In particular, the increase in child benefits and the reduction in pre-primary school costs are expected to produce a positive effect. Gammage et al. (2019) show that increasing the government expenditure on pre-primary education by 1 percentage point of GDP can reduce the labor force participation gap by about 10 points. In fact, the solutions proposed by the OECD for developing countries are in the line of increasing the offer of public services, infrastructures and social protection policies and of promoting the shared responsibility within the household (OECD 2019). In addition, the introduction of flexible work schedules (Goldin 2014) would also reduce the current effects of discrimination.

A more gender-balanced distribution of family commitments can be achieved by increasing the incentives to share the jobs related to reproduction. This may be a difficult endeavor because of the differences between male and female preferences and attitudes (Bertrand et al. 2010; Gneezy et al. 2003; Iriberri and Rey-Biel 2021) and especially the female preference for maternity or their preference for dedicating time and effort to non-professional activities related to unpaid care work: caring for household members and doing domestic chores (Azmat and Ferrer 2019; Kleven et al. 2019).

Paternity leave has been implemented in many countries (Patnaik 2019) with the intention of inducing the share of the jobs related to reproduction but again the effects have been found to have a negligible to a small positive impact (Olivetti and Petrongolo 2017; González and Zoaby 2021). In particular, this measure did not affect the parents' longer-term leave taking but only delayed higher-order births (Farré and González 2019). Thus, its effectiveness in increasing the long-term involvement of fathers in childcare and the household work has not been confirmed.

Explicit education about the relevance of the care and nurture issues is important to try in order to change the existing social values and social norms. Bertrand (2019) finds that childhood exposure to a non-traditional family (a working married mother, a married mother that is the primary breadwinner or a non-married mother) affects gender role attitudes in young adulthood. And regarding the transmission of values, Farré and Vella (2013) show that a mother's attitudes have a statistically significant effect on those of her children, while Fernandez et al. (2004) indicate that wives of men whose mothers worked are themselves significantly more likely to work.

However, it is not clear that these measures will completely correct the currently unbalanced gender labor market. In particular, extended maternity leave mandates have been found to increase female labor force participation at the cost of lower wages and less Economies **2024**, 12, 101 12 of 18

presence of women in high-profile occupations and they induce a more traditional division of tasks within the family (Farré 2016). Thus, it is very important to find a different way to compensate women for the costs induced by maternity issues.

6. Concluding Remarks

The analysis proposed in this paper can be extended in several ways. One important way is to consider factors that affect the demand of labor that produce additional gender discrimination. Some of them have been mentioned in the introduction: taste-based discrimination and statistical discrimination. In addition, the possibility of maternity leaves increases the risk and the expected costs associated with hiring women relative to those of hiring men. These larger costs also induce employers to hire men rather than women (Bertrand and Mullainathan 2014; Goldin 2014; Bertrand and Duflo 2017). The smaller labor demand for women also produces a reduction in their labor supply: since women have a lower probability of being selected, their relative benefits of pursuing a job search are smaller and their incentives to participate in the labor market are also smaller.

Social norms may be considered as an additional cause of both lower supply and demand of females in the labor market. The fact that it is more naturally and more generally accepted that males participate in the labor market than females is one such social norm that negatively affects women's participation in the labor market because it induces employers (mostly men) to be more reluctant to hire women and at the same time it induces women to be more reluctant to search for jobs (Farré and Vella 2013; Bertrand 2019; Fernandez et al. 2004).

Another important extension is to introduce imperfect information in hiring that affects the individual's decision to look for a job and that has a negative effect especially on women in the labor market because of their lower expected probability of being hired. The present model can also be extended to include not only the individual choice, but also the family choice, as in (Francois 1998). In particular, the analysis of the family choices about the labor market could help us to better understand the possible effects of paternity leave over time. These extensions would complement the present study and offer a more complete analysis of gender discrimination in the labor market. Finally, the current model and its possible extensions could be replicated over time. With a repeated version of the basic model, it would be possible to figure out the dynamics associated with the discrimination in the labor market.

Additional possible extensions are related to the generalization of some of the main assumptions of the present formal model. If instead of a linear income function we consider a more general income function of the time or effort devoted to work our results may be affected quantitatively but not qualitatively. Including a fixed cost in the model may induce some individual types to decide not to participate in the labor market and this is a feature that may be desirable because it is what we observe in the real world. Adding a fixed wage in the current model may reduce labor market participation and also the total cost borne by the society overall.

Finally, in order to obtain the aggregated economic variables, we have assumed a uniform distribution of types. The main reason behind this choice is that it is not clear what kind of distribution of costs related to family commitments exists in the real world. If the empirical literature could obtain a specific shape for such a distribution, it would be interesting to apply it to our model to obtain more accurate predictions for the effects of the discrimination in the labor market. Up to our knowledge at this point, there are some studies that have investigated this question but the results are still not conclusive (Beneria 1999).

