Iceberg transport technologies in spatial competition. Hotelling reborn*

Xavier Martinez-Giralt
Institut d'Anàlisi Econòmica (CSIC) and
Universitat Autònoma de Barcelona

José M. Usategui
Universidad del Pais Vasco

October 2009

Abstract

Transport costs in address models of differentiation are usually modeled as separable of the consumption commodity and with a parametric price. However, there are many sectors in an economy where such modeling is not satisfactory either because transportation is supplied under oligopolistic conditions or because there is a difference (loss) between the amount delivered at the point of production and the amount received at the point of consumption. This paper is a first attempt to tackle these issues proposing to study competition in spatial models using an iceberg-like transport cost technology allowing for concave and convex melting functions.

Keywords: Spatial Competition, Iceberg transport costs.

JEL Classification: L12, D42, R32

*We thank Pedro P. Barros, Javier Casado, M. Paz Espinosa, Carlos Gutiérrez-Hita, Inés Macho-Stadler, Rosella Nicolini, and David Pérez-Castrillo, for helpful comments and suggestions. We also acknowledge partial support from the Barcelona Economics Program of CREA and projects 2009SGR-169, SEJ2006-00538-ECON, and Consolider-Ingenio 2010 (X. Martinez-Giralt), and SEJ2006-06309 and Departamento de Educación, Universidades e Investigación del Gobierno Vasco (IT-313-07) (J.M. Usategui). Xavier Martinez-Giralt is a research fellow of MOVE (Markets, Organizations and Votes in Economics). The usual disclaimer applies.

CORRESPONDENCE ADDRESSES:

Xavier Martinez-Giralt
Institut d'Anàlisi Econòmica
CSIC
Campus Universitat Autònoma de Barcelona
08193 Bellaterra
Spain.
Fax: +34935812461
e-mail:xavier.martinez.giralt@uab.cat

José M. Usategui
Dpt. de Fdtos. del Análisis Económico II
Universidad del Pais Vasco
Av. Lehendakari Aguirre, 83
48015 Bilbao
Spain.
+34946013774
josemaria.usategui@ehu.es

1 Introduction.

Transportation is an essential element in the modeling of address models of differentiation. Two common assumptions in this regard are (i) the separability between the demand of the consumption good and of transportation services, and (ii) a constant transport cost function.

Such an approach can be challenged on two grounds. On the one hand, in many sectors of an economy we find an oligopolistic structure in the market of transportation services; on the other hand, there are sectors in the economy where transport costs are dependent on the demand (i.e. price) of the consumption good transported. Such is the case for instance, in the supply of electricity and of water. In both cases, there is a difference (loss) between the amount delivered at the point of production and the amount received at the point of consumption. Such loss depends (among other factors) of the geographical distance between producers and consumers. Several interpretations can be put forward. We can think of this phenomenon as a loss of quality, or, in a temporal interpretation, as a lag between the buying of the commodity and its consumption.

The approach we propose to tackle this issue has been extensively used in other domains of research, such as urban economics (see e.g. Fujita, 1995, Abdel-Raman, 1994, Abdel-Rahman and Anas, 2004) or in general equilibrium models of international trade (see e.g. Krugman, 1991a,b, 1992, Helpman and Krugman, 1988). Curiously enough, in those areas the way to cope with the difference between the amount of good delivered and consumed has been different. There, transport costs are formulated in terms of the transported commodity. This modeling was formalized by Samuelson (1954, 1983) as "iceberg transport costs" taking up an idea originated in von Thünen (1930). Excellent surveys of this literature (known under the heading of the new economic geography) are Fujita and Krugman (2004), Fujita et al. (2000), Neary (2001), Ottaviano and Thisse (2004) and Schmutzler (1999). A usual interpretation of the iceberg transport cost in the so-called New Economic Geography, is that it embodies information costs, institutional and trade barriers, quality standards, and cultural and linguistic differences[1], in addition to the pure transport costs (Ottaviano and Thisse, 2004; Fujita *et al.*, 1999).

A common technical feature of the iceberg and traditional transport technologies is that both give rise to a delivered price function convex with distance. However, empirical evidence points towards concave delivered price schemes rather than convex with distance. This leads McCann (2005) to raise a warning against the use of such transport costs functions to provide real-world insights as the properties of such transport costs are largely implausible when compared with the available empirical evidence. In this respect de Frutos *et al.* (2002) show that for any convex transport cost function there exists a concave one such that the location-then-price games induced by these functions are strategically equivalent. Our proposal also overcomes McCann's criticism as it is robust to concave and convex transport cost functions.

A different approach to the use of iceberg transport cost functions follows Krugman (1998) where the spatial iceberg assumption is considered as a pure technical device for avoiding the need to model a two-sector economy with a consumption good industry and a transport industry. Following this line of reasoning, it can be argued that the iceberg approach vis-a-vis the Hotelling approach in the modeling of transport costs allows for an endogenous determination of the provision of transportation services. Also, as the price of transportation per unit of distance is assumed constant, implicitly it amounts to assume that the transport sector is perfectly competitive. No empirical evidence supports such assumption. However, as we will see, the technical simplicity of Hotelling analysis contrasts with the complexity that the iceberg formulation introduces in the analysis.

