Single-Majoron emission in μ decay

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The $\mu \rightarrow e\theta$ and $\mu \rightarrow e\rho_L$ decays, where θ is a Nambu-Goldstone boson associated with the $B-L$ breakdown and ρ_L is a very light neutral Higgs boson, are evaluated in the framework of the triplet model of Gelmini and Roncadelli. It is shown that the widths of these decay modes may be comparable to the $\mu \rightarrow e\gamma$ one.

At the level of the electroweak $SU(2)_L \otimes U(1)$ gauge theory, an appealing way to generate neutrino Majorana masses is the spontaneous breaking of a global $B-L$ symmetry. In such a case a Nambu-Goldstone boson, called the Majoron, should arise. Though no new long-range force has been detected so far, the existence of such a massless particle is not ruled out by present experiments due to the weakness of its coupling to matter.¹

An interesting model of this kind has been proposed by Gelmini and Roncadelli.² In this model there are no right-handed neutrinos and Majorana masses are acquired through the Yukawa coupling with a scalar triplet which develops a very small vacuum expectation value v ($v < 1$ MeV).³ Due to the smallness of v , the scalar bosons of the theory can play an important role via the Yukawa couplings proportional to m_ν/v . In fact, the charged-Higgs-boson sector, containing a single-charged scalar and a double-charged scalar, dominates the electromagnetic μ decay⁴ ($\mu \rightarrow e\gamma$).

In this paper we are interested in the relevance of the neutral-Higgs-boson sector. The model has three neutral scalars: a heavy one ρ_H , which corresponds to the neutral Higgs boson of the standard theory, a light scalar ρ_L , with mass proportional to v [$m\rho_L \sim O(\lambda_1 v)$, λ_1 being the parameters of the Higgs potential], and the massless Nambu-Goldstone boson θ associated with the $B-L$ breakdown. The last two have a gauge coupling to the Z^0 that contributes to the Z^0 decay, $Z^0 \rightarrow \theta\rho_L$. The partial decay width into this channel is precisely twice its decay width into each neutrino-antineutrino³ (two-neutrino for Majorana neutrinos) mode. This result is also valid for the $\theta\rho_L$ decay of a virtual Z^0 , so there would be an ambiguity in all the proposed experiences to measure the number of generations via the production of neutrino pairs,⁵⁻⁷ if the resulting counting modes are more than four. It is then of great importance to look for evidence for or against the existence of these two scalars. Unfortunately, these particles have proven to be very evasive, because they are weakly coupled to charged leptons ($g_{\theta l+l-} \sim g_{\rho_L l+l-} \sim m_l/u$, where u is the vacuum expectation value of the standard scalar doublet).

The ρ_L and θ have also diagonal Yukawa couplings to neutrinos, which are proportional to m_ν/v . This fact has been used in Majoron bremsstrahlung from external neutrino lines to put bounds on the Yukawa couplings.⁸ One-loop diagrams involving internal neutrino lines will also result in Majoron emission, leading to processes such as $\mu \rightarrow e\theta\theta$ and $\mu \rightarrow \rho_L\rho_L e$ which may be relevant in precision measurements of the parameters in muon decay.⁹

In the same philosophy we study the decays $\mu \rightarrow e\theta$ and $\mu \rightarrow e\rho_L$, occurring through one-loop diagrams with neutrinos

in the internal lines. In Fig. 1 we show the contributing diagrams at the leading order in the Feynman gauge. Additional diagrams with the emitted scalar coming from the charged-lepton lines or from other trilinear bosonic vertices are negligible. The Yukawa couplings of the relevant charged scalars, including the nonphysical one, are given by

$$\begin{aligned} \mathcal{L}_{CY} = & 2 \left(\frac{G_F}{\sqrt{2}} \right)^{1/2} \bar{\nu} (\tan \alpha U D_L R + \cot \alpha D_\nu U L) e \omega^{(+)} \\ & - 2 \left(\frac{G_F}{\sqrt{2}} \right)^{1/2} \bar{\nu} (U D_L R - D_\nu U L) e \sigma^{(+)} + \text{H.c.} , \quad (1) \end{aligned}$$

where ν and e are neutrino and charged-lepton matrices in

FIG. 1. Diagrams contributing in the Feynman gauge to the process $\mu^- \rightarrow e^- + \theta$ and $\mu^- \rightarrow e^- + \rho_L$.

