

TAU PHYSICS*

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ABSTRACT

The pure leptonic or semileptonic character of τ decays makes them a good laboratory to test the structure of the weak currents and the universality of their couplings to the gauge bosons. The hadronic τ decay modes constitute an ideal tool for studying low-energy effects of the strong interactions in very clean conditions; a well-known example is the precise determination of the QCD coupling from τ -decay data. New physics phenomena, such as a non-zero m_{ν_τ} or violations of (flavour / CP) conservation laws can also be searched for with τ decays.

1. INTRODUCTION

The τ lepton is a member of the third generation which decays into particles belonging to the first and second ones. Thus, τ physics could provide some clues to the puzzle of the recurring families of leptons and quarks. In fact, one naïvely expects the heavier fermions to be more sensitive to whatever dynamics is responsible for the fermion-mass generation.

The pure leptonic or semileptonic character of τ decays provides a clean laboratory to test the structure of the weak currents and the universality of their couplings to the gauge bosons. Moreover, the τ is the only known lepton massive enough to decay into hadrons; its semileptonic decays are then an ideal tool for studying strong interaction effects in very clean conditions.

Since its discovery¹ in 1975 at the SPEAR e^+e^- storage ring, the τ lepton has been a subject of extensive experimental study.^{2,3,4,5,6,7,8,9,10,11,12,13} However, it has been during the last few years when τ physics has reached its maturity level. The very clean sample of boosted $\tau^+\tau^-$ events accumulated at the Z peak, together with the large statistics collected in the Υ region, have not only considerably improved the statistical accuracy of the τ measurements but, more importantly, have brought a new level of systematic understanding. Many of the small ($\sim 2\sigma$) discrepancies which were plaguing before⁸ the τ data have been already resolved, allowing now to make sensible tests of the τ properties. The improved quality of the data has motivated a growing interest on the τ particle, reflected in a series of workshops^{14,15,16,17} devoted entirely to the τ .

On the theoretical side, a lot of effort has been invested recently to improve our understanding of the τ dynamics. The basic τ properties were already known, before its actual discovery,¹ thanks to the pioneering paper of Tsai.¹⁸ The detailed study of higher-order electroweak corrections and QCD contributions, performed during the last few years, has promoted the physics of the τ lepton to the level of precision tests. There is now an ample recognition among the physics community of the unique properties of the τ for testing the Standard Model, both in the electroweak and the strong sectors.

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All experimental results obtained so far confirm the Standard Model scenario, in which the τ is a sequential lepton with its own quantum number and associated neutrino. With the increased sensitivities achieved recently, interesting limits on possible new physics contributions to the τ decay amplitudes start to emerge. In the following, the present knowledge on the τ lepton is analyzed. Rather than given a detailed review of experimental results, the emphasis is put on the physics which can be investigated with the τ data. Exhaustive information on more experimental aspects can be found in Refs. 17 and 19.

2. CHARGED-CURRENT UNIVERSALITY

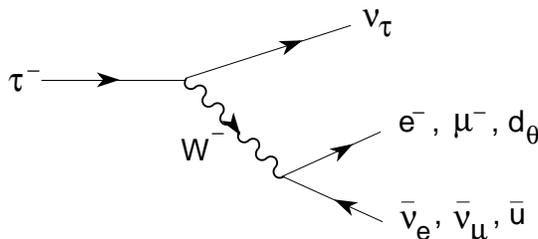


Figure 1: Feynman diagram for the decay of the τ lepton.

Within the Standard Model, the τ lepton decays via the W -emission diagram shown in Figure 1. Since the W coupling to the charged current is of universal strength,

$$\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \left\{ \sum_l \bar{\nu}_l \gamma^\mu (1 - \gamma_5) l + \bar{u} \gamma^\mu (1 - \gamma_5) d_\theta \right\} + \text{h.c.}, \quad (2.1)$$

there are five equal contributions (if final masses and gluonic corrections are neglected) to the τ decay width. Two of them correspond to the leptonic decay modes $\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e$ and $\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu$, while the other three are associated with the three possible colours of the quark-antiquark pair in the $\tau^- \rightarrow \nu_\tau d_\theta \bar{u}$ decay mode ($d_\theta \equiv \cos \theta_C d + \sin \theta_C s$). Hence, the branching ratios for the different channels are expected to be approximately:

$$B_l \equiv \text{Br}(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l) \simeq \frac{1}{5} = 20\% \quad (l = e, \mu), \quad (2.2)$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \simeq N_C = 3, \quad (2.3)$$

which should be compared with the present experimental averages^{17,19} in Table 1. The agreement is fairly good. Notice that the measured τ hadronic width provides strong evidence for the colour degree of freedom. We will discuss later whether the QCD dynamics is able to explain the (20%) difference between the measured value of R_τ and the lowest-order prediction $R_\tau = N_C$.

The leptonic decays $\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau$ ($l = e, \mu$) are theoretically understood at the level of the electroweak radiative corrections.²⁰ Within the Standard Model (neutrinos are assumed to be massless),

$$\Gamma_{\tau \rightarrow l} \equiv \Gamma(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192\pi^3} f\left(\frac{m_l^2}{m_\tau^2}\right) r_{EW}, \quad (2.4)$$

Table 1: Average values^{17,19} of some basic τ parameters. h^- stands for either π^- or K^- .

m_τ	$(1777.00^{+0.30}_{-0.27})$ MeV
τ_τ	(290.21 ± 1.15) fs
$\text{Br}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)$	$(17.786 \pm 0.072)\%$
$\text{Br}(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu)$	$(17.317 \pm 0.078)\%$
R_τ	3.649 ± 0.014
$\text{Br}(\tau^- \rightarrow \nu_\tau \pi^-)$	$(11.01 \pm 0.11)\%$
$\text{Br}(\tau^- \rightarrow \nu_\tau K^-)$	$(0.692 \pm 0.028)\%$
$\text{Br}(\tau^- \rightarrow \nu_\tau h^-)$	$(11.70 \pm 0.11)\%$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$. The factor r_{EW} takes into account radiative corrections not included in the Fermi coupling constant G_F , and the non-local structure of the W propagator; these effects²⁰ are quite small [$\alpha(m_\tau) = 1/133.3$]:

$$r_{EW} = \left[1 + \frac{\alpha(m_\tau)}{2\pi} \left(\frac{25}{4} - \pi^2 \right) \right] \left[1 + \frac{3}{5} \frac{m_\tau^2}{M_W^2} - 2 \frac{m_l^2}{M_W^2} \right] = 0.9960. \quad (2.5)$$

Using the value of G_F measured in μ decay, Eq. (2.4) provides a relation between the τ lifetime and the leptonic branching ratios:

$$B_e = \frac{B_\mu}{0.972564 \pm 0.000010} = \frac{\tau_\tau}{(1632.1 \pm 1.4) \times 10^{-15} \text{ s}}. \quad (2.6)$$

The quoted errors reflect the present uncertainty of 0.3 MeV in the value of m_τ .

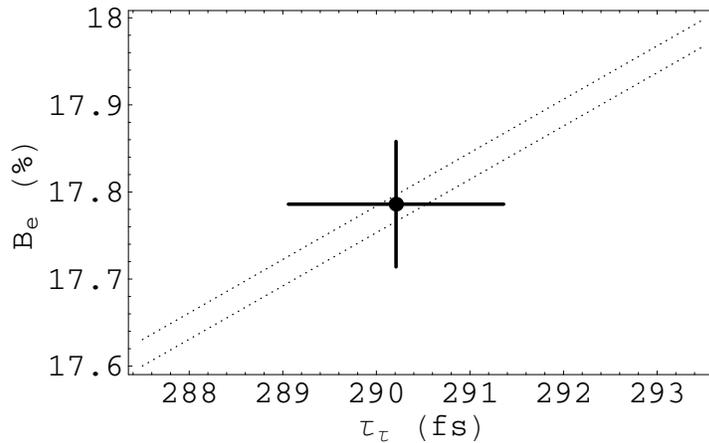


Figure 2: Relation between B_e and τ_τ . The dotted band corresponds to Eq. (2.6).

The predicted value of B_μ/B_e is in perfect agreement with the measured ratio $B_\mu/B_e = 0.974 \pm 0.006$. As shown in Figure 2, the relation between B_e and τ_τ is also well satisfied by the present data. Notice, that this relation is very sensitive to the value of the τ mass

Table 2: Present constraints¹³ on $|g_\mu/g_e|$.

	B_μ/B_e	$R_{\pi \rightarrow e/\mu}$	$\sigma \cdot B_{W \rightarrow \mu/e}$
$ g_\mu/g_e $	1.0005 ± 0.0030	1.0017 ± 0.0015	1.01 ± 0.04

Table 3: Present constraints¹³ on $|g_\tau/g_\mu|$.

	$B_e \tau_\mu/\tau_\tau$	$R_{\tau/\pi}$	$R_{\tau/K}$	$\sigma \cdot B_{W \rightarrow \tau/\mu}$
$ g_\tau/g_\mu $	1.0001 ± 0.0029	1.005 ± 0.005	0.984 ± 0.020	0.99 ± 0.05

$[\Gamma_{\tau \rightarrow l} \propto m_\tau^5]$. The most recent measurements of τ_τ , B_e and m_τ have consistently moved the world averages in the correct direction, eliminating the previous ($\sim 2\sigma$) disagreement.⁸ The experimental precision (0.4%) is already approaching the level where a possible non-zero ν_τ mass could become relevant; the present bound²¹ $m_{\nu_\tau} < 24$ MeV (95% CL) only guarantees that such effect^a is below 0.14%.

