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Top Quark Mass from Radiative Corrections to the $Z \to b\bar{b}$ Decay

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ABSTRACT

The one-loop correction to the $Zb\bar{b}$ vertex presents a quadratic dependence on the top quark mass, which has its origin in the spontaneous symmetry breaking mechanism of the Standard Model. We study the possibility of fixing the top mass by comparing LEP measurements with theoretical predictions. Using the $\overline{MS}$ renormalization scheme, we calculate the top mass dependence of the $Zb\bar{b}$ vertex. For all Z -widths, we give simple approximate formulae which work at the 0.05% level. It is found that if some branching ratios involving the b quark are measured at the 0.5% level the top mass will be determined with an error of 30 GeV using only LEP1 experiments.

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1. Introduction.

The first results from LEP [1] have given a precise determination of the Z resonance parameters, the Z mass and the Z widths. In particular, the Z mass together with the inclusion of radiative corrections allows the calculation of the Weinberg mixing angle θ_W. However this calculation is marred by our ignorance of the top and Higgs masses. Predictions of m_t have been given by requiring agreement between this prediction for the Weinberg angle and the experimental results obtained in other experiments like the mass of the W vector boson measured at the $p-\bar{p}$ colliders or the mixing angle obtained from deep inelastic $\nu - N$ neutral current data. Clearly, this requires the comparison of experimental results obtained at very different energies and a complete control of the evolution of the radiative corrections at the different scales, which in principle will depend on all the parameters of the standard theory (and other parameters if models beyond the standard theory are considered). Thus, it seems of great interest to find a way of obtaining information on the top quark mass by using *only* LEP measurements around the Z peak. There remain, however, two basic quantities that are obtained with great precision only at low energy, the Fermi constant G_μ obtained in μ decay and the electromagnetic coupling α obtained from the Thompson cross section. But these quantities are common to all kinds of processes and disappear when appropriate ratios are taken.

The possibility of bounding the top mass through radiative corrections in the standard model comes about because of the violation of the decoupling theorem [2] in spontaneously broken gauge theories, which implies radiative corrections proportional to m_t^2. For processes relevant at LEP these appear only in two places: i) Radiative corrections to the gauge boson mass matrix [3], *i.e.*, the effects on the neutral current phenomena and ii) Radiative corrections to the vertex † $Zb\bar{b}$ [6,7]. The first effect is common to all LEP processes and can be included in a common factor and in a redef-

†The non-decoupling effect in the vertex is also present for the flavour-changing $Z \to b\bar{s}$ decay [4,5].

inition of the Weinberg angle; the second one is particularly interesting because it is specific to the $b\bar{b}$ channel and does not contain any dependence on the Higgs mass.

Detailed analyses [8] of present available data from LEP and non-LEP results provide a value for the top mass of about $m_t = 140 \pm 50\ GeV$, when analyzed within the standard model. This makes use of the first effect, the self-energy non-decoupling, where the unknown value of the Higgs mass plays an important role (it gives an error of about 20 GeV in m_t if m_H is allowed to vary between 40 and 1000 GeV). The precise measurement of the Z -mass has been the cornerstone for this approach. The second source of non-decoupling is apparent in the top mass dependence of the width $\Gamma(Z \to b\bar{b})$. Explicit calculations of this width have been performed in Ref. 7 in the unitary gauge and in Refs. 6,9 in the renormalizable gauge.

It is quite important to notice that there are substantial differences between these two sources of non-decoupling. While any kind of new heavy particle coupling to the gauge bosons would contribute to the W and Z self-energies, the possible new physics contributions to the $Zb\bar{b}$ vertex are much more restricted [10] and in any case different from the contributions to the self-energies. Therefore, an independent experimental test of the two effects, self-energies and the $Zb\bar{b}$ vertex, would be very valuable in order to disentangle the possible new physics contributions from the Standard Model corrections. In addition, the non-decoupling effect in the $Zb\bar{b}$ vertex offers a unique test of the spontaneous symmetry breaking mechanism of the standard theory: the leading (quadratic) dependence on m_t is produced by the exchange of longitudinal W's [6]. Our purpose in this paper is to isolate from appropriate combinations of LEP data the top mass effects associated with this vertex.

In the last years it has become popular to express the results of complicated radiative corrections in terms of the so-called *improved Born approximation*. In a few words, it consists in expressing the relevant physical quantities formally like the results obtained in the Born approximation but written in terms of an effective Weinberg angle that contains the leading radiative corrections depending on the top quark mass. We find this language particularly useful in discussing the Z decay widths and bounds on the top quark mass.

To express all relevant quantities in terms of the effective Weinberg angle has, however, some drawbacks. First it is not in general a gauge invariant parameter, and second it is not clear how to incorporate nonleading corrections when needed. Recently it has been suggested [11,12] that an appropriate quantity to play the role of the effective Weinberg angle would be the Weinberg angle defined in a $\overline{MS}$ renormalization scheme ($\sin^2\hat\theta_W(m_Z^2) \equiv \hat s_W^2$). Degrassi, Fanchiotti and Sirlin have shown that the difference between $\hat s_W^2$ and other definitions of the effective Weinberg angle are minimal. In particular it absorbs the leading dependences of self-energy diagrams on the top quark and Higgs masses. In addition it is a gauge invariant quantity and the remaining non-leading corrections can be incorporated in a very definite way and are in general small. Finally this is the relevant quantity to be compared with the predictions coming from grand unified models [13]. For all those reasons we have chosen the $\overline{MS}$ renormalization scheme to calculate the relevant widths, in particular the Z decay width to $b\bar b$ which was not considered in Ref. 12.

The plan of this paper is as follows. In section 2 we discuss the $\overline{MS}$ scheme as proposed in Ref. 11 and give the relevant relations. In section 3 we present the formulae needed to compute Z decay widths and branching ratios; the most interesting δ_b-parameter which contains the top quark mass effect in the $Zb\bar b$ vertex is calculated. In section 4 we discuss the results and possible bounds on the top quark and Higgs masses from LEP experiments. Finally in section 5 we summarize the main conclusions and prospects of this study.

