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QCD FORMULATION OF THE TAU DECAY AND DETERMINATION OF $\Lambda_{\overline{MS}}$

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ABSTRACT

We present a simple formulation of the inclusive and exclusive semi-hadronic decays of the tau-lepton using QCD-duality Finite Energy Sum Rules (FESR). We find that the tau-decay is a good laboratory for measuring the QCD scale Λ. Within the present experimental accuracy, we obtain $\Lambda_{\overline{MS}} \simeq$ (100-200)MeV to four loops. This prediction can be sensibly improved once the experimental situation will be clarified.

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Since its discovery in 1975, the tau lepton has been a subject of extensive study. Besides its intrinsic interest as a sequential lepton in the standard model of electroweak interactions, the tau is the only presently known lepton heavy enough to decay into hadrons. Therefore, the semi-leptonic tau decay appears to be an ideal laboratory for studying the algebra of currents of QCD. Contrary to the well-known $e^+e^- \to \gamma \to$ Hadrons process which only tests the electromagnetic current, the tau semihadronic decay modes offer the possibility to study the properties of both vector and axial currents.

Inversely, we can use our present QCD understanding of these weak currents in order to make predictions on the semi-inclusive tau decay properties. We shall be concerned with the ratio :

$$R_H \equiv \Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right) \Big/ \Gamma\left(\tau^- \to \nu_\tau\, e^-\, \bar{\nu}_e\right) , \tag{1}$$

which can be expressed as an integral over the invariant mass of the final hadrons :

$$R_H = 12\,\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left\{\left(1 + 2\,\frac{s}{M_\tau^2}\right) \mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right\} . \tag{2}$$

The spectral functions $(k = 0,1)$

$$\begin{aligned} \mathrm{Im}\,\Pi^{(k)}(s) \equiv \cos^2\theta_c &\left\{\mathrm{Im}\,\Pi_V^{(k)}(s)^{12} + \mathrm{Im}\,\Pi_A^{(k)}(s)^{12}\right\} \\ + \sin^2\theta_c &\left\{\mathrm{Im}\,\Pi_V^{(k)}(s)^{13} + \mathrm{Im}\,\Pi_A^{(k)}(s)^{13}\right\} \end{aligned} \tag{3}$$

are related to the hadronic correlators

$$\begin{aligned} \left(\Pi^{\mu\nu}_{V(A)}\right)^{ij} &\equiv i \int d^4x\, e^{iqx} \left\langle 0 \left| T \left(J^\mu_{V(A)}(x)^{ij} \left(J^\nu_{V(A)}(0)^{ij}\right)^+\right) \right| 0\right\rangle \\ &\equiv (-g^{\mu\nu} q^2 + q^\mu q^\nu)\, \Pi^{(1)}_{V(A)}(q^2)^{ij} + q^\mu q^\nu\, \Pi^{(0)}_{V(A)}(q^2)^{ij} , \end{aligned} \tag{4}$$

where $J^\mu_{V(A)}(x)^{ij} \equiv \bar{q}_i\, \gamma^\mu\, (\gamma_5)\, q_j$ are the charged vector (axial) currents and i,j denote the quark flavours.

R_H has been already considered in the past within QCD for the estimate of the hadron continuum[1] above the A_1-resonance and more recently in Ref.2. A full QCD estimate of R_H has been given in Ref 3 by using the Shankar contour method[4] which relates the integration on the physical cut appearing in Eq. (2) to integrals around a circle of radius $|s| = M_\tau^2$:

$$R_H = 6\pi i \int_{C_{M_\tau^2}} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2 \left\{\left(1 + 2\frac{s}{M_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right\}. \tag{5}$$

The integration on the circle $C_{M_\tau^2}$ is done by using the short-distance QCD expression of the $\Pi^{(k)}(s)$ correlators.

Though conceptually very similar to the Shankar method, Finite Energy Sum Rules (FESR)[4,5] in the way derived in Ref 6) can provide an elegant and quite rigorous method for a QCD evaluation of Eq (2), including the non-perturbative effects parametrized à la SVZ[7]. We shall deal here with the QCD expression of the moments :

$$\mathcal{M}_n^{(k)}\left(M_\tau^2\right)^{ij} = \int_0^{M_\tau^2} ds\ s^n\ \mathrm{Im}\ \Pi^{(k)}_{V(A)}\ (s)^{ij} \tag{6}$$

for n=0,1,2,3 ; k = 0,1 and will estimate within FESR the contribution of each J^{P-} channel to R_H.