These measurements can be used to test the universality of the W couplings to the leptonic charged currents. The B_μ/B_e ratio constraints $|g_\mu/g_e|$, while the B_e/τ_τ relation provides information on $|g_\tau/g_\mu|$. The present results are shown in Tables 2 and 3, together with the values obtained from the π -decay ratio²³ $R_{\pi \rightarrow e/\mu} \equiv \Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)/\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$, and from the comparison of the $\sigma \cdot B$ partial production cross-sections for the various $W^- \rightarrow l^- \bar{\nu}_l$ decay modes at the $p\bar{p}$ colliders.^{24,25,26}

The decay modes $\tau^- \rightarrow \nu_\tau \pi^-$ and $\tau^- \rightarrow \nu_\tau K^-$ can also be used to test universality through the ratios

$$R_{\tau/\pi} \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau \pi^-)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = \left| \frac{g_\tau}{g_\mu} \right|^2 \frac{m_\tau^3}{2m_\pi m_\mu^2} \frac{(1 - m_\pi^2/m_\tau^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} (1 + \delta R_{\tau/\pi}), \quad (2.7)$$

$$R_{\tau/K} \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau K^-)}{\Gamma(K^- \rightarrow \mu^- \bar{\nu}_\mu)} = \left| \frac{g_\tau}{g_\mu} \right|^2 \frac{m_\tau^3}{2m_K m_\mu^2} \frac{(1 - m_K^2/m_\tau^2)^2}{(1 - m_\mu^2/m_K^2)^2} (1 + \delta R_{\tau/K}), \quad (2.8)$$

where the dependence on the hadronic matrix elements (the so-called decay constants $f_{\pi,K}$) factors out. Owing to the different energy scales involved, the radiative corrections to the $\tau^- \rightarrow \nu_\tau \pi^-/K^-$ amplitudes are however not the same than the corresponding effects in $\pi^-/K^- \rightarrow \mu^- \bar{\nu}_\mu$. The size of the relative correction has been estimated^{27,28} to be:

$$\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%, \quad \delta R_{\tau/K} = (0.90 \pm 0.22)\%. \quad (2.9)$$

Using these numbers, the measured $\tau^- \rightarrow \pi^- \nu_\tau$ and $\tau^- \rightarrow K^- \nu_\tau$ decay rates imply the $|g_\tau/g_\mu|$ ratios given in Table 3. The inclusive sum of both decay modes provides a slightly more accurate determination: $|g_\tau/g_\mu| = 1.004 \pm 0.005$.

The present data verify the universality of the leptonic charged-current couplings to the 0.15% (e/μ) and 0.30% (τ/μ) level. The precision of the most recent τ -decay measurements is becoming competitive with the more accurate π -decay determination. It is important to realize the complementarity of the different universality tests. The pure leptonic decay

^a The preliminary ALEPH bound,²² $m_{\nu_\tau} < 18.2$ MeV (95% CL), implies a correction smaller than 0.08%.

modes probe the charged–current couplings of a transverse W . In contrast, the decays $\pi/K \rightarrow l\bar{\nu}$ and $\tau \rightarrow \nu_\tau\pi/K$ are only sensitive to the spin–0 piece of the charged current; thus, they could unveil the presence of possible scalar–exchange contributions with Yukawa–like couplings proportional to some power of the charged–lepton mass. One can easily imagine new–physics scenarios which would modify differently the two types of leptonic couplings.²⁹ For instance, in the usual two–Higgs doublet model, charged–scalar exchange generates a correction to the ratio B_μ/B_e , but $R_{\pi\rightarrow e/\mu}$ remains unaffected. Similarly, lepton mixing between the ν_τ and an hypothetical heavy neutrino would not modify the ratios B_μ/B_e and $R_{\pi\rightarrow e/\mu}$, but would certainly correct the relation between B_l and the τ lifetime.

3. LORENTZ STRUCTURE OF THE CHARGED CURRENT

Let us consider the leptonic decays $l^- \rightarrow \nu_l l'^- \bar{\nu}_{l'}$, where the lepton pair (l, l') may be (μ, e) , (τ, e) , or (τ, μ) . The most general, local, derivative–free, lepton–number conserving, four–lepton interaction Hamiltonian, consistent with locality and Lorentz invariance,^{30,31,32,33,34,35,36}

$$\mathcal{H} = 4 \frac{G_{l'l}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g_{\epsilon\omega}^n \left[\bar{l}'_\epsilon \Gamma^n (\nu_l)_\sigma \right] \left[\overline{(\nu_l)_\lambda} \Gamma_n l_\omega \right], \quad (3.1)$$

contains ten complex coupling constants or, since a common phase is arbitrary, nineteen independent real parameters which could be different for each leptonic decay. The subindices $\epsilon, \omega, \sigma, \lambda$ label the chiralities (left–handed, right–handed) of the corresponding fermions, and n the type of interaction: scalar (I), vector (γ^μ), tensor ($\sigma^{\mu\nu}/\sqrt{2}$). For given n, ϵ, ω , the neutrino chiralities σ and λ are uniquely determined.

Taking out a common factor $G_{l'l}$, which is determined by the total decay rate, the coupling constants $g_{\epsilon\omega}^n$ are normalized to³⁴

$$1 = \frac{1}{4} \left(|g_{RR}^S|^2 + |g_{RL}^S|^2 + |g_{LR}^S|^2 + |g_{LL}^S|^2 \right) + 3 \left(|g_{RR}^T|^2 + |g_{LR}^T|^2 \right) \\ + \left(|g_{RR}^V|^2 + |g_{RL}^V|^2 + |g_{LR}^V|^2 + |g_{LL}^V|^2 \right). \quad (3.2)$$

In the Standard Model, $g_{LL}^V = 1$ and all other $g_{\epsilon\omega}^n = 0$.

For an initial lepton polarization \mathcal{P}_l , the final charged lepton distribution in the decaying lepton rest frame is usually parametrized in the form^{31,32}

$$\frac{d^2\Gamma_{l\rightarrow l'}}{dx d\cos\theta} = \frac{m_l \omega^4}{2\pi^3} G_{l'l}^2 \sqrt{x^2 - x_0^2} \left\{ F(x) - \frac{\xi}{3} \mathcal{P}_l \sqrt{x^2 - x_0^2} \cos\theta A(x) \right\}, \quad (3.3)$$

where θ is the angle between the l^- spin and the final charged–lepton momentum, $\omega \equiv (m_l^2 + m_{l'}^2)/2m_l$ is the maximum l'^- energy for massless neutrinos, $x \equiv E_{l'}/\omega$ is the reduced energy, $x_0 \equiv m_{l'}/\omega$ and

$$F(x) = x(1-x) + \frac{2}{9}\rho \left(4x^2 - 3x - x_0^2 \right) + \eta x_0(1-x), \\ A(x) = 1 - x + \frac{2}{3}\delta \left(4x - 4 + \sqrt{1 - x_0^2} \right). \quad (3.4)$$

For unpolarized l' s, the distribution is characterized by the so-called Michel³⁰ parameter ρ and the low-energy parameter η . Two more parameters, ξ and δ , can be determined when the initial lepton polarization is known. If the polarization of the final charged lepton is also measured, 5 additional independent parameters¹⁹ (ξ' , ξ'' , η'' , α' , β') appear.

For massless neutrinos, the total decay rate is given by³⁶

$$\Gamma_{l \rightarrow l'} = \frac{\widehat{G}_{l'l}^2 m_l^5}{192\pi^3} f\left(\frac{m_{l'}^2}{m_l^2}\right) r_{\text{EW}}, \quad (3.5)$$

where

$$\widehat{G}_{l'l} \equiv G_{l'l} \sqrt{1 + 4\eta \frac{m_{l'}}{m_l} \frac{g(m_{l'}^2/m_l^2)}{f(m_{l'}^2/m_l^2)}}, \quad (3.6)$$

$g(z) = 1 + 9z - 9z^2 - z^3 + 6z(1+z) \ln z$, and the Standard Model radiative correction^b r_{EW} has been included. Thus, the normalization $G_{e\mu}$ corresponds to the Fermi coupling G_F , measured in μ decay. The B_μ/B_e and $B_e\tau_\mu/\tau_\tau$ universality tests, discussed in the previous section, actually prove the ratios $|\widehat{G}_{\mu\tau}/\widehat{G}_{e\tau}|$ and $|\widehat{G}_{e\tau}/\widehat{G}_{e\mu}|$, respectively. An important point, emphatically stressed by Fetscher and Gerber,³⁵ concerns the extraction of $G_{e\mu}$, whose uncertainty is dominated by the uncertainty in $\eta_{\mu \rightarrow e}$.

In terms of the $g_{e\omega}^n$ couplings, the shape parameters in Eqs. (3.3) and (3.4) are:

$$\begin{aligned} \rho &= \frac{3}{4}(\beta^+ + \beta^-) + (\gamma^+ + \gamma^-), \\ \xi &= 3(\alpha^- - \alpha^+) + (\beta^- - \beta^+) + \frac{7}{3}(\gamma^+ - \gamma^-), \\ \xi\delta &= \frac{3}{4}(\beta^- - \beta^+) + (\gamma^+ - \gamma^-), \\ \eta &= \frac{1}{2}\text{Re} \left[g_{LL}^V g_{RR}^{S*} + g_{RR}^V g_{LL}^{S*} + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*}) + g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) \right], \end{aligned} \quad (3.7)$$

where³⁷

$$\begin{aligned} \alpha^+ &\equiv |g_{RL}^V|^2 + \frac{1}{16}|g_{RL}^S + 6g_{RL}^T|^2, & \alpha^- &\equiv |g_{LR}^V|^2 + \frac{1}{16}|g_{LR}^S + 6g_{LR}^T|^2, \\ \beta^+ &\equiv |g_{RR}^V|^2 + \frac{1}{4}|g_{RR}^S|^2, & \beta^- &\equiv |g_{LL}^V|^2 + \frac{1}{4}|g_{LL}^S|^2, \\ \gamma^+ &\equiv \frac{3}{16}|g_{RL}^S - 2g_{RL}^T|^2, & \gamma^- &\equiv \frac{3}{16}|g_{LR}^S - 2g_{LR}^T|^2, \end{aligned} \quad (3.8)$$

are positive-definite combinations of decay constants, corresponding to a final right-handed (α^+ , β^+ , γ^+) or left-handed (α^- , β^- , γ^-) lepton. In the Standard Model, $\rho = \delta = 3/4$, $\eta = \eta'' = \alpha' = \beta' = 0$ and $\xi = \xi' = \xi'' = 1$.