Funding: This research was funded by Generalitat de Catalunya grant number 2021-SGR-00416; from the Severo Ochoa Programme for Centres of Excellence in R&D (Barcelona School of Economics CEX2019-000915-S), funded by MCIN/AEI/10.13039/501100011033; and from grant PID2021-126200NB-I00 funded by MCIN/AEI/10.13039/501100011033 and by ERDF A way of making Europe.

Economies **2024**, 12, 101 13 of 18

Informed Consent Statement: Not applicable.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Conflicts of Interest: The author declares no conflict of interests.

Appendix A

Proof of Proposition 1. The optimization of $U(s)=s-\frac{\gamma}{2}(t-s)^2$ with respect to $s\in[0,1]$ produces a first-order condition given by $\frac{\partial U(s)}{\partial s}=1+\gamma(t-s)=0$ which implies that the optimal value of s is given by $s^*(t,\gamma)=\frac{1}{\gamma}+t$. Notice that the second-order condition $\frac{\partial^2 U(s)}{\partial s^2}=-\gamma<0$ is always satisfied.

We have that $s^*(t,\gamma)=\frac{1}{\gamma}+t>0$ for all types and for all parameter values. And we also have that $s^*(t,\gamma)=\frac{1}{\gamma}+t\leq 1$ if and only if $t\leq \frac{\gamma-1}{\gamma}$. This implies that for $t>\frac{\gamma-1}{\gamma}$ we must have $s^*(t,\gamma)=1$.

Thus

$$s^*(t,\gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \le \frac{\gamma - 1}{\gamma} \\ 1 & if \quad t \ge \frac{\gamma - 1}{\gamma} \end{cases}$$

Proof of Proposition 2. Given
$$s^*(t,\gamma) = \left\{ \begin{array}{ccc} \frac{1}{\gamma} + t & if & t \leq \frac{\gamma-1}{\gamma} \\ 1 & if & t \geq \frac{\gamma-1}{\gamma} \end{array} \right.$$

we have that
$$W^*(t,\gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \le \frac{\gamma - 1}{\gamma} \\ 1 & if \quad t \ge \frac{\gamma - 1}{\gamma} \end{cases}$$
$$C^*(t,\gamma) = \begin{cases} \frac{1}{2\gamma} & if \quad t \le \frac{\gamma - 1}{\gamma} \\ \frac{\gamma}{2}(1 - t)^2 & if \quad t \ge \frac{\gamma - 1}{\gamma} \end{cases}$$
$$U^*(t,\gamma) = \begin{cases} \frac{1}{2\gamma} + t & if \quad t \le \frac{\gamma - 1}{\gamma} \\ 1 - \frac{\gamma}{2}(1 - t)^2 & if \quad t \ge \frac{\gamma - 1}{\gamma} \end{cases}$$

Comparative statics:

Comparative statics.
$$\frac{\partial W^*(t,\gamma)}{\partial t} = \begin{cases} 1 & \text{if} \quad t \leq \frac{\gamma-1}{\gamma} \\ 0 & \text{if} \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial C^*(t,\gamma)}{\partial t} = \begin{cases} 0 & \text{if} \quad t \leq \frac{\gamma-1}{\gamma} \\ -\gamma(1-t) & \text{if} \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial U^*(t,\gamma)}{\partial t} = \begin{cases} 1 & \text{if} \quad t \leq \frac{\gamma-1}{\gamma} \\ \gamma(1-t) & \text{if} \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial W^*(t,\gamma)}{\partial \gamma} = \begin{cases} -\frac{1}{\gamma^2} & \text{if} \quad t \leq \frac{\gamma-1}{\gamma} \\ 0 & \text{if} \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial C^*(t,\gamma)}{\partial \gamma} = \begin{cases} -\frac{1}{2\gamma^2} & \text{if} \quad t \leq \frac{\gamma-1}{\gamma} \\ \frac{1}{2}(1-t)^2 & \text{if} \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial U^*(t,\gamma)}{\partial \gamma} = \begin{cases} -\frac{1}{2\gamma^2} & \text{if} \quad t \leq \frac{\gamma-1}{\gamma} \\ -\frac{1}{2}(1-t)^2 & \text{if} \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

Proof of Proposition 3.

$$T^*(\gamma) = TW^*(\gamma) = \int_0^1 W^*(t; \gamma) dt = \int_0^{\frac{\gamma - 1}{\gamma}} \frac{1}{\gamma} + t dt + \int_{\frac{\gamma - 1}{\gamma}}^1 dt =$$