generation space, ω and σ represent the physical charged Higgs boson and the nonphysical charged scalar, respectively, $L = \frac{1}{2}(1 + \gamma_5)$ and $R = \frac{1}{2}(1 - \gamma_5)$ are the left and right projection operators, and $\tan\alpha \equiv \sqrt{2}v/u \ll 1$. D_l and D_ν are the diagonal mass matrices of the charged leptons and physical neutrinos, respectively, and U is a unitary mixing matrix parametrized in the Kobayashi-Maskawa manner. In fact, U is the same leptonic mixing matrix which appears in the charged gauge sector.¹⁰ In this parametrization the additional phases characteristic of Majorana neutrinos have been included in the definition of the neutrino fields, i.e., $\nu = -A\nu^c$, A being a diagonal matrix of phases.¹¹

The Yukawa couplings of the neutral light scalars to neutrinos are given by

$$\mathcal{L}_{\text{NY}} \approx -\frac{1}{2v} \bar{\nu} D_\nu \nu \rho_L - \frac{i}{2v} \bar{\nu} D_\nu \gamma_5 \nu \theta, \quad (2)$$

and the relevant gauge couplings of the neutral light scalars, charged physical Higgs bosons, and charged gauge bosons are

$$\mathcal{L}_{\text{GH}} \approx i\frac{g}{\sqrt{2}} W_\mu^{(+)} (\omega^{(-)} \bar{\nu} \rho_L) - \frac{g}{\sqrt{2}} W_\mu^{(+)} (\omega^{(-)} \bar{\nu} \theta) + \text{H.c.} \quad (3)$$

The similar vertices with a $\sigma^{(-)}$ replacing the $\omega^{(-)}$ are suppressed by a factor v/u .

In the Feynman gauge there are couplings to the nonphysical charged scalar, coming from the Higgs potential, which contribute to the leading order. They are

$$\mathcal{L}_{\text{HP}} \approx \frac{\lambda_4 u}{2\sqrt{2}} \sigma^{(+)} \omega^{(-)} \rho_L + i\frac{\lambda_4 u}{2\sqrt{2}} \sigma^{(+)} \omega^{(-)} \theta + \text{H.c.}, \quad (4)$$

where λ_4 is the coupling constant of the Higgs potential that

characterizes the mass of the physical charged scalar:

$$M_\omega^2 = \lambda_4 M_W^2 / g^2 = \frac{\lambda_4}{4\sqrt{2}G_F}. \quad (5)$$

Our calculation of the processes $\mu \rightarrow e \rho_L$ and $\mu \rightarrow e \theta$ gives a finite result. In fact, the divergent parts of diagrams (d) and (e) of Fig. 1 are, in the Feynman gauge, independent of neutrino masses, and, therefore, they vanish due to the unitarity of the mixing matrix. Note that for a diagonal leptonic amplitude this divergent part would not cancel. In this gauge, diagrams (a), (f), and (g) are finite, whereas the divergences of (b) and (c) cancel among themselves. In order to check our results we have also performed the calculation in the unitary gauge. Here the situation is quite different. Diagrams (c), (f), and (g) are not present, and the divergent contribution of (b) is canceled with the divergences coming from the transverse part of the W propagator in diagrams (a), (d), and (e). The sum of diagrams gives the same result as in the Feynman gauge, as it should.

Due to the renormalizability of the theory, we can understand these cancellations as follows. In this model, the Lagrangian expressed¹⁰ in terms of the fields of definite mass has all the neutral couplings diagonal. This property of the model is a consequence of the fact that each component of the lepton doublets gets its mass from only one vacuum expectation value. In particular, the Yukawa couplings of charged leptons and neutrinos to neutral Higgs bosons are both diagonal. So, we can argue that the one-loop calculation of the processes $\mu \rightarrow e \rho_L$ and $\mu \rightarrow e \theta$ should be finite because there are no counterterms available to absorb the divergences.