The normalization constraint (3.2) is equivalent to $\alpha^+ + \alpha^- + \beta^+ + \beta^- + \gamma^+ + \gamma^- = 1$. It is convenient to introduce³⁴ the probabilities $Q_{e\omega}$ for the decay of an ω -handed l^- into

^b Since we assume that the Standard Model provides the dominant contribution to the decay rate, any additional higher-order correction beyond the effective Hamiltonian (3.1) would be a subleading effect.

an ϵ -handed daughter lepton,

$$\begin{aligned}
Q_{LL} &= \beta^- = \frac{1}{4}|g_{LL}^S|^2 + |g_{LL}^V|^2 = \frac{1}{4} \left(-3 + \frac{16}{3}\rho - \frac{1}{3}\xi + \frac{16}{9}\xi\delta + \xi' + \xi'' \right), \\
Q_{RR} &= \beta^+ = \frac{1}{4}|g_{RR}^S|^2 + |g_{RR}^V|^2 = \frac{1}{4} \left(-3 + \frac{16}{3}\rho + \frac{1}{3}\xi - \frac{16}{9}\xi\delta - \xi' + \xi'' \right), \\
Q_{LR} &= \alpha^- + \gamma^- = \frac{1}{4}|g_{LR}^S|^2 + |g_{LR}^V|^2 + 3|g_{LR}^T|^2 = \frac{1}{4} \left(5 - \frac{16}{3}\rho + \frac{1}{3}\xi - \frac{16}{9}\xi\delta + \xi' - \xi'' \right), \\
Q_{RL} &= \alpha^+ + \gamma^+ = \frac{1}{4}|g_{RL}^S|^2 + |g_{RL}^V|^2 + 3|g_{RL}^T|^2 = \frac{1}{4} \left(5 - \frac{16}{3}\rho - \frac{1}{3}\xi + \frac{16}{9}\xi\delta - \xi' - \xi'' \right).
\end{aligned} \tag{3.9}$$

Upper bounds on any of these (positive-semidefinite) probabilities translate into corresponding limits for all couplings with the given chiralities.

For μ decay, where precise measurements of the polarizations of both μ and e have been performed, there exist³⁴ upper bounds on Q_{RR} , Q_{LR} and Q_{RL} , and a lower bound on Q_{LL} . They imply corresponding upper bounds on the 8 couplings $|g_{RR}^n|$, $|g_{LR}^n|$ and $|g_{RL}^n|$. The measurements of the μ^- and the e^- do not allow to determine $|g_{LL}^S|$ and $|g_{LL}^V|$ separately.^{34,38} Nevertheless, since the helicity of the ν_μ in pion decay is experimentally known³⁹ to be -1 , a lower limit on $|g_{LL}^V|$ is obtained³⁴ from the inverse muon decay $\nu_\mu e^- \rightarrow \mu^- \nu_e$. The present (90% CL) bounds¹⁹ on the μ -decay couplings are shown in Figure 3. These limits show nicely that the bulk of the μ -decay transition amplitude is indeed of the predicted V–A type.

The experimental analysis of the τ -decay parameters is necessarily different from the one applied to the muon, because of the much shorter τ lifetime. The measurement of the τ polarization and the parameters ξ and δ is still possible due to the fact that the spins of the $\tau^+ \tau^-$ pair produced in $e^+ e^-$ annihilation are strongly correlated.^{18,40,41,42,43,44,45,46,47,48,49} Another possibility is to use the beam polarization, as done by SLD.⁵⁰ However, the polarization of the charged lepton emitted in the τ decay has never been measured. In principle, this could be done for the decay $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ by stopping the muons and detecting their decay products.⁴⁶ An alternative method would be⁵¹ to use the radiative decays $\tau \rightarrow l^- \bar{\nu}_l \nu_\tau \gamma$ ($l = e, \mu$), since the distribution of the photons emitted by the daughter lepton is sensitive to the lepton polarization. The measurement of the inverse decay $\nu_\tau l^- \rightarrow \tau^- \nu_l$ looks far out of reach.

The present experimental status¹⁷ on the τ -decay Michel parameters is shown in Table 4. For comparison, the values measured in μ decay¹⁹ are also given. The improved accuracy of the most recent experimental analyses has brought an enhanced sensitivity to the different shape parameters, allowing the first measurements^{17,50,52,53,54} of $\eta_{\tau \rightarrow \mu}$, $\xi_{\tau \rightarrow e}$, $\xi_{\tau \rightarrow \mu}$, $(\xi\delta)_{\tau \rightarrow e}$ and $(\xi\delta)_{\tau \rightarrow \mu}$, without any e/μ universality assumption.

The determination of the τ -polarization parameters allows us to bound the total probability for the decay of a right-handed τ ,⁴⁶

$$Q_{\tau_R} \equiv Q_{RR} + Q_{LR} = \frac{1}{2} \left[1 + \frac{\xi}{3} - \frac{16}{9}(\xi\delta) \right]. \tag{3.10}$$

One finds¹³ (ignoring possible correlations among the measurements):

$$Q_{\tau_R}^{\tau \rightarrow \mu} = 0.05 \pm 0.10 < 0.20 \quad (90\% \text{ CL}),$$

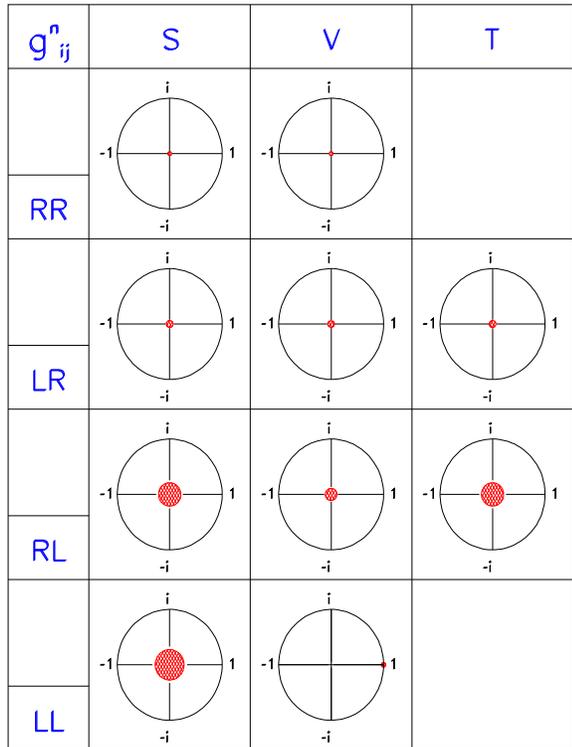


Figure 3: 90% CL experimental limits¹⁹ for the normalized μ -decay couplings $g_{e\omega}^n \equiv g_{e\omega}^n/N^n$, where $N^n \equiv \max(|g_{e\omega}^n|) = 2, 1, 1/\sqrt{3}$ for $n = S, V, T$. (Taken from Ref. 55).

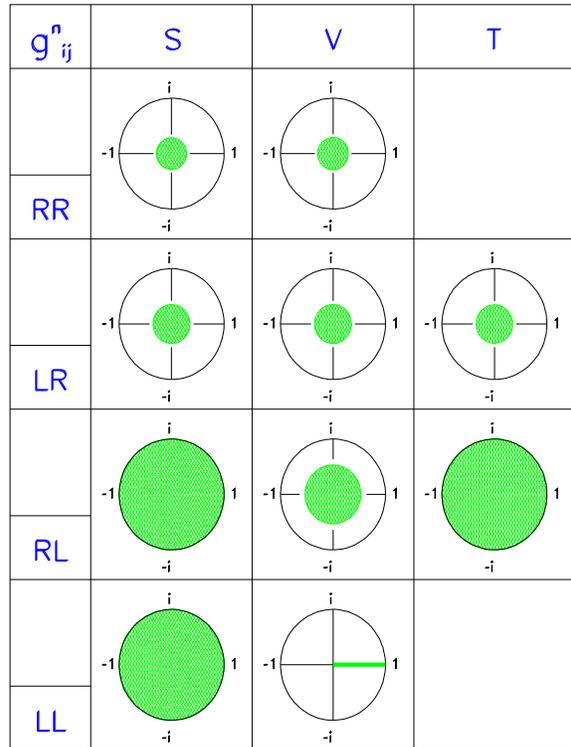


Figure 4: 90% CL experimental limits¹³ for the normalized τ -decay couplings $g_{e\omega}^n \equiv g_{e\omega}^n/N^n$, assuming e/μ universality.

$$\begin{aligned}
Q_{\tau_R}^{\tau \rightarrow e} &= -0.03 \pm 0.16 < 0.25 \quad (90\% \text{ CL}), \\
Q_{\tau_R}^{\tau \rightarrow l} &= 0.02 \pm 0.06 < 0.12 \quad (90\% \text{ CL}),
\end{aligned} \tag{3.11}$$

where the last value refers to the τ decay into either $l = e$ or μ , assuming identical e/μ couplings. Since these probabilities are positive-semidefinite quantities, they imply corresponding limits on all $|g_{RR}^n|$ and $|g_{LR}^n|$ couplings.

A measurement of the final lepton polarization could be even more efficient, since the total probability for the decay into a right-handed lepton depends on a single Michel parameter:

$$Q_{l_R}' \equiv Q_{RR} + Q_{RL} = \frac{1}{2}(1 - \xi'). \tag{3.12}$$

Thus, a single polarization measurement could bound the five RR and RL complex couplings.

Another useful positive-semidefinite quantity is⁵⁵

$$\rho - \xi\delta = \frac{3}{2}\beta^+ + 2\gamma^-, \tag{3.13}$$

which provides direct bounds on $|g_{RR}^V|$ and $|g_{RR}^S|$. A rather weak upper limit on γ^+ is obtained from the parameter ρ . More stringent is the bound on α^+ obtained from $(1 - \rho)$, which is also positive-semidefinite; it implies a corresponding limit on $|g_{RL}^V|$.

