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QCD-ENHANCEMENT OF $|\Delta I| = \frac{1}{2}$ TRANSITIONS

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A B S T R A C T

The interplay of strong interactions in $\Delta S = 1$ non-leptonic weak transitions is analyzed. In addition to the known short-distance enhancement present in the $|\Delta I| = \frac{1}{2}$ Wilson coefficients at the leading logarithmic approximation, it is found that huge α_s corrections to the $|\Delta I| = \frac{1}{2}$ two-point functions show up at the next-to-leading order, with the appropriate sign to produce the phenomenologically required enhancement.

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The origin of the empirically observed enhancement of strangeness-changing non-leptonic weak amplitudes with isospin transfer $|\Delta I| = \frac{1}{2}$ is a long-standing question in particle physics, which has not yet been given a satisfactory explanation within the framework of the standard model. The short-distance analysis of the product of weak hadronic currents [1-6] results in an effective $\Delta S = 1$ Hamiltonian which is a sum of local four-quark operators, constructed with the light (u,d,s) quark fields only, modulated by Wilson coefficients which are functions of the heavy (W,t,b,c) masses and an overall renormalization scale μ:

$$H^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\, s_1 c_1 c_3 \quad \{c_+(\mu)\ Q_+ + c_-(\mu)\ Q_- + c_6(\mu)\ Q_6 + \ldots\}$$

$$Q_\pm = 4\ \{(\bar{s}_L\gamma^\mu u_L)\ (\bar{u}_L\gamma_\mu d_L) \pm (\bar{s}_L\gamma^\mu d_L)(\bar{u}_L\gamma_\mu u_L)\ \}$$

$$Q_6 = -8 \sum_{q=u,d,s} (\bar{s}_L q_R)(\bar{q}_R d_L). \tag{1}$$

Here, G_F is the Fermi coupling constant and $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ are the conventional Cabibbo-Kobayashi-Maskawa factors.

In the absence of strong interactions, $c_+(\mu) = c_-(\mu) = \frac{1}{2}$ and all other Wilson coefficients vanish. The standard electroweak model gives rise then to $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = 3/2$ amplitudes of nearly equal size (Q_- is a pure $|\Delta I| = \frac{1}{2}$ operator, and Q_+ induces both $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = 3/2$ transitions), while experimentally the ratio between both amplitudes is a factor of twenty! To solve this big discrepancy, QCD effects should be enormous.

The leading gluonic corrections give indeed, for μ-values around 1 GeV, an enhancement by a factor two to three of the $c_-(\mu)$ Wilson coefficient with respect to $c_+(\mu)$ [1,2]. Moreover, new $|\Delta I| = \frac{1}{2}$ operators like Q_6 are generated by gluonic exchanges through the so-called "Penguin" diagrams [3-6]. Although the Wilson coefficients of these "Penguin" operators are small (order α_s), it was suggested [3] that Q_6 could have large hadronic matrix elements due to its left-right helicity structure.

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The evaluation of hadronic matrix elements of local four-quark operators is a difficult problem due to the fact that they are governed by the long-distance behaviour of the strong interactions; i.e., the confinement regime of QCD. Moreover, these operators have non-zero anomalous dimensions and, therefore, their matrix elements depend on the renormalization scale μ. Since physical amplitudes are renormalization-scale independent quantities, this μ-dependence should exactly cancel the one appearing in the Wilson coefficients. In order to keep control of the renormalization-scale dependence, and therefore to get a meaningful result, a full QCD calculation of the relevant matrix elements is required. This is a highly non-trivial task.