We aim at providing a direct comparison of the "costs" and "benefits" of using both approaches in a common framework. Thus, we propose to study price and location equilibria in a duopoly model à la Hotelling where transport costs are

[1]This interpretation of the iceberg formulation of transport costs may not be very rigorous. The iceberg formulation implies that transport cost are dependent of the price of the transported commodity. It is not always easy to establish the link between some of those arguments and the price of the transported good.

modeled in the iceberg tradition. In this sense we want to contribute to the debate between the modeling of transport costs in the spatial location and price competition models and in the new economic geography tradition.

Section 2 introduces the concepts of melting function and melting rate associated with the iceberg transport technology. It also clarifies the difference induced in demand as compared to the traditional modeling of transport costs.

To illustrate our point, in section 3 we propose a monopoly and a general melting function. By assuming away competition, we can concentrate in the consequences of modeling transports costs in the iceberg fashion. Next, we apply our analysis to particular specifications of the melting function. The driving force behind the results is that under the iceberg formulation, transportation is done by oligopolistic producers. Accordingly, transport costs represent a transfer of resources to the firms, thus inducing a more elastic demand. Should producers be perfectly competitive, under the iceberg formulation each unit would be sold at a price equal to the marginal cost of production. The transport cost would be equal to the production costs of the units of commodity lost in transportation (i.e. melted away). Thus, there would not be any transfer of resources to the firms.

Section 4 extends the analysis with an illustration of the effects of the melting function approach in a competitive framework defined by a symmetric duopoly. In section 5 we perform a welfare analysis where we take into account the role of the resources devoted to transportation in the agents' optimization behavior. A section with conclusion closes the paper.

2 Melting vs. Transport Costs.

Let δ be the distance between consumer x and a firm and let $\mu \in (0,1)$ be a constant positively linked to the melting rate per unit of distance.[2] We denote by $M(\delta;\mu)$ the rate at which the commodity melts away per unit of distance. Given $M(\delta\,;\mu)$ we have to distinguish between the demand an individual addresses to

[2] A detailed analysis of the properties of the iceberg transport cost functions is found in McCann (2005).

a firm from his consumption. Denote by $q(\delta;\mu)$ the quantity of the commodity a consumer located at a distance δ of the firm needs to buy to consume exactly one unit of the good. With this notation, we can formally define the melting rate M as,

$$M(\delta;\mu) = \frac{\partial q}{\partial \delta}\frac{1}{q}.$$

Definition 1 (Generalized melting function)**.** *A generalized melting function, $h(\delta;\mu)$, specifies the additional demand addressed by an individual located at a distance δ from the firm to be able to consume one unit of the commodity. It is given by,*

$$h(\delta;\mu) = q(\delta;\mu) - 1, \tag{1}$$

where

$$h(\delta;\mu) > 0,\ \frac{\partial h}{\partial \delta} > 0,\ \frac{\partial h}{\partial \mu} > 0,\ \frac{\partial^2 h}{\partial \delta^2} \geq 0,\ \frac{\partial^2 h}{\partial \mu^2} \geq 0. \tag{2}$$

Thus, the melting rate is given by,

$$M(\delta;\mu) = \frac{\partial h}{\partial \delta}\frac{1}{1 + h(\delta;\mu)}.$$

We could also think of specifying the melting as a function of the distance and the amount of commodity transported. In this case we would have $q(\delta;\mu)f(\delta;\mu)) = q(\delta;\mu) - 1$. However, we can rewrite this latter additional demand as the former with $h(\delta;\mu) = \dfrac{f(\delta;\mu)}{1 - f(\delta;\mu)}$. Therefore, both cases are formally equivalent.

To ease a proper understanding of the role of the melting in the modeling, Figure 1 illustrates the standard transportation cost and the melting function approaches.

Consider the standard spatial model with convex transport costs. The unit price at distance δ of the firm is given by $P(\delta;t,p) = p + t\delta^\alpha$, where p denotes the unit f.o.b. price and $\alpha > 1$. Note that in this case an increase in p translates in exactly the same way to all consumers, i.e. the impact is positive but independent of δ. This situation is depicted in part (a) of Figure 1 for a firm located in a and consumers located between 0 and L.

The price paid by a consumer located at a distance δ of the firm is given, according to the proposed general melting function, by $P(\delta;\mu,p) = p(1 + h(\delta;\mu))$.

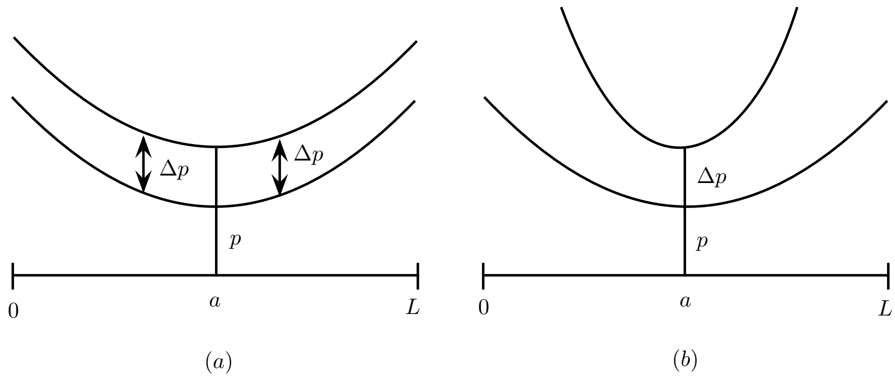

Figure 1: Delivered prices in a spatial model.