1. 1^{--} Vector Channel

For these modes, the moments $\mathcal{M}_n^{(k)}\left(M_\tau^2\right)^{ij}$ for n=0,1,2,3 have been derived in Ref 5e) and to three loops in Ref 6). Including the new four-loop calculation of Ref 8), one gets in the $\overline{MS}$-scheme :

$$R_H\ (1^-)^{ij} = \xi^{ij}\ \frac{3}{2}\left\{1 + a_s + a_s^2\left(F_3 - \frac{19}{24}\beta_1 - \frac{\beta_2}{\beta_1}L\right) + a_s^3\left[F_4 - \frac{19}{12}F_3\ \beta_1\right.\right.$$

$$\left. - \frac{19}{24}\beta_2 + \frac{265}{288}\beta_1^2 - 2\frac{\beta_2}{\beta_1}L\left(F_3 - \frac{19}{24}\beta_1\right) + \left(\frac{\beta_2}{\beta_1}\right)^2 (L^2 - L - 1) + \frac{\beta_3}{\beta_1}\right] +$$

$$+\, 2\, \frac{C_2\langle O_2\rangle_V}{M_\tau^2} - 6\, \frac{C_6\langle O_6\rangle_V}{M_\tau^6} - 4\, \frac{C_8\langle O_8\rangle}{M_\tau^8}\Bigg\} \tag{7a}$$

where :

$$\beta_1 = -\frac{11}{2} + \frac{n}{3} \quad , \quad \beta_2 = -\frac{51}{4} + \frac{19}{12}\, n$$

$$\beta_3 = \frac{1}{32}\left(-\frac{2857}{2} + \frac{5033}{18}\, n - \frac{325}{54}\, n^2\right) \quad \text{for} \quad SU(3)_C \times SU(n)_F \quad ;$$

$$a_s \equiv -\frac{1}{\beta_1 \log M_\tau/\Lambda} \quad ; \quad L \equiv \log\log M_\tau^2/\Lambda^2 \quad ;$$

$$F_3 = 1.986 - 0.115\, n$$

$$F_4 = 70.985 - 1.200\, n - 0.005\, n^2 \quad . \tag{7b}$$

The factor ξ^{1j} takes into account the different Cabibbo mixing structure of the $|\Delta S| = 0,1$ channels, i.e. :

$$\xi^{12} = \cos^2\theta_c \qquad \xi^{13} = \sin^2\theta_c \quad . \tag{8}$$

The leading quark mass corrections are given by :

$$C_2\langle O_2\rangle = -\, 3\left(\bar{m}_i^2 + \bar{m}_j^2\right) + \frac{17}{4}\left(\bar{m}_i - \bar{m}_j\right)^2 \quad , \tag{9}$$

where $\bar{m}_i(M_\tau) \equiv \hat{m}_i \Big/ (\log M_\tau/\Lambda)^{2/-\beta_1}$ are the running quark masses evaluated at the M_τ-scale. The invariant light quark masses have been determined to be[9]:

$$\hat{m}_u = (8.6 \pm 1.5)\ \text{MeV} \; ; \; \hat{m}_d = (15.2 \pm 2.7)\ \text{MeV} \; ; \; \hat{m}_s = (272 \pm 47)\ \text{MeV} \; . \tag{10}$$