Making the $m_\nu, m_l, m_{\rho_L} \ll M_W, M_\omega$ approximation, the amplitudes are given to leading order by

$$T(\mu \rightarrow e \theta) \approx \frac{iG_F}{4\sqrt{2}\pi^2 v} \sum_l U_l^* U_{l\mu} m_{\nu_l}^2 F_l^\theta \bar{u}(p_e) (m_\mu R - m_e L) u(p_\mu), \quad (6)$$

$$T(\mu \rightarrow e \rho_L) \approx \frac{-G_F}{4\sqrt{2}\pi^2 v} \sum_l U_l^* U_{l\mu} m_{\nu_l}^2 F_l^{\rho_L} \bar{u}(p_e) (m_\mu R + m_e L) u(p_\mu),$$

where

$$F_l^\theta \approx 3 + 2 \ln \eta_l - \cot^2 \alpha \eta_l (1 + \ln \eta_l) + \frac{3}{2} \frac{1+r_\omega}{1-r_\omega} \ln r_\omega, \quad (7)$$

$$F_l^{\rho_L} \approx 1 + \ln r_\omega + \frac{1}{2} \eta_l \cot^2 \alpha,$$

with

$$r_\omega \equiv M_\omega^2 / M_W^2, \quad \eta_l \equiv m_{\nu_l}^2 / M_\omega^2. \quad (8)$$

The neutrino-mass dependence appearing in the amplitudes of Eq. (6) has two different origins. In the diagrams with a single Yukawa coupling one gets a neutrino-mass factor (accompanied by v in the denominator) from this source, the other m_ν factor coming from the neutrino propagator in the internal line. When several Yukawa couplings are present, one gets contributions having directly m_ν^2 factors from the vertices [see Eq. (1)], in addition to the contributions described above. These neutrino-mass factors avoid the existence of Glashow-Iliopoulos-Maiani-type (GIM-type) cancellations, so the corresponding loop in-

tegrations do not lead to m_ν^2 / M_W^2 suppressing factors.

The same absence of GIM-type cancellations occurs⁴ in the scalar couplings relevant for $\mu \rightarrow e \gamma$ decay, contrary to the standard gauge contribution to it. This situation is different from the one present, for example, in the $\nu_2 \rightarrow \nu_1 \gamma$ decay amplitude, where the dominant Yukawa couplings give neutrino-mass factors of the fixed external lines. As a consequence, these amplitudes mediated by scalars suffer the same GIM suppression as that induced by the gauge couplings.

Using the amplitudes (6), one obtains immediately the widths

$$\Gamma(\mu^- \rightarrow e^- \theta) \approx \frac{G_F^2 m_\mu^3}{2^9 \pi^5 v^2} \left| \sum_l U_l^* U_{l\mu} m_{\nu_l}^2 F_l^\theta \right|^2, \quad (9)$$

$$\Gamma(\mu^- \rightarrow e^- \rho_L) \approx \frac{G_F^2 m_\mu^3}{2^9 \pi^5 v^2} \left| \sum_l U_l^* U_{l\mu} m_{\nu_l}^2 F_l^{\rho_L} \right|^2.$$

Adding the two contributions, the branching ratio for μ decay into an electron and a neutral Higgs boson is given by

$$R \equiv \frac{\Gamma(\mu \rightarrow e\theta) + \Gamma(\mu \rightarrow e\rho_L)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} \approx \frac{3}{8m_\mu^2 v^2 \pi^2} \left[\left| \sum_i U_{ie}^* U_{i\mu} m_{\nu_i}^2 F_i^\theta \right|^2 + \left| \sum_i U_{ie}^* U_{i\mu} m_{\nu_i}^2 F_i^{\rho L} \right|^2 \right]. \quad (10)$$

It seems reasonable to assume that the mixings are dominated by the muon-neutrino mass. In this case we obtain a simplified expression

$$R \approx \frac{3\beta^2 m_{\nu_\mu}^4}{8m_\mu^2 v^2 \pi^2} (|F_\mu^\theta|^2 + |F_\mu^{\rho L}|^2), \quad (11)$$

in terms of the $e - \mu$ mixing $\beta \equiv |U_{e\mu}|$.