Economies **2024**, 12, 101 14 of 18

$$\begin{split} & \left[\frac{1}{\gamma}t+\frac{t^2}{2}\right]_0^{\frac{\gamma-1}{\gamma}}+[t]_{\frac{\gamma-1}{\gamma}}^1=\frac{\gamma^2+2\gamma-1}{2\gamma^2} \\ & TC^*(\gamma)=\int_0^1C^*(t;\gamma)dt=\int_0^{\frac{\gamma-1}{\gamma}}\frac{1}{2\gamma}dt+\int_{\frac{\gamma-1}{\gamma}}^{\frac{\gamma}{2}}\frac{\gamma}{2}(1-t)^2dt=\\ & \left[\frac{1}{2\gamma}t\right]_0^{\frac{\gamma-1}{\gamma}}+\left[-\frac{\gamma}{6}(1-t)^3\right]_{\frac{\gamma-1}{\gamma}}^1=\frac{3\gamma-2}{6\gamma^2} \\ & TU^*(\gamma)=1-\frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2-\frac{1}{2\gamma^2}\left(\gamma-\frac{2}{3}\right)=\frac{3\gamma^2+3\gamma-1}{6\gamma^2} \\ & \text{Comparative statics:} \\ & \frac{\partial TW^*(\gamma)}{\partial \gamma}=\frac{1}{2}\frac{(2\gamma+2)2\gamma-4\left(\gamma^2-1+2\gamma\right)}{\gamma^3}=\frac{-2(\gamma-1)}{\gamma^3}<0 \\ & \frac{\partial^2TW^*(\gamma)}{\partial \gamma^2}=\frac{4\gamma-6}{\gamma^4}>0 \text{ iff } \gamma>\frac{3}{2} \\ & \lim_{\gamma\to\infty}TW^*(\gamma)=\frac{1}{2} \\ & \frac{\partial TC^*(\gamma)}{\partial \gamma}=\frac{1}{6}\frac{3\gamma-2(3\gamma-2)}{\gamma^3}=\frac{4-3\gamma}{6\gamma^3}>0 \text{ iff } \gamma<\frac{4}{3} \\ & \frac{\partial^2TC^*(\gamma)}{\partial \gamma^2}=\frac{\gamma-2}{\gamma^4}>0 \text{ iff } \gamma>2 \\ & \lim_{\gamma\to\infty}TC^*(\gamma)=0 \\ & \frac{\partial TU^*(\gamma)}{\partial \gamma}=\frac{2-3\gamma}{6\gamma^3}<0 \text{ iff } \frac{2}{3}<\gamma \text{ which always holds since } \gamma>1. \\ & \frac{\partial^2TU^*(\gamma)}{\partial \gamma^2}=\frac{\gamma-1}{\gamma^4}>0 \\ & \lim_{\gamma\to\infty}TU^*(\gamma)=\frac{1}{2}. \\ & \Box \end{split}$$

Proof of Proposition 4. We consider two separate cases depending on whether $\gamma \gtrless 2$.

Case 1: For $\gamma \le 2$ we have $\frac{\gamma - 1}{\gamma} \le \frac{1}{2}$ and:

Case 2: For $\gamma \geq 2$ we have $\frac{\gamma - 1}{\gamma} \geq \frac{1}{2}$ and:

$$T^{F*}(\gamma) = TW^{F*}(\gamma) = \int_0^{\frac{1}{2}} W^{F*}(t;\gamma) = \int_0^{\frac{\gamma-1}{\gamma}} \left(\frac{1}{\gamma} + t\right) dt + \int_{\frac{\gamma-1}{\gamma}}^{\frac{1}{2}} dt = \\ \left[\frac{1}{\gamma}t + \frac{t^2}{2}\right]_0^{\frac{\gamma-1}{\gamma}} + \left[t\right]_{\frac{\gamma-1}{\gamma}}^{\frac{1}{2}} = \frac{2\gamma-1}{2\gamma^2} \\ TC^{F*}(\gamma) = \int_0^{\frac{1}{2}} C^{F*}(t;\gamma) = \int_0^{\frac{\gamma-1}{\gamma}} \frac{1}{2\gamma} dt + \int_{\frac{\gamma-1}{\gamma}}^{\frac{1}{2}} \frac{\gamma}{2} (1-t)^2 dt = \\ \left[\frac{1}{2\gamma}t\right]_0^{\frac{\gamma-1}{\gamma}} + \left[-\frac{\gamma}{6}(1-t)^3\right]_{\frac{\gamma-1}{\gamma}}^{\frac{1}{2}} = \frac{24\gamma-16-\gamma^3}{48\gamma^2} \\ TU^{F*}(\gamma) = \frac{1}{2} - \frac{1}{2} \left(\frac{\gamma-1}{\gamma}\right)^2 - \frac{3\gamma-2}{6\gamma^2} + \frac{\gamma}{48} = \frac{\gamma^3+24\gamma-8}{48\gamma^2} \\ \text{Comparative statics:} \\ \frac{\partial TW^{F*}(\gamma)}{\partial \gamma} = \frac{1}{2\gamma^3} < 0 \\ \frac{\partial^2 TW^{F*}(\gamma)}{\partial \gamma^2} = \frac{2\gamma-3}{\gamma^4} > 0 \text{ iff } \gamma < \frac{3}{2} \\ \frac{\partial TC^{F*}(\gamma)}{\partial \gamma} = \frac{4-3\gamma}{6\gamma^3} - \frac{1}{48} > 0 \text{ iff } \gamma < \frac{3}{\gamma} < \frac{4}{3} \text{ because} \\ \text{if } \frac{4}{3} < \gamma \text{ we have that } \frac{\partial TC^{F*}(\gamma)}{\partial \gamma} = \frac{4-3\gamma}{6\gamma^3} - \frac{1}{48} < 0 \text{ iff } 0 < \gamma^3 + 24\gamma - 32 \\ \text{thus there is a } 1 < \overline{\gamma} < \frac{4}{3} \text{ such that } \frac{\partial TC^{F*}(\gamma)}{\partial \gamma} = \frac{1}{2\gamma^2} \left(\frac{4}{3\gamma} - 1\right) - \frac{1}{48} < 0 \text{ iff } \gamma > \overline{\gamma} \\ \frac{\partial^2 TC^{F*}(\gamma)}{\partial \gamma^2} = \frac{-12}{6\gamma^3} < 0 \\ \frac{\partial TU^{F*}(\gamma)}{\partial \gamma} = \frac{2-12}{6\gamma^3} + \frac{1}{48} < 0 \text{ iff } \gamma^3 - 24\gamma + 16 < 0 \\ \text{Since } \gamma^3 - 24\gamma + 16 \text{ is a decreasing function of } \gamma \text{ and it holds for } \gamma = 1 \text{ then it also holds for all } 1 < \gamma < 2. \\ \frac{\partial^2 TU^{F*}(\gamma)}{\partial \gamma^2} = \frac{\gamma-1}{\gamma^4} < 0. \\ \end{cases}$$