One should note that only $C_6\langle O_6\rangle$ and $C_8\langle O_8\rangle$ non-perturbative contributions due to dimension-six and eight vacuum condensates enter in (7),but there is no contribution of dimension-four condensates to this order. The effects of condensates are known and can be estimated to be :

$$C_6\langle O_6\rangle_V = -\frac{896}{81}\pi^3\ \alpha_s\left\langle\bar\psi\ \psi\right\rangle^2\ .\ \rho$$

$$C_8\langle O_8\rangle_V \simeq \frac{\pi^2}{648}\ .\ (26 \sim 39)\left\langle\alpha_s\ G^2\right\rangle^2 \qquad (11)$$

where $\rho = 1$ corresponds to the usual vacuum saturation assumption[7] for the four-quark condensate. We will use $\rho \simeq 2$-4 as suggested by different phenomenological QCD sum rules analyses[10]. The dimension-eight operator contribution in Eq (11) comes from the result of Ref 11). The strength of these dimension - eight operators has been estimated by using the modified factorization of Ref 12). Using the range $\left\langle\alpha_s G^2\right\rangle \simeq (1 \sim 2).\ (0.04\ \mathrm{GeV}^4)$, one can realize that the contribution of the dimension-eight operators is negligible, indicating a good convergence of the Operator Product Expansion at the M_τ^2-scale. The biggest non-perturbative effects come from the dimension-six condensates which are 2.2 to 4.5% for the range of ρ-values given previously*). Quark mass corrections are negligible for the $\Delta S = 0$ case and amount to 2-3% of the leading-term in the Cabibbo suppressed channels.

The most important corrections are due to the radiative gluonic contributions. For $\Lambda \simeq$ 100 MeV, the a_s, a_s^2 and a_s^3- terms contribute in Eq (7a) by 7.7, 1.2 and 3.5% of the lowest order term. One can notice that the a_s^3 -correction turns out to be relatively important due to the large value of F_4 found in the four-loop calculation of Ref 8). In fact, for larger values of Λ say 300 MeV, this a_s^3-term gives the dominant correction. The effects of the a_s, a_s^2 and a_s^3-terms are in this case 12.5, 4.6 and 16.0 % respectively. Here, we should notice that the sign of the a_s^2-contribution in Ref 3) is incorrect.

In the chiral limit and to four-loops, we find for the ratio $R_H(1^-)$ of the 1^{--} channel**) :

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*) *Radiative corrections due to the dimension-four condensates are much smaller than the uncertainty in the* $C_6\langle O_6\rangle$ *contribution.*

**) *We have not considered the electro-weak radiative corrections to our result. We might expect that these effects are smaller than the QCD ones discussed previously. We plan to analyze this point in a future publication.*

$$R_H(1^-) \simeq \begin{pmatrix} 1.72 - 1.75 \\ 1.85 - 1.88 \\ 2.03 - 2.06 \end{pmatrix} \quad \text{for} \quad \Lambda \simeq \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix} \text{ GeV} \tag{12}$$

The value of $R(1^-)^{11}$ for massive quarks is given in the Table for each Cabibbo favoured and suppressed channel , which we can compare with the data of exclusive decay modes given in the Table. The $R(1^-)^{12}$ channel is estimated phenomenologically as a source of even number of pions using the data of Refs 13), 14) and 15). Here, we have used the CVC argument of Ref 16) for the estimate of the 4π final states from the $e^+e^- \to 4\pi$ data. One can notice from the Table that a good agreement between QCD and the data is obtained for :

$$\Lambda \simeq 100 \text{ MeV} . \tag{13}$$

The $R(1^-)^{13}$ channel is estimated from the data $\tau \to K\,\nu_\tau$ + pions of Refs 13) and 17). The data are inaccurate for a good determination of Λ.

2. 0^{++} Channels

Due to the conservation of the vector current, the associated divergence is proportional to the light quark mass difference :

$$\partial_\mu V^\mu = (m_i - m_j)\, \bar{\psi}_i (i)\, \psi_j \; : \tag{14}$$

The tau decay width into the 0^{++} modes is dominated by the $a_0(980)$ for $\Delta S = 0$ and by the $K_0^*(1350)$ for $|\Delta S| = 1$. These widths were estimated in Ref 18) and are unobservable. The smallness of these widths is due to the fact that unlike f_π , the decay constants of the a_0 and K_0^* are proportional to the light quark mass differences so that they are negligible.