We observe that the impact on $P(\delta;\mu,p)$ of an increase in p now is a positive function of δ. That is, $\dfrac{\partial^2 P(\delta;\mu,p)}{\partial p \partial \delta} > 0$. Also, $P(\delta;\mu,p)$ is increasing with δ. Generically, this situation is presented in part (b) of Figure 1.

In the standard spatial model, a decrease in price increases demand because the demand is downward sloping with respect to price. When transport costs are modeled in the iceberg fashion, a decrease in price has an additional effect on demand. Demand increases not only because it is negatively related to the price but because the extra quantity demanded ("transport cost") is also cheaper. Although both approaches are not directly comparable, we can illustrate one difference by saying that the iceberg transport technology induces a more elastic demand system than the traditional Hotelling model.

Note that production and transportation are two different activities with different technologies. Although independent, they are linked through consumers' gross demand. Firms use the same technology to produce all units of the good. However, the additional demand due to melting depends on the transport technology and not on the production technology.

3 Analysis of Monopoly.

Consider a spatial market described by a line segment of length L. Consumers are evenly distributed on the market with unit density. They are identical in all respects but for their location. A consumer is denoted by $x \in [0, L]$. All consumers have a common reservation price $\overline{p}$. Consumers adjust their demands so that, if positive, they consume exactly one unit of the commodity.

There is a monopolist in the market located at a distance a from the left end of the market. It produces a homogeneous product with a constant returns to scale technology represented by a constant marginal cost c. Assume, without loss of generality $a \leq L/2$.

The assumptions on $h(\delta; \mu)$ given by (2), imply that the demand addressed by consumer x given by (1), is a symmetric and increasing function around the firm's location and convex in both δ and μ.

The consumers indifferent between buying one unit from the monopolist or stay out of the market (denote them by $z \in [0, a]$ and $y \in [a, L]$) are given by the solution of the following equations:

$$p\big(1 + h(a - z; \mu)\big) = \overline{p} = p\big(1 + h(y - a; \mu)\big). \tag{3}$$

A direct inspection of (3), tells us that since $\dfrac{\partial h}{\partial \delta} > 0$

$$a - z = y - a. \tag{4}$$

In equilibrium, the consumers located at the extremes of the interval describing the market covered by the monopolist must obtain no surplus.

If the monopolist charges a f.o.b. price $p = \overline{p}$, it obtains zero demand regardless of its location. For prices $p < \overline{p}$, we can identify prices so that all the market is covered, but it is also possible to identify prices leaving some consumers unattended. In this case, for each of those prices there is a continuum of locations yielding the same demand. Among them, there is always one such that the indifferent consumer x is located at zero. We can, thus, characterize the demand captured by the monopolist by the set of consumers with its left bound at zero.

Demand addressed to the monopolist is given by,

$$\begin{aligned} D(p) &= \int_z^a \big(1 + h(a - w; \mu)\big) dw + \int_a^y \big(1 + h(w - a; \mu)\big) dw \\ &= 2(a - z) + H(a - z; \mu) + H(y - a; \mu) - 2H(0; \mu), \end{aligned}$$

where we have made use of (4) and $H(\delta;\mu) = \int h(\delta;\mu)d\delta$. From (4) it follows $H(a-x;\mu) = H(y-a;\mu)$ so that

$$D(p) = 2(a - z + H(a-z;\mu) - H(0;\mu)). \tag{5}$$

The price that makes the consumer located at zero indifferent between buying the commodity or staying out of the market is,

$$\tilde{p}(a) = \frac{\overline{p}}{1 + h(a;\mu)} = \frac{\overline{p}}{q(\delta,\mu)}. \tag{6}$$

Evaluating firm's profits at $\tilde{p}(a)$, we obtain,

$$\Pi(a) = 2(\tilde{p}(a) - c)\big(a + H(a;\mu) - H(0;\mu)\big). \tag{7}$$

Proposition 1. *(i) The monopoly profit maximizing price is,*

$$\tilde{p}(a) = \frac{\overline{p}}{1 + h(a;\mu)}.$$

(ii) A monopolist will cover all the market if, for all $a \leq L/2$,

$$\big(1 + h(a;\mu)\big)^2\big(1 - \frac{c}{\overline{p}}(1 + h(a;\mu))\big) \geq \frac{\partial h}{\partial \delta}(a;\mu)\big(a + H(a;\mu) - H(0;\mu)\big).$$

Among all these locations, profits are maximum at $a = L/2$.

Proof. From (7) we derive

$$\frac{\partial \Pi(a)}{\partial a} = 2\overline{p}\left(1 - \frac{\frac{\partial h}{\partial \delta}(a;\mu)(a + H(a;\mu) - H(0;\mu))}{(1 + h(a;\mu))^2}\right) - 2c(1 + h(a;\mu)), \tag{8}$$

This first order condition (8) is strictly positive for all $a < L/2$, and is equal to zero at $a = L/2$ □

As particular cases of melting functions consider the following:

Definition 2 (Melting with Quantity and Distance: MQD)**.** *We say that melting is MQD when it is proportional to the product of the quantity bought and the distance traveled. Formally:*

$$q(\delta;\mu) - \mu\delta q(\delta;\mu) = 1 \tag{9}$$

and $h(\delta;\mu) = \frac{\mu\delta}{1-\mu\delta}$.

