CERN-TH.5102/88

QCD-DUALITY APPROACH TO NON-LEPTONIC WEAK TRANSITIONS: TOWARDS AN UNDERSTANDING OF THE $|\Delta I| = \frac{1}{2}$ RULE

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ABSTRACT

The progress done in the QCD-duality analysis of strangeness changing non-leptonic weak transitions is reviewed. The method combines the information provided by the effective chiral realization of QCD at long distances with its perturbative short-distance behaviour, via a duality approach. Special attention is given to the failure of the earlier leading-logarithm calculations in explaining the observed $|\Delta I| = \frac{1}{2}$ $K \to \pi\pi$ amplitude. It is shown that the next-to-leading α_s corrections to the $|\Delta I| = \frac{1}{2}$ correlator are enormous, and with the appropriate sign to produce the required enhancement.

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Talk given at the Ringberg Workshop on Hadronic matrix Elements and Weak Decays Ringberg Castle, Germany, April 18-22, 1988

CERN-TH.5102/88
July 1988

1. INTRODUCTION

It is well known experimentally that the strangeness changing non-leptonic weak decays with isospin transfer $|\Delta I| = \frac{1}{2}$ are enhanced, by more than one order of magnitude in amplitude, with respect to the $|\Delta I| = 3/2$ transitions. However, a satisfactory explanation of this $|\Delta I| = \frac{1}{2}$ "selection rule" within the framework of the standard model is still lacking.

The short-distance analysis of the product of weak hadronic currents[1] results in an effective $\Delta S = 1$ Hamiltonian which is a sum of local four-quark operators, constructed with the light (u,d,s) quark fields only, modulated by Wilson coefficients which are functions of the heavy (W,t,b,c) masses and an overall renormalization scale μ. The Wilson coefficients are calculated by successively removing the heavy fields from explicitly appearing in the Lagrangian, and by summing the leading logarithms of heavy masses using renormalization group techniques. It is then found that the effect of the leading QCD gluonic corrections indeed gives an enhancement by a factor two to three of the Wilson coefficient of one of the $|\Delta I| = \frac{1}{2}$ operators. Nevertheless, this by itself is not enough to explain the experimentally observed rates, without simultaneously appealing to a further enhancement in the hadronic matrix element of at least some of the isospin $\frac{1}{2}$ four-quark operators.

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The evaluation of hadronic matrix elements of local four-quark operators is a difficult problem due to the fact that they are governed by the long-distance behaviour of the strong interactions; i.e., the confinement regime of QCD. Moreover, these operators have non-zero anomalous dimensions and, therefore, their matrix elements depend on the renormalization scale μ. Since physical amplitudes are renormalization-scale independent quantities, this μ-dependence should exactly cancel the one appearing in the Wilson coefficients. In order to keep control of the renormalization-scale dependence, and therefore to get a meaningful result, a full QCD calculation of the relevant matrix elements is required. This is a highly non-trivial task.

A method to evaluate pseudoscalar matrix elements, which has the virtue of being completely defined within the framework of the standard model, was presented in Ref. 2 and applied to $K \to \pi\pi$ decays in Refs. 3 and 4. The basic idea is to combine the information provided by the effective chiral realization of QCD at long distances with its perturbative short-distance behaviour, via a duality approach using finite energy sum rules (FESRs). When applied to the $|\Delta I| = 3/2$ Hamiltonian, the method has been extremely successful in reproducing[3] the measured $K^+ \to \pi^+\pi^0$ amplitude. However, the attempt done in Ref. 4 to analyze the $|\Delta I| = \frac{1}{2}$ Hamiltonian in a similar way failed - by an order of magnitude - to explain the observed enhancement of the $K \to \pi\pi$ $|\Delta I| = \frac{1}{2}$ amplitude.

After giving (Sections 2 to 4) a brief review of the work done in Refs. 2,3 and 4, I will try in the following to analyze the reasons of this puzzling failure. Using a simplified analysis[5], it will be shown in Section 5 that the blame can be easily put on the short-distance part of the calculation, where something fundamental should be missing.

Section 6 will be devoted to the evaluation of radiative gluon corrections to the results of Ref. 4. Using some formal (but perfectly well defined) limits, to simplify as much as possible an otherwise involved computation, I will show that huge α_s corrections show up in the $|\Delta I| = \frac{1}{2}$ sector, with the appropriate sign to produce the required enhancement. The enormous size of these gluonic contributions provides a strong and rigorous evidence of an indeed peculiar dynamical behaviour of the $|\Delta I| = \frac{1}{2}$ weak transitions within the standard model. The results will be summarized in Section 7.

2. $\Delta S = 1$ SHORT-DISTANCE HAMILTONIAN

The standard short-distance analysis of the product of weak hadronic currents results in the effective $|\Delta S| = 1$ Hamiltonian[1]

$$H^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} s_1c_1c_3 \sum_i c_i(\mu)\, Q_i \tag{1}$$

where G_F is the Fermi coupling constant and $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ are the conventional Cabibbo-Kobayashi-Maskawa factors. Q_i (i=1,...,6) is a complete basis of four-quark operators, which is usually taken as[1e]

$$\begin{aligned}
Q_1 &= 4(\bar{s}_L\gamma^\mu d_L)(\bar{u}_L\gamma_\mu u_L)\\
Q_2 &= 4\ \bar{s}^\alpha_L\gamma^\mu d^\beta_L\ \ \bar{u}^\beta_L\gamma_\mu u^\alpha_L\\
Q_3 &= 4(\bar{s}_L\gamma^\mu d_L)[\bar{u}_L\gamma_\mu u_L+\bar{d}_L\gamma_\mu d_L+\bar{s}_L\gamma_\mu s_L]\\
Q_4 &= 4\ \bar{s}^\alpha_L\gamma^\mu d^\beta_L\ [\bar{u}^\beta_L\gamma_\mu u^\alpha_L+\bar{d}^\beta_L\gamma_\mu d^\alpha_L+\bar{s}^\beta_L\gamma_\mu s^\alpha_L]\\
Q_5 &= 4(\bar{s}_L\gamma^\mu d_L)[\bar{u}_R\gamma_\mu u_R+\bar{d}_R\gamma_\mu d_R+\bar{s}_R\gamma_\mu s_R]\\
Q_6 &= 4\ \bar{s}^\alpha_L\gamma^\mu d^\beta_L\ [\bar{u}^\beta_R\gamma_\mu u^\alpha_R+\bar{d}^\beta_R\gamma_\mu d^\alpha_R+\bar{s}^\beta_R\gamma_\mu s^\alpha_R]
\end{aligned} \tag{2}$$

where $q_L = \frac{1}{2}(1-\gamma_5)q$ and $q_R = \frac{1}{2}(1+\gamma_5)q$; α and β are colour indices and $(\bar{q}_L\gamma^\mu q_L) = \Sigma_\alpha \bar{q}^\alpha_L\gamma^\mu q^\alpha_L$. Only five of these operators are independent, since $Q_1+Q_4 = Q_2+Q_3$. Following the standard convention, we shall choose Q_1, Q_2, Q_3, Q_5 and Q_6 as a basis.