2. The $\overline{MS}$ scheme for the Weinberg mixing angle.

The most natural definition for the renormalized Weinberg angle is that associated with the on-shell renormalization scheme introduced by Sirlin [14] $\sin^2\theta_W = 1 - m_W^2/m_Z^2$ (denoted s_W^2 from now on). The reason is obvious: the clear physical meaning of this quantity. However, as commented previously, in computing Z mediated amplitudes there are large corrections that depend on the top and Higgs masses which are not absorbed in $\sin^2\theta_W$. This does not happen with the definition of the Weinberg angle associated with the $\overline{MS}$ renormalization scheme. However the physical meaning is less obvious. The $\overline{MS}$ definition of the Weinberg angle $\hat s_W^2$ is obtained just by requiring that the counterterms present in the bare mixing $\sin^2\theta_W^0$ cancel the terms involving $1/(n-4) + (\gamma - \ln(4\pi))/2$ —n is the number of dimensions and γ the Euler constant— in the Z mediated amplitude and setting the 't Hooft mass scale μ to the value m_Z. In Ref. 11 it has been shown that at the one-loop level the value of $\hat s_W^2$ can be related to the on mass-shell definition through

$$\hat s_W^2 = (1 - \frac{c_W^2}{s_W^2}\,\mathrm{Re}\{\frac{A_{ZZ}(m_Z^2)}{m_Z^2} - \frac{A_{WW}(m_W^2)}{m_W^2}\}_{\overline{MS}})s_W^2 \,, \tag{1}$$

where A_{ZZ} and A_{WW} are unrenormalized self-energies. From the $\overline{MS}$ definition one can write a formula to compute $\hat s_W^2$ in terms of m_Z^2

$$\hat s_W^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4A^2}{m_Z^2\,(1-\Delta\hat r)}}\right). \tag{2}$$

Here $A = \sqrt{\pi\alpha/(\sqrt{2}G_\mu)} = (37.2803 \pm 0.0003)\ GeV$ and $\Delta\hat r$ contains all the radiative corrections and can be computed from Ref. 11, where explicit formulae and tables have been given for this correction including all sort of nonleading corrections. For many purposes it is enough to use an approximate formula. We took their results and fit them to the leading behaviour, obtaining a very simple expression for $\Delta\hat r$,

$$\Delta\hat r \approx 0.06842 - 0.00247\frac{m_t^2}{m_Z^2} + 0.00073\ln\frac{m_h^2}{m_Z^2} \,, \tag{3}$$

which agrees with the exact result better than 0.5% of the correction, for top masses between 90 and 230 GeV and Higgs masses between 40 and 1000 GeV. The error in this approximation translates into an absolute error in $\hat{s}^2_W$ of about 0.00005, which is enough for the level of precision of present (and near future!!) experiments. We have also checked that the coefficients of the fit do not change, at the level of accuracy quoted, when m_Z changes within the present experimental error band ($m_Z = 91.18 \pm 0.03\, GeV$).

3. Z decay widths in the $\overline{MS}$ scheme.

A complete $\overline{MS}$ calculation of the Z decay partial widths, excluding the decay to $b\bar{b}$, has been performed recently by Degrassi and Sirlin [12]. The decay width can be written in the following way (without QED and QCD corrections)

$$\Gamma(Z \to f\bar{f}) = \frac{N_c^f}{48}\frac{\hat{\alpha}}{\hat{s}^2_W\hat{c}^2_W} m_Z\, |\rho_f| \sqrt{1-\mu_f^2}\left(1-\mu_f^2 + [1-4\,|Q_f|\,\hat{\kappa}_f\hat{s}^2_W]^2\,(1+\frac{1}{2}\mu_f^2)\right). \quad (4)$$

The form factors ρ_f and $\hat{\kappa}_f$ are in general complex and contain the remainder of the corrections not already included in $\hat{s}^2_W$. As a first approximation they can be set to the value one; this introduces errors at the 1% level in the worst case. Further corrections to this approximation will be given below. $\hat{\alpha}$ is the electromagnetic fine structure constant defined at the m_Z scale; it has a slight dependence on the top quark mass [11] ,

$$\hat{\alpha}^{-1} = \alpha^{-1}(1 - 0.0675 + \frac{4\alpha}{9\pi}\ln\frac{m_t^2}{m_Z^2}). \quad (5)$$

In $\hat{\alpha}^{-1}$ there is an error, due to the computation of the hadronic contribution from the experimental data, of about 0.1 in absolute value. The terms dependent on $\mu_f^2 \equiv 4m_f^2/m_Z^2$ are kinematical effects only important for heavy fermions. Since the deviation from unity of ρ_f and $\hat{\kappa}_f$ is small, it is possible to rewrite the widths in the form of a Born-like expression; thus, if $\rho_f = 1 + \delta\rho_f$ and $\hat{\kappa}_f = 1 + \delta\hat{\kappa}_f$, we can write

$$\Gamma(Z \to f\bar{f}) = \frac{N_c^f}{48}\frac{\hat{\alpha}}{\hat{s}^2_W\hat{c}^2_W} m_Z \sqrt{1-\mu_f^2}\left(|\hat{a}_f + \delta\hat{a}_f|^2\,(1-\mu_f^2) + |\hat{v}_f + \delta\hat{v}_f|^2\,(1+\frac{1}{2}\mu_f^2)\right)$$

$$= \frac{N_c^f}{48}\frac{\hat{\alpha}}{\hat{s}^2_W\hat{c}^2_W} m_Z \sqrt{1-\mu_f^2}\left(|\hat{a}_f|^2\,(1-\mu_f^2) + |\hat{v}_f|^2\,(1+\frac{1}{2}\mu_f^2)\right)(1+\delta_f^{(0)}), \quad (6)$$

where

$$\begin{aligned} \hat{v}_f &= 2I_3^f - 4Q_f\hat{s}^2_W = 2I_3^f(1-4\,|Q_f|\,\hat{s}^2_W) \\ \hat{a}_f &= 2I_3^f \\ \delta\hat{v}_f &= 2I_3^f(\frac{1}{2}\delta\rho_f - 4\,|Q_f|\,(\delta\hat{\kappa}_f + \frac{1}{2}\delta\rho_f)\hat{s}^2_W) \\ \delta\hat{a}_f &= 2I_3^f\frac{1}{2}\delta\rho_f, \end{aligned} \quad (7)$$

and

$$\delta_f^{(0)} = 2\frac{\hat{v}_f\,\mathrm{Re}\{\delta\hat{v}_f\} + \hat{a}_f\,\mathrm{Re}\{\delta\hat{a}_f\}}{\hat{v}_f^2 + \hat{a}_f^2}. \quad (8)$$