Definition 3 (Melting with Distance: MD)**.** *We refer to MD melting as the situation where the melting is proportional to a power (α) of the distance. Formally:*

$$q(\delta;\mu) - \mu\delta^{\alpha} = 1 \tag{10}$$

and $h(\delta;\mu) = \mu\delta^{\alpha}$.

Definition 4 (Krugman-Samuelson Melting: KSM)**.** *KSM is obtained wen the melting rate is constant with* $h(\delta;\mu) = e^{\mu\delta} - 1$ *and*

$$q(\delta;\mu) = e^{\mu\delta} \tag{11}$$

As it is shown in Proposition 1 the greater $\overline{p}/c$, the more likely will the monopolist locate at $a = \frac{L}{2}$ and cover all the market. In the following Corollary we analyze the intensity of the preference of the monopolist for covering the market when $c = 0$ in the particular melting functions introduced in definitions 2 to 4.[3]

Corollary 1. *(i) Under MQD, the monopolist covers the whole market[4] and locates at its center iff* $\dfrac{L}{2} \leq \dfrac{e-1}{e\mu}$

(ii) Under MD, the monopolist locates at the center and covers all the market if $\alpha \leq 2$.

(iii) Under KSM, the monopolist locates at the center and covers all the market.

Proof. See appendix □

4 Melting in oligopolistic markets

We now extend our analysis to the case of oligopolistic markets. We intend to illustrate the effect of the iceberg transport approach in the study of oligopoly. We retain the same model as in the monopoly case, but now we introduce competition between two identical firms. We characterize transport costs by a MD-type melting

[3]Note that under MQD we have $\tilde{p}(\frac{L}{2}) > 0 \Leftrightarrow \mu L < 2$. Hence, when $\mu L > 2$ coverage of the whole market is not feasible. However, under MD or under Samuelson melting coverage of the whole market is always feasible.

[4]Note that $\mu L < 2$ is a necessary condition to obtain full market coverage by the monopolist. Note also that $\mu L \leq 2(e-1)/e \approx 1.264$ is a more stringent condition for full market coverage.

function. We assume that first firms decide simultaneously their locations and then, they compete in prices by selecting simultaneously their mill prices.

To be precise, we introduce first the necessary notation.

Let $L = 1$ without loss of generality. Two firms a, and b are located at points a and b where $a \in [0, b)$. Let p_a and p_b denote their respective prices, and $\bar{p}$ the common reservation price for consumers. We assume $\bar{p}$ to be high enough (but finite) so that all consumers can afford purchasing from one of the firms (in other words, the market is fully covered). Let $\alpha = 1$ in the MD melting function[5] and $\mu > 0$. Under the MD regime, the price paid by a consumer located at a distance δ from firm i is given by $p_i(1 + \mu\delta)$.

4.1 Indifferent consumers

To construct the contingent demand system, we will distinguish three regions in the market and define their corresponding indifferent consumers. A first region is given by the segment $[0, a]$. Denote an indifferent consumer in that region as x_1. The second region is the segment $[a, b]$, ant the corresponding indifferent consumer will be denoted as x_2. Finally, the interval $[b, 1]$ describes the third region and x_3 denotes its indifferent consumer.

Indifferent consumer x_1. A consumer in $[0, a]$ is indifferent between patronizing either firm if

$$p_a(1 + \mu(a - x)) = p_b(1 + \mu(b - x))$$

or

$$x_1(p_a, p_b; a, b) = \frac{p_b(1 + \mu b) - p_a(1 + \mu a)}{\mu(p_b - p_a)} \in [0, a] \qquad (12)$$

Note that firm b may only capture consumers in firm a's hinterland if $p_b < p_a$. In turn, this implies that the denominator of (12) is negative. Therefore, x_1 is well-defined only if the numerator is also negative.

[5] We can develop the analysis for a concave transport function taking for instance $\alpha = 1/2$ with analogous results. Namely, the firm quoting the lower price would be able to capture a non-connected market share. In this respect our analysis is robust to McCann (2005) criticisms. Note also that $\alpha = 1/2$ corresponds to the transport cost function proposed in McCann (1993, 1998).

Indifferent consumer x_2. A consumer in $[a, b]$ is indifferent between patronizing either firm if

$$p_a(1 + \mu(x - a)) = p_b(1 + \mu(b - x))$$

or

$$x_2(p_a, p_b; a, b) = \frac{p_b(1 + \mu b) - p_a(1 - \mu a)}{\mu(p_b + p_a)} \in [a, b] \tag{13}$$

Indifferent consumer x_3. A consumer in $[b, 1]$ is indifferent between patronizing either firm if

$$p_a(1 + \mu(x - a)) = p_b(1 + \mu(x - b))$$

or

$$x_3(p_a, p_b; a, b) = \frac{-p_b(1 - \mu b) + p_a(1 - \mu a)}{\mu(p_b - p_a)} \in [b, 1] \tag{14}$$

Note that firm a may only capture consumers in firm b's hinterland if $p_a < p_b$.

To easy notation we will refer to the indifferent consumers as x_1, x_2, x_3 when such notation will not induce confusion.

Remark 1. *Note that indifferent consumer functions are continuous at a and b. That is,*

$$x_1|_a = x_2|_a \qquad \textit{and} \qquad x_2|_b = x_3|_b.$$

4.2 Firm a's contingent demand

Before computing the contingent demand captured by firm a, let us identify four critical prices.