The problem can be simplified by looking to the $\Delta S = 1$ transitions in an inclusive way. In order to do this, it is convenient to split the $\Delta S = 1$ short-distance Hamiltonian in three pieces with definite transformation properties under the chiral group $SU(3)_L \times SU(3)_R$ and isospin,

$$H^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 \left\{ H_{\underline{8}}^{(\frac{1}{2})} + H_{\underline{27}}^{(\frac{1}{2})} + H_{\underline{27}}^{(3/2)} \right\} . \tag{2}$$

$H_{\underline{27}}^{(\frac{1}{2})}$ and $H_{\underline{27}}^{(3/2)}$ contain the $(27_L, 1_R)$ $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = 3/2$ components of Q_+, while all other operators are in the $(8_L, 1_R)$ Hamiltonian $H_{\underline{8}}^{(\frac{1}{2})}$.
The key objects to consider are the two-point correlators [7-9]

$$\psi_{\underline{R}}^{(I)}(q^2) \equiv i \int d^4x \, e^{iqx} \langle 0 | T(H_{\underline{R}}^{(I)}(x) \, H_{\underline{R}}^{(I)}(0)^+) | 0 \rangle . \tag{3}$$

The spectral function associated with $\psi_{\underline{R}}^{(I)}(q^2)$ describes, in an inclusive (and averaged) way, how the weak $H_{\underline{R}}^{(I)}$ operator couples the vacuum to physical states of a given invariant mass. Since the $|\Delta I| = \frac{1}{2}$ enhancement is an intrinsic property of the octet operator, and not of a particular final state, it should obviously show up at the inclusive level; i.e., the strength of the octet spectral function should be much bigger than that of the $\underline{27}$ ones.

At large q^2, one can use perturbative QCD (with leading $1/q^2$ power corrections incorporated) to study these two-point functions. A complete (μ-independent) calculation at the leading logarithm approximation was done in Refs. [8] and [9], showing that the only enhancement present at this order is the one coming from the Wilson coefficients (i.e., a factor of about two). Moreover, using chiral perturbation theory techniques, to analyze the low q^2 region, and analyticity, it was possible to estimate the $K \to \pi\pi$ amplitudes implied by this short-distance calculation. While the computed $|\Delta I| = 3/2$ amplitude [8] agrees surprisingly well with the measured one, the result obtained for the $|\Delta I| = \frac{1}{2}$ channel [9] fails by an order of magnitude to explain the observed enhancement!

In order to understand this puzzling result, one should investigate the size of the next-to-leading gluonic effects. The non-logarithmic α_s-correction to the $\underline{27}$-correlators was also computed in Ref. [7], and it was found to be moderate. The corresponding calculation for the octet case is, however, much more involved due to the fact that there are several operators ($Q_-, Q_6, \ldots$) which mix under renormalization. One needs to compute, at the four-loop level, all possible two-point functions built with the different octet operators

$$\psi_{ij}(q^2) \equiv i \int d^4x \; e^{iqx} \; \langle 0|T(Q_i(x)Q_j(0)^+)|0\rangle, \tag{4}$$

i.e., a matrix correlator, which must be renormalized in matrix form. Figure 1 shows the kind of diagrams contributing to this order

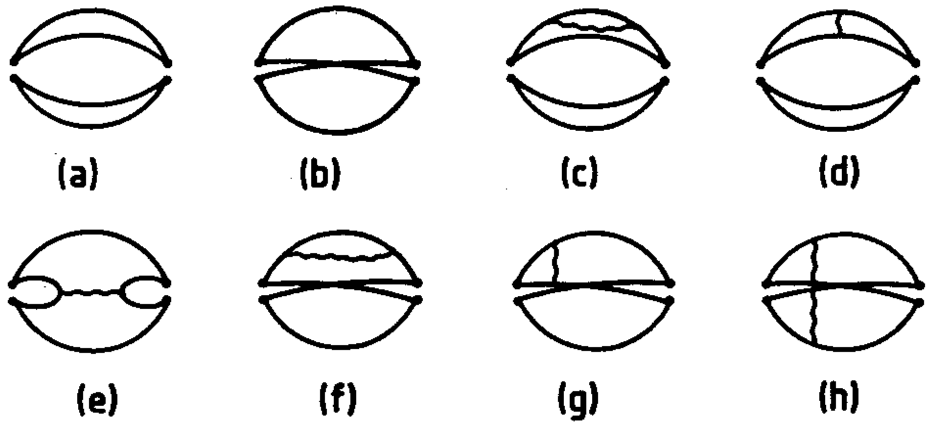

Fig. 1 Feynman diagrams contributing to the short-distance calculation of the two-point correlators $\psi_{\underline{R}}^{(I)}(q^2)$, at lowest order (a,b) and order α_s(c-h).