$\delta_f^{(0)}$ depends only on the quantum numbers of the fermion f, but not on fermion masses because masses have been neglected in computing it. It contains some vertex corrections for massless quarks running in the loop and also some of the remainder of the gauge boson self-energies not included in $\hat{s}^2_W$. A fit to the exact results gives

$$\begin{aligned} \delta_b^{(0)} = \delta_s^{(0)} = \delta_d^{(0)} &= 10^{-3}(-2.7 + 1.1\ln\frac{m_t^2}{m_Z^2} + 2.8\arctan\frac{m_h^2}{m_Z^2}), \\ \delta_c^{(0)} = \delta_u^{(0)} &= 10^{-3}(-3.9 + 1.1\ln\frac{m_t^2}{m_Z^2} + 2.8\arctan\frac{m_h^2}{m_Z^2}), \\ \delta_e^{(0)} = \delta_\mu^{(0)} = \delta_\tau^{(0)} &= 10^{-3}(-4.4 + 1.1\ln\frac{m_t^2}{m_Z^2} + 2.8\arctan\frac{m_h^2}{m_Z^2}), \\ \delta_\nu^{(0)} &= 10^{-3}(-1.4 + 1.0\ln\frac{m_t^2}{m_Z^2} + 2.8\arctan\frac{m_h^2}{m_Z^2}). \end{aligned} \quad (9)$$

At most they give a correction at the 0.5% level. In addition, as clearly seen in the results of the fit, the top and Higgs mass dependences are independent of the particular process in good approximation. Therefore those dependences will cancel in all the branching ratios.

All these formulae apply to the decay to all fermions but the b quark. In the decay of the Z to $b\bar{b}$, the top quark mass enters in the loop correction to the vertex mediated by the W gauge boson. Due to the spontaneous symmetry breaking effects discussed in section 1, the top mass cannot be neglected in the calculation. In fact, there is a top mass dependence that grows like m_t^2. We have calculated [6] that correction given by the diagrams of Fig. 1. The additional contribution to the $Zb\bar{b}$ vertex, due to the nonzero value of the top quark mass, can be written as follows

$$\Delta\Gamma_b^\mu = i\frac{\hat{e}}{4\hat{s}_W\hat{c}_W}\gamma^\mu(1-\gamma_5)\delta_{b-vertex}, \tag{10}$$

where $\hat{e}$ is the renormalized electric charge at $q^2 = m_Z^2$ $(\hat{\alpha} \equiv \hat{e}^2/(4\pi))$ and

$$\delta_{b-vertex} = -\frac{\hat{\alpha}}{4\pi\hat{s}_W^2}F_b(m_t). \tag{11}$$

The function $F_b(m_t)$ is obtained directly from the unrenormalized diagrams of Fig. 1 , subtracting the zero mass contribution. The renormalized vertex correction for massless internal quarks is already contained in $\delta_b^{(0)}$. In computing this form factor we have always used for the couplings the renormalized couplings in the $\overline{MS}$ scheme (*i.e.*for the Z coupling $\hat{e}/(\hat{s}_W\hat{c}_W)$) and for m_W we have used its calculated value in terms of the Z mass [11]. This makes small differences but it takes into account correctly some of the α^2 corrections. In Appendix A we give analytic formulae for the form factor $F_b(m_t)$ and in Table 1 we give its value for several top masses and a Higgs mass of 100 GeV. The dependence on the Higgs mass is however completely negligible. From eq. (10) it is easy to see that the effects of the top quark in the $b\bar{b}$ vertex can be included just with the substitution

$$\begin{aligned} \delta\hat{v}_b &= \delta\hat{v}_s + \delta_{b-vertex}, \\ \delta\hat{a}_b &= \delta\hat{a}_s + \delta_{b-vertex}, \end{aligned} \tag{12}$$

and the width can be written as

$$\Gamma(Z \to b\bar{b}) = \frac{3}{48}\frac{\hat{\alpha}}{\hat{s}_W^2\hat{c}_W^2}m_Z\sqrt{1-\mu_b^2}\left(|\hat{a}_b|^2(1-\mu_b^2) + |\hat{v}_b|^2(1+\frac{1}{2}\mu_b^2)\right)(1+\delta_b^{(0)})(1+\delta_b), \tag{13}$$

where

$$\delta_b = 2\frac{\hat{v}_b + \hat{a}_b}{\hat{v}_b^2 + \hat{a}_b^2}\,\mathrm{Re}\{\delta_{b-vertex}\}. \tag{14}$$

δ_b contains all the interesting effects of the top quark mass in the diagrams of Fig. 1, and $\delta_b^{(0)}$ contains all the corrections common to the other quarks with the same quantum numbers (the s and the d quarks). In Table 1 we give the values obtained for δ_b. We have also fitted our results to a very simple function that contains the main behaviour,

$$\delta_b \approx 10^{-2}(-\frac{1}{2}\frac{m_t^2}{m_Z^2} + \frac{1}{5}). \tag{15}$$

This formula works better than 1% of the correction in the range of m_t between 90 and 230 GeV. (The Higgs mass dependence is completely negligible)

3.1. QED and QCD corrections.

In addition to the genuine electroweak corrections, there are some final state QED corrections and also some important QCD corrections that must be taken into account. Those are global corrections that in general can be factorized. We also found it convenient to factorize the small effects of the external fermion masses. Finally, we have to include some second order QCD corrections that depend on the top quark mass, which could be important [15]. To take into account all those effects, we will write the decay widths as

$$\Gamma(Z \to f\bar{f}) = \frac{N_c^f}{48}\frac{\hat{\alpha}}{\hat{s}_W^2\hat{c}_W^2}m_Z(|\hat{a}_f|^2 + |\hat{v}_f|^2)$$
$$(1+\delta_f^{(0)})(1+\delta_{QED}^f)(1+\delta_{QCD})(1+\delta_\mu^f)(1+\delta_{tQCD}^f)(1+\delta_b). \tag{16}$$

$\delta_f^{(0)}$, as explained before, contains the small electroweak corrections not absorbed in $\hat{s}_W^2$; they are given in eq. (9).

δ_{QED}^f gives small final state QED corrections that depend on the charge of the final fermion,

$$\delta_{QED}^f = \frac{3\alpha}{4\pi}Q_f^2. \tag{17}$$

It is very small (0.2% for charged leptons, 0.08% for u-type quarks and 0.02% for d-type quarks).

δ_{QCD} gives the QCD corrections common to all quarks (obviously it has to be included only for quarks!). It is given by

$$\delta_{QCD} = \frac{\alpha_s}{\pi} + 1.41\left(\frac{\alpha_s}{\pi}\right)^2 . \qquad (18)$$

α_s is the QCD coupling constant taken at the m_Z scale, *i.e.* $\alpha_s = \alpha_s(m_Z^2) = 0.12$. If the error in the determination of α_s is about 0.01, the error in the hadronic widths will be of about 0.3%.