Firm a will capture no demand at all when, given a certain price p_b of firm b, firm a calls a price p_a such that $x_1 = a$. Naturally, for even higher prices firm a will remain inactive in the market. Let us denote such price as p_a^{max}. Its expression is given by the solution of $x_1 = a$, i.e.

$$p_a^{max} = p_b(1 + \mu(b - a)). \tag{15}$$

Figure 2 illustrates this scenario.

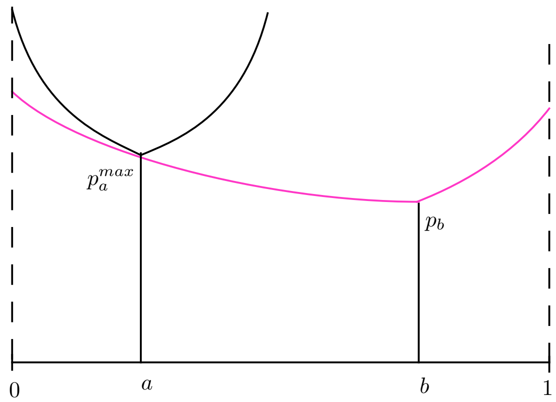

Figure 2: Firm a captures no demand.

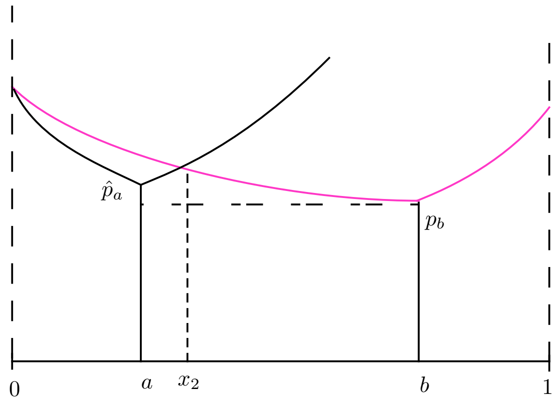

Figure 3: Firm a captures its hinterland.

As firm a lowers the price, it starts capturing consumers in the neighborhood of its location a. Firm a captures all consumers in its hinterland when, given a certain price p_b of firm b, firm a calls a price p_a such that $x_1 = 0$. Let us denote such price as $\hat{p}_a$. It is given by,

$$\hat{p}_a = p_b \frac{1 + \mu b}{1 + \mu a}. \tag{16}$$

Note that $\hat{p}_a > p_b$. Figure 3 illustrates.

Further reductions of the price, allows to increase demand from consumers located to the right to a. Also, at a price $\tilde{p}_a$ (given p_b), firm a will start to capture consumers in the hinterland of firm b. This price is given by the solution of $x_3 = 1$, and its expression is,

$$\tilde{p}_a = p_b \frac{1 + \mu(1 - b)}{1 + \mu(1 - a)}. \tag{17}$$

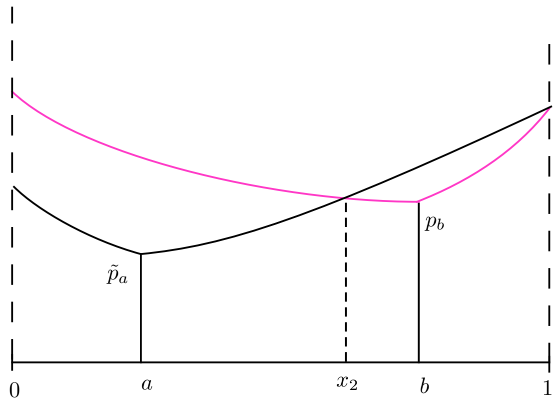

Figure 4: Firm a's demand at $\tilde{p}_a$.

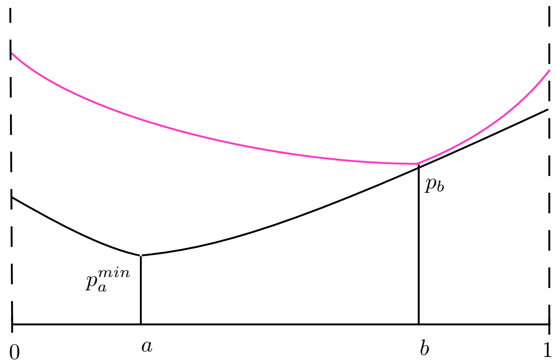

Figure 5: Firm a captures all the market.

Note that, $\tilde{p}_a < p_b$. Figure 4 illustrates.

Finally, firm a captures all consumers in the market when, given a certain price p_b of firm b, firm a calls a price p_a such that $x_3 = b$. Naturally, for even lower prices firm a's demand will not expand as all consumers are already patronizing it. Let us denote such price as p_a^{min}. Its expression is given by the solution of $x_3 = 1$, i.e.

$$p_a^{min} = p_b \frac{1}{1 + \mu(b - a)}. \tag{18}$$

Figure 5 illustrates.