All the information on the heavy mass scales, which have been already integrated out, is contained in the Wilson coefficients $c_i(\mu)$. The numerical values of these coefficients can be found in Ref. 4.

From the point of view of chiral $SU(3)_L\times SU(3)_R$ and isospin quantum numbers, Q_2-Q_1, Q_3, Q_5 and Q_6 transform like $(8_L,1_R)$ and induce $|\Delta I| = \frac{1}{2}$ transitions; while $Q_1+2/3\ Q_2-1/3\ Q_3 \equiv \tilde{Q}_4$ transforms like $(27_L,1_R)$ and induces both $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = 3/2$ transitions via its components

$$\tilde{Q}_4 = 4\left(\frac{1}{9}\, O_4^{(\frac{1}{2})} + \frac{5}{9}\, O_4^{(3/2)}\right) \tag{3}$$

where

$$\begin{aligned}
O_4^{(\frac{1}{2})} &= (\bar{s}_L\gamma^\mu d_L)(\bar{u}_L\gamma_\mu u_L) + (\bar{s}_L\gamma^\mu u_L)(\bar{u}_L\gamma_\mu d_L)\\
&\quad + 2(\bar{s}_L\gamma^\mu d_L)(\bar{d}_L\gamma_\mu d_L) - 3(\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu s_L)
\end{aligned} \tag{4}$$

$$O_4^{(3/2)} = (\bar{s}_L\gamma^\mu d_L)(\bar{u}_L\gamma_\mu u_L) - (\bar{s}_L\gamma^\mu d_L)(\bar{d}_L\gamma_\mu d_L) + (\bar{s}_L\gamma^\mu u_L)(\bar{u}_L\gamma_\mu d_L)$$

It is then convenient to split the ΔS = 1 short-distance Hamiltonian in three pieces with definite transformation properties:

$$H^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \{H_{\underline{8}}^{(\frac{1}{2})} + H_{\underline{27}}^{(\frac{1}{2})} + H_{\underline{27}}^{(3/2)}\} \quad (5)$$

3. EFFECTIVE CHIRAL LAGRANGIAN FORMULATION

With chiral symmetry in the u,d,s flavour sector realized à la Nambu-Goldstone, there is an effective chiral Lagrangian formulation which represents the combined effect of the strong interactions and the ΔS = 1 first-order non-leptonic weak transitions of the octet of pseudoscalar particles at low energies

$$\mathcal{L} = \mathcal{L}_{strong} + \frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \mathcal{L}^{\Delta S=1} \quad (6)$$

To leading order in derivatives and light quark masses ($m_u = m_d = 0$, $m_s \neq 0$), the effective chiral realization of QCD at long-distances is described by the non-linear sigma model Lagrangian ($f_\pi = 93.3$ MeV),

$$\mathcal{L}_{strong} = \frac{f_\pi^2}{4} \{Tr(\partial_\mu U \partial^\mu U^+) + 2M_K^2 (U+U^+)_{33}\} \quad (7)$$

where

$$U \equiv \exp\{\frac{i\sqrt{2}}{f\pi}\Phi\}$$

denotes the 3×3 special unitary matrix, which incorporates the pseudo-Goldstone fields Φ, i.e., the pseudoscalar octet

$$\Phi \equiv \frac{\vec{\lambda}}{\sqrt{2}} \cdot \vec{\phi} = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{bmatrix} \quad (8)$$

The weak perturbation in Eq. (6) can be written most conveniently as a combination of bilinear products of the 3×3 matrix which represents the octet of V-A currents, $L_\mu = if_\pi^2 U \partial_\mu U^+$, in the following way[4]:

$$\mathcal{L}^{\Delta S=1} = g_{\underline{8}}^{(\frac{1}{2})} \mathcal{L}_{\underline{8}}^{(\frac{1}{2})} + g_{\underline{27}}^{(\frac{1}{2})} \mathcal{L}_{\underline{27}}^{(\frac{1}{2})} + g_{\underline{27}}^{(3/2)} \mathcal{L}_{\underline{27}}^{(3/2)} \quad (9)$$

where

$$\mathcal{L}_{\underline{8}}^{(\frac{1}{2})} = (L_\mu L^\mu)_{23}$$

$$\mathcal{L}_{\underline{27}}^{(\frac{1}{2})} = L_{\mu 13}L^\mu_{21} + \tfrac{1}{5}\, L_{\mu 23}(9L^\mu_{11+22} - L^\mu_{11-22}) \qquad (10)$$

$$\mathcal{L}_{\underline{27}}^{(3/2)} = L_{\mu 13}L^\mu_{21} + L_{\mu 23}L^\mu_{11-22}$$

The transformation properties under isospin and chiral rotations of these three terms are the same as those of the corresponding four-quark operators in Eq. (5).

$g_{\underline{8}}^{(\frac{1}{2})}$, $g_{\underline{27}}^{(\frac{1}{2})}$ and $g_{\underline{27}}^{(3/2)}$ are coupling constants which are not fixed by chiral symmetry requirements alone. From the observed $K \to \pi\pi$ decay rates, it is possible to extract the values

$$|g_{\underline{8}}^{(\frac{1}{2})} + g_{\underline{27}}^{(\frac{1}{2})}|_{exp.} \simeq 5.1$$

$$|g_{\underline{27}}^{(3/2)}|_{exp.} \simeq 0.16 \qquad (11)$$

clearly indicating the enhancement of the $|\Delta I| = \frac{1}{2}$ transitions.