δ_μ^f contains the kinematical effects of the external fermion masses, including some mass dependent QCD radiative corrections [16]. It is only important for the b quark (0.5%) and to a lesser extent for the τ lepton (0.2%) and the c quark (0.05%). It is given by (the α_s terms should obviously be removed for leptons)

$$\delta_\mu^f = \frac{3\hat{\mu}_f^2}{\hat{v}_f^2 + \hat{a}_f^2}\left(-\frac{1}{2}\hat{a}_f^2(1 + \frac{8\alpha_s}{3\pi}) + \hat{v}_f^2\frac{\alpha_s}{\pi}\right), \qquad (19)$$

where $\hat{\mu}_f^2 \equiv 4\bar{m}_f^2(m_Z^2)/m_Z^2$. We will take for the "on-shell" mass of the b quark the value $m_b = (4.6 \pm 0.1)$ GeV, and use the known relation [17] between the "on-shell" and the $\overline{MS}$ schemes to compute the running mass $\bar{m}_b(m_Z^2)$ at the Z scale. The quoted error in the b-quark mass will translate into a 0.05% error in the $Z \to b\bar{b}$ width. We have used the same treatment for the c-quark, where 1.5 GeV has been taken for its "on-shell" mass.

It has been noted recently [15] that the large mass splitting between the bottom and top quarks gives rise, through triangle quark loops, to an additional $O(\alpha_s^2)$ correction to the axial-vector current induced rate

$$\delta_{tQCD}^f = -\frac{\hat{a}_t\hat{a}_f}{\hat{v}_f^2 + \hat{a}_f^2}\left(\frac{\alpha_s}{\pi}\right)^2 f(\mu_t). \qquad (20)$$

For top quark masses between 90 and 230 GeV, the function $f(\mu_t)$ is well parametrized by

$$f(\mu_t) = \log(4/\mu_t^2) - 3.083 + 0.346(1/\mu_t^2) + 0.211(1/\mu_t^4), \qquad (21)$$

and varies from ≈ -3 to ≈ -5. Note that this top-mass induced correction has opposite sign for up and down type quarks ($\delta_{tQCD}^d \approx -\delta_{tQCD}^u \approx 10^{-3} f(\mu_t)$).

Finally δ_b gives the large top mass dependence to be included only in the partial width to $b\bar{b}$.

This parametrization of the corrections to the decay widths is particularly convenient for the discussion of branching ratios which is our next point.

3.2. Branching ratios concerning Z decay into $b\bar{b}$.

We are interested in isolating the large top mass dependences occurring in the $Zb\bar{b}$ vertex δ_b. This can be done by taking appropriate branching ratios. We found convenient the following quantities:

i)

$$R_s \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to s\bar{s})} \simeq 0.9949(1 + \delta_b). \qquad (22)$$

The factor 0.9949 is due to the b mass effects. All other corrections cancel exactly in this branching ratio except the correction to the $Zb\bar{b}$ vertex which, at the level of precision considered, only depends on the top quark mass.

ii)

$$R_c \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to c\bar{c})} \simeq 0.9960\frac{(1 + \hat{v}_b^2)}{(1 + \hat{v}_c^2)}\frac{(1 + \delta_{tQCD}^b)}{(1 + \delta_{tQCD}^c)}(1 + \delta_b). \qquad (23)$$

The constant factor 0.9960 includes, as before, b and c mass effects. But now it also includes some weak corrections not included in $\hat{s}_W^2$, the ratio $(1 + \delta_b^{(0)})/(1 + \delta_c^{(0)})$ which does not cancel. However the top and Higgs mass dependences that appear individually in $\delta_f^{(0)}$ do cancel when branching ratios are taken. This allows us to absorb those corrections in a constant factor. Another important point in this branching ratio is the presence of the second order QCD corrections depending on the top quark mass which, in this case, not only do not cancel but are added (remember that these corrections are similar but opposite in sign for up and down type quarks); in addition the final correction has the same sign as the pure weak correction to the $Zb\bar{b}$ vertex, δ_b, making R_c very sensitive to the top quark mass.

iii)

$$R_h \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to hadrons)} = (1 + 2/R_s + 1/R_c + 1/R_u)^{-1}. \tag{24}$$

Here $R_u \equiv \Gamma(Z \to b\bar{b})/\Gamma(Z \to u\bar{u})$ is slightly different from R_c because of the small corrections dependent on the c mass. For R_u we can use eq. (23), but changing 0.9960 to 0.9955. By taking the normalization to the hadronic width, we keep some of the advantages of the previous branching ratios ,like cancellation of QCD corrections and top and Higgs mass dependences in $\delta_f^{(0)}$. In addition the hadronic width is much better known. However, we lose some of the sensitivity to the top quark mass because the $b\bar{b}$ channel represents an important fraction of the hadronic decays.

iv)

$$R_\mu \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \mu\bar{\mu})} \simeq 3.1052 \frac{(1+\hat{v}_b^2)}{(1+\hat{v}_\mu^2)}(1+\delta^b_{tQCD})(1+\delta_b). \tag{25}$$

Here 3.1052 contains all corrections which are independent of the top quark mass, m_b effects, $\delta_f^{(0)}$ corrections and now also the δ_{QCD} corrections.

We have used that $\hat{a}_f^2 = 1$ for all fermions and ,in computing the QCD corrections, we took $\alpha_s(m_Z^2) = 0.12 \pm 0.01$; however only the last branching ratio, normalized to the leptonic width, depends strongly on the chosen value. It is also interesting to note that the $\delta_f^{(0)}$ dependences on the top and Higgs masses cancel to a large extent in the branching ratios. This can be understood because the top and Higgs mass dependences in $\delta_f^{(0)}$ do not come from vertex corrections, which are the process dependent ones, but from the corrections to the gauge boson propagator not included in $\hat{s}_W^2$, which are process independent.

4. Results

In Table 1 we give the relevant function that characterizes the large top dependence in the $b\bar{b}$ vertex and the correction to the width δ_b. In Tables 2,3,4 we give the decay widths of the Z gauge boson to fermions. For d quarks the result is the same as for

s quarks. For u quarks the result is slightly different from the one obtained for the c quark, because of the small effect of the c quark mass. The same comment applies to the τ lepton. We expect these results for the widths to hold at least at the 0.05% level, if the uncertainties in the parameters are not considered. We have to remember that the error in $\hat{\alpha}$ due to the experimental error in the determination of the hadronic contribution is 0.08%, the error in the Z mass is about 0.03% and the error in the widths coming from the error in α_s is around 0.3%. These errors cancel to a large extent in the branching ratios considered (except in R_μ where QCD corrections do not cancel). In addition, there is a 0.05% error in the $Z \to b\bar{b}$ width, due to the uncertainties in the value of m_b.