Table 4: World average^{17,19} Michel parameters. The last column ($\tau \rightarrow l$) assumes identical couplings for $l = e, \mu$. $\xi_{\mu \rightarrow e}$ refers to the product $\xi_{\mu \rightarrow e} \mathcal{P}_\mu$, where $\mathcal{P}_\mu \approx 1$ is the longitudinal polarization of the μ from π decay.

	$\mu \rightarrow e$	$\tau \rightarrow \mu$	$\tau \rightarrow e$	$\tau \rightarrow l$
ρ	0.7518 ± 0.0026	0.733 ± 0.031	0.734 ± 0.016	0.741 ± 0.014
η	-0.007 ± 0.013	-0.04 ± 0.20	—	0.047 ± 0.076
ξ	1.0027 ± 0.0085	1.19 ± 0.18	1.09 ± 0.16	1.04 ± 0.09
$\xi\delta$	0.7506 ± 0.0074	0.73 ± 0.11	0.80 ± 0.18	0.73 ± 0.07

Table 5 gives¹³ the resulting (90% CL) bounds on the τ -decay couplings. The relevance of these limits can be better appreciated in Figure 4, where e/μ universality has been assumed.

Table 5: 90% CL limits for the $g_{e\omega}^n$ couplings.¹³

	$\mu \rightarrow e$	$\tau \rightarrow \mu$	$\tau \rightarrow e$	$\tau \rightarrow l$
$ g_{RR}^S $	< 0.066	< 0.71	< 0.83	< 0.57
$ g_{LR}^S $	< 0.125	< 0.90	< 1.00	< 0.70
$ g_{RL}^S $	< 0.424	≤ 2	≤ 2	≤ 2
$ g_{LL}^S $	< 0.55	≤ 2	≤ 2	≤ 2
$ g_{RR}^V $	< 0.033	< 0.36	< 0.42	< 0.29
$ g_{LR}^V $	< 0.060	< 0.45	< 0.50	< 0.35
$ g_{RL}^V $	< 0.110	< 0.56	< 0.54	< 0.53
$ g_{LL}^V $	> 0.96	≤ 1	≤ 1	≤ 1
$ g_{LR}^T $	< 0.036	< 0.26	< 0.29	< 0.20
$ g_{RL}^T $	< 0.122	$\leq 1/\sqrt{3}$	$\leq 1/\sqrt{3}$	$\leq 1/\sqrt{3}$

If lepton universality is assumed, the leptonic decay ratios B_μ/B_e and $B_e\tau_\mu/\tau_\tau$ provide limits on the low-energy parameter η . The best sensitivity⁵⁶ comes from $\hat{G}_{\mu\tau}$, where the term proportional to η is not suppressed by the small m_e/m_l factor. The measured B_μ/B_e ratio implies then:

$$\eta_{\tau \rightarrow l} = 0.005 \pm 0.027 . \quad (3.14)$$

This determination is more accurate than the one in Table 4, obtained from the shape of the energy distribution, and is comparable to the value measured in μ decay.

A non-zero value of η would show that there are at least two different couplings with opposite chiralities for the charged leptons. Assuming the V–A coupling g_{LL}^V to be dominant, the second one would be⁴⁶ a Higgs-type coupling g_{RR}^S . To first order in new physics contribu-

tions, $\eta \approx \text{Re}(g_{RR}^S)/2$; Eq. (3.14) puts then the (90% CL) bound: $-0.08 < \text{Re}(g_{RR}^S) < 0.10$.

3.1. Model-Dependent Constraints

The sensitivity of the present data is not good enough to get strong constraints from a completely general analysis of the four-fermion Hamiltonian. Nevertheless, better limits can be obtained within particular models, as shown in Tables 6 and 7.

Table 6: 90% CL limits for the couplings $g_{\epsilon\omega}^n$, assuming that there are no tensor couplings.

	$\mu \rightarrow e$	$\tau \rightarrow \mu$	$\tau \rightarrow e$	$\tau \rightarrow l$
$ g_{RR}^S $	< 0.066	< 0.71	< 0.70	< 0.55
$ g_{LR}^S $	< 0.125	< 0.71	< 0.70	< 0.55
$ g_{RL}^S $	< 0.424	≤ 2	≤ 2	≤ 2
$ g_{LL}^S $	< 0.55	≤ 2	≤ 2	≤ 2
$ g_{RR}^V $	< 0.033	< 0.35	< 0.35	< 0.27
$ g_{LR}^V $	< 0.060	< 0.29	< 0.23	< 0.20
$ g_{RL}^V $	< 0.047	< 0.20	< 0.20	< 0.16
$ g_{LL}^V $	> 0.96	≤ 1	≤ 1	≤ 1

Table 6 assumes that there are no tensor couplings, i.e. $g_{\epsilon\omega}^T = 0$. This condition is satisfied in any model where the interactions are mediated by vector bosons and/or charged scalars.³⁶ In this case, the quantities $(1 - \frac{4}{3}\rho)$, $(1 - \frac{4}{3}\xi\delta)$ and $(1 - \frac{4}{3}\rho) + \frac{1}{2}(1 - \xi)$ reduce to sums of $|g_{\epsilon\omega}^n|^2$, which are positive semidefinite; i.e. , in the absence of tensor couplings, $\rho \leq \frac{3}{4}$, $\xi\delta \leq \frac{3}{4}$ and $(1 - \xi) > 2(\frac{4}{3}\rho - 1)$.

If one only considers W -mediated interactions, but admitting the possibility that the W couples non-universally to leptons of any chirality,³⁶ the stronger limits in Table 7 are obtained. In this case, the $g_{\epsilon\omega}^V$ constants factorize into the product of two leptonic W couplings, implying⁵⁷ additional relations among the couplings, such as $g_{LR}^V g_{RL}^V = g_{LL}^V g_{RR}^V$, which hold within any of the three channels, (μ, e) , (τ, e) , and (τ, μ) . Moreover, there are additional equations relating different processes, such as³⁶ $g_{\mu L \tau L}^V g_{e L \tau R}^V = g_{\mu L \tau R}^V g_{e L \tau L}^V$. The normalization condition (3.2) provides lower bounds on the g_{LL}^V couplings.

For W -mediated interactions, the hadronic τ -decay modes can also be used to test the

Table 7: 90% CL limits on the $g_{\epsilon\omega}^V$ couplings, assuming that (non-standard) W -exchange is the only relevant interaction.

	$\mu \rightarrow e$	$\tau \rightarrow \mu$	$\tau \rightarrow e$
$ g_{RR}^V $	< 0.0028	< 0.017	< 0.011
$ g_{LR}^V $	< 0.060	< 0.29	< 0.23
$ g_{RL}^V $	< 0.047	< 0.060	< 0.047
$ g_{LL}^V $	> 0.997	> 0.95	> 0.97

structure of the $\tau\nu_\tau W$ vertex, if one assumes that the W coupling to the light quarks is the Standard Model one:^c The \mathcal{P}_τ dependent part of the decay amplitude is then proportional to (twice) the mean ν_τ helicity

$$h_{\nu_\tau} \equiv \frac{|g_R|^2 - |g_L|^2}{|g_R|^2 + |g_L|^2}, \quad (3.15)$$

which plays a role analogous to the leptonic-decay parameter ξ . The analysis of $\tau^+\tau^-$ decay correlations in leptonic-hadronic and hadronic-hadronic decay modes, using the π , ρ and a_1 hadronic final states, gives^{19,50,59,60}

$$h_{\nu_\tau} = -1.003 \pm 0.022 . \quad (3.16)$$

This implies $|g_R/g_L|^2 < 0.017$ (90% CL).

3.2. Expected Signals in Minimal New-Physics Scenarios

All experimental results obtained so far are consistent with the Standard Model. Clearly, the Standard Model provides the dominant contributions to the τ -decay amplitudes. Future high-precision measurements of allowed τ -decay modes should then look for small deviations of the Standard Model predictions and find out the possible source of any detected discrepancy.

In a first analysis, it seems natural to assume³⁶ that new physics effects would be dominated by the exchange of a single intermediate boson, coupling to two leptonic currents. Table 8 summarizes the expected changes on the measurable shape parameters,³⁶ in different new physics scenarios. The four general cases studied correspond to adding a single intermediate boson exchange, V^+ , S^+ , V^0 , S^0 (charged/neutral, vector/scalar), to the Standard Model contribution.

Table 8: Changes in the Michel parameters induced by the addition of a single intermediate boson exchange (V^+ , S^+ , V^0 , S^0) to the Standard Model contribution.³⁶

	V^+	S^+	V^0	S^0
$\rho - 3/4$	< 0	0	0	< 0
$\xi - 1$	\pm	< 0	< 0	\pm
$\delta\xi - 3/4$	< 0	< 0	< 0	< 0
η	0	\pm	\pm	\pm

4. NEUTRAL-CURRENT COUPLINGS

In the Standard Model, tau pair production in e^+e^- annihilation proceeds through the electromagnetic and weak neutral-current interactions,

$$e^+e^- \rightarrow \gamma, Z \rightarrow \tau^+\tau^-. \quad (4.1)$$

^c A more general analysis of the process $e^+e^- \rightarrow \tau^+\tau^- \rightarrow (\bar{\nu}_\tau\pi^+\pi^0)(\nu_\tau\pi^-\pi^0)$, which includes scalar-like couplings, can be found in Ref. 58.

At low energies ($s \ll M_Z^2$), the production cross-section is only sensitive to the coupling of the τ to the photon. From the energy dependence of the production cross-section near threshold, the spin of the τ has been determined^{61,62} to be 1/2 and its mass has been measured to be⁶³ $m_\tau = 1776.96_{-0.21}^{+0.18+0.25}_{-0.17}$ MeV.