3. $(1^{++} + 0^{-+})$ Channels

Unlike the case of 1^{--} and 0^{++}, it is convenient to work with the sum of the axial and pseudoscalar channels. Experimentally, this is justified as they both are responsible for odd numbers of pions in the tau decay. From the theoretical side, the difficulty in separating these two channels is due to the fact that a Goldstone (zero-mass) pion contributes to the lowest moment in (6) like f_π^2 where we know that f_π^2 does not vanish in the chiral limit. A separate QCD estimate of the 0^{-+} channels requires the theoretical knowledge of the subtracting constant $\frac{\partial \Pi^{(0)}}{\partial q^2}$ at $q^2 = 0$, which remains outside the QCD control at present. On the other hand, in the chiral limit $m_i = 0$, only one invariant function $\Pi_A(q^2)$ appears in the Lorentz decomposition of the axial-axial correlator:

$$\Pi_A^{\mu\nu}(q^2) \equiv -(g^{\mu\nu} q^2 - q^\mu q^\nu)\, \Pi_A(q^2) \tag{15a}$$

with :

$$\Pi_A(q^2) = \Pi_A^{(1)}(q^2) + \Pi_A^{(0)}(q^2)\ , \tag{15b}$$

which corresponds to the sum of the 1^{++} and 0^{-+} invariants.
From (15) and chiral invariance, we can use (7a) in the massless quark limit with the change :

$$C_6 \langle O_6 \rangle_{1+0} = -\frac{11}{7}\, C_6 \langle O_6 \rangle_V\ , \tag{16}$$

due to the γ_5-flip.
Therefore, one obtains in the chiral limit :

$$R(1^+ + 0^-) = \begin{pmatrix} 1.58 - 1.64 \\ 1.71 - 1.77 \\ 1.89 - 1.94 \end{pmatrix} \quad \text{for} \quad \Lambda = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix} \text{GeV}\ , \tag{17}$$

where we have used the same F_4 for the vector and axial correlators. This property is supported by the general theorem of Ref 19). The inclusion of the mass correction corresponds to :

$$C_2 \langle O_2 \rangle_{1+0} = -3\left(m_i^2 + m_j^2\right) - (m_i + m_j)^2\ . \tag{18}$$

We give the mass corrected ratio in the Table for each ΔS channel together with the corresponding experimental value. For $R(1^+ + 0^-)^{12}$, we use the data on odd number of pions given in Refs 13), 14), 15), 17) and 20), namely $\tau \to (\pi,\ 3\pi,\ K\bar{K}\pi,\ 5\pi) + \nu_\tau$. For the 13 channel, we use the data[13,17] on $\tau \to (K,\ K\pi\pi) + \nu_\tau$. As in the vector channel, a small value of Λ in the range given by Eq (13) is favoured. One should notice again the inaccuracy of the data in the $|\Delta S| = 1$ channel.

4. Electronic and Total Widths

Let us add Eqs (12) and (17). We obtain the tau-hadronic width in the chiral limit :

$$R_H \equiv R(1^-) + R(1^+ + 0^-) = \begin{pmatrix} 3.34 - 3.36 \\ 3.60 - 3.62 \\ 3.95 - 3.97 \end{pmatrix} \text{ for } \Lambda = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix} \text{ GeV} \tag{19}$$

The inclusion of the mass corrections decreases (19) by about 0.5% after adding the different widths in the Table.
R_H is related to the total tau width through the equation :

$$\Gamma(\tau^-) = \Gamma\left(\tau \to \nu_\tau \; e \; \bar{\nu}_e\right) \left\{1 + f\left(\left(\frac{m_\mu}{M_\tau}\right)^2\right) + R_H\right\} , \tag{20a}$$

where :

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12\,x^2 \log x \,. \tag{20b}$$

We can use the experimental electronic branching ratio[13] :

$$B_e \equiv \frac{\Gamma\left(\tau \to \nu_\tau \; e \; \bar{\nu}_e\right)}{\Gamma(\tau^-)} \simeq (17.7 \pm 0.4)\% \tag{21}$$

in Eq (20) and deduce the experimental value of R_H:

$$R_H^{EXP} \simeq 3.68 \pm 0.13 \,. \tag{22}$$

One should notice the well known discrepancy between this inclusive determination of the semihadronic width and the sum of the experimental exclusive widths in the Table $R_H = 3.22 \pm 0.10$, i.e more than 6% of the measured one-prong inclusive width seems to be missing in the exclusive measurements.
Equivalently, one can insert the QCD prediction R_H into (20) and deduce the QCD estimate of the electronic branching ratio B_e. One predicts :