In Fig. 2 we give, as a function of m_t, the branching ratio of $Z \to b\bar{b}$ with several normalizations: Fig. 2.a normalized to $Z \to s\bar{s}$, Fig. 2.b normalized to $Z \to c\bar{c}$, Fig. 2.c normalized to the hadronic width and Fig. 2.d normalized to the leptonic width. The two lines are for Higgs masses of 40 and 1000 GeV. The best one, from the theoretical point of view, is the $s\bar{s}$ normalization because in that branching ratio all the factors with some uncertainty cancel (except for δ_μ^b) and the only remaining effect is just the large top-dependent contribution from the vertex. The normalization to $c\bar{c}$ is in principle also very good even though there are some dependences on $\hat{s}_W^2$. However they are completely under control and do not present any additional difficulty. Another characteristic of this branching ratio is the QCD correction dependent on the top quark mass. On one side it complicates the analysis because the large m_t dependence in R_c comes not only from δ_b but also from this QCD correction, but, on the other side, the top-dependent QCD correction increases the sensitivity of R_c to the top quark mass. The normalization to the hadronic width has the problem that some of the sensitivity to the vertex correction is lost. Finally the leptonic normalization, even good in principle, has the problem that the large QCD corrections do not cancel and thus, it has a potentially large error (0.3%) due to the error in the measurement of α_s.

In Fig. 3, we give in the plane m_t -$\hat{s}_W^2$ the possible constraints one could obtain if the previously introduced branching ratios were measured at the 1% (0.5%) level –dashed lines (dotted lines)–. We assumed central values of 0.98 for R_s, 1.255 for R_c, 0.2155 for R_h and 4.48 for the leptonic branching ratio R_μ. We have also plotted the allowed region (the horizontal dash-dotted lines) in the case where $\hat{s}^2_W$ is measured from the asymmetries to be 0.232 ± 0.001[‡]. Finally, we show for comparison the prediction for $\hat{s}^2_W$ obtained from the measurement of the Z mass (solid lines). The band is obtained by varying the Higgs mass between 40 and 1000 GeV.

From Figs. 2 and 3 it is clear that the determination of these branching ratios (specially with the normalizations to $s\bar{s}$ or $c\bar{c}$) with an error at the 0.5% level would represent a measurement of the top mass with an error of $25 - 30\ GeV$.

5. Conclusions

The manifestation of a heavy top quark through its virtual effects provides a crucial test of the standard model at the quantum level. Hard radiative corrections, proportional to the square of the top-mass, are predicted to be present, which constitutes an explicit evasion of the decoupling theorem [2] in spontaneously broken gauge theories. Two different sources of m_t^2-corrections can be identified:

i) The large splitting between the top and bottom masses produces a hard breaking of the custodial $SU(2)$-symmetry of the scalar sector. This generates different (and large) corrections to the self-energies of the charged and neutral $SU(2)_L$ gauge bosons [3].

ii) The exchange of longitudinal W's between the external bottom legs gives rise to

[‡]Strictly speaking, there are some nonleading corrections in the asymmetries which depend on the top quark mass and which are not included in $\hat{s}^2_W$. Thus, the constraints coming from the measurement of asymmetries will not be exactly horizontal lines. The lines in the figure are only indicative of what one could obtain.

hard m_t^2-corrections to the $Zb\bar{b}$ vertex [6]. This effect can be easily understood in the renormalizable gauges, where the hard mass term originates from the exchange of the unphysical charged scalar . It is rather natural to find that behaviour in these gauges, because the Yukawa coupling of the charged scalar to fermions is proportional to the fermion mass. Indeed, in the Feynman gauge the final correction proportional to m_t^2 comes from diagram (2a) in Fig. 1.

The first source of non-decoupling (the self-energy effect) has been extensively discussed in the literature, and has already been used to get information on the top-mass from present data. In this paper, we have focused on the second source of non-decoupling, trying to isolate from appropriate combinations of observables the top mass effects associated with the $Zb\bar{b}$ vertex. It is important to stress that the possible contamination of these corrections by any kind of new physics is much more restricted than (and in any case different from) the self-energy one. Therefore, accurate experimental information on the $Zb\bar{b}$ vertex would be very valuable in order to disentangle possible new physics contributions. Moreover, the m_t^2-correction to the $Zb\bar{b}$ vertex does not contain any dependence on the unknown Higgs mass. Even if the top-mass is finally measured in collider experiments, it is extremely important to study experimentally this m_t^2 effect, since it provides a unique test of the spontaneous symmetry breaking mechanism of the standard model.

We have studied different ways of normalizing the $Z \to b\bar{b}$ width, in order to isolate the vertex correction we are interested in. The cleaner quantity is the ratio R_s, defined in eq. (22), where all other corrections exactly cancel; unfortunately, this is probably not the best quantity to be accurately measured. R_c, defined in eq. (23), is slightly contaminated by other vertex corrections, including an interesting second order QCD-correction which also depends on the top quark mass; this additional correction increases somewhat the sensitivity to m_t. The normalization to the total hadronic width, R_h, although a much cleaner experimental quantity, is less sensitive to m_t. Finally, the leptonic normalization used in R_μ has a potentially large error coming from the value used for α_s.

In order to be sensitive to the interesting m_t^2-correction, one needs to measure the branching ratios involving the b-quark at the 1% level. A better precision of 0.5% would allow us to determine the top quark mass with an error of about 30 GeV, which would constitute an extremely interesting test of the standard model. Although this is certainly not an easy experimental task, the obvious importance of checking the spontaneous symmetry breaking mechanism requires a strong effort in this direction. The prospects for future improvements in the identification of bottom quarks at LEP in fact look quite promising for making an accurate measurement [18,19].

Acknowledgements

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Appendix A. The form factor $F_b(m_t)$.