At high energies, where the Z contribution is important, the study of the production cross-section allows to extract information on the lepton electroweak parameters. The Z coupling to the neutral lepton current is given by

$$\mathcal{L}_{\text{NC}}^Z = \frac{g}{2 \cos \theta_W} Z_\mu \sum_l \bar{l} \gamma^\mu (v_l - a_l \gamma_5) l, \quad (4.2)$$

where $v_l = T_3^l(1 - 4|Q_l| \sin^2 \theta_W)$ and $a_l = T_3^l$; i.e., the weak neutral couplings are predicted to be the same for all leptons with equal electric charge.

For unpolarized e^+ and e^- beams, the differential $e^+e^- \rightarrow l^+l^-$ cross-section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8s} \left\{ A(1 + \cos^2 \theta) + B \cos \theta - h_l [C(1 + \cos^2 \theta) + D \cos \theta] \right\}, \quad (4.3)$$

where $h_l (= \pm 1)$ is (twice) the l^- helicity and θ is the scattering angle between e^- and l^- . At lowest order,

$$\begin{aligned} A &= 1 + 2v_e v_l \text{Re}(\chi) + (v_e^2 + a_e^2)(v_l^2 + a_l^2) |\chi|^2, \\ B &= 4a_e a_l \text{Re}(\chi) + 8v_e a_e v_l a_l |\chi|^2, \\ C &= 2v_e a_l \text{Re}(\chi) + 2(v_e^2 + a_e^2) v_l a_l |\chi|^2, \\ D &= 4a_e v_l \text{Re}(\chi) + 4v_e a_e (v_l^2 + a_l^2) |\chi|^2, \end{aligned} \quad (4.4)$$

and χ contains the Z propagator

$$\chi = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \frac{s}{s - M_Z^2 + is\Gamma_Z/M_Z}. \quad (4.5)$$

The coefficients A , B , C and D can be experimentally determined, by measuring the total cross-section, the forward-backward asymmetry, the polarization asymmetry and the forward-backward polarization asymmetry, respectively:

$$\begin{aligned} \sigma(s) &= \frac{4\pi\alpha^2}{3s} A, \\ \mathcal{A}_{\text{FB}}(s) &\equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3B}{8A}, \\ \mathcal{A}_{\text{Pol}}(s) &\equiv \frac{\sigma^{(h_l=+1)} - \sigma^{(h_l=-1)}}{\sigma^{(h_l=+1)} + \sigma^{(h_l=-1)}} = -\frac{C}{A}, \\ \mathcal{A}_{\text{FB,Pol}}(s) &\equiv \frac{N_F^{(h_l=+1)} - N_F^{(h_l=-1)} - N_B^{(h_l=+1)} + N_B^{(h_l=-1)}}{N_F^{(h_l=+1)} + N_F^{(h_l=-1)} + N_B^{(h_l=+1)} + N_B^{(h_l=-1)}} = -\frac{3D}{8A}. \end{aligned} \quad (4.6)$$

Here, N_F and N_B denote the number of l^- 's emerging in the forward and backward hemispheres, respectively, with respect to the electron direction.

For $s = M_Z^2$, the real part of the Z propagator vanishes and the photon exchange terms can be neglected in comparison with the Z -exchange contributions ($\Gamma_Z^2/M_Z^2 \ll 1$). Eqs. (4.6) become then,

$$\begin{aligned}\sigma^{0,l} \equiv \sigma(M_Z^2) &= \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_l}{\Gamma_Z^2}, & \mathcal{A}_{\text{FB}}^{0,l} \equiv \mathcal{A}_{\text{FB}}(M_Z^2) &= \frac{3}{4} \mathcal{P}_e \mathcal{P}_l, \\ \mathcal{A}_{\text{Pol}}^{0,l} \equiv \mathcal{A}_{\text{Pol}}(M_Z^2) &= \mathcal{P}_l, & \mathcal{A}_{\text{FB,Pol}}^{0,l} \equiv \mathcal{A}_{\text{FB,Pol}}(M_Z^2) &= \frac{3}{4} \mathcal{P}_e,\end{aligned}\quad (4.7)$$

where Γ_l is the Z partial decay width to the l^+l^- final state, and

$$\mathcal{P}_l \equiv \frac{-2v_l a_l}{v_l^2 + a_l^2} \quad (4.8)$$

is the average longitudinal polarization of the lepton l^- , which only depends on the ratio of the vector and axial-vector couplings. \mathcal{P}_l is a sensitive function of $\sin^2 \theta_W$.

The Z partial decay width to the l^+l^- final state,

$$\Gamma_l \equiv \Gamma(Z \rightarrow l^+l^-) = \frac{G_F M_Z^3}{6\pi\sqrt{2}} (v_l^2 + a_l^2) \left(1 + \frac{3\alpha}{4\pi}\right), \quad (4.9)$$

determines the sum $(v_l^2 + a_l^2)$, while the ratio v_l/a_l is derived from the asymmetries.^d The signs of v_l and a_l are fixed by requiring $a_e < 0$.

The measurement of the final polarization asymmetries can (only) be done for $l = \tau$, because the spin polarization of the τ 's is reflected in the distorted distribution of their decay products. Therefore, \mathcal{P}_τ and \mathcal{P}_e can be determined from a measurement of the spectrum of the final charged particles in the decay of one τ , or by studying the correlated distributions between the final products of both τ 's.⁴⁸

With polarized e^+e^- beams, one can also study the left-right asymmetry between the cross-sections for initial left- and right-handed electrons. At the Z peak, this asymmetry directly measures the average initial lepton polarization, \mathcal{P}_e , without any need for final particle identification:

$$\mathcal{A}_{\text{LR}}^0 \equiv \mathcal{A}_{\text{LR}}(M_Z^2) = \frac{\sigma_L(M_Z^2) - \sigma_R(M_Z^2)}{\sigma_L(M_Z^2) + \sigma_R(M_Z^2)} = -\mathcal{P}_e. \quad (4.10)$$

Tables 9 and 10 show the present experimental results for the leptonic Z -decay widths and asymmetries. The data are in excellent agreement with the Standard Model predictions and confirm the universality of the leptonic neutral couplings.^e There is however a small ($\sim 2\sigma$) discrepancy between the \mathcal{P}_e values obtained⁶⁶ from $\mathcal{A}_{\text{FB,Pol}}^{0,\tau}$ and $\mathcal{A}_{\text{LR}}^0$. Assuming lepton universality, the combined result from all leptonic asymmetries gives

$$\mathcal{P}_l = -0.1500 \pm 0.0025. \quad (4.11)$$

^d The asymmetries determine two possible solutions for $|v_l/a_l|$. This ambiguity can be solved with lower-energy data or through the measurement of the transverse spin-spin correlation⁴⁷ of the two τ 's in $Z \rightarrow \tau^+\tau^-$, which requires^{64,65} $|v_\tau/a_\tau| \ll 1$.

^e A small 0.2% difference between Γ_τ and $\Gamma_{e,\mu}$ is generated by the m_τ corrections.

Table 9: Measured values⁶⁶ of $\Gamma_l \equiv \Gamma(Z \rightarrow l^+l^-)$ and the leptonic forward–backward asymmetries. The last column shows the combined result (for a massless lepton) assuming lepton universality.

	e	μ	τ	l
Γ_l (MeV)	83.96 ± 0.15	83.79 ± 0.22	83.72 ± 0.26	83.91 ± 0.11
$\mathcal{A}_{\text{FB}}^{0,l}$ (%)	1.60 ± 0.24	1.62 ± 0.13	2.01 ± 0.18	1.74 ± 0.10

Table 10: Measured values⁶⁶ of the different polarization asymmetries.

$\mathcal{A}_{\text{Pol}}^{0,\tau} = \mathcal{P}_\tau$	$\frac{4}{3}\mathcal{A}_{\text{FB,Pol}}^{0,\tau} = \mathcal{P}_e$	$-\mathcal{A}_{\text{LR}}^0 = \mathcal{P}_e$	$-\{\frac{4}{3}\mathcal{A}_{\text{FB}}^{0,l}\}^{1/2} = P_l$
-0.1401 ± 0.0067	-0.1382 ± 0.0076	-0.1542 ± 0.0037	-0.1523 ± 0.0044

The measurement of $\mathcal{A}_{\text{Pol}}^{0,\tau}$ and $\mathcal{A}_{\text{FB,Pol}}^{0,\tau}$ assumes that the τ decay proceeds through the Standard Model charged–current interaction. A more general analysis should take into account the fact that the τ decay width depends on the product $\xi\mathcal{P}_\tau$, where ξ is the corresponding Michel parameter in leptonic decays, or the equivalent quantity $\xi_h (= -h_{\nu_\tau})$ in the semileptonic modes. A separate measurement of ξ and \mathcal{P}_τ has been performed by ALEPH⁶⁷ ($\mathcal{P}_\tau = -0.139 \pm 0.040$) and L3⁵⁹ ($\mathcal{P}_\tau = -0.154 \pm 0.022$), using the correlated distribution of the $\tau^+\tau^-$ decays.

The combined analysis of all^f leptonic observables from LEP and SLD ($\mathcal{A}_{\text{LR}}^0$) results in the effective vector and axial–vector couplings given⁶⁶ in Table 11. The corresponding 68% probability contours in the a_l – v_l plane are shown in Figure 5. The measured ratios of the e , μ and τ couplings provide a test of charged–lepton universality in the neutral–current sector.

The neutrino couplings can be determined from the invisible Z decay width, by assuming three identical neutrino generations with left–handed couplings (i.e., $v_\nu = a_\nu$), and fixing the sign from neutrino scattering data.⁶⁹ The resulting experimental value,⁶⁶ given in Table 11, is in perfect agreement with the Standard Model. Alternatively, one can use the Standard Model prediction for $\Gamma_{\text{inv}}/\Gamma_l$ to get a determination of the number of (light) neutrino flavours⁶⁶:

$$N_\nu = 2.989 \pm 0.012. \quad (4.12)$$

The universality of the neutrino couplings has been tested with $\nu_\mu e$ scattering data, which fixes⁷⁰ the ν_μ coupling to the Z : $v_{\nu_\mu} = a_{\nu_\mu} = 0.502 \pm 0.017$.