4. QCD-DUALITY CONSTRAINTS

In the short-distance formulation of the $\Delta S = 1$ non-leptonic weak Hamiltonian, one is confronted with the problem of the evaluation of hadronic matrix elements of the various local four-quark operators. On the other hand, in the chiral formulation, we are confronted with the fact that the coupling constants g's of the various terms which appear are not fixed by chiral symmetry requirements alone. The two pictures are however complementary. Equations (5) and (9) correspond to effective realizations of the same theory in different regimes. Therefore, a formulation of duality, which spells out the consistency of both pictures, can be used to constrain the various effective parameters[6]. The key objects to consider are the two-point functions

$$\psi_{\underline{R}}^{(I)}(q^2) \equiv i \int d^4x\, e^{iqx} \langle 0|T(H_{\underline{R}}^{(I)}(x)\, H_{\underline{R}}^{(I)}(0)^+)|0\rangle \qquad (12)$$

The QCD-duality approach consists in writing down a system of FESRs which relate integrals of the corresponding hadronic spectral functions $1/\pi\ \mathrm{Im}\ \psi_{\underline{R}}^{(I)}(t)_{hadrons}$ to their QCD counterparts,

$$F_{\underline{R}}^{(I)}(s_0,n) \equiv \int_0^{s_0} dt\ t^n \frac{1}{\pi} \operatorname{Im}\phi_{\underline{R}}^{(I)}(t)_{\text{hadrons}} \equiv |g_{\underline{R}}^{(I)}|^2 \int_0^{s_0} dt\ t^n \sum_\Gamma |\langle 0|\mathcal{L}_{\underline{R}}^{(I)}|\Gamma\rangle|^2$$

$$= \frac{s_0^{n+5}}{(n+5)(16\pi^2)^3} \left\{ a_{\underline{R}}^{(I)} + b_{\underline{R}}^{(I)} \frac{m_s(s_0)^2}{s_0} + \frac{c_{\underline{R}}^{(I)}}{s_0^2} + \dots \right\} \tag{13}$$

The right-hand sides of these sum rules are the result of a short-distance QCD evaluation. The first term corresponds to the leading asymptotic behaviour from the three-loop diagrams of the type shown in Fig. 1a. The second term is the leading effect of the finite strange quark mass shown in Fig. 1b; and the third term is the leading non-perturbative power correction from vacuum condensates à la Shifman, Vainshtein and Zakharov[7], shown in Figs. 1c and 1d.

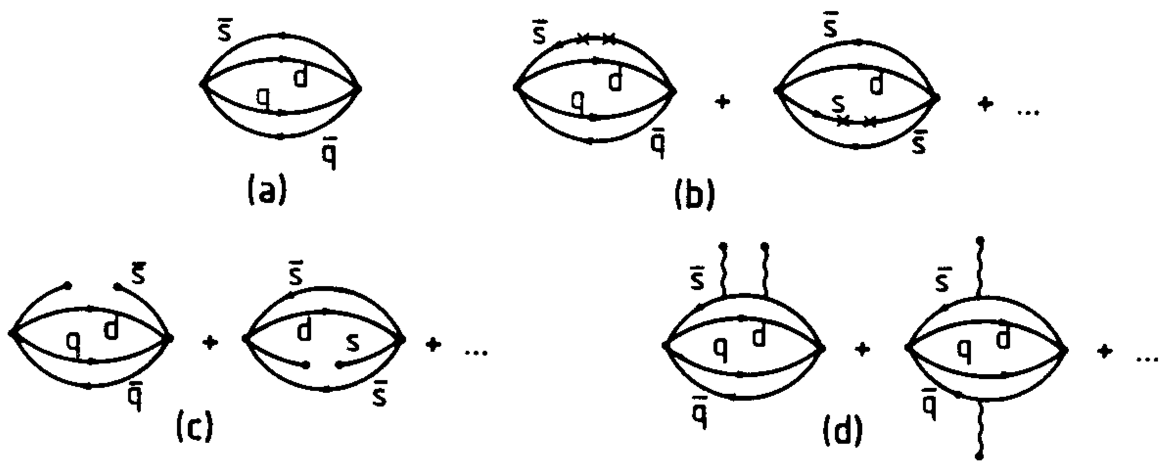

FIGURE 1
Feynman diagrams which have been taken into account in the evaluation of the QCD two-point functions. Figure 1a represents the typical three-loop diagram of the lowest order (asymptotic freedom); Figure 1b represents the correction from finite strange quark leading mass insertions; Figure 1c and 1d the leading non-perturbative power corrections à la SVZ from the light quark condensate, Fig. 1c; and from the gluon-condensate, Fig. 1d.

The effective chiral Lagrangian in Eq. (6) gives a precise framework to evaluate the threshold behaviour of the possible intermediate pseudoscalar states ($\Gamma = K\pi, K\pi\pi, \dots$) which contribute to the hadronic spectral function in the left-hand side of the FESRs. The coupling constant $g_{\underline{R}}^{(I)}$ we want to determine appears then as an overall normalization factor. The value of s_0 (the onset of the QCD continuum) in the upper limit of the hadronic integrals, has to be chosen sufficiently high so that the QCD asymptotic expansion in the right-hand side of the FESRs is meaningful. Therefore, we also need a parametrization of the hadronic spectral function which reproduces well the intermediate energy region where resonances are produced. In fact, what we need precisely is a description of the final state strong interactions between pseudoscalar states which can lead to the formation of resonances. To a good approximation this can be done by modulating the various spectral function subchannels which appear in the course of the chiral evaluation of $1/\pi\ \mathrm{Im}\psi_{\underline{R}}^{(I)}(t)_{\mathrm{hadrons}}$, with appropriate Breit-Wigner like factors which incorporate phenomenologically the masses and widths of the observed 1^-, 0^+, 1^+ and 0^- resonant states with strangeness S = 1 and S = 0. The precise way this has been done can be found in Ref. 4.

Two different FESRs (n=0,1) are needed to determine s_0 and the coupling constant $g_{\underline{R}}^{(I)}$ we are looking for. The ratio of both sum rules

$$<t_{\underline{R}}^{(I)}> \equiv F_R^{(I)}(s_0,1)/F_{\underline{R}}^{(I)}(s_0,0), \tag{14}$$

does not depend on $g_{\underline{R}}^{(I)}$ and therefore can be used as an eigenvalue equation to fix the duality region in the s_0 variable.