The correction to the vertex due to the diagrams of Fig. 1. can be written as

$$\Delta\Gamma_b^\mu = i\frac{\hat{e}}{4\hat{s}_W\hat{c}_W}\gamma^\mu(1-\gamma_5)\delta_{b-vertex}, \tag{A1}$$

with[§]

$$\delta_{b-vertex} = -\frac{\hat{\alpha}}{4\pi\hat{s}_W^2}F_b(m_t). \tag{A2}$$

The function $F_b(m_t)$ is obtained directly from the diagrams of Fig. 1 after subtraction of the zero mass contribution,

$$F_b(m_t) = \sum_j (I_j(x,y) - I_j(0,y)), \tag{A3}$$

where $x \equiv (m_t/m_W)^2$, $y \equiv (m_Z/m_W)^2 = 1/c_W^2$ and the contributions for each of the diagrams in Fig. 1 are

$$\begin{aligned}
I_{1a}(x,y) &= (\frac{1}{2}-\frac{2}{3}\hat{s}_W^2)\left(\frac{3}{2}+f_{0a}(x,y)+(y+\frac{1}{2}(1-x))f_{2a}(x,y)-yf_{1a}(x,y)\right)\\
&+ \frac{2}{3}x\hat{s}_W^2 f_{1a}(x,y)),\\
I_{1b}(x,y) &= -3\hat{c}_W^2\left(-\frac{1}{2}+f_{0b}(y)+\frac{1}{3}(y+\frac{1}{2}(1-x))f_{2b}(x,y)+\frac{2}{3}xf_{1b}(x,y)\right),\\
I_{1cd}(x,y) &= -\hat{s}_W^2 x f_{1b}(x,y),\\
I_{1ef}(x,y) &= (\frac{1}{2}-\frac{1}{3}\hat{s}_W^2)\left(\frac{x}{1-x}+\frac{x^2\ln x}{(1-x)^2}\right),\\
I_{2a}(x,y) &= \hat{s}_W^2 x\left(-\frac{1}{6}-\frac{1}{3}f_{0a}(x,y)-\frac{1}{6}(1-x)f_{2a}(x,y)\right)\\
&- \frac{1}{2}x^2(\frac{1}{2}-\frac{2}{3}\hat{s}_W^2)f_{1a}(x,y),\\
I_{2b}(x,y) &= (\frac{1}{4}-\frac{1}{2}\hat{s}_W^2)x\left(\frac{3}{2}-f_{0b}(y)-\frac{1}{2}(1-x)f_{2b}(x,y)-xf_{1b}(x,y)\right),\\
I_{2cd}(x,y) &= (\frac{1}{4}-\frac{1}{6}\hat{s}_W^2)x\left(-1+\frac{x}{1-x}+\frac{x^2\ln x}{(1-x)^2}\right).
\end{aligned} \tag{A4}$$

[§]The function $F_b(m_t)$ defined here is also the relevant function for the flavour changing process $Z \to b\bar{s}$ [5] and differs from the function F used in Ref. 6 in a factor $4\hat{s}_W^2$.

The I's are defined up to a constant term in m_t . The relevant functions in eq. (A4) are given by

$$f_{0a}(x,y) = \begin{cases} \ln x - 2 + \sqrt{4x/y-1}(\pi - 2\arctan\sqrt{4x/y-1}) & \text{if } 4x \geq y \\ \ln x - 2 + \sqrt{1-4x/y}\ln\frac{1+\sqrt{1-4x/y}}{1-\sqrt{1-4x/y}} - i\pi\sqrt{1-4x/y} & \text{otherwise} \end{cases}, \quad \text{(A5)}$$

$$f_{1a}(x,y) = \frac{2}{y}\left\{Li_2\left(\frac{1}{r_1}\right) + Li_2\left(\frac{1}{r_2}\right) - Li_2\left(\frac{1}{r_1+xr_2}\right) - Li_2\left(\frac{1}{r_2+xr_1}\right)\right.$$
$$\left. + Li_2\left(\frac{x}{r_1+xr_2}\right) + Li_2\left(\frac{x}{r_2+xr_1}\right) - \ln x \ln\left(1+\frac{y}{(1-x)^2}\right)\right\}, \quad \text{(A6)}$$

$$f_{2a}(x,y) = -\frac{4}{y}\left(f_{0a}(x,y) + 1 + \frac{x\ln x}{1-x} + \frac{1}{2}(1-x)f_{1a}(x,y)\right). \quad \text{(A7)}$$

Here,

$$r_1 \equiv \begin{cases} \frac{1}{2}(1+i\sqrt{4x/y-1}) & \text{if } 4x \geq y \\ \frac{1}{2}(1+\sqrt{1-4x/y}) + i\eta & \text{otherwise} \end{cases}, r_2 \equiv \begin{cases} \frac{1}{2}(1-i\sqrt{4x/y-1}) & \text{if } 4x \geq y \\ \frac{1}{2}(1-\sqrt{1-4x/y}) - i\eta & \text{otherwise} \end{cases}$$

and $Li_2(z) = -\int_0^z dx \ln(1-x)/x$ is the dilogarithm or Spence function. The 'b' functions can be related to the 'a' functions through

$$\begin{aligned} f_{0b}(y) &= f_{0a}(1,y), \\ f_{1b}(x,y) &= \frac{1}{x}f_{1a}(1/x, y/x), \\ f_{2b}(x,y) &= \frac{1}{x}f_{2a}(1/x, y/x). \end{aligned} \quad \text{(A8)}$$

In calculating the value of the form factor , we have used the values of m_W and $\hat{s}_W^2$ computed in terms of the top, Higgs and Z masses, as described in Ref. 11. This is the reason why the form factor, which is only a function of $\hat{s}_W^2$ and $(m_t/m_W)^2$, becomes slightly dependent on the Higgs mass.

m_t (GeV)	Re$\{F_b(m_t)\}$	Im$\{F_b(m_t)\}$	δ_b (%)
90.	-0.473	0.732	-0.29
110.	-0.869	0.731	-0.53
130.	-1.354	0.730	-0.83
150.	-1.915	0.729	-1.18
170.	-2.540	0.728	-1.56
190.	-3.227	0.727	-1.99
210.	-3.969	0.725	-2.46
230.	-4.764	0.724	-2.96

Table 1: The form factor F_b as a function of m_t and the δ_b vertex correction for the b quark ($m_{Higgs} = 100$ GeV).

m_t	90.	110.	130.	150.	170.	190.	210.	230.
u	297.0	297.7	298.6	299.5	300.6	301.8	303.1	304.5
c	296.8	297.6	298.4	299.4	300.5	301.6	302.9	304.4
s	380.9	381.5	382.1	383.0	384.0	385.2	386.5	387.9
b	377.9	377.5	377.0	376.6	376.1	375.6	375.0	374.5
e	83.4	83.5	83.7	83.8	84.0	84.3	84.5	84.8
τ	83.2	83.3	83.5	83.7	83.9	84.1	84.3	84.6
ν	166.3	166.5	166.7	167.0	167.4	167.7	168.2	168.6
Had.	1734.	1736.	1738.	1742.	1745.	1749.	1754.	1759.
Tot.	2482.	2486.	2489.	2494.	2499.	2505.	2512.	2519.