The measured leptonic asymmetries can be used to obtain the effective electroweak mixing angle in the charged–lepton sector:⁶⁶

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} \equiv \frac{1}{4} \left(1 - \frac{v_l}{a_l} \right) = 0.23114 \pm 0.00031. \quad (4.13)$$

Including also the hadronic asymmetries, one gets⁶⁶ $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23165 \pm 0.00024$ with a $\chi^2/\text{d.o.f.} = 12.8/6$.

^f Not yet included is the recent SLD measurement⁶⁸ of the leptonic forward–backward left–right asymmetries: $\mathcal{P}_e = -0.152 \pm 0.012 \pm 0.001$, $\mathcal{P}_\mu = -0.102 \pm 0.034 \pm 0.002$, $\mathcal{P}_\tau = -0.195 \pm 0.034 \pm 0.003$.

Table 11: Effective vector and axial–vector lepton couplings derived from LEP and SLD data.⁶⁶

	Without Lepton Universality	
	LEP	LEP + SLD
v_e	-0.0368 ± 0.0015	-0.03828 ± 0.00079
v_μ	-0.0372 ± 0.0034	-0.0358 ± 0.0030
v_τ	-0.0369 ± 0.0016	-0.0367 ± 0.0016
a_e	-0.50130 ± 0.00046	-0.50119 ± 0.00045
a_μ	-0.50076 ± 0.00069	-0.50086 ± 0.00068
a_τ	-0.50116 ± 0.00079	-0.50117 ± 0.00079
v_μ/v_e	1.01 ± 0.11	0.935 ± 0.085
v_τ/v_e	1.001 ± 0.062	0.959 ± 0.046
a_μ/a_e	0.9989 ± 0.0018	0.9993 ± 0.0017
a_τ/a_e	0.9997 ± 0.0019	1.0000 ± 0.0019
	With Lepton Universality	
	LEP	LEP + SLD
v_l	-0.03688 ± 0.00085	-0.03776 ± 0.00062
a_l	-0.50115 ± 0.00034	-0.50108 ± 0.00034
$a_\nu = v_\nu$	$+0.5009 \pm 0.0010$	$+0.5009 \pm 0.0010$

5. ELECTROMAGNETIC AND WEAK MOMENTS

A general description of the electromagnetic coupling of a spin- $\frac{1}{2}$ charged lepton to the virtual photon involves three different form factors:

$$T[l\bar{l}\gamma^*] = e \varepsilon_\mu(q) \bar{l} \left[F_1(q^2) \gamma^\mu + i \frac{F_2(q^2)}{2m_l} \sigma^{\mu\nu} q_\nu + \frac{F_3(q^2)}{2m_l} \sigma^{\mu\nu} \gamma_5 q_\nu \right] l, \quad (5.1)$$

where q^μ is the photon momentum. Owing to the conservation of the electric charge, $F_1(0) = 1$. At $q^2 = 0$, the other two form factors reduce to the lepton magnetic dipole moment, $\mu_l \equiv (e/2m_l) (g_l^\gamma/2) = e(1+F_2(0))/2m_l$, and electric dipole moment $d_l^\gamma = eF_3(0)/2m_l$. Similar expressions can be defined for the $l\bar{l}$ coupling to a virtual Z .

The $F_i(q^2)$ form factors are sensitive quantities to a possible lepton substructure. Moreover, $F_3(q^2)$ violates T and P invariance; thus, the electroweak dipole moments $d_l^{\gamma,Z}$, which vanish in the Standard Model, constitute a good probe of CP violation. Owing to their chiral–changing structure, the dipole moments may provide important insights on the mechanism responsible for mass generation. In general, one expects²⁹ that a fermion of mass m_f (generated by physics at some scale $M \gg m_f$) will have induced dipole moments proportional to some power of m_f/M . Therefore, heavy fermions such as the τ should be a good testing ground for this kind of effects.

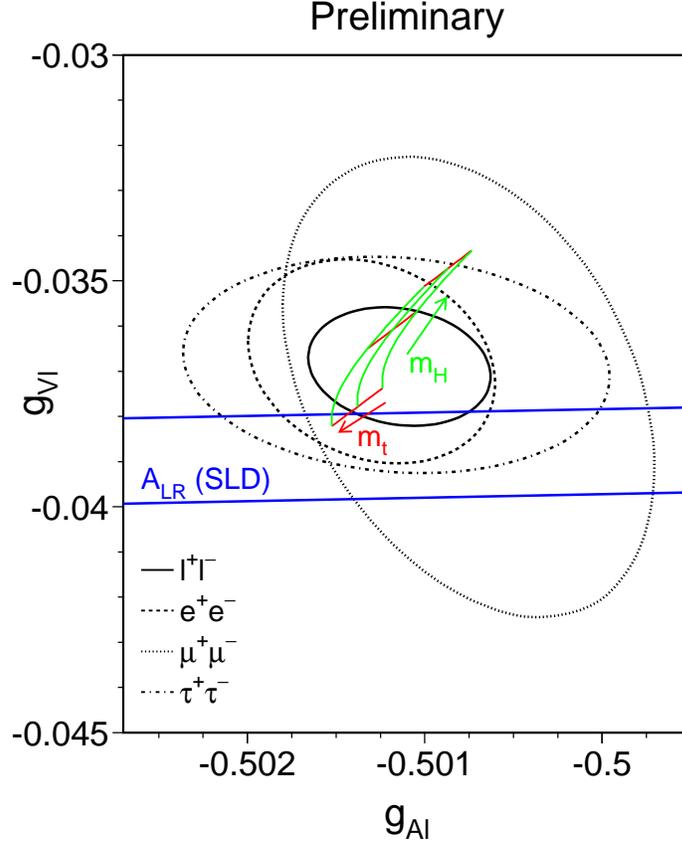


Figure 5: 68% probability contours in the a_l-v_l plane from LEP measurements.⁶⁶ The solid contour assumes lepton universality. Also shown is the 1σ band resulting from the A_{LR}^0 measurement at SLD. The grid corresponds to the Standard Model prediction.

Information on the τ electroweak form factors can be obtained by measuring the $e^+e^- \rightarrow \tau^+\tau^-$ cross-section. Their $q^2 = 0$ values can be tested in $e^+e^- \rightarrow \tau^+\tau^-\gamma$ and in the decay $Z \rightarrow \tau^+\tau^-\gamma$.⁷¹ A general analysis of the τ electroweak form factors has never been performed. The existing experimental tests only provide limits on a single F_i assuming the other form factors to take their Standard Model values.

At low energies, where the Z contribution is very small, the deviations from the QED prediction are usually parametrized through $F_1(s) = [1 \mp s/(s - \Lambda_{\pm}^2)]$. The cut-off parameters Λ_{\pm} characterize the validity of QED and measure the point-like nature of the τ . From PEP and PETRA data, one finds⁷² $\Lambda_+(\tau) > 285$ GeV and $\Lambda_-(\tau) > 246$ GeV (95% CL), which correspond to upper limits on the τ charge radius of 10^{-3} fm.

The same PEP/PETRA data can be used to extract limits on the τ anomalous magnetic moment,^{73,74} $a_{\tau}^{\gamma} \equiv (g_{\tau}^{\gamma} - 2)/2$, or electric dipole moment⁷⁵; one finds: $|a_{\tau}^{\gamma}| < 0.023$ (95% CL), $|d_{\tau}^{\gamma}| < 1.6 \times 10^{-16}$ e cm (90% CL). These limits actually probe the corresponding form factors $F_2(s)$ and $F_3(s)$ at $s \sim 35$ GeV. More direct bounds at $q^2 = 0$ have been extracted⁷⁶

from the decay^g $Z \rightarrow \tau^+\tau^-\gamma$:

$$|a_\tau^\gamma| < 0.0104, \quad |d_\tau^\gamma| < 5.8 \times 10^{-17} \text{ e cm} \quad (95\% \text{ CL}). \quad (5.2)$$

Slightly better, but more model-dependent, limits have been derived⁷⁸ from the $Z \rightarrow \tau^+\tau^-$ decay width: $-0.004 < a_\tau^\gamma < 0.006$, $|d_\tau^\gamma| < 2.7 \times 10^{-17} \text{ e cm}$ (95% CL); these bounds would be invalidated in the presence of any CP-conserving contribution to $\Gamma(Z \rightarrow \tau^+\tau^-)$ interfering destructively with the Standard Model amplitude.