To illustrate how this procedure works, we have plotted[3] in Fig. 2 the ratio $<t_{\underline{27}}^{(3/2)}>$, normalized to its asymptotic freedom behaviour $5\ s_0/6$, versus s_0. The continuous line represents the QCD prediction; it approaches the asymptotic freedom limit (the dashed line) at large s_0 values. The dotted line shows the behaviour obtained with the hadronic effective parametrization. There is a clear overlapping region of both descriptions in the range 8 $GeV^2 < s_0 <$ 11 GeV^2. With s_0 fixed in this duality range, one finds from either of the two sum rules in Eq. (13)

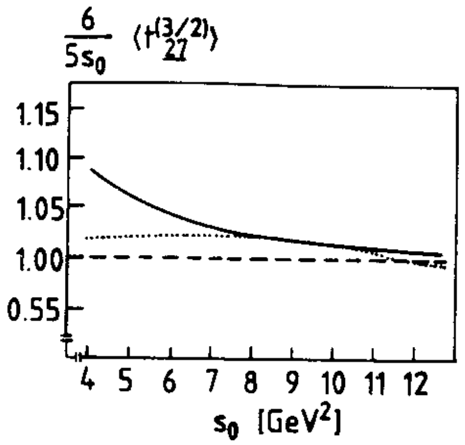

FIGURE 2
The ratio $6/5s_0 < t_{\underline{27}}^{(3/2)}>$ is plotted versus s_0. The continuous line represents the QCD behaviour. It approaches the asymptotic freedom limit (the dashed line) at large values of s_0. The dotted curve shows the behaviour obtained with the hadronic effective parametrization.

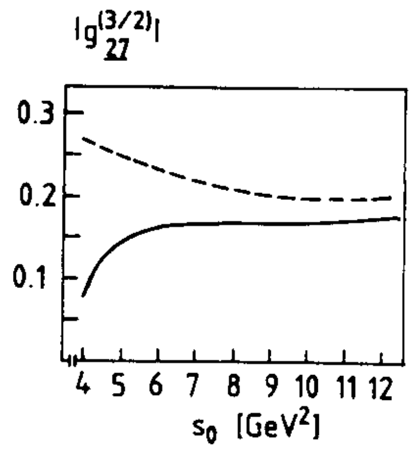

FIGURE 3
The quantity $|g_{\underline{27}}^{(3/2)}|$ is plotted versus s_0. The continuous line corresponds to the result with the inclusion of the calculated QCD corrections. The dashed curve corresponds to the result with the asymptotic freedom term only.

$$|g_{\underline{27}}^{(3/2)}| \simeq 0.17 \tag{15}$$

to be compared with the experimental value given in Eq. (11). The agreement is quite remarkable. The stability of this result can best be seen by plotting $|g_{\underline{27}}^{(3/2)}|$, as obtained from the first FESR (n = 0), versus s_0. This is shown in Fig. 3 (the solid line) where for the sake of comparison, the result obtained in the asymptotic freedom limit (the dashed line) is also shown.

For the second $(27_L, 1_R)$ coupling constant one finds similarly[4]

$$|g_{\underline{27}}^{(\frac{1}{2})}| \simeq 3.2\cdot10^{-2} \tag{16}$$

showing as expected that the $|\Delta I| = \frac{1}{2}$ enhancement cannot come from the $\underline{27}$ operator. This result is in perfect agreement with the relation $g_{\underline{27}}^{(\frac{1}{2})} = (1/5)\, g_{\underline{27}}^{(3/2)}$ predicted in the exact SU(3) limit.

The same method has also been applied to the determination of the so-called B-parameter which governs the K^0-$\bar{K}^0$ matrix element of the $\Delta S = 2$ Hamiltonian (another $\underline{27}_L$-operators), with the result[2]

$$\alpha_s(\mu^2)^{-2/9}\, |B| \simeq 0.33 \tag{17}$$

This value agrees with the one extracted phenomenologically[8] from the decay $K^+ \to \pi^+\pi^0$ (i.e., from $g_{\underline{27}}^{(3/2)}$) using SU(3) symmetry.

In spite of these successful predictions for all the $\underline{27}$ couplings, the result obtained in Ref. 4 for the octet coupling constant fails dramatically to reproduce the observed enhancement of the $K \to \pi\pi$ $|\Delta I| = \frac{1}{2}$ amplitude. It was found that

$$|g_{\underline{8}}^{(\frac{1}{2})}| \simeq 0.4, \tag{18}$$

which is an order of magnitude smaller than the experimental value quoted in Eq. (11). The small enhancement exhibited in Eq. (18) is essentially due to the short-distance enhancement in the Wilson coefficient of the Q_2-Q_1 four-quark operator[1a,1b]. No additional enhancement was found. In particular, looking at the separate contribution of the different four-quark operators to the octet correlator, it was realized[9] that the Penguin operators contribute a very small amount to the result in Eq. (18).

5. CHIRAL BOUND ON $g_{\underline{8}}^{(\frac{1}{2})}$

The results obtained so far are really puzzling. We have applied exactly the same method to four different sectors of the strangeness changing Hamiltonian. For the three $(27_L,1_R)$ operators [the $O_4^{(\frac{1}{2})}$ and $O_4^{(3/2)}$ pieces of $H^{\Delta S=1}$, and the $\Delta S = 2$ effective Hamiltonian) we have been extremely successful, while at the same time we fail dramatically in the $(8_L,1_R)$ case. Why is it so, if after all the different calculations are conceptually quite similar? In fact, this is actually the problem: in our analysis we have not found any crucial difference (aside from the fact that the octet sector is technically more complicated because of operator mixing) between $H_{\underline{27}}^{(3/2)}$ and $H_{\underline{8}}^{(\frac{1}{2})}$. In order to explain the experimental octet enhancement (a factor 20 in amplitude!) something should be really different. We are then missing some fundamental ingredient.

In our duality approach, the coupling constant $g_{\underline{8}}^{(\frac{1}{2})}$ appears, roughly speaking, as a ratio of the short-distance behaviour of a two-point function which we evaluate using QCD, to an integral of the corresponding hadronic spectral function. Since we got a far too small value for $|g_{\underline{8}}^{(\frac{1}{2})}|$, either the hadronic integral in the denominator has been grossly overestimated, or the QCD numerator has been badly underestimated. The following simple argument[5] shows that our short distance calculation is in fact unable to explain the required enhancement independently of any hadronic parametrization used.

