RARE K-DECAYS AND CP-VIOLATION

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ABSTRACT

Chiral perturbation theory provides a useful framework to analyze rare kaon decays, where long-distance effects are expected to play an important rôle. The theoretical predictions, obtained within this framework for radiative decay modes are reviewed, together with the present experimental status. Special consideration is given to the $K_L \rightarrow \pi^0 e^+ e^-$ decay, which appears as an ideal candidate to look for new signals of CP violation within the standard model.

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ABSTRACT
Chiral perturbation theory provides a useful framework to analyze rare kaon decays, where long-distance effects are expected to play an important rôle. The theoretical predictions, obtained within this framework, for radiative decay modes are reviewed, together with the present experimental status. Special consideration is given to the $K_L \to \pi^+ e^+ e^-$ decay, which appears as an ideal candidate to look for new signals of CP violation within the standard model.

A new generation of high precision experiments on rare kaon decays is already under way at BNL1, CERN2, KEK3 and Fermilab4. The sensitivity of these experiments will allow for substantial improvements in the branching ratios, reaching down as low as $10^{-10}$ and even $10^{-12}$ in some cases. Furthermore, several proposals for future kaon factories are presently being discussed (BNL, EHF, JINR, KEK, LAMPF, TRIUMPH,...), which could push the possible experimental sensitivities even more.

There is an obvious motivation for this big experimental interest. Rare kaon decays offer the exciting possibility of unravelling new physics beyond the standard model. Searching for forbidden flavour-changing processes at the $10^{-10}$ level, one is actually exploring energy-scales above the 10 TeV region. The study of allowed (but highly suppressed) decay modes provides, at the same time, very interesting tests of the standard model itself. Electromagnetic-induced non-leptonic weak transitions and higher-order weak processes are a useful tool to improve our understanding of the interplay among electromagnetic, weak and strong interactions. In addition, new signals of CP-violation, which would help to elucidate the source of CP-violating phenomena, can be looked for in these high precision experiments.

Since the kaon mass is a very low scale, the standard short-distance approach to weak transitions is unable, at the moment, to make accurate predictions for non-leptonic K-decays. The general problem of dealing with unknown hadronic matrix elements of quark operators becomes even more involved in the case of radiative modes, due to the additional presence of the electromagnetic interactions. However, these difficulties can be bypassed by following a completely different approach, which takes advantage of the fact that the pseudoscalar mesons are the lowest energy modes of the hadronic spectrum; they correspond to the octet of Goldstone bosons associated with the spontaneous chiral symmetry breaking of QCD, $SU(3)_L \times SU(3)_R \to SU(3)_V$.

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The pseudoscalar octet $\phi^a (a = 1, \ldots, 8)$ is incorporated through the $3 \times 3$ special unitary matrix

$$U(x) = \exp \left( \frac{i}{f} \sum_a \lambda^a \phi^a(x) \right)$$

which transforms linearly under the chiral group

$$U(x) + s_L U(x) s_R^T, \quad s_L, s_R \in SU(3)_L \times SU(3)_R$$

At low energies, it is possible to work out the consequences of the chiral symmetry properties of the underlying constituent QCD-theory, by writing the more general effective Lagrangian involving the matrix $U$, which is consistent with the $SU(3)_L \times SU(3)_R$ symmetry. Moreover, we can organize the Lagrangian in terms of increasing powers of momentum or, equivalently, in terms or increasing number of derivatives. In the low energy domain we are interested in, the terms with a minimum number of derivatives will dominate, additional powers of momentum being associated with inverse powers of some dimensional parameter (the chiral symmetry breaking scale $\sim 4\pi f$) that characterizes the convergence of the expansion.

The lowest-dimensional effective chiral Lagrangian is uniquely given by

$$L_2 = \frac{f^2}{4} \text{Tr}(D_\mu U \ U^{\mu*}) + \nu \text{Tr}((MU + U^TM)$$

where the second term is an explicit breaking of chiral symmetry due to the presence of the quark mass matrix $M = \text{diag}(m_u, m_d, m_s)$ in the QCD Lagrangian. The parameter $\nu$ relates the squares of the pseudoscalar meson masses to the quark masses and the covariant derivative

$$D_\mu U = \partial_\mu U - ie A_\mu [Q, U]$$

accounts for the coupling to electromagnetism with the charge matrix $Q = 1/3 \text{diag}(2, -1, -1)$.