In the Standard Model, d_τ^γ vanishes, while the overall value of a_τ^γ is dominated by the second order QED contribution,⁷⁹ $a_\tau^\gamma \approx \alpha/2\pi$. Including QED corrections up to $O(\alpha^3)$, hadronic vacuum polarization contributions and the corrections due to the weak interactions (which are a factor 380 larger than for the muon), the τ anomalous magnetic moment has been estimated to be^{80,81}

$$a_\tau^\gamma|_{th} = (1.1773 \pm 0.0003) \times 10^{-3}. \quad (5.3)$$

The first direct limit on the weak anomalous magnetic moment has been obtained by L3, by using correlated azimuthal asymmetries of the $\tau^+\tau^-$ decay products.⁸² The preliminary (95% CL) result of this analysis is⁸³:

$$-0.016 < a_\tau^Z < 0.011. \quad (5.4)$$

The possibility of a CP-violating weak dipole moment of the τ has been investigated at LEP, by studying T -odd triple correlations^{84,85} of the final τ -decay products in $Z \rightarrow \tau^+\tau^-$ events. The present (95% CL) limits are⁷⁶:

$$\begin{aligned} |\text{Re } d_\tau^Z(M_Z^2)| &\leq 3.6 \times 10^{-18} \text{ e cm}, \\ |\text{Im } d_\tau^Z(M_Z^2)| &\leq 1.1 \times 10^{-17} \text{ e cm}. \end{aligned} \quad (5.5)$$

These limits provide useful constraints on different models of CP violation.^{84,86,87,88}

T -odd signals can be also generated through a relative phase between the vector and axial-vector couplings of the Z to the $\tau^+\tau^-$ pair,⁴⁷ i.e. $\text{Im}(v_\tau a_\tau^*) \neq 0$. This effect, which in the Standard Model appears⁸⁹ at the one-loop level through absorptive parts in the electroweak amplitudes, gives rise⁴⁷ to a spin-spin correlation associated with the transverse (within the production plane) and normal (to the production plane) polarization components of the two τ 's. A preliminary analysis of this transverse-normal spin correlation has been reported by ALEPH.⁶⁴

6. CP VIOLATION

In the three-generation Standard Model, the violation of the CP symmetry originates from the single phase naturally occurring in the quark mixing matrix⁹⁰. Therefore, CP violation is predicted to be absent in the lepton sector (for massless neutrinos). The present

^g The present upper limit on $|a_\tau^\gamma|$ has been extracted from the dependence of $\Gamma(Z \rightarrow \tau^+\tau^-\gamma)$ on $|a_\tau^\gamma|^2$, neglecting the interference terms, which are linear in $|a_\tau^\gamma|$ but are suppressed by a factor m_τ^2/M_Z^2 . This approximation is no longer justified if the limit is better than a few per cent.⁷⁷

experimental observations are in agreement with the Standard Model; nevertheless, the correctness of the Kobayashi—Maskawa mechanism is far from being proved. Like fermion masses and quark mixing angles, the origin of the Kobayashi—Maskawa phase lies in the most obscure part of the Standard Model Lagrangian: the scalar sector. Obviously, CP violation could well be a sensitive probe for new physics.

Up to now, CP violation in the lepton sector has been investigated mainly through the electroweak dipole moments. Violations of the CP symmetry could also happen in the τ decay amplitude. In fact, the possible CP-violating effects can be expected to be larger in τ decay than in $\tau^+\tau^-$ production.⁹¹ Since the decay of the τ proceeds through a weak interaction, these effects could be $\mathcal{O}(1)$ or $\mathcal{O}(10^{-3})$, if the leptonic CP violation is *weak* or *milliweak*.⁹¹

With polarized electron (and/or positron) beams, one could use the longitudinal polarization vectors of the incident leptons to construct T-odd rotationally invariant products. CP could be tested by comparing these T-odd products in τ^- and τ^+ decays. In the absence of beam polarization, CP violation could still be tested through $\tau^+\tau^-$ correlations. In order to separate possible CP-odd effects in the $\tau^+\tau^-$ production and in the τ decay, it has been suggested to study the final decays of the τ -decay products and build the so-called *stage-two spin-correlation functions*.⁹² For instance, one could study the chain process $e^+e^- \rightarrow \tau^+\tau^- \rightarrow (\rho^+\bar{\nu}_\tau)(\rho^-\nu_\tau) \rightarrow (\pi^+\pi^0\bar{\nu}_\tau)(\pi^-\pi^0\nu_\tau)$. The distribution of the final pions provides information on the ρ polarization, which allows to test for possible CP-violating effects in the $\tau \rightarrow \rho\nu_\tau$ decay.

CP violation could also be tested through rate asymmetries, i.e. comparing the partial fractions $\Gamma(\tau^- \rightarrow X^-)$ and $\Gamma(\tau^+ \rightarrow X^+)$. However, this kind of signal requires the presence of strong final-state interactions in the decay amplitude. Another possibility would be to study T-odd (CPT-even) asymmetries in the angular distributions of the final hadrons in semileptonic τ decays.⁹³ Explicit studies of the decay modes⁹⁴ $\tau^- \rightarrow K^-\pi^-\pi^+\nu_\tau$, $\pi^-K^-K^+\nu_\tau$ and⁹⁵ $\tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau$ show that sizeable CP-violating effects could be generated in some models of CP violation involving several Higgs doublets or left-right symmetry.

7. LEPTON-NUMBER VIOLATION

In the minimal Standard Model with massless neutrinos, there is a separately conserved additive lepton number for each generation. All present data are consistent with this conservation law. However, there are no strong theoretical reasons forbidding a mixing among the different leptons, in the same way as happens in the quark sector. Many models in fact predict lepton-flavour or even lepton-number violation at some level.^{96,97,98,99,100} Experimental searches for these processes can provide information on the scale at which the new physics begins to play a significant role.

K , π and μ decays, together with μ - e conversion, neutrinoless double beta decays and neutrino oscillation studies, have put already stringent limits¹⁹ on lepton-flavour and lepton-number violating interactions. However, given the present lack of understanding of the origin of fermion generations, one can imagine different patterns of violation of this conservation law for different mass scales. Moreover, the larger mass of the τ opens the possibility of

new types of decay which are kinematically forbidden for the μ .

Table 12: Present limits^{19,101} on the branching ratios of lepton–flavour and lepton–number violating decays of the τ . All bounds are at 90% CL, except the l^-G^0 modes which are at 95% CL (G^0 denotes an unobservable neutral particle).

Decay mode	Upper limit	Decay mode	Upper limit
$\tau^- \rightarrow e^-e^+e^-$	3.3×10^{-6}	$\tau^- \rightarrow \mu^- \mu^+ \mu^-$	1.9×10^{-6}
$\tau^- \rightarrow e^- \mu^+ \mu^-$	3.6×10^{-6}	$\tau^- \rightarrow \mu^- e^+ e^-$	3.4×10^{-6}
$\tau^- \rightarrow e^+ \mu^- \mu^-$	3.5×10^{-6}	$\tau^- \rightarrow \mu^+ e^- e^-$	3.4×10^{-6}
$\tau^- \rightarrow e^- \pi^+ \pi^-$	4.4×10^{-6}	$\tau^- \rightarrow \mu^- \pi^+ \pi^-$	7.4×10^{-6}
$\tau^- \rightarrow e^+ \pi^- \pi^-$	4.4×10^{-6}	$\tau^- \rightarrow \mu^+ \pi^- \pi^-$	6.9×10^{-6}
$\tau^- \rightarrow e^- \rho^0$	4.2×10^{-6}	$\tau^- \rightarrow \mu^- \rho^0$	5.7×10^{-6}
$\tau^- \rightarrow e^- \pi^+ K^-$	7.7×10^{-6}	$\tau^- \rightarrow \mu^- \pi^+ K^-$	8.7×10^{-6}
$\tau^- \rightarrow e^- \pi^- K^+$	4.6×10^{-6}	$\tau^- \rightarrow \mu^- \pi^- K^+$	1.5×10^{-5}
$\tau^- \rightarrow e^+ \pi^- K^-$	4.5×10^{-6}	$\tau^- \rightarrow \mu^+ \pi^- K^-$	2.0×10^{-5}
$\tau^- \rightarrow e^- K^{*0}$	6.3×10^{-6}	$\tau^- \rightarrow \mu^- K^{*0}$	9.4×10^{-6}
$\tau^- \rightarrow e^- \bar{K}^{*0}$	1.1×10^{-5}	$\tau^- \rightarrow \mu^- \bar{K}^{*0}$	8.7×10^{-6}
$\tau^- \rightarrow e^- K^0$	1.3×10^{-3}	$\tau^- \rightarrow \mu^- K^0$	1.0×10^{-3}
$\tau^- \rightarrow e^- \gamma$	2.7×10^{-6}	$\tau^- \rightarrow \mu^- \gamma$	3.0×10^{-6}
$\tau^- \rightarrow e^- \pi^0$	3.7×10^{-6}	$\tau^- \rightarrow \mu^- \pi^0$	4.0×10^{-6}
$\tau^- \rightarrow e^- \eta$	8.2×10^{-6}	$\tau^- \rightarrow \mu^- \eta$	9.6×10^{-6}
$\tau^- \rightarrow e^- \pi^0 \pi^0$	6.5×10^{-6}	$\tau^- \rightarrow \mu^- \pi^0 \pi^0$	1.4×10^{-5}
$\tau^- \rightarrow e^- \eta \eta$	3.5×10^{-5}	$\tau^- \rightarrow \mu^- \eta \eta$	6.0×10^{-5}
$\tau^- \rightarrow e^- \pi^0 \eta$	2.4×10^{-5}	$\tau^- \rightarrow \mu^- \pi^0 \eta$	2.2×10^{-5}
$\tau^- \rightarrow e^- G^0$	2.7×10^{-3}	$\tau^- \rightarrow \mu^- G^0$	5×10^{-3}
$\tau^- \rightarrow \bar{p} \gamma$	2.9×10^{-4}	$\tau^- \rightarrow \bar{p} \pi^0$	6.6×10^{-4}
$\tau^- \rightarrow \bar{p} \eta$	1.3×10^{-3}		

The present upper limits on lepton–flavour and lepton–number violating decays of the τ are given in Table 12. These limits are in the range of 10^{-4} to 10^{-6} , which is far away from the impressive (90% CL) bounds¹⁹ obtained in μ decay:

$$\begin{aligned}
\text{Br}(\mu^- \rightarrow e^- \gamma) &< 4.9 \times 10^{-11}, \\
\text{Br}(\mu^- \rightarrow e^- e^+ e^-) &< 1.0 \times 10^{-12}, \\
\text{Br}(\mu^- \rightarrow e^- \gamma \gamma) &< 7.2 \times 10^{-11}.
\end{aligned}
\tag{7.1}$$

With future τ –decay samples of 10^7 events per year, an improvement of one to two orders of magnitude seems possible.