 $TW^{F*}(\gamma) = \int_0^{\frac{1}{2}} W^{F*}(t;\gamma) = \int_0^{\frac{1}{2}} \left(\frac{1}{\gamma} + t\right) dt = \left[\frac{1}{\gamma}t + \frac{t^2}{2}\right]_0^{\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{\gamma} + \frac{1}{4}\right) = \frac{4+\gamma}{8\gamma}$

Economies **2024**, 12, 101 15 of 18

$$TC^{F*}(\gamma) = \int_0^{\frac{1}{2}} C^{F*}(t;\gamma) = \int_0^{\frac{1}{2}} \frac{1}{2\gamma} dt = \left[\frac{1}{2\gamma} t\right]_0^{\frac{1}{2}} = \frac{1}{4\gamma}$$

$$TU^{F*}(\gamma) = \frac{1}{2} \left(\frac{1}{\gamma} + \frac{1}{4}\right) - \frac{1}{4\gamma} = \frac{1}{4} \left(\frac{1}{2} + \frac{1}{\gamma}\right) = \frac{2+\gamma}{8\gamma}$$
Comparative statics:
$$\frac{\partial TW^{F*}(\gamma)}{\partial \gamma} = -\frac{1}{2\gamma^2} < 0$$

$$\frac{\partial^2 TW^{F*}(\gamma)}{\partial \gamma^2} = \frac{1}{\gamma^3} > 0$$

$$\lim_{\gamma \to \infty} TW^{F*}(\gamma) = \frac{1}{8}$$

$$\frac{\partial TC^{F*}(\gamma)}{\partial \gamma} = -\frac{1}{4\gamma^2} < 0$$

$$\frac{\partial^2 TC^{F*}(\gamma)}{\partial \gamma^2} = \frac{1}{2\gamma^3} > 0$$

$$\lim_{\gamma \to \infty} TC^{F*}(\gamma) = 0$$

$$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} = -\frac{1}{4\gamma^2} < 0$$

$$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} = \frac{1}{2\gamma^3} > 0$$

$$\lim_{\gamma \to \infty} TU^{F*}(\gamma) = \frac{1}{8}.$$
Overall, we have that:
$$T^{F*}(\gamma) = TW^{F*}(\gamma) = \begin{cases} \frac{2\gamma-1}{2\gamma^2} & \text{if } \gamma \leq 2\\ \frac{4+\gamma}{8\gamma} & \text{if } \gamma \geq 2 \end{cases}$$

$$TC^{F*}(\gamma) = \begin{cases} \frac{24\gamma-16-\gamma^3}{48\gamma^2} & \text{if } \gamma \leq 2\\ \frac{1}{4\gamma} & \text{if } \gamma \geq 2 \end{cases}$$

$$TU^{F*}(\gamma) = \begin{cases} \frac{\gamma^3+24\gamma-8}{48\gamma^2} & \text{if } \gamma \leq 2\\ \frac{2+\gamma}{8\gamma} & \text{if } \gamma \geq 2 \end{cases}$$

$$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} < 0$$

$$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} < 0$$

$$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} < 0 \text{ off } \gamma > \overline{\gamma} \text{ for some } 1 < \overline{\gamma} < \frac{4}{3}$$

$$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} < 0.$$

Proof of Proposition 5. We consider two separate cases depending on whether $\gamma \ge 2$.