The two-point function $\psi_{\underline{8}}^{(\frac{1}{2})}(q^2)$ obeys a dispersion relation in $Q^2 = -q^2$ up to an arbitrary polynomial in Q^2 of degree four. Five derivatives are thus required to get rid of this arbitrariness, with the result

$$F(Q^2) \equiv -\frac{\partial^5\psi_{\underline{8}}^{(\frac{1}{2})}(Q^2)}{(\partial Q^2)^5} = 5! \int_0^\infty \frac{dt}{(t+Q^2)^6}\frac{1}{\pi}\,\mathrm{Im}\psi_{\underline{8}}^{(\frac{1}{2})}(t) \qquad (19)$$

Let us split the full domain of integration in three regions: a very low region $0 \leq t \leq \Lambda^2$, with Λ some chiral cut-off sufficiently low to justify a chiral perturbation theory evaluation of the spectral function; an intermediate region $\Lambda^2 < t < Q^2$; and a very high energy region $Q^2 \leq t < \infty$, where we can use perturbative QCD with leading $1/Q^2$ power corrections incorporated. Since the spectral function is a positive semi-definite quantity at all t-values, we can write the following inequality

$$F(Q^2)-5!\int_{Q^2}^\infty \frac{dt}{(t+Q^2)^6}\frac{1}{\pi}\,\mathrm{Im}\psi_{\underline{8}}^{(\frac{1}{2})}(t) \geq 5!\int_0^{\Lambda^2}\frac{dt}{(t+Q^2)^6}\frac{1}{\pi}\,\mathrm{Im}\psi_{\underline{8}}^{(\frac{1}{2})}(t) \qquad (20)$$

This inequality becomes an identity in the extreme case where the spectral function vanishes in the intermediate region, i.e.

$$\frac{1}{\pi}\, \mathrm{Im}\psi_{\underline{8}}^{(\frac{1}{2})}(t) = 0 \text{ for } \Lambda^2 < t < Q^2 \tag{21}$$

The left-hand side of Eq. (20) can be easily evaluated from our short-distance calculation in Ref. 4. Putting only the contribution of the lowest $K\pi$ and $K\eta$ intermediate states in the hadronic spectral function of the right-hand side, Eq. (20) gives an upper bound on the coupling $|g_{\underline{8}}^{(\frac{1}{2})}|$. With the cut-off values $\Lambda^2 = M_\rho^2 \simeq 0.59$ GeV^2 and $Q^2 = 4$ GeV^2, and doing for simplicity all computations in the chiral limit, one gets $|g_{\underline{8}}^{(\frac{1}{2})}| < 4.3$ GeV, already smaller than the empirical determination in Eq. (11). Additional intermediate states in the hadronic parametrization could only decrease this bound, due to the positivity of the spectral function. A similar result is obtained when mass effects are carefully taken into account[5].

Admittedly, we can find upper bounds for $|g_{\underline{8}}^{(\frac{1}{2})}|$ compatible with the experimental value in Eq. (11) by lowering Λ^2 and/or raising Q^2. However, it is very striking that for $|g_{\underline{8}}^{(\frac{1}{2})}|$ to reach the upper bound value, an extremely peculiar behaviour of the spectral function is required: it has to vanish in a large intermediate energy region $\Lambda^2 < t < Q^2$, where resonance production is copious. The more sensible conclusion to extract from this exercise is that the left-hand side of Eq. (20) is, by far, too small to reproduce the empirical octet enhancement.

6. GLUONIC CORRECTIONS TO THE TWO-POINT FUNCTIONS

The spectral function associated with the $\psi_R^{(I)}(q^2)$ correlator describes, in an inclusive (and averaged) way, how the weak $H_R^{(I)}$ operator couples the vacuum to physical states of a given invariant mass. Since the $|\Delta I| = \frac{1}{2}$ enhancement is an intrinsic property of the octet operator, and not of a particular final state, it should obviously show up at the inclusive level; i.e., the strength of the octet spectral function should be much bigger than that of the $\underline{27}$ one. This is not, however, what we found in our short-distance calculation.

Let us try to investigate what happens when gluonic corrections to the two-point functions (12) are incorporated. In fact, we already took into account the leading $[\alpha_s(\mu^2)\ell n(t/\mu^2)]^n$ contributions, by doing the rescaling $\mu^2 = t$ which sums all leading logarithms into the Wilson coefficients. The question is now what is the size of the next-to-leading effects?

The non-logarithmic α_s-correction to the $\underline{27}$-correlators was also computed in Ref. 2, and it was found to be moderate. The corresponding calculation for the octet case is, however, much more involved due to the fact that we have to deal now with several operators (Q_2-Q_1, Q_3, Q_5, Q_6) which mix under renormalization, i.e.

$$Q_i^{(0)}(\varepsilon) = \sum_j Z_{ij}(\varepsilon,\mu) Q_j^R(\mu) \tag{22}$$

where $Q_i^{(0)}(\varepsilon)$ are the bare operators and $Q_j^R(\mu)$ the renormalized ones. (Here, $\varepsilon = (D-4)/2$ and μ is the $\overline{\mathrm{MS}}$ renormalization scale.) We need then to compute, at the four-loop level, all possible two point functions built with the four octet operators

$$\psi_{ij}(q^2) \equiv i\int d^4x\, e^{iqx} \langle 0|T(Q_i(x)Q_j(0)^+)|0\rangle, \tag{23}$$

i.e., a 4×4 matrix correlator, which must be renormalized in matrix form. Figure 4 shows the kind of diagrams contributing to this order

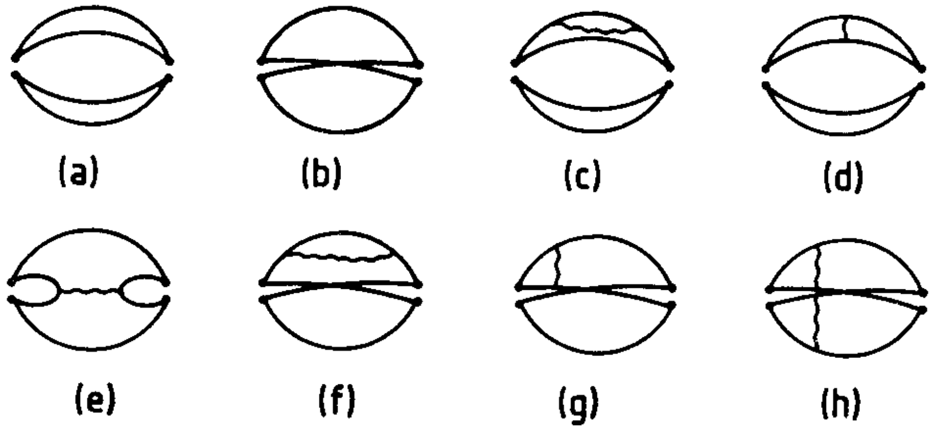

FIGURE 4
Feynman diagrams contributing to the short-distance calculation of the two-point correlators $\psi_{\underline{R}}^{(I)}(q^2)$, at lowest order (a,b) and order α_s (c-h).