The calculation can be considerably simplified by using two approximations [10] which eliminate the mixing among operators, while keeping at the same time the important physical effects.

1) In the absence of "Penguin" diagrams, the only operators appearing in the short-distance Hamiltonian are Q_+ and Q_-, and they are then multiplicatively renormalizable (i.e., no mixing). Therefore, if we do not take into account the contribution of diagram (e) (it would originate mixing with the "Penguin" operators), we can study the two-point functions associated with the operators $Q_\pm$ separately.

2) The interesting Q_6 operator can be isolated by noting that in the large-N limit (N = number of colours) the anomalous dimension matrix γ of the set of operators Q_i

becomes zero, but for γ_{66}, i.e., in this limit there is no mixing among operators and only Q_6 gets renormalized. [Note that the large-N limit is not appropriate to study the operators $Q_\pm$, since $\gamma_{++} = \gamma_{--} = 0$ implies $c_+(\mu) = c_-(\mu) = \frac{1}{2}$, and the short-distance enhancement is therefore lost.]

With these approximations, one obtains [10]

$$\phi_-(t,\mu^2) \equiv [\alpha_s(\mu^2)^{2\gamma_-^{(1)}/\beta^{(1)}} \frac{1}{\pi} \mathrm{Im}\phi_{--}(t)]_{\text{no penguin}} =$$

$$= \alpha_s(\mu^2)^{8/9} \frac{16}{15} \frac{t^4}{(16\pi^2)^3} \{1 + \frac{\alpha_s(\mu^2)}{\pi} [-2\ln(t/\mu^2) + \frac{47}{5}] + O((\frac{\alpha_s}{\pi})^2)\}$$

$$\phi_+(t,\mu^2) \equiv [\alpha_s(\mu^2)^{2\gamma_+^{(1)}/\beta^{(1)}} \frac{1}{\pi} \mathrm{Im}\phi_{++}(t)]_{\text{no penguin}} =$$

$$= \alpha_s(\mu^2)^{-4/9} \frac{32}{15} \frac{t^4}{(16\pi^2)^3} \{1 + \frac{\alpha_s(\mu^2)}{\pi} [\ln(t/\mu^2) - \frac{49}{20}] + O((\frac{\alpha_s}{\pi})^2)\}$$

$$\phi_6(t,\mu^2) \equiv [\alpha_s(\mu^2)^{2\gamma_{66}^{(1)}/\beta^{(1)}} \frac{1}{\pi} \mathrm{Im}\phi_{66}(t)]_{1/N} =$$

$$= \alpha_s(\mu^2)^{18/11} \frac{12}{5} \frac{t^4}{(16\pi^2)^3} \{1 + \frac{\alpha_s(\mu^2)}{\pi} [-\frac{9}{2} \ln(t/\mu^2) + \frac{423}{20}] + O((\frac{\alpha_s}{\pi})^2)\} , \tag{5}$$

where I have included the $\alpha_s(\mu^2)^{2\gamma^{(1)}/\beta^{(1)}}$ factors coming from the Wilson coefficients in order to have a meaningful, μ-independent (to the computer order) quantity. ($\gamma^{(1)}$ and $\beta^{(1)}$ are the first coefficients of γ and the QCD β-function respectively.)

The coefficients of the logarithmic terms are the known anomalous dimension factors as it should. The amazing thing is the very big positive factors one gets for the non-logarithmic α_s-corrections to the octet correlators (47/5 and 423/20 for Q_- and Q_6 respectively), while for ϕ_{++} this correction is moderate and negative. If one does the usual $\mu^2 = t$ rescaling to eliminate all logarithms in the spectral function, summing them into the Wilson coefficients, the remaining α_s-terms produce corrections to the octet correlators which are bigger than 100%, even at values of momentum transfer as high as 10 GeV^2. The perturbative calculation has therefore blown up!