Case 1: For
$$\gamma \le 2$$
 we have $\frac{\gamma - 1}{\gamma} \le \frac{1}{2}$ and:
$$T^{M*}(\gamma) = TW^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} W^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} dt = [t]_{\frac{1}{2}}^{1} = 1 - \frac{1}{2} = \frac{1}{2}$$

 $TU^{M*}(\gamma) = \frac{3}{8} + \frac{\gamma - 1}{2\gamma^2} - \frac{3\gamma - 4}{12\gamma^2} = \frac{9\gamma^2 + 6\gamma - 4}{24\gamma^2}$

$$TC^{M*}(\gamma) = TW \quad (\gamma) = J_{\frac{1}{2}}W \quad (\gamma) = J_{\frac{1}{2}}ut = [t]_{\frac{1}{2}} = 1 - \frac{1}{2} - \frac{1}{2}$$

$$TC^{M*}(\gamma) = \int_{\frac{1}{2}}^{1}C^{M*}(\gamma) = \int_{\frac{1}{2}}^{1}\frac{\gamma}{2}(1-t)^{2}dt = \left[-\frac{\gamma}{6}(1-t)^{3}\right]_{\frac{1}{2}}^{1} = \frac{\gamma}{48}$$

$$TU^{M*}(\gamma) = \frac{1}{2} - \frac{\gamma}{48} = \frac{24-\gamma}{48}$$
Comparative statics:
$$\frac{\partial TW^{M*}(\omega,\gamma)}{\partial \gamma} = 0$$

$$\frac{\partial TC^{M*}(\omega,\gamma)}{\partial \gamma} = \frac{1}{48} > 0$$

$$\frac{\partial TU^{M*}(\omega,\gamma)}{\partial \gamma} = -\frac{1}{48} < 0.$$
Case 2: For $\gamma \geq 2$ we have $\frac{\gamma-1}{\gamma} \geq \frac{1}{2}$ and:
$$TW^{M*}(\gamma) = \int_{\frac{1}{2}}^{1}W^{M*}(\gamma) = \int_{\frac{1}{2}}^{\gamma-1}\left(\frac{1}{\gamma}+t\right)dt + \int_{\frac{\gamma-1}{\gamma}}^{1}dt = \left[\frac{1}{\gamma}t + \frac{t^{2}}{2}\right]_{\frac{\gamma}{2}}^{\frac{\gamma-1}{\gamma}} + [t]_{\frac{\gamma-1}{\gamma}}^{1} = \frac{3\gamma^{2}+4\gamma-4}{8\gamma^{2}}$$

$$TC^{M*}(\gamma) = \int_{\frac{1}{2}}^{1}C^{M*}(\gamma) = \int_{\frac{1}{2}}^{\frac{\gamma-1}{\gamma}}\frac{1}{2\gamma}dt + \int_{\frac{\gamma-1}{\gamma}}^{1}\frac{\gamma}{2}(1-t)^{2}dt = \left[\frac{1}{2\gamma}t\right]_{\frac{1}{3}}^{\frac{\gamma-1}{\gamma}} + \left[-\frac{\gamma}{6}(1-t)^{3}\right]_{\frac{\gamma-1}{2}}^{1} = \frac{3\gamma-4}{12\gamma^{2}}$$

Economies 2024, 12, 101 16 of 18

$$\begin{array}{l} \frac{\partial TW^{M*}(\gamma)}{\partial \gamma} = \frac{2\gamma^2 - 4\gamma(\gamma - 1)}{2\gamma^2} = \frac{2 - \gamma}{2\gamma^3} < 0 \text{ iff } \gamma > 2 \text{ which always holds in this case.} \\ \frac{\partial^2 TW^{M*}(\gamma)}{\partial \gamma^2} = \frac{\gamma - 3}{\gamma^4} > 0 \text{ iff } \gamma > 3 \\ \lim_{\gamma \to \infty} TW^{M*}(\gamma) = \frac{3}{8} \\ \frac{\partial TC^{M*}(\gamma)}{\partial \gamma} = \frac{3\gamma - 2(3\gamma - 4)}{12\gamma^3} = \frac{8 - 3\gamma}{12\gamma^3} < 0 \text{ iff } \gamma > \frac{8}{3} \\ \frac{\partial^2 TC^{M*}(\gamma)}{\partial \gamma} = \frac{\gamma - 4}{2\gamma^4} > 0 \text{ iff } \gamma > 4 \\ \lim_{\gamma \to \infty} TC^{M*}(\gamma) = 0 \\ \frac{\partial TU^{M*}(\gamma)}{\partial \gamma} = \frac{4 - 3\gamma}{12\gamma^3} < 0 \text{ iff } \gamma > \frac{4}{3} \text{ which always holds in this case.} \\ \frac{\partial^2 TU^{M*}(\gamma)}{\partial \gamma} = \frac{\gamma - 2}{2\gamma^4} > 0 \text{ iff } \gamma > 2 \\ \lim_{\gamma \to \infty} TU^{M*}(\gamma) = \frac{3}{8} \\ \text{Overall, we have that:} \\ T^{M*}(\gamma) = TW^{M*}(\gamma) = \begin{cases} \frac{\gamma}{48} & \text{if } \gamma \leq 2 \\ \frac{3\gamma^2 + 4\gamma - 4}{8\gamma^2} & \text{if } \gamma \geq 2 \end{cases} \\ TC^{M*}(\gamma) = \begin{cases} \frac{\gamma}{48} & \text{if } \gamma \leq 2 \\ \frac{3\gamma - 4}{48} & \text{if } \gamma \leq 2 \end{cases} \\ \frac{24 - \gamma}{48} & \text{if } \gamma \leq 2 \\ \frac{2\gamma}{24\gamma^2} & \text{if } \gamma \geq 2 \end{cases} \\ \frac{\partial TC^{M*}(\gamma)}{\partial \gamma} \leq 0 \text{ if } \gamma \geq \frac{8}{3} \\ \frac{\partial TU^{M*}(\gamma)}{\partial \gamma} < 0 \text{ for all } \gamma. \end{cases}$$