$$B \equiv \frac{\Gamma(\mu \rightarrow e\theta) + \Gamma(\mu \rightarrow e\rho_L)}{\Gamma(\mu \rightarrow e\gamma)} \approx \frac{9\lambda_4^2 v^2}{25\alpha\pi m_\mu^2} \frac{\left| \sum_i U_{ie}^* U_{i\mu} m_{\nu_i}^2 F_i^\theta \right|^2 + \left| \sum_i U_{ie}^* U_{i\mu} m_{\nu_i}^2 F_i^{\rho L} \right|^2}{\left| \sum_i U_{ie}^* U_{i\mu} m_{\nu_i}^2 \right|^2}. \quad (13)$$

In this expression we have leptonic mixings in both the numerator and the denominator. Assuming again dominance of the muon-neutrino mass, these mixing factors simplify and we get

$$B \approx \frac{9\lambda_4^2 v^2}{25\alpha\pi m_\mu^2} (|F_\mu^\theta|^2 + |F_\mu^{\rho L}|^2) < 9.5 \times 10^{-3} (|F_\mu^\theta|^2 + |F_\mu^{\rho L}|^2), \quad (14)$$

where the bounds for λ_4 [$\lambda_4 < 2.6$ (Ref. 4)], coming from the known limits for the doubly-charged-scalar mass^{3,12} and for v have been used. The upper bounds given in Eqs. (12) and (14) are related to each other by the present experimental limit on the $\mu \rightarrow e\gamma$ branching ratio:^{13,14}

$$\Gamma(\mu \rightarrow e\gamma)/\Gamma(\mu \rightarrow e\nu\bar{\nu}) < 1.7 \times 10^{-10}.$$

The F_μ^θ function appearing in these expressions is dominated by the terms in $\ln\eta_\mu$ and gives a contribution to the ratios which is much bigger than that of $F_\mu^{\rho L}$. From Eq. (7) we have

$$|F_\mu^\theta|^2 + |F_\mu^{\rho L}|^2 \approx |F_\mu^\theta|^2 \approx \left| 2\ln\eta_\mu \left[1 - \frac{m_{\nu_\mu}^2}{\lambda_4 v^2} \right] \right|^2. \quad (15)$$

Therefore, the μ decay to a single neutral Higgs boson occurs mainly through the Majoron channel.

From the limit¹² on the Yukawa coupling to the muon

Using here the bound $m_{\nu_\mu}^2 \beta / v^2 < 7 \times 10^{-4}$, coming from $\mu \rightarrow e\gamma$ decay,⁴ and the known upper limit on the triplet vacuum expectation value, we have

$$R < 1.7 \times 10^{-12} (|F_\mu^\theta|^2 + |F_\mu^{\rho L}|^2). \quad (12)$$

It is worthwhile to compare these decay modes with the $\mu \rightarrow e\gamma$ one. Using the results of Ref. 4, we can write

neutrino, we can safely neglect the second term within the parentheses of Eq. (15). The enhancement of the logarithmic factor can produce a width for the Majoron channel comparable to the $\mu \rightarrow e\gamma$ one. This is illustrated by the upper bound (14), which can be written as

$$B \lesssim 50 \left[1 - \frac{\ln(m_{\nu_\mu}/1 \text{ keV})}{20} \right]^2, \quad (16)$$

so it increases for smaller neutrino masses. To get an actual value and not an upper bound we estimate, for $\lambda_4 = 1$ ($M_\omega = 123 \text{ GeV}$), $v = 100 \text{ keV}$, and $m_{\nu_\mu} = 1 \text{ keV}$, the value $B \approx 8 \times 10^{-2}$.

In conclusion, we can expect a width for the Majoron mode of the order of 10^{-1} – 10^2 times the $\mu \rightarrow e\gamma$ one, and therefore a branching ratio to the $\mu \rightarrow e\nu\bar{\nu}$ channel smaller than 10^{-8} . Unfortunately, as the only signal of this decay would be a little bump at the end of the electron energy spectrum (the Majoron scarcely interacts with ordinary matter), the detection of this channel, suppressed by more than eight orders of magnitude, seems to be difficult for the present experimental possibilities.

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