Before starting such a formidable calculation, it would be useful to see if there is any limit or approximation which allows a much simpler computation, in order to get a feeling of what is going on. The relevant quantity to look at is the anomalous dimension matrix of the set of operators Q_i (i=1,...,6) which is known at the one-loop level. For three flavours, one has[1e,10]

$$\gamma \equiv \mu Z^{-1} \frac{dZ}{d\mu} = \frac{\alpha_s N}{2\pi} \begin{bmatrix} -\frac{3}{N^2} & \frac{3}{N} & 0 & 0 & 0 & 0 \\ \frac{3}{N} & -\frac{3}{N^2} & \frac{-1}{3N^2} & \frac{1}{3N} & \frac{-1}{3N^2} & \frac{1}{3N} \\ 0 & 0 & -\frac{11}{3N^2} & \frac{11}{3N} & -\frac{2}{3N^2} & \frac{2}{3N} \\ 0 & 0 & \frac{3}{N}-\frac{1}{N^2} & \frac{1}{N}-\frac{3}{N^2} & -\frac{1}{N^2} & \frac{1}{N} \\ 0 & 0 & 0 & 0 & \frac{3}{N^2} & -\frac{3}{N} \\ 0 & 0 & -\frac{1}{N^2} & \frac{1}{N} & -\frac{1}{N^2} & \frac{1}{N}-3(1-\frac{1}{N^2}) \end{bmatrix} \tag{24}$$

where the dependence on the number of colours N has been explicitly displayed. Note that in the large N limit all entries are zero but for γ_{66}, i.e., in this limit there is no mixing among operators, and only Q_6 gets renormalized.

Working at leading order in 1/N , we can then try to compute the α_s-corrections to the penguin two-point function $\psi_{66}(q^2)$, without worrying about the other operators. Moreover, in this limit only diagrams 4c and 4d (together with the lowest order contribution of diagram 4a) need to be computed because all the others are 1/N-suppressed. One gets

$$\begin{aligned} \phi_6(t,\mu^2) &\equiv [\alpha_s(\mu^2)^{2\gamma_{66}^{(1)}/\beta^{(1)}} \frac{1}{\pi} \operatorname{Im}\psi_{66}(t)]_{1/N} = \\ &= \alpha_s(\mu^2)^{18/11} \frac{12}{5} \frac{t^4}{(16\pi^2)^3} \{1 + \frac{\alpha_s(\mu^2)}{\pi} [-\frac{9}{2} \ln(t/\mu^2) + \frac{423}{20}] + 0((\frac{\alpha_s}{\pi})^2)\} \end{aligned} \tag{25}$$

where I have included the $\alpha_s(\mu^2)^{18/11}$ factor coming from the Wilson coefficient in order to have a meaningful, μ^2-independent (to the computer order) quantity. Here, $\gamma^{(1)}$ and $\beta^{(1)}$ are the first coefficients of γ and the QCD β-function respectively ($\gamma \equiv \alpha_s/\pi\ \gamma^{(1)} + \ldots$, $\beta \equiv \alpha_s/\pi\ \beta^{(1)} + \ldots$).

The coefficient of the logarithmic term is $\gamma_{66}^{(1)}|_{1/N} = -9/2$ as it should, which provides a check of the calculation. The amazing thing is the very big positive factor, 423/20, one gets for the non-logarithmic α_s correction. If one does the usual $\mu^2 = t$ rescaling, to eliminate all logarithms in the spectral function, the remaining α_s-term amounts to a 110% (140%) correction, for Λ = 100 (200) MeV, at values of momentum transfer as high as t = 10 GeV^2. The perturbative calculation (with the scaling μ^2 = t) has therefore blown up.

At this point, one should ask if this is a peculiar phenomenon of the penguin, or if something similar happens with the other four-quark operators. The same 1/N-calculation can also be done for the other correlators ψ_{ii}, however the fact that their anomalous dimensions are zero in this limit can make us already suspect that this is not a sensible approximation in this case, because the short-distance octet enhancement in the Wilson coefficient just disappears in this limit. Doing the computation for the octet operator $Q_- \equiv Q_2 - Q_1$ and its orthogonal combination $Q_+ \equiv Q_2 + Q_1$, one easily gets, to leading order in 1/N,

$$\left[\frac{1}{\pi} \mathrm{Im}\psi_{--}(t) \right]_{1/N} = \left[\frac{1}{\pi} \mathrm{Im}\psi_{++}(t) \right]_{1/N} = \frac{8}{5} \frac{t^4}{(16\pi^2)^3} \left\{ 1 + \frac{9}{4} \frac{\alpha_s(\mu)}{\pi} + O\left(\left(\frac{\alpha_s}{\pi} \right)^2 \right) \right\} \tag{26}$$

showing that in this case the α_s-correction not only is moderate, but in addition is exactly the same for the $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = 3/2$ channels.

It is then clear that for the non-penguin operators we should compute the 1/N-suppressed contributions, in order to keep the important physics (the short-distance enhancement). However, there is still some useful simplification that one can do. The mixing of Q_2 with Q_j (j=3,4,5,6) is generated by the penguin diagram shown in Fig. 5.

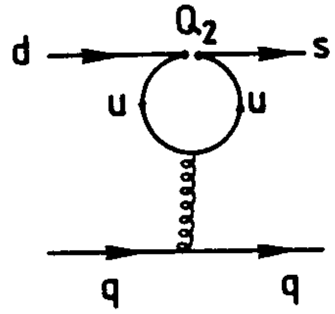

FIGURE 5
"Penguin" diagram responsible for the mixing of Q_2 with the operators Q_3, Q_4, Q_5 and Q_6.