Proof of Proposition 6. We consider two separate cases depending on whether $\gamma \geq 2$. Case 1: For $\gamma \leq 2$ we have:

$$\frac{T^{F*}(\gamma)}{T^{M*}(\gamma)} = \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = \frac{\frac{2\gamma-1}{2\gamma^2}}{\frac{1}{2}} = \frac{2\gamma-1}{\gamma^2} \in \left[\frac{3}{4}, 1\right] < 1$$

$$\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = \frac{\frac{24\gamma-16-\gamma^3}{48\gamma^2}}{\frac{\gamma}{48}} = \frac{24\gamma-16-\gamma^3}{\gamma^3} \in \left[3, 7\right]$$

$$\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{\frac{24\gamma-8+\gamma^3}{48\gamma^2}}{\frac{24+\gamma}{48}} = \frac{24\gamma-8+\gamma^3}{24\gamma^2-\gamma^3} \in \left[\frac{6}{11}, \frac{17}{23}\right]$$
and
$$\frac{\partial \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)}}{\partial \gamma} = \frac{2(1-\gamma)}{\gamma^3} < 0; \frac{TW^{F*}(1)}{TW^{M*}(1)} = 1; \frac{TW^{F*}(2)}{TW^{M*}(2)} = \frac{3}{4}$$

$$\frac{\partial \frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)}}{\partial \gamma} = \frac{48(1-\gamma)}{\gamma^4} < 0; \frac{TC^{F*}(1)}{TC^{M*}(1)} = 7; \frac{TC^{F*}(2)}{TC^{M*}(2)} = 3$$

$$\frac{TU^{F*}(1)}{TU^{M*}(1)} = \frac{17}{23}; \frac{TC^{F*}(2)}{TC^{M*}(2)} = \frac{6}{11}$$

$$\frac{\partial \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)}}{\partial \gamma} = 24\gamma \frac{\gamma^3+2\gamma^2-25\gamma+16}{(24\gamma^2-\gamma^3)^2} < 0 \text{ iff } \gamma^3+2\gamma^2-25\gamma+16 < 0$$
Notice that for $\alpha = 1$ we have that $\alpha^3 + 2\alpha^2 = 25\alpha + 16 = 6$

Notice that for $\gamma=1$ we have that $\gamma^3+2\gamma^2-25\gamma+16=-6<0$; for $\gamma=2$ we have that $\gamma^3+2\gamma^2-25\gamma+16=-18<0$ and in addition we have that $\gamma^3+2\gamma^2-25\gamma+16$ decreases with γ for $1<\gamma<2$.

Case 2: For
$$\gamma \geq 2$$
 we have:
$$\frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = \frac{\frac{4+\gamma}{8\gamma}}{\frac{3\gamma^2 + 4\gamma - 4}{8\gamma^2}} = \frac{4\gamma + \gamma^2}{3\gamma^2 + 4\gamma - 4} \in \left[\frac{1}{3}, \frac{3}{4}\right]$$
$$\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = \frac{\frac{1}{4\gamma}}{\frac{3\gamma - 4}{12\gamma^2}} = \frac{3\gamma}{3\gamma - 4} \in [1, 3]$$