$$B_e = \begin{pmatrix} 18.79 - 18.89 \\ 17.94 - 18.01 \\ 16.88 - 16.94 \end{pmatrix}\% \text{ for } \Lambda = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix} \text{ GeV} \tag{23}$$

compared to the data in (21). The best agreement between QCD prediction and experiment corresponds to :

$$\Lambda \simeq (200)\ \mathrm{MeV}\ . \tag{24}$$

Finally, using the theoretical expression of the electronic width :

$$\Gamma\left(\tau \to \nu_\tau\ e\ \bar{\nu}_e\right) \simeq \frac{G_F^2}{192\pi^3} M_\tau^5 \tag{25}$$

we can use our prediction for R_H to estimate the τ-lifetime.
We obtain :

$$\tau_\tau \simeq \begin{pmatrix} 2.99 - 3.01 \\ 2.86 - 2.87 \\ 2.69 - 2.70 \end{pmatrix} 10^{-13}\mathrm{sec} \quad \text{for} \quad \Lambda \simeq \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix} \mathrm{GeV} \tag{26a}$$

compared to the new world average quoted by the TASSO group[21]

$$\tau_\tau = (3.06 \pm 0.09)\ 10^{-13}\ \mathrm{sec}\ . \tag{26b}$$

A comparison of the experimental and QCD predicted τ-lifetime requires :

$$\Lambda \simeq 100\ \mathrm{MeV}\ . \tag{27}$$

However, one should note that the value of the τ-lifetime deduced by using the electronic width in (20) is found to be $\tau_\tau \simeq (2.82 \pm 0.06) 10^{-13}$ sec which is significantly lower than the average of direct measurements quoted in Eq. (26b). In this case, the agreement with QCD predictions would be obtained for $\Lambda \simeq 0.2 \sim 0.25$ GeV.

We conclude from our previous discussions that precise measurements of the τ electronic and hadronic widths and lifetime can provide a good determination of Λ. At present, the value of Λ deduced from the above two observables is

$$\Lambda \simeq 100 - 200\ \mathrm{MeV}, \tag{28}$$

the lowest value being preferred by the lifetime and exclusive decays measurements, while the experimental electronic branching ratio seems to favour the higher one. The determination of Λ can be sensibly improved once the experimental situation (B_e-lifetime and exclusive-inclusive discrepancies) will be clarified.

The value in (28) fits nicely with the new estimate of Λ from $e^+e^- \rightarrow$ Hadrons at $\sqrt{s}$ = 34 GeV with the inclusion of the four-loop corrections[8] into the QCD estimate of the cross-section. A small value of Λ is also favoured by QCD sum rules analyses of the isovector part of the $e^+e^- \rightarrow$ Hadrons data [22,10c], heavy[7] and heavy-light[23] quarks systems. This value of Λ is also in good agreement with the one from deep inelastic muon scattering data[24].

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TABLE :

QCD predictions and experimental (exclusive) values of the τ-hadronic widths in units of $\Gamma\left(\tau \to \nu_\tau\ e\ \bar{\nu}_e\right)^{*)}$

$\Lambda_{\overline{MS}}$ (MeV)	$\lvert\Delta S\rvert = 0$ 1^-	$1^+ + 0^-$	$\lvert\Delta S\rvert = 1$ 1^-	$1^+ + 0^-$	Total Width
100	1.64 - 1.67	1.50 - 1.55	0.086 - 0.089	0.070 - 0.076	3.22 - 3.35
200	1.76 - 1.79	1.63 - 1.68	0.092 - 0.096	0.074 - 0.082	3.58 - 3.60
300	1.93 - 1.96	1.80 - 1.85	0.101 - 0.105	0.081 - 0.090	3.93 - 3.95
EXPERIMENT	1.62 ± 0.06	1.44 ± 0.08	0.102 ± 0.017	0.062 ± 0.026	3.22 ± 0.10

*) We have not included in the Table the 0^{++} contributions,which are negligible[18].

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