At the same order $p^2$, the effect of strangeness changing non-leptonic weak interactions with $\Delta S = 1$ is incorporated as a perturbation to the strong interaction Lagrangian (3), which is dominated by a term transforming as a $8_L \times 8_R$ operator under chiral $SU(3)$ rotations,

$$L_2^S = \frac{G_F}{\sqrt{2}} s_1 c_3 \ g_8 \ (L_\mu L^{\mu})_{23} + \text{h.c.} + \text{non-octet terms}$$

The matrix $L_\mu = if^2 UD_\mu \ U^{\mu*}$ represents the octet of $V-A$ currents, and $g_8$ is a dimensionless coupling constant which can be extracted from $K \rightarrow 2\pi$ decays, $|g_8| = 5.1$.

Using the Lagrangians (3) and (5), the rates for decays like $K \rightarrow 3\pi$ or $K \rightarrow \pi\pi\pi$ can be predicted at $O(p^2)$ through a trivial tree-level calculation. However, the experimental data are already accurate enough for the next-order corrections to be sizeable. Moreover, due to a mismatch between the minimum number of powers of momenta required by gauge invariance and the powers of momenta that the lowest-order effective Lagrangian can provide, the amplitude for any non-leptonic radiative $K$ decay with at most one pion in the final state ($K \rightarrow \gamma\gamma, K \rightarrow \gamma\pi^+\pi^-$, $K \rightarrow \pi\pi\gamma, K \rightarrow \pi^+\pi^-\gamma$, ...) vanishes to lowest order in chiral perturbation
theory (CHPT), i.e., $O(p^2)$. These decays are then sensitive to the nontrivial quantum field theory aspects of CHPT.

At the one-loop level, corresponding to $O(p^4)$, we need to add to the effective Lagrangian all possible terms with four powers of momenta, satisfying the symmetry constraints. Each term will introduce an additional coupling constant, not fixed by chiral symmetry. These constants can be seen as remnants of the fundamental theory after quarks and gluons have been integrated out; they contain both long- and short-distance information, and some of them (like $g_9$) have in addition a CP-violating imaginary part. Since the one-loop divergences are reabsorbed by the $O(p^n)$ couplings, these constants will depend, in general, on an arbitrary renormalization scale. The determination of the free parameters of CHPT proceeds by using either experimental information$^5,8$, or additional theoretical input$^7$ (resonance saturation, $1/N_C$, etc.).

It is natural to classify the $O(p^6)$ decay amplitudes in three categories$^7$:

I) the symmetry constraints do not allow any contribution from $O(p^4)$ counterterms, and therefore the loop amplitudes are necessarily finite. Both the rates and the spectra are unambiguously predicted;

II) the loop amplitude is still finite, but there is in addition a scale-independent counterterm amplitude;

III) the loop amplitude diverges implying the existence of a scale-dependent counterterm amplitude.

Following this classification, the results obtained at $O(p^4)$ for several radiative decay modes are given in the Table, together with the present experimental status. The recent measurement$^{11}$ of the $K^+ \rightarrow \gamma \gamma$ decay is in very good agreement with the chiral prediction$^{10}$, providing an encouraging check of the CHPT techniques. From the remaining Class I channels, the more interesting is the decay $K_L \rightarrow \pi^0 \gamma \gamma$, which should be within reach of the E731 and NA31 experiments, provided the relevant signal can be disentangled from the $K_L \rightarrow 2\pi^0$, $3\pi^0$ backgrounds$^{15}$. The spectrum in the invariant mass of the two photons is predicted$^5$ to have a very characteristic behaviour, as shown in the Figure.

![Figure: Normalized $q^2$ distribution for $K_L \rightarrow \pi^0 \gamma \gamma$ (full curve). Also shown for comparison is the phase-space distribution (dashed curve). $q^2$ is the invariant mass-squared of the photon pair.](image)