If this diagram were absent, $\gamma_{2j}^{(1)} = 0$ (j = 3,4,5,6), and we would only need to consider the operators Q_1 and Q_2. The associated 2×2 anomalous dimension matrix has the eigenvalues $\gamma_+^{(1)} = 1$ and $\gamma_-^{(1)} = -2$ which correspond to the operators Q_+ and Q_- defined before. Although the penguin diagram is obviously there, we can take the (maybe academic) attitude of not taking it into account; the only justification being that this simplifies a lot the calculation (no mixing) and still keeps the important effects coming from the anomalous dimension factors. For the two-point function calculation, this means that we will not consider the contribution of diagram 4e. With this prescription, I get

$$\phi_-(t,\mu^2) \equiv [\alpha_s(\mu^2)^{2\gamma_-^{(1)}/\beta^{(1)}} \frac{1}{\pi} \mathrm{Im}\psi_{--}(t)]_{\text{no penguin}} = \tag{27a}$$
$$= \alpha_s(\mu^2)^{8/9} \frac{16}{15} \frac{t^4}{(16\pi^2)^3} \{1 + \frac{\alpha_s(\mu^2)}{\pi} [-2\ell n(t/\mu^2) + \frac{47}{5}] + O((\frac{\alpha_s}{\pi})^2)\}$$

$$\phi_+(t,\mu^2) \equiv [\alpha_s(\mu^2)^{2\gamma_+^{(1)}/\beta^{(1)}} \frac{1}{\pi} \mathrm{Im}\psi_{++}(t)]_{\text{no penguin}} = \tag{27b}$$
$$= \alpha_s(\mu^2)^{-4/9} \frac{32}{15} \frac{t^4}{(16\pi^2)^3} \{1 + \frac{\alpha_s(\mu^2)}{\pi} [\ell n(t/\mu^2) - \frac{49}{20}] + O((\frac{\alpha_s}{\pi})^2)\}$$

which look completely different from the 1/N results of Eq. (26).

For ψ_{++} we have obtained the already known* moderate and negative non-logarithmic α_s-correction of the 27-correlators (the penguin diagram is purely octet, so this is in fact an exact result). However, as in the ψ_{66} case, we have got a big and positive non-logarithmic α_s-correction for the octet ψ_{--} correlator. Note that although this correction was a factor of about two bigger for ψ_{66}, the one in ψ_{--} will be more important phenomenologically due to the bigger Wilson coefficient of the Q_- operator.

7. DISCUSSION AND OUTLOOK

Preliminary results of an exact calculation to order α_s, taking the full mixing structure into account, confirm the behaviour found in the previous section and show that Eqs. (25) and (27) are very good approximations to the true results. As a byproduct, this computation also shows that the 1/N results obtained for the $Q_\pm$ operators in Eq. (26) are completely misleading; leading 1/N calculations for weak amplitudes, should therefore not be trusted in the cases where the anomalous dimension of the relevant operator becomes zero at leading order in 1/N. However, as exemplified by the ψ_{66} calculation, the 1/N approximation[11] can give good (and easy) estimates when the important physical ingredient is already present at the leading 1/N order.

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* The α_s-correction in Eq. (27b) differs slightly from the one quoted in Refs. 2 and 3, due to an additional contribution which was missing there.

The blowing-up of α_s-corrections to the $|\Delta I| = \frac{1}{2}$ two-point functions provides a nice indication of a dynamical mechanism for the $|\Delta I| = \frac{1}{2}$ rule within the standard model. The $O(\alpha_s)$ calculation also indicates, however, that a quantitative estimate of the $g_8^{(\frac{1}{2})}$ coupling will require much more effort. When the scaling $\mu^2 = t$ is used, perturbation theory seems to break down, even at quite high values of momentum transfer. Full non-perturbative methods, like lattice calculations, appear to be therefore the ideal way of dealing with the problem. Nevertheless, lattice simulations present their own technical difficulties[12] and some time will be needed to have them well under control. In any case, other approaches giving complementary information should be pursued.

Let us try to see if something more can be learned from our approximate results of the previous section. Equations (25) and (27) are telling us that the usual scaling $\mu^2 = t$ leads to α_s-corrections bigger than 100% and therefore corresponds to a non-perturbative regime. This scaling is a natural one in the sense that it cancels, to all orders in the strong coupling constant, all logarithmic corrections to the two-point functions, summing them into the Wilson coefficients (at the level these coefficients have been computed) via renormalization group equations. However, this is not the only choice. Since the physical amplitudes are renormalization scale independent quantities, one can try to see if there is some region of μ^2 where the α_s-corrections are small and, therefore, a perturbative calculation still makes sense. Surprisingly, there is one selection of scale

$$\mu^2 = \tilde{\mu}(t)^2 \equiv e^{-47/10} t \tag{28}$$

which cancels the α_s-terms both in $\phi_6(t,\mu^2)$ and $\phi_-(t,\mu^2)$, while leaving a correction of normal size in $\phi_+(t,\mu^2)$. Dropping the free quark prediction for these correlators [i.e., $\phi_i(t,\mu^2) \equiv A_i t^4 \tilde{\phi}_i(t,\mu^2)/(16\pi^2)^3$, $A_6 = 12/5$, $A_- = 16/15$, $A_+ = 32/15$], one has

$$\begin{aligned}
\tilde{\phi}_6(t,\tilde{\mu}^2) &= \alpha_s(\tilde{\mu}^2)^{18/11} \\
\tilde{\phi}_-(t,\tilde{\mu}^2) &= \alpha_s(\tilde{\mu}^2)^{8/9} \\
\tilde{\phi}_+(t,\tilde{\mu}^2) &= \alpha_s(\tilde{\mu}^2)^{-4/9} \left\{1 + \frac{9}{4} \frac{\alpha_s(\tilde{\mu}^2)}{\pi}\right\}
\end{aligned} \tag{29}$$

Although the reasons for this simultaneous cancellation are still not very clear to me, it is hard to believe that this is just a coincidence. I have checked numerically that in the exact calculation of the 4×4 matrix $|\Delta I| = \frac{1}{2}$ correlator, all α_s-corrections become small when the scaling (28) is used.