To construct the (contingent) demand of firm a, lets us start by assuming that it quotes a price $p_a \geq p_a^{max}$. Then,

$$D_a(p_a, p_b) = 0, \text{ if } p_a \geq p_a^{max}$$

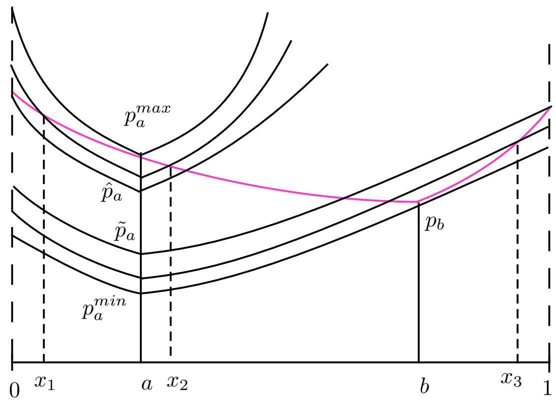

Figure 6: Constructing firm a's contingent demand.

For prices below p_a^{max}, firm a starts capturing consumers on both sides. At $\hat{p}_a$, it will capture all consumers in $[0, a]$ (and, of course some more consumers to its right). Therefore, the demand function in this domain of prices will be,

$$D_a(p_a, p_b) = \int_{x_1}^{a} (1 + \mu(a - s))ds + \int_{a}^{x_2} (1 + \mu(s - a))ds, \text{ if } p_a^{max} \geq p_a \geq \hat{p}_a.$$

As firm a quotes prices lower than $\hat{p}_a$ its demand expands from its right hand side in $[a, b]$ only up to the price $\tilde{p}_a$. Therefore, for prices until $\tilde{p}_a$ demand is given by,

$$D_a(p_a, p_b) = \int_{0}^{a} (1 + \mu(a - s))ds + \int_{a}^{x_2} (1 + \mu(s - a))ds, \text{ if } \hat{p}_a \geq p_a \geq \tilde{p}_a.$$

From this point, lower prices allows firm a to start stealing consumers in firm b's hinterland, so that its demand will be given by

$$D_a(p_a, p_b) = \int_{0}^{a} (1+\mu(a-s))ds + \int_{a}^{x_2} (1+\mu(s-a))ds + \int_{x_3}^{1} (1+\mu(s-a))ds, \text{ if } \tilde{p}_a \geq p_a \geq p_a^{min}.$$

At p_a^{min} firm a captures all the market, so that further reductions of its price does not contribute any revenues. Thus,

$$D_a(p_a, p_b) = \int_{0}^{a} (1 + \mu(a - s))ds + \int_{a}^{1} (1 + \mu(s - a))ds, \text{ if } p_a \leq p_a^{min}$$

Figure 6 summarizes the discussion.

Formally, the contingent demand of firm a is,

$$D_a(p_a,p_b)=\begin{cases}0, & \text{if } p_a\geq p_a^{max}\\ \int_{x_1}^{a}(1+\mu(a-s))ds+\int_a^{x_2}(1+\mu(s-a))ds, & \text{if } p_a^{max}\geq p_a\geq \hat{p}_a\\ \int_0^{a}(1+\mu(a-s))ds+\int_a^{x_2}(1+\mu(s-a))ds, & \text{if } \hat{p}_a\geq p_a\geq \tilde{p}_a\\ \int_0^{a}(1+\mu(a-s))ds+\int_a^{x_2}(1+\mu(s-a))ds & \\ \qquad +\int_b^{x_3}(1+\mu(s-a))ds, & \text{if } \tilde{p}_a\geq p_a\geq p_a^{min}\\ \int_0^{a}(1+\mu(a-s))ds+\int_a^{1}(1+\mu(s-a))ds, & \text{if } p_a^{min}\geq p_a\end{cases} \tag{19}$$

It is straightforward to verify the continuity of this contingent demand from the continuity of the indifferent consumer functions (see remark 1).

5 Characterizing a symmetric price equilibrium

Given the demand function just identified, if a symmetric equilibrium exists, it must lie in the domain of prices $(\hat{p}_a,\tilde{p}_a)$. In this domain, the relevant piece of the demand function is,

$$D_a(p_a,p_b)=\int_0^{a}(1+\mu(a-s))ds+\int_a^{x_2}(1+\mu(s-a))ds,\ \text{if } \hat{p}_a\geq p_a\geq \tilde{p}_a.$$

or

$$D_a(p_a,p_b)=a+\frac{a^2\mu}{2}-\frac{(p_a+(-1+a\mu-b\mu)p_b)(p_a+(3-a\mu+b\mu)p_b)}{2\mu(p_a+p_b)^2} \tag{20}$$

Assuming without loss of generality that marginal production costs are constant and denoted by $c>0$ for both firms, firm a's profits in the domain of prices relevant to the analysis are

$$\begin{aligned}\Pi_a(p_a,p_b)&=(p_a-c)D_a(p_a,p_b)\\ &=(p_a-c)\Big(a+\frac{a^2\mu}{2}-\frac{(p_a+(-1+a\mu-b\mu)p_b)(p_a+(3-a\mu+b\mu)p_b)}{2\mu(p_a+p_b)^2}\Big)\end{aligned}$$

The first order condition is given by

$$\begin{aligned}\frac{\partial\Pi_a}{\partial p_a}=\frac{1}{2\mu(p_a+p_b)^3}\Big(&(p_a^3+3p_a^2p_b)(a^2\mu^2+2a\mu-1)\\ &+p_ap_b^2(2a^2\mu^2+2a\mu(5+b\mu)-b^2\mu^2-4b\mu-7)\\ +p_b^2(2-a\mu+b\mu)2c&+p_b^3(2a^2\mu^2-2a\mu(1+b\mu)+b^2\mu^2+4b\mu+3)\Big)\end{aligned} \tag{21}$$