For the Class II decay, $K^\pm \rightarrow \pi^\mp \gamma \gamma$, one can only derive$^7$ a lower bound on the branching ratio, due to the existence of a possible counterterm contribution. Nevertheless, CHPT gives, up to a twofold ambiguity, a
<table>
<thead>
<tr>
<th>Mode</th>
<th>Br\text{th}</th>
<th>Br\text{exp}</th>
</tr>
</thead>
<tbody>
<tr>
<td>I) $K_S \rightarrow \gamma \gamma$</td>
<td>$2.0 \times 10^{-6}$</td>
<td>$(2.4 \pm 1.2) \times 10^{-6}$</td>
</tr>
<tr>
<td>$K_L \rightarrow \pi^0 \gamma \gamma$</td>
<td>$6.8 \times 10^{-7}$</td>
<td>&lt; $2.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>$K_S \rightarrow \pi^0 \gamma \gamma$</td>
<td>$3.8 \times 10^{-8}$</td>
<td>6</td>
</tr>
<tr>
<td>$K_S \rightarrow \gamma \mu^+ \mu^-$</td>
<td>$3.2 \times 10^{-8}$</td>
<td>7</td>
</tr>
<tr>
<td>$K_S \rightarrow \gamma \mu^+ \mu^-$</td>
<td>$7.5 \times 10^{-10}$</td>
<td>7</td>
</tr>
<tr>
<td>II) $K^\pm \rightarrow \pi^\pm \gamma \gamma$</td>
<td>$\geq 4 \times 10^{-7}$</td>
<td>&lt; $8 \times 10^{-6}$</td>
</tr>
<tr>
<td>III) $K^+ \rightarrow \pi^+ e^+ e^-$</td>
<td>fit</td>
<td>$(2.7 \pm 0.5) \times 10^{-7}$</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^0 \mu^+ \mu^-$</td>
<td>$6.1 \times 10^{-8}$</td>
<td>5</td>
</tr>
<tr>
<td>$K_S \rightarrow \pi^0 e^+ e^-$</td>
<td>$4.5 \times 10^{-8}$</td>
<td>&lt; $2.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>$K_S \rightarrow \pi^0 e^+ e^-$</td>
<td>$4.8 \times 10^{-10}$</td>
<td>5</td>
</tr>
<tr>
<td>$K_S \rightarrow \pi^0 e^+ e^-$</td>
<td>$4.9 \times 10^{-9}$</td>
<td>&lt; $4.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ \mu^+ \mu^-$</td>
<td>$1.0 \times 10^{-10}$</td>
<td>5</td>
</tr>
<tr>
<td>$K_L \rightarrow \pi^0 e^+ e^-$</td>
<td>$1.0 \times 10^{-9}$</td>
<td>7</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ e^+ e^-$</td>
<td>$10^{-11} - 10^{-12}$</td>
<td>7</td>
</tr>
</tbody>
</table>

precise correlation between the rate and the spectrum. The imaginary part of a certain counterterm coupling constant, generated by the electromagnetic Penguin diagram, can be estimated in the large $N_c$ limit, suggesting a CP-violating charge asymmetry of order $10^{-3}$ for this mode.

Using the measured $K^+ \rightarrow \pi^0 e^+ e^-$ decay width to fix one unknown counterterm amplitude, one can predict the rates (and $q^2$-distributions) of the other Class III modes quoted in the Table. Two solutions are, however, possible, and therefore one additional experimental information (or theoretical input) is needed in order to resolve this ambiguity.

Especially interesting is the $K_L \rightarrow \pi^0 e^+ e^-$ mode. At $O(P^4)$ the CP-conserving two-photon exchange amplitude is strongly suppressed due to its helicity structure (it is proportional to $m_e$), and therefore the decay is completely dominated by the CP violating one-photon exchange amplitude. Moreover, the contribution due to direct CP-violation in the decay amplitude is expected to be comparable or even bigger than the usual $\epsilon$-like CP-violation from the $K^+ - K^0$ mass matrix. The situation is rather
different for the \( K_L \rightarrow \pi^0 \mu^+ \mu^- \) mode where the CP-violating and CP-conserving amplitudes are comparable; the interference between the two amplitudes produces then a large transverse muon polarization\(^7\) as a spectacular signal of CP violation.

The helicity suppression of the two-photon exchange contribution to \( K_L \rightarrow \pi^0 e^+ e^- \) is, however, no longer true at the next order in the chiral expansion, \( P^6 \), because an additional tensor structure is then allowed. Although no rigorous CHPT calculation exists at this order, naive chiral power counting suggests\(^7\) that the contribution of this \( P^6 \) amplitude to the branching ratio is at least two orders of magnitude smaller than the corresponding one-photon exchange result. This estimate has been recently challenged\(^17\) on the basis of a vector-meson dominance mechanism producing a \( P^6 \) two-photon exchange amplitude as large as the CP-violating \( P^4 \) contribution; this would give rise to large interference effects such as a pronounced \( e^+ e^- \) energy asymmetry. However, this possibility is not supported by a recent investigation\(^9\) on the role of low-lying resonances in CHPT, which seems to indicate the existence of an additional chiral suppression factor in the vector-meson exchange mechanism and, therefore, a smaller CP-conserving amplitude in agreement with the chiral power counting estimate. In any case, since a big \( P^6 \) two-photon exchange amplitude would produce a large distortion in the photon spectrum shown in the Figure for the \( K_L \rightarrow \pi^0 \gamma \gamma \) mode, forthcoming experimental data on this decay should certainly settle any controversy.

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1. BNL experiments E777, E780, E787, E791 and E845.
2. CERN experiments NA31 and PS195.
3. KEK experiments E137 and E162.
4. Fermilab experiments E621 and E731.