Table 2: Z decay widths (in MeV) as a function of m_t (in GeV) and $m_{Higgs} = 100$ GeV

m_t	90.	110.	130.	150.	170.	190.	210.	230.
u	296.3	297.1	297.9	298.9	300.0	301.1	302.4	303.9
c	296.2	296.9	297.8	298.7	299.8	301.0	302.3	303.7
s	380.2	380.8	381.5	382.3	383.3	384.5	385.8	387.3
b	377.2	376.8	376.4	375.9	375.4	374.9	374.4	373.9
e	83.3	83.4	83.6	83.7	83.9	84.2	84.4	84.7
τ	83.1	83.2	83.4	83.6	83.8	84.0	84.2	84.5
ν	166.2	166.4	166.6	166.9	167.2	167.6	168.0	168.5
Had.	1730.	1732.	1735.	1738.	1742.	1746.	1751.	1756.
Tot.	2478.	2481.	2485.	2490.	2495.	2501.	2508.	2516.

Table 3: Z decay widths (in MeV) as a function of m_t (in GeV) and $m_{Higgs} = 500$ GeV

m_t	90.	110.	130.	150.	170.	190.	210.	230.
u	295.8	296.5	297.4	298.3	299.4	300.6	301.9	303.3
c	295.6	296.4	297.2	298.2	299.3	300.5	301.8	303.2
s	379.6	380.1	380.8	381.7	382.7	383.9	385.2	386.7
b	376.6	376.2	375.7	375.3	374.8	374.3	373.8	373.3
e	83.2	83.3	83.5	83.6	83.8	84.1	84.3	84.6
τ	83.0	83.1	83.3	83.4	83.6	83.9	84.1	84.4
ν	166.0	166.2	166.4	166.7	167.1	167.4	167.9	168.3
Had.	1727.	1729.	1732.	1735.	1739.	1743.	1748.	1753.
Tot.	2474.	2478.	2481.	2486.	2491.	2497.	2504.	2512.

Table 4: Z decay widths (in MeV) as a function of m_t (in GeV) and m_{Higgs} = 1000 GeV

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Figure Captions

Fig. 1 Diagrams contributing to the $F_b(m_t)$ form factor in Z decay to $b\bar{b}$.

Fig. 2 Branching ratios of $Z \to b\bar{b}$ with several normalizations. a) normalization to $Z \to s\bar{s}$, b) same but for $c\bar{c}$, c) normalization to the hadronic width and d) normalization to the leptonic width. The two lines are for Higgs masses of 40 and 1000 GeV.

Fig. 3 Possible constraints one could obtain if the branching ratios of Fig. 2 were measured at the 1% (0.5%) level –dashed lines (dotted lines)–. We assumed central values of 0.98 for R_s, 1.255 for R_c, 0.2155 for R_h and 4.48 for R_μ. We have also drawn the allowed region (the horizontal dash-dotted lines) in the case where $\hat{s}^2_W$ is measured from the asymmetries to be 0.232 ± 0.001. Finally for comparison we show the prediction for $\hat{s}^2_W$ obtained from the measurement of the Z mass (solid lines). The band is obtained by varying the Higgs mass between 40 and 1000 GeV.

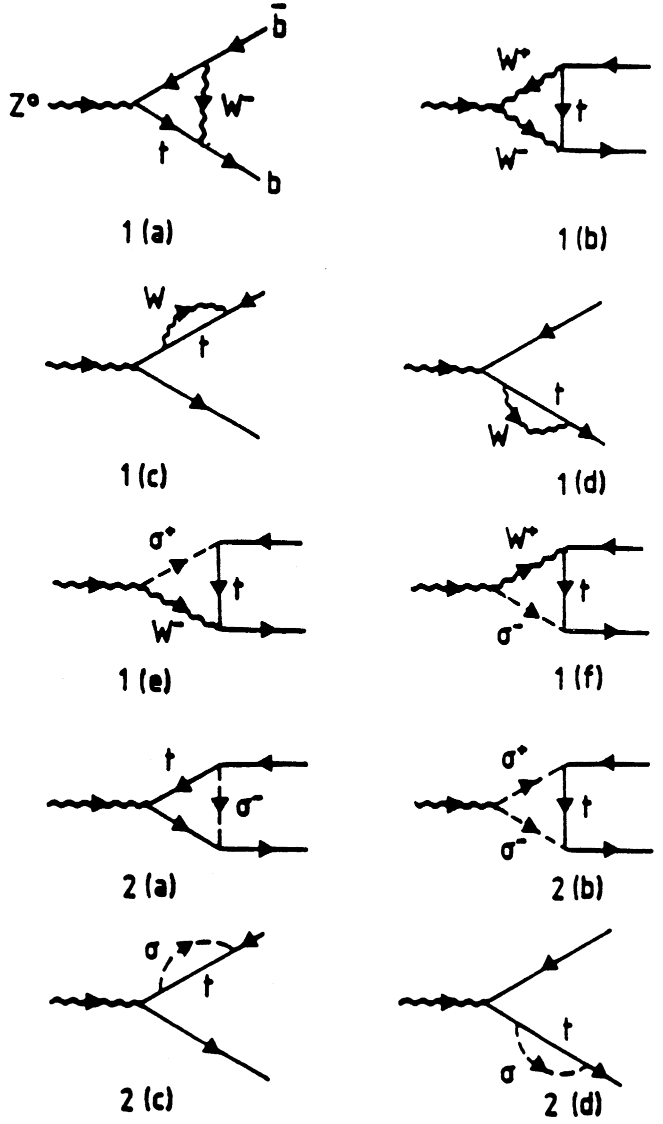
Z°
b̄
b
W⁻
t
W⁺
W
σ⁺
σ⁻
σ
1 (a)
1 (b)
1 (c)
1 (d)
1 (e)
1 (f)
2 (a)
2 (b)
2 (c)
2 (d)

Fig. 1.