Let us characterize a symmetric equilibrium in prices, $p_b = p_a$. Then, expression (21) simplifies to

$$\frac{\partial \Pi_a}{\partial p_a}\Big|_{p_b=p_a} = \frac{c(2 - a\mu + b\mu)^2}{8p_a\mu} + \frac{a^2\mu^2 + 2a\mu - 1}{2\mu},$$

so that the (candidate) equilibrium price is

$$p^* = c\frac{(2 - a\mu + b\mu)^2}{4(1 - 2a\mu - a^2\mu^2)} \tag{22}$$

Remark 2. *The (candidate) symmetric equilibrium price is above the marginal cost c.*

Note that

$$\frac{(2 - a\mu + b\mu)^2}{4(1 - 2a\mu - a^2\mu^2)} > 1 \iff 4\mu(a + b) + 4a^2\mu^2 + \mu^2(b - a)^2 > 0$$

that is always verified.

Also, the symmetric candidate equilibrium price is well-defined if the denominator of (22) is positive.

Remark 3.

$$1 - 2a\mu - a^2\mu^2 > 0 \iff a\mu < \sqrt{2} - 1 \tag{23}$$

A sufficient condition to satisfy the condition in (23) for all values of $a \in [0, 1]$ is

$$\mu < \sqrt{2} - 1.$$

Summarizing the discussion, we can state the following result,

Proposition 2. *Let $\mu < \sqrt{2} - 1$. Then, there exists a unique symmetric price equilibrium. It is given by*

$$p^* = c\frac{(2 - a\mu + b\mu)^2}{4(1 - 2a\mu - a^2\mu^2)}.$$

Note incidentally, that p^* is increasing in μ.

6 Location

We now tackle the first stage of the overall game dealing with the location choice of the firms. By substituting the symmetric equilibrium price (22), into the profit functions, we can express profits in terms of the location parameters a and b. Firm a's demand evaluated at the equilibrium price, using (5) becomes

$$D_a(a,b) = \frac{5a^2\mu + 2a(2-b\mu) + b(4+b\mu)}{8}.$$

Also,

$$p - c = c\mu\Big(\frac{5a^2\mu + 2a(2-b\mu) + b(4+b\mu)}{4(1-2a\mu - a^2\mu^2)}\Big), \tag{24}$$

so that firm a's profits in the location (sub)game are given by

$$\Pi_a(a,b) = \frac{c\mu(5a^2\mu + 2a(2-b\mu) + b(4+b\mu))^2}{32(1-2a\mu - a^2\mu^2)} \tag{25}$$

The first order condition after some simplifications can be written as

$$\frac{\partial \Pi_a}{\partial a} = \frac{c\mu(5a^2\mu + 2a(2-b\mu) + b(4+b\mu))\Lambda(a,b)}{16(1-2a\mu + a^2\mu^2)^2}$$

where

$$\Lambda(a,b) = 4 + 2b\mu + b^2\mu^2 - 15a^2\mu^2 - 5a^3\mu^3 + a\mu(6 + 6b\mu + b^2\mu^2)$$

Note that $\Lambda(a,b) > 0$, because given the restrictions on a and μ it follows that

$$6a\mu > 5a^3\mu^3, \text{ and}$$
$$4 + 2b\mu + b^2\mu^2 + a\mu(6b\mu + b^2\mu^2) > 15a^2\mu^2$$

Using (24), we can write the first order condition as

$$\frac{\partial \Pi_a}{\partial a} = \frac{(p-c)\Lambda(a,b)}{4(1-2a\mu + a^2\mu^2)}.$$

The sign of this derivative is always positive. Therefore,

Proposition 3. *The symmetric equilibrium price induces a symmetric location around the center of the market.*

7 Conclusion

Address models of differentiation are characterized by the exogeneity of the transport cost function. Implicitly, this amounts to assume that transport services are delivered under perfectly competitive conditions. However, the market of transportation often is oligopolistic. Also there are markets where transport costs are dependent on demand such as the supply of electricity, or water. We propose to approach this issue by using an iceberg transport cost technology. We show that the iceberg transport technology induces a more elastic demand system than the traditional Hotelling model. We propose to study price and location equilibria in a duopoly model à la Hotelling where transport costs are modeled in the iceberg fashion. In this sense we want to contribute to the debate between the modeling of transport costs in the spatial location and price competition models and in the new economic geography tradition. We characterize a symmetric price equilibrium when melting is proportional to a power of distance. Then we show that a symmetric equilibrium price induces a symmetric location around the center of the market, thus reproducing the principle of minimum differentiation.

We argue that our analysis is robust to both concave and convex transport cost functions. Also, it is easy to extend our analysis considering melting and traditional transport cost functions together. In this sense, we could capture a competitive transport market and a quality dimension by which higher quality products convey higher transport costs. In other words, we would model the delivered price as the combination of an *ad valorem* and a per unit component. An example of this modeling could be $p(1+\mu\delta^{\alpha})+t\delta$. Note that this formulation implies that the slope of the delivered price function increases in an amount equal to the transport cost per unit of distance. Accordingly, our qualitative results would remain unchanged.

References

Abdel-Rahman, H.M., (1994), Economies of Scope in Intermediate Goods and a System of Cities, *Regional Science and Urban Economics*, 24: 497-524.