Economies 2024, 12, 101 17 of 18

$$\begin{array}{l} \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{\frac{2+\gamma}{8\gamma}}{\frac{9\gamma^2+6\gamma-4}{24\gamma^2}} = \frac{6\gamma+3\gamma^2}{9\gamma^2+6\gamma-4} \in \left[\frac{1}{3},\frac{6}{11}\right] \\ \text{and} \\ \frac{\partial}{\partial \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)}} = -4\frac{2+\gamma+\gamma^2}{(3\gamma^2+4\gamma-4)^2} < 0 < 0; \frac{TW^{F*}(2)}{TW^{M*}(2)} = \frac{3}{4}; \lim_{\gamma \to \infty} \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = \frac{1}{3} \\ \frac{\partial}{\partial \gamma} \frac{TC^{F*}(\gamma)}{\partial \gamma} = \frac{-12}{(3\gamma-4)^2} < 0; \frac{TC^{F*}(2)}{TC^{M*}(2)} = 3; \lim_{\gamma \to \infty} \frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = 1 \\ \frac{\partial}{\partial \gamma} \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = -2\frac{2+2\gamma+3\gamma^2}{(9\gamma^2+6\gamma-4)^2} < 0; \frac{TU^{F*}(2)}{TU^{M*}(2)} = \frac{6}{11}; \lim_{\gamma \to \infty} \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{1}{3} \\ \text{Overall, we have:} \\ \frac{T^{F*}(\gamma)}{T^{M*}(\gamma)} = \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} \in \left[\frac{1}{3},1\right] \\ \frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} \in \left[1,7\right] \\ \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} \in \left[\frac{1}{3},\frac{17}{23}\right] \\ \text{and} \\ \frac{\partial}{TU^{M*}(\gamma)} \frac{TW^{F*}(\gamma)}{\partial \gamma} < 0; \frac{\partial}{\partial \gamma} \frac{TC^{F*}(\gamma)}{\partial \gamma} < 0; \frac{\partial}{\partial \gamma} \frac{TU^{F*}(\gamma)}{\partial \gamma} < 0. \end{array}$$

Proof of Proposition 7. For $\gamma \leq 1$ we have that $s^*(t, \gamma) = 1$ and

$$W^*(t,\gamma) = 1 C^*(t,\gamma) = \frac{\gamma}{2}(1-t)^2 U^*(t,\gamma) = 1 - \frac{\gamma}{2}(1-t)^2.$$

For females types we have:

$$T^{F*}(\gamma) = TW^{F*}(\gamma) = \int_0^{\frac{1}{2}} W^{F*}(t;\gamma) = \int_0^{\frac{1}{2}} dt = [t]_0^{\frac{1}{2}} = \frac{1}{2}$$

$$TC^{F*}(\gamma) = \int_0^{\frac{1}{2}} C^{F*}(t;\gamma) = \int_0^{\frac{1}{2}} \frac{\gamma}{2} (1-t)^2 dt = \left[-\frac{\gamma}{6} (1-t)^3 \right]_0^{\frac{1}{2}} = \frac{7\gamma}{48}$$

$$TU^{F*}(\gamma) = \frac{1}{2} - \frac{7\gamma}{48} = \frac{24-7\gamma}{48}$$
For mole types we have:

For male types we have:

$$T^{M*}(\gamma) = TW^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} W^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} dt = [t]_{\frac{1}{2}}^{1} = \frac{1}{2}$$

$$TC^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} C^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} \frac{\gamma}{2} (1-t)^{2} dt = \left[-\frac{\gamma}{6} (1-t)^{3} \right]_{\frac{1}{2}}^{1} = \frac{\gamma}{48}$$

$$TU^{M*}(\gamma) = \frac{1}{2} - \frac{\gamma}{48} = \frac{24-\gamma}{48}$$
Comparing female and male types we obtain:

Comparing remain and male types we obtain:
$$\frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = \frac{\frac{7\gamma}{48}}{\frac{48}{7}} = 7$$

$$\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{\frac{24-7\gamma}{48}}{\frac{24-\gamma}{48}} = \frac{24-7\gamma}{24-\gamma}$$
 with
$$\frac{TU^{F*}(\gamma)}{\gamma} = \frac{-144}{(24-\gamma)^2} < 0, \frac{TU^{F*}(1)}{TU^{M*}(1)} = \frac{17}{23}, \text{ and } \frac{TU^{F*}(0)}{TU^{M*}(0)} = 1.$$

References

Albanesi, Stefania, and Claudia Olivetti. 2009. Home production, market production and the gender wage gap: Incentives and expectations. Review of Economic Dynamics 12: 80–107. [CrossRef]

Arrow, Kenneth. 1973. The Theory of Discrimination. In Discrimination in Labor Markets. Edited by Orley C. Ashenfelter and Albert Everett Rees. Princeton: Princeton University Press, pp. 3–33.

Azmat, Ghazala, and Rosa Ferrer. 2019. Gender Gaps in Performance: Evidence from Young Lawyers. Journal of Political Economy 125: 1306-55. [CrossRef]

Becker, Gary S. 1957. The Economics of Discriminatio. Chicago: University of Chicago Press.

Becker, Gary S. 1971. The Economics of Discrimination, 2nd ed. Chicago: University of Chicago Press.

Beneria, Lourdes. 1999. The enduring debate over unpaid labour. International Labour Review 138: 287–309. [CrossRef]

Economies **2024**, 12, 101

Bertrand, Marianne. 2019. The Gender Socialization of Children Growing up in Non-Traditional Families. *American Economic Association: Papers and Proceedings* 109: 115–21.

Bertrand, Marianne. 2020. Gender in the Twenty-First, Century. *American Economic Association: Papers and Proceedings* 110: 1–24. [CrossRef]

Bertrand, Marianne, and Esther Duflo. 2017. Field Experiments on Discrimination. In *Handbook of Field Experiments*. Edited by Abhijit Banerjee and Esther Duflo. Amsterdam: North Holland.