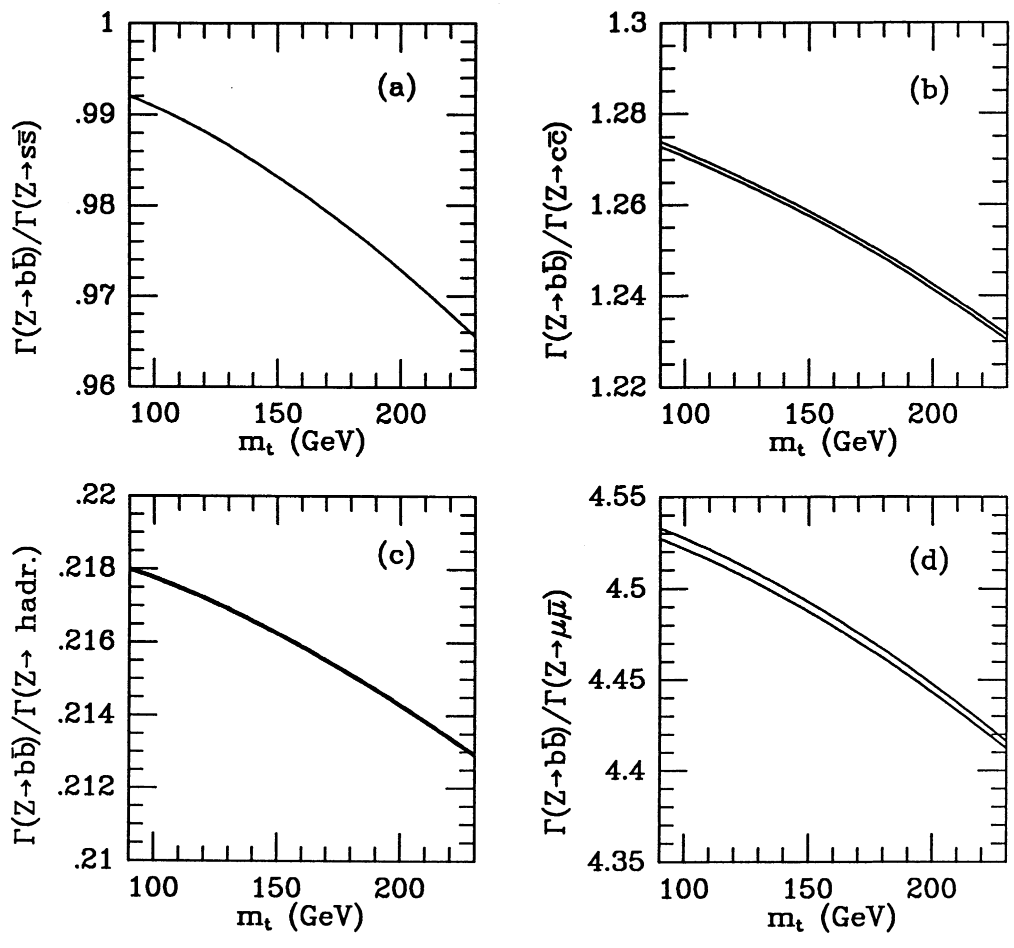
(a)
(b)
(c)
(d)
Γ(Z→bb̄)/Γ(Z→ss̄)
Γ(Z→bb̄)/Γ(Z→cc̄)
Γ(Z→bb̄)/Γ(Z→ hadr.)
Γ(Z→bb̄)/Γ(Z→μμ̄)
m_t (GeV)
1
.99
.98
.97
.96
1.3
1.28
1.26
1.24
1.22
.22
.218
.216
.214
.212
.21
4.55
4.5
4.45
4.4
4.35
100
150
200

Fig. 2

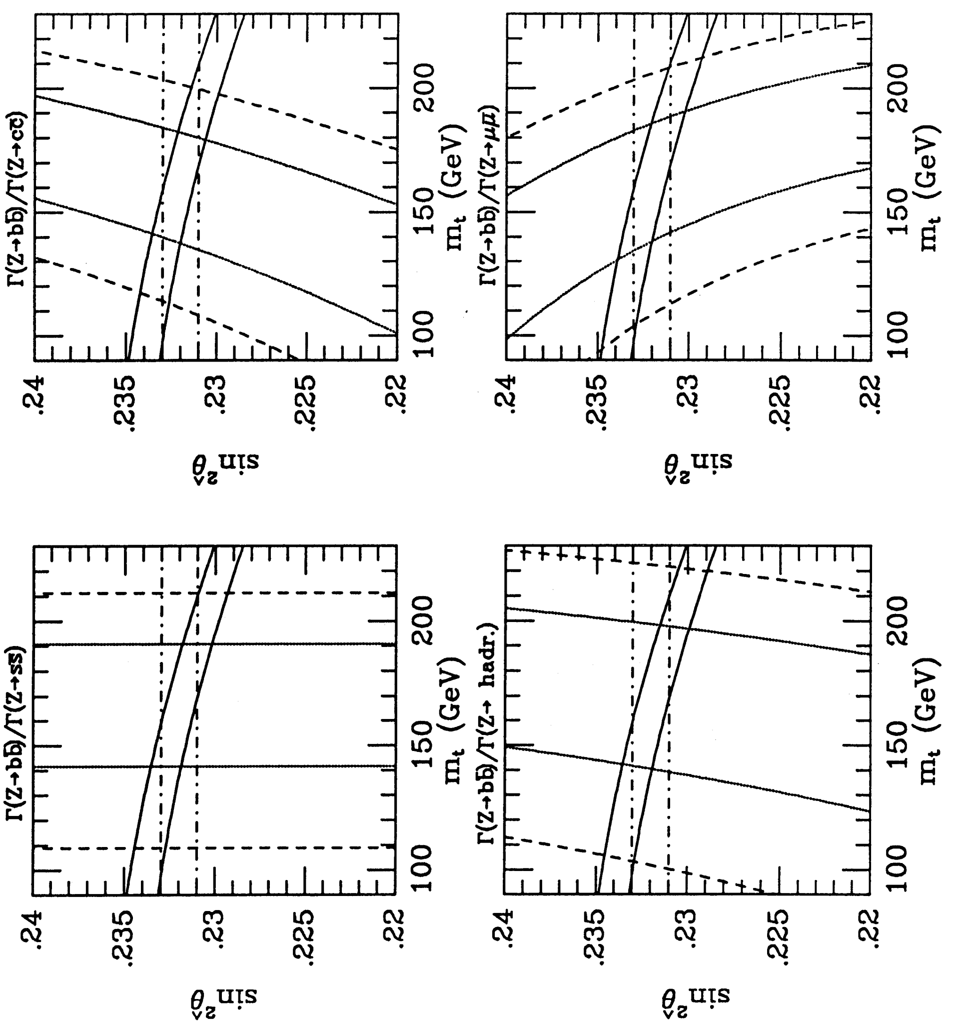
Γ(Z→bb̄)/Γ(Z→ss̄)
Γ(Z→bb̄)/Γ(Z→cc̄)
Γ(Z→bb̄)/Γ(Z→ hadr.)
Γ(Z→bb̄)/Γ(Z→μμ̄)
sin²θ̂
m_t (GeV)
.24
.235
.23
.225
.22
100
150
200

Fig. 3