Since $e^{-47/10} \sim 10^{-2}$ is a very small factor, it is necessary to go to high values of momentum transfer in order that the scale $\tilde{\mu}(t)$ makes sense for a perturbative calculation. If we are interested in the behaviour of the two-point function at not too big values of t, this means that we should choose the renormalization scale as low as possible, and therefore, we are going to obtain big values for $\tilde{\Phi}_6(t,\mu^2)$ and $\tilde{\Phi}_-(t,\mu^2)$ because the $|\Delta I| = \frac{1}{2}$ Wilson coefficients increase substantially when lowering μ. In fact, a small value of μ has been frequently used in the literature to "fit" the experimental $|\Delta I| = \frac{1}{2}$ amplitude. It is surprising that such ad hoc and a priori meaningless selection of scale happens to be the one required in order to minimize the α_s-corrections to the matrix elements.

In the following table, I present values for the ratios

$$R_i(t) \equiv \tilde{\Phi}_i(t,\tilde{\mu}^2)/\alpha_s(t)^{a_i}, \quad (i = 6,-,+)$$
$$a_6 = 18/11; \quad a_- = 8/9; \quad a_+ = -4/9 \qquad (30)$$

which compare the result (29), obtained by doing naively the scaling $\mu = \tilde{\mu}(t)$ with the leading logarithm calculation at $\mu^2 = t$, used in Ref. 4:

Λ (GeV)	t (GeV^2)	$\alpha_s(\tilde{\mu}^2)/\pi$	R_6	R_-	R_+
0.1	2	0.74	35.5	6.9	1.01
"	5	0.29	10.1	3.5	0.89
"	10	0.20	6.5	2.8	0.88
"	20	0.15	4.8	2.4	0.88
0.2	5	3.46	378.6	25.1	1.75
"	10	0.54	22.6	5.4	0.95
"	20	0.29	10.1	3.5	0.89

The size of the relevant expansion parameter, $\alpha_s(\tilde{\mu}^2)/\pi$ is also given to illustrate in which region of momentum transfer the results begin to make sense (for instance, $\Lambda = 0.2$ GeV and $t = 5$ GeV^2 is clearly meaningless). For R_6, the strong coupling should be reduced by a factor 9/11 due to the different value of the β-function in the 1/N limit. It is apparent from the Table, that not only substantial enhancement factors (R_6 and R_-) appear in the octet sector, but in addition the predicted 27-spectral function turns out to be completely stable ($R_+ \sim 0.9$) under the rescaling.

The numbers given in the Table show a very nice pattern and could offer indeed a qualitative understanding of the $\Delta S = 1$ dynamics. However, they should be taken with a lot of care. There are two points that one should investigate before using this kind of rescaling for an actual evaluation of the octet coupling constant:

i) It is not fully justified to play with the renormalization scale because the functions $\tilde{\phi}_i(t,\mu^2)$ are not really μ^2-independent at next-to-leading order. The point is that, although we have done a complete $O(\alpha_s)$ calculation of the spectral functions, we are still using the leading logarithm approximation for the Wilson coefficients. The next-to-leading order corrections to these coefficients are needed for consistency; however, such a calculation has still not been done. Nevertheless, partial results exist in the literature which, fortunately, can be directly applied to our approximate expressions.

The two-loop anomalous dimensions of the $Q_\pm$ operators, in the absence of "penguin"-like diagrams are already known[13]. Inclusion of next-to-leading effects in the $Q_\pm$ Wilson coefficients, amounts to the following redefinition of the $\tilde{\phi}_\pm(t,\mu^2)$ functions

$$\tilde{\phi}_\pm(t) \equiv [1 + 2\,\frac{\alpha_s(\mu^2)}{\pi}\,\binom{-0.443}{+1.27}]\;\tilde{\phi}_\pm(t,\mu^2) \qquad (31)$$

The additional α_s-correction is moderate and therefore will not change too much our previous results. Note, however, that it further reinforces the enhancement (suppression) of the octet (27) two-point functions.

For the penguin operator it can be easily shown[14] that, at leading order in 1/N, its Wilson coefficient scales as the square of the running quark mass, i.e., $c_6(\mu) \sim m(\mu)^2$. Therefore, the known two-loop anomalous dimension of the running mass[15] implies the redefinition

$$\tilde{\phi}_6(t) \equiv [1 + \frac{3027}{968}\,\frac{\alpha_s(\mu^2)}{\pi}]\;\tilde{\phi}_6(t,\mu^2) \qquad (32)$$

which again provides an additional enhancement factor.

ii) Since $O(\alpha_s^2)$ corrections to the spectral functions are not known, one does not have any control on what happens to the higher order terms, when the $O(\alpha_s)$ corrections are minimized by doing an appropriate rescaling. It could happen, for instance, that the selection of renormalization scale in Eq. (28) would generate huge α_s^2-corrections, invalidating the use of perturbation theory also in this region.

Thanks to the factorization property of the quark currents in the large N limit, the $O(\alpha_s^2)$ correction to $\tilde{\Phi}_6(t)$ can be easily computed, because this correlator can be expressed as a convolution of simpler two-point functions which are already known at this order[16]. This calculation, which will be reported elsewhere[14], could provide a valuable test of the quality of a given rescaling for improving the perturbative expansion. In addition, it will be interesting to see if the behaviour found in the $O(\alpha_s)$ corrections, generalizes to higher orders.

In conclusion, the calculated α_s-corrections to the $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = 3/2$ correlators clearly show that a dynamical enhancement mechanism appears in the octet weak amplitudes, as a consequence of the interplay of the strong interactions. However, while some $|\Delta I| = \frac{1}{2}$ enhancement is found in one Wilson coefficient, already in the leading logarithmic approximation, it is necessary to go to the next-to-leading order to see an additional enhancement in the matrix elements of the four-quark operators (the corresponding two-point functions in our approach). It remains to be seen whether this strong qualitative evidence for an explanation of the $|\Delta I| = \frac{1}{2}$ rule within the standard model, can be translated into a quantitative estimate of the octet $K \to \pi\pi$ amplitude.

ACKNOWLEDGEMENTS

I would like to thank the organizers for the charming atmosphere of this workshop, and E. de Rafael for a very enjoyable and rewarding collaboration. I am also very thankful to G. Altarelli, G. Martinelli, A. De Rújula and R. Tarrach for useful discussions and comments. This work has been partly supported by C.A.I.C.Y.T., Spain, under grand No. AE-0021.

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