Preliminary results [11] of an exact calculation to $O(\alpha_s)$, taking the full mixing structure into account, show that Eqs. (5) are very good approximations to the true results. The size of these α_s corrections provides a nice indication of a dynamical mechanism for the $|\Delta I| = \frac{1}{2}$ rule within the standard model. Moreover, this qualitative understanding of the $\Delta S = 1$ dynamics is further reinforced by the following considerations.

(i) One can try to see if there is some region of μ^2 where the α_s-corrections are small and, therefore, a perturbative calculation still makes sense. Surprisingly, there is one selection of scale, $\mu^2 = \tilde{\mu}(t)^2 \equiv e^{-47/10}t$, which cancels the α_s-terms both in $\phi_6(t,\mu^2)$ and $\phi_-(t,\mu^2)$, while leaving a correction of normal size in $\phi_+(t,\mu^2)$. Since $e^{-47/10} \sim 10^{-2}$ is a very small factor, it is necessary to go to high values of momentum transfer in order that the scale $\tilde{\mu}(t)$ makes sense for a perturbative calculation. If we are interested in the behaviour of the two-point function at not too big values of t, this means that we should choose the renormalization scale as low as possible, and therefore, we are going to obtain big values for $\phi_6(t,\mu^2)$ and $\phi_-(t,\mu^2)$ because the $|\Delta I| = \frac{1}{2}$ Wilson coefficients increase substantially when lowering μ. In fact, a small value of μ has been frequently used in the literature to "fit" the experimental $|\Delta I| = \frac{1}{2}$ amplitude. It is surprising that such ad hoc and a priori meaningless selection of scale happens to be the one required in order to minimize the α_s-corrections to the matrix elements.

Comparing The results obtained by doing naively the scaling $\mu = \tilde{\mu}(t)$, with the leading logarithm calculation at $\mu^2 = t$, one realizes [10] that not only substantial enhancement factors appear in the octet sector, but in addition the predicted $\underline{27}$-correlator turns out to be completely stable under the rescaling.

(ii) In order to have a μ-independent result at the next-to-leading logarithm order, we still need to include the corresponding corrections to the Wilson coefficients at this order. This amounts to the redefinition [10]

$$\tilde{\phi}_\pm(t) \equiv [1 + 2\,\frac{\alpha_s(\mu^2)}{\pi}\,\binom{-0.443}{+1.27}]\;\phi_\pm(t,\mu^2)$$

$$\tilde{\phi}_6(t) \equiv [1 + \frac{3027}{968}\,\frac{\alpha_s(\mu^2)}{\pi}]\;\phi_6(t,\mu^2)\;. \tag{6}$$

The additional α_s corrections, though moderate, increase the enhancement (suppression) pattern of the octet (27) two-point functions.

(iii) Thanks to the factorization property of the quark currents in the large N limit, the $O(\alpha_s^2)$ correction to $\tilde{\phi}_6(t)$ can be easily computed, because this correlator can be expressed as a convolution of simpler two-point functions which are already known at this order. One finds [11]

$$\tilde{\Phi}_6(t) \sim \alpha_s(t)^{18/11} \left\{1 + 24.3 \frac{\alpha_s(t)}{\pi} + 459.0 \left(\frac{\alpha_s(t)}{\pi}\right)^2 + \ldots\right\} , \quad (7)$$

showing that the behaviour found in the $O(\alpha_s)$ corrections, generalizes to higher orders.

In conclusion, the calculated α_s-corrections to the $|\Delta I| = \frac{1}{2}$ and $|\Delta I| = 3/2$ correlators clearly show that a dynamical enhancement mechanism appears in the octet weak amplitudes, as a consequence of the interplay of the strong interactions. However, while some $|\Delta I| = \frac{1}{2}$ enhancement is found in one Wilson coefficient, already in the leading logarithmic approximation, it is necessary to go to the next-to-leading order to see an additional enhancement in the matrix elements of the four-quark operators (the corresponding two-point functions in our approach). It remains to be seen whether this strong qualitative evidence for an explanation of the $|\Delta I| = \frac{1}{2}$ rule within the standard model can be translated into a quantitative estimate of the octet $K \to \pi\pi$ amplitude.

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