Abdel-Rahman, H.M., and A. Anas (2004), Theories of Systems of Cities, in *Handbook of Regional and Urban Economics*, edited by J.V. Henderson and J.F. Thisse, Amsterdam, North-Holland, chapter 52: 2293-2339.

de Frutos, M.A., H. Hamoudi, and X. Jarque, 2002, Spatial competition with concave transport costs, *Regional Science and Urban Economics*, 32:531-540.

Fujita, M., (1995), Increasing Returns and Economic Geography: Recent Advances, mimeo.

Fujita, M., and P. Krugman, 2004, The New Economic Geography: past, present and future, *Papers on Regional Science*, 83: 139-164.

Fujita, M., P. Krugman, and A. Venables, 2000, *The Spatial Economy: Cities, Regions, and International Trade*, Cambridge (Mass.), The MIT Press.

Helpman, E. and P. Krugman, 1988, *Market Structure and Foreign Trade*, Cambridge, Mass, MIT Press.

Krugman, P., 1991a, Increasing Returns and Economic Geography, *Journal of Political Economy*, 99: 483-499.

Krugman, P., 1991b, *Geography and Trade*, Cambridge (Mass.), The MIT Press.

Krugman, P., 1992, A Dynamic Spatial Model, NBER, working paper No. 4219.

Krugman, P., 1998, Space: the final frontier, *Journal of Economic Perspectives*, 12: 161-174.

McCann, P., 2005, Transport Costs and New Economic Geography, *Journal of Economic Geography*, 5: 305-318.

Neary, P., 2001, Of Hype and Hyperbolas: Introducing the New Economic Geography, *International Economic Review*, 43: 409-416.

Ottaviano, G., and J.F. Thisse, 2004, Agglomeration and Economic Geography, in *Handbook of Regional and Urban Economics*, edited by J.V. Henderson and J.F. Thisse, Amsterdam, North-Holland, chapter 58: 2563-2608.

Samuelson, P.A., (1954), The Transfer Problem and Transport Costs II: Analysis of Effects of Trade Impediments, *The Economic Journal*, 64: 264-289.

Samuelson, P.A. (1983), Thünen at Two Hundred, *American Economic Review*, 21: 1468-1488.

Schmutzler, A., 1999, The New Economic Geography, *Journal of Economic Surveys*, 13: 355-379.

Thünen, J.H. von, 1930, *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie*, third ed., Ed. Heinrich Waentig, Jena: Gustav Fisher (english translation Peter Hall (ed.) *von Thünen's Isolated State*, London, Pergamon Press, 1966.)

Appendix

Proof of Corollary 1

i) Under MQD melting we have:

$$\frac{\partial h(\delta,\mu)}{\partial \delta} = \frac{\mu}{(1-\mu\delta)^2}$$

and

$$H(\delta,\mu) = \frac{1}{\mu}(-\mu\delta - \ln(1-\mu\delta)).$$

Hence,

$$(1+h(a,\mu))^2 = \frac{1}{(1-\mu a)^2}$$

and

$$\frac{\partial h(a,\mu)}{\partial \delta}(a + H(a,\mu) - H(0,\mu)) = \frac{-\ln(1-\mu a)}{(1-\mu a)^2}.$$

As $\ln(1-\mu a) < 0$ and $\dfrac{d\ln(1-\mu a)}{da} < 0$, if expression (8) in Proposition 1 holds for $a = \frac{L}{2}$, it will also hold for $a < \frac{L}{2}$.

Given that

$$\frac{1}{(1-\mu\frac{L}{2})^2} > \frac{-\ln(1-\mu\frac{L}{2})}{(1-\mu\frac{L}{2})^2} \Leftrightarrow e > \frac{1}{1-\mu\frac{L}{2}} \Leftrightarrow 2\frac{e-1}{e\mu} > L,$$

we obtain, from Proposition 1, the result.

ii) Under MD melting we have:

$$\frac{\partial h(\delta,\mu)}{\partial \delta} = \mu\alpha\delta^{\alpha-1}$$

and

$$H(\delta,\mu) = \frac{\mu\delta^{\alpha+1}}{\alpha+1}.$$

Hence, as

$$(1+h(a,\mu))^2 = 1 + 2\mu a^{\alpha} + \mu^2 a^{2\alpha}$$

and

$$\frac{\partial h(a,\mu)}{\partial \delta}(a + H(a,\mu) - H(0,\mu)) = \alpha\mu a^{\alpha} + \frac{\alpha}{\alpha+1}\mu^2 a^{2\alpha},$$

the inequality (8) in Proposition 1 holds for $\alpha \le 2$. Accordingly, for $\alpha \le 2$ the monopolist locates at the center and covers all the market.

iii) Under Samuelson melting we have:

$$\frac{\partial h(\delta,\mu)}{\partial \delta} = \mu e^{\mu\delta}$$

and

$$H(\delta,\mu) = \frac{e^{\mu\delta}}{\mu} - \delta.$$

Hence, given that

$$(1 + h(a,\mu))^2 = e^{2\mu a}$$

and

$$\frac{\partial h(a,\mu)}{\partial \delta}.(a + H(a,\mu) - H(0,\mu)) = e^{2\mu a} - e^{\mu a},$$

from Proposition 1 we obtain that the monopolist locates at the center and covers all the market. □