Bertrand, Marianne, and Sendhil Mullainathan. 2014. Are Emily and Greg More Employable than Lakisha and Jamal? A Field Experiment on Labor Market Discrimination. *American Economic Review* 94: 991–1013. [CrossRef]

Bertrand, Marianne, Rema Hanna, and Sendhil Mullainathan. 2010. Affirmative Action in Education: Evidence from Engineering College Admissions in India. *Journal of Public Economics* 94: 16–29. [CrossRef]

Cella, Michela, and Elena Manzoni. 2023. Gender Bias and Women's Political Performance. *European Journal of Political Economy* 77: 102314. [CrossRef]

Dolado, Juan J., Cecilia García-Peñalosa, and Sara De La Rica. 2013. On Gender Gaps and Self-Fulfilling Expectations: Alternative Implications of Paid-For Training. *Economic Inquiry* 51: 1829–48. [CrossRef]

Farré, Lídia. 2016. Parental Leave Policies and Gender Equality: A Survey of the Literature. *Estudios de Economía Aplicada* 34: 45–60. [CrossRef]

Farré, Lídia, and Francis Vella. 2013. The Intergenerational Transmission of Gender Role Attitudes and its Implications for Female Labour Force Participation. *Economica* 80: 219–47. [CrossRef]

Farré, Lídia, and Libertad González. 2019. Does Paternity Leave Reduce Fertility? *Journal of Public Economics* 172: 52–66. [CrossRef] Fernandez, Raquel, Alessandra Fogli, and Claudia Olivetti. 2004. Mothers and Sons: Preference Formation and Female Labor Force

Folbre, Nancy. 2001. The Invisible Heart: Economics and Family Values. New York: The New Press.

Dynamics. Quarterly Journal of Economics 119: 1249–99. [CrossRef]

Francois, Patrick. 1998. Gender discrimination without genderdifference: Theory and policy responses. *Journal of Public Economics* 68: 1–32. [CrossRef]

Gammage, Sarah, Naziha Sultana, and Manon Mouron. 2019. The Hidden Costs of Unpaid Caregiving. In *Finance and Development*. Washington: International Monetary Fund.

Gneezy, Uri, Muriel Niederle, and Aldo Rustichini. 2003. Performance in Competitive Environments: Gender Differences. *Quarterly Journal of Economics* 118: 1049–74. [CrossRef]

Goldin, Claudia. 2014. A Grand Gender Convergence: Its Last Chapter. American Economic Review 104: 1091–19. [CrossRef]

González, Libertad, and Hosny Zoaby. 2021. *Does Paternity Leave Promote Gender Equality within Households?* CESifo Working Paper n. 9430. Munich: Munich Society for the Promotion of Economic Research—CESifo GmbH.

Iriberri, Nagore, and Pedro Rey-Biel. 2021. Brave Boys and Play-it-Safe Girls: Gender Differences in Willingness to Guess in a Large Scale Natural Field Experiment. *European Economic Review* 131: 103–603. [CrossRef]

Isen, Adam, Maya Rossin-Slater, and Reed Walker. 2017. Relationship Between Season of Birth, Temperature Exposure, and Later Life Well-Being. *Proceedings of the National Academy of Sciences* 114: 13447–52. [CrossRef]

Kleven, Henrik, Camille Landais, Johanna Posch, Andreas Steinhauer, and Josef Zweimuller. 2019. Child Penalties Across Countries: Evidence and Explanations. *American Economic Association: Papers and Proceedings* 109: 122–26.

OECD. 2019. Enabling Women's Economic Empowerment: New Approaches to Unpaid Care Work in Developing Countries. OECD Report.

Olivetti, Claudia, and Barbara Petrongolo. 2017. The Economic Consequences of Family Policies: Lessons from a Century of Legislation in High-Income Countries. *Journal of Economic Perspectives* 31: 205–30. [CrossRef]

Patnaik, Ankita. 2019. Reserving Time for Daddy: The Consequences of Fathers' Quotas. *Journal of Labor Economics* 37: 1009–59. [CrossRef]

Peski, Marcin, and Balazs Szentes. 2013. Spontaneous Discrimination. American Economic Review 103: 2412–36. [CrossRef]

Phelps, Edmund. 1972. The Statistical Theory of Racism and Sexism. American Economic Review 62: 659-61.

Samman, Emma, Elisabeth Presler-Marshall, and Nicola Jones. 2016. *Women's Work: Mothers, Children and the Global Childcare Crisis*. Research Report. London: Overseas Development Institute.

Schelling, Thomas C. 1971. Dynamic Models of Segregation. Journal of Mathematical Sociology 1: 143–86. [CrossRef]

Spence, Michael. 1973. Job Market Signalling. Quarterly Journal of Economics 87: 355–64. [CrossRef]

UN Women. 2022. A Toolkid on Paid and Unpaid Care Work. New York: Economic Empowerment Section UN Women.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.