The lepton–flavour violating couplings of the Z boson can be investigated at LEP. The present (95% CL) limits are¹⁰²

$$\text{Br}(Z \rightarrow e^\pm \mu^\mp) < 1.7 \times 10^{-6};$$

$$\begin{aligned}\text{Br}(Z \rightarrow e^\pm \tau^\mp) &< 9.8 \times 10^{-6}; \\ \text{Br}(Z \rightarrow \mu^\pm \tau^\mp) &< 1.7 \times 10^{-5}.\end{aligned}\tag{7.2}$$

Below the Z pole, the search for the lepton-flavour violating processes $e^+e^- \rightarrow e^+\tau^-$ and $e^+e^- \rightarrow \mu^+\tau^-$ has given the (95% CL) upper bounds¹⁰³:

$$\begin{aligned}\sigma_{e\tau}/\sigma_{\mu\mu} &< 1.8 \times 10^{-3}; \\ \sigma_{\mu\tau}/\sigma_{\mu\mu} &< 6.1 \times 10^{-3}.\end{aligned}\tag{7.3}$$

8. THE TAU NEUTRINO

All observed τ decays are supposed to be accompanied by neutrino emission, in order to fulfil energy-momentum conservation requirements. As seen in Sections 3 and 4, the present data are consistent with the ν_τ being a conventional sequential neutrino. Since taus are not produced by ν_e or ν_μ beams, we know that ν_τ is different from the electronic and muonic neutrinos, and an upper limit can be set on the couplings of the tau to ν_e and ν_μ :¹⁰⁴

$$|g_{\tau\nu_e}| < 0.073, \quad |g_{\tau\nu_\mu}| < 0.002, \quad (90\% \text{ CL}).\tag{8.1}$$

These limits can be interpreted in terms of $\nu_e/\nu_\mu \rightarrow \nu_\tau$ oscillations, to exclude a region in the neutrino mass-difference and neutrino mixing-angle space. In the extreme situations of large δm^2 or maximal mixing, the present limits are¹⁰⁴

$$\begin{aligned}\nu_\mu \rightarrow \nu_\tau : \quad \sin^2 2\theta_{\mu,\tau} &< 0.004 \quad (\text{large } \delta m_{\mu,\tau}^2), \\ \delta m_{\mu,\tau}^2 &< 0.9 \text{ eV}^2 \quad (\sin^2 2\theta_{\mu,\tau} = 1);\end{aligned}\tag{8.2}$$

$$\begin{aligned}\nu_e \rightarrow \nu_\tau : \quad \sin^2 2\theta_{e,\tau} &< 0.12 \quad (\text{large } \delta m_{e,\tau}^2), \\ \delta m_{e,\tau}^2 &< 9 \text{ eV}^2 \quad (\sin^2 2\theta_{e,\tau} = 1).\end{aligned}\tag{8.3}$$

The new CHORUS¹⁰⁵ and NOMAD¹⁰⁶ experiments, presently running at CERN, and the future Fermilab E803 experiment¹⁰⁷ are expected to improve the $\nu_\mu \rightarrow \nu_\tau$ oscillation limits by at least an order of magnitude.

LEP and SLC have confirmed⁶⁶ the existence of three (and only three) different light neutrinos, with standard couplings to the Z (see Section 4). However, no direct observation of ν_τ , that is, interactions resulting from neutrinos produced in τ decay, has been made so far.

The expected source of τ neutrinos in beam dump experiments is the decay of D_s mesons produced by interactions in the dump; i.e., $p + N \rightarrow D_s + \dots$, followed by the decays $D_s^- \rightarrow \tau^- \bar{\nu}_\tau$ and $\tau^- \rightarrow \nu_\tau + \dots$. Several experiments¹⁰⁸ have searched for $\nu_\tau + N \rightarrow \tau^- + \dots$ interactions with negative results; therefore, only an upper limit on the production of ν_τ 's has been obtained. The direct detection of the ν_τ should be possible¹⁰⁹ at the LHC, thanks to the large charm-production cross-section of this collider.

The possibility of a non-zero neutrino mass is obviously a very important question in particle physics. There is no fundamental principle requiring a null mass for the neutrino. On the contrary, many extensions of the Standard Model predict non-vanishing neutrino masses, which could have, in addition, important implications in cosmology and astrophysics.

The first attempts to place a limit on m_{ν_τ} were done by studying the endpoint of the momentum spectrum of charged leptons from the decays $\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l$ ($l = e, \mu$). The precision which can be achieved is limited by the experimental momentum resolution of fastest particles, which deteriorates with increasing centre-of-mass energy. Better limits have been set by studying the endpoint of the hadronic mass spectrum of high multiplicity τ decays. The limiting factor is then the resolution of the effective hadronic-mass determination. The strongest bound up to date is the preliminary ALEPH limit,²²

$$m_{\nu_\tau} < 18.2 \text{ MeV} \quad (95\% \text{ CL}), \quad (8.4)$$

obtained from a two-dimensional likelihood fit of the visible energy and the invariant-mass distribution of $\tau^- \rightarrow (3\pi)^-\nu_\tau, (5\pi)^-\nu_\tau$ events.

For comparison, the present limits on the muon and electron neutrinos are¹⁹ $m_{\nu_\mu} < 170$ KeV (90% C.L.) and $m_{\nu_e} < 15$ eV. Note, however, that in many models a mass hierarchy among different generations is expected, with the neutrino mass being proportional to some power of the mass of its charged lepton partner. Assuming for instance the fashionable relation $m_{\nu_\tau}/m_{\nu_e} \sim (m_\tau/m_e)^2$, the bound (8.4) would be equivalent to a limit of 1.5 eV for m_{ν_e} . A relatively crude measurement of m_{ν_τ} may then imply strong constraints on neutrino-mass model building.

More stringent (but model-dependent) bounds on m_{ν_τ} can be obtained from cosmological considerations. A stable neutrino (or an unstable neutrino with a lifetime comparable to or longer than the age of the Universe) must not overclose the Universe. Therefore, measurements of the age of the Universe exclude stable neutrinos in the range^{110,111} $200 \text{ eV} < m_\nu < 2 \text{ GeV}$. Unstable neutrinos with lifetimes longer than 300 sec could increase the expansion rate of the Universe, spoiling the successful predictions for the primordial nucleosynthesis of light isotopes in the early universe¹¹²; the mass range $0.5 \text{ MeV} < m_{\nu_\tau} < 30 \text{ MeV}$ has been excluded in that case.^{112,113,114,115,116} For neutrinos of any lifetime decaying into electromagnetic daughter products, it is possible to exclude the same mass range, combining the nucleosynthesis constraints with limits based on the supernova SN 1987A and on BEBC data.^{115,116} Light neutrinos ($m_{\nu_\tau} < 100 \text{ keV}$) decaying through $\nu_\tau \rightarrow \nu_\mu + G^0$, are also excluded by the nucleosynthesis constraints, if their lifetime is shorter than 10^{-2} sec.¹¹⁴

The astrophysical and cosmological arguments lead indeed to quite stringent limits; however, they always involve (plausible) assumptions which could be relaxed in some physical scenarios.^{117,118,119} For instance, in deriving the abundance of massive ν_τ 's at nucleosynthesis, it is always assumed that τ neutrinos annihilate at the rate predicted by the Standard Model. Moreover, the present observational situation is rather unclear, due to the existence of inconsistent sets of data on the primordial abundances of light isotopes¹²⁰; therefore, one cannot be confident in the reliability of such limits.

A ν_τ mass in the few MeV range (i.e. the mass sensitivity which can be achieved in the foreseeable future) could have a host of interesting astrophysical and cosmological consequences¹¹⁶: relaxing the big-bang nucleosynthesis bound to the baryon density and the number of neutrino species; allowing big-bang nucleosynthesis to accommodate a low (< 20%) ^4He mass fraction or high ($> 10^{-4}$) deuterium abundance; improving significantly the agreement between the cold dark matter theory of structure formation and observations¹²¹;

and helping to explain how type II supernovae explode.

The electromagnetic structure of the ν_τ can be tested through the process $e^+e^- \rightarrow \nu_\tau\bar{\nu}_\tau\gamma$. The combined data from PEP and PETRA implies¹²² the following 90% CL upper bounds on the magnetic moment and charge radius of the ν_τ ($\mu_B \equiv e\hbar/2m_e$): $|\mu(\nu_\tau)| < 4 \times 10^{-6} \mu_B$; $\langle r^2 \rangle(\nu_\tau) < 2 \times 10^{-31} \text{ cm}^2$. A better limit on the ν_τ magnetic moment,

$$|\mu(\nu_\tau)| < 5.4 \times 10^{-7} \mu_B \quad (90\% \text{ CL}), \quad (8.5)$$

has been placed by the BEBC experiment,¹²³ by searching for elastic $\nu_\tau e$ scattering events, using a neutrino beam from a beam dump which has a small ν_τ component.

A big ν_τ magnetic moment of about $10^{-6} \mu_B$ has been suggested, in order to make the τ neutrino an acceptable cold dark matter candidate. For this to be the case, however, the ν_τ mass should be in the range $1 \text{ MeV} < m_{\nu_\tau} < 35 \text{ MeV}$.¹²⁴ The same region of m_{ν_τ} has been suggested in trying to understand the baryon–antibaryon asymmetry of the universe.¹²⁵

9. HADRONIC DECAYS

The τ is the only presently known lepton massive enough to decay into hadrons. Its semileptonic decays are then an ideal laboratory for studying the hadronic weak currents in very clean conditions. The decay modes $\tau^- \rightarrow \nu_\tau H^-$ probe the matrix element of the left-handed charged current between the vacuum and the final hadronic state H^- ,

$$\langle H^- | \bar{d}_\theta \gamma^\mu (1 - \gamma_5) u | 0 \rangle. \quad (9.1)$$

Contrary to the well-known process $e^+e^- \rightarrow \gamma \rightarrow \text{hadrons}$, which only tests the electromagnetic vector current, the semileptonic τ -decay modes offer the possibility to study the properties of both vector and axial-vector currents.

For the decay modes with lowest multiplicity, $\tau^- \rightarrow \nu_\tau \pi^-$ and $\tau^- \rightarrow \nu_\tau K^-$, the relevant matrix elements are already known from the measured decays $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ and $K^- \rightarrow \mu^- \bar{\nu}_\mu$,

$$\begin{aligned} \langle \pi^-(p) | \bar{d} \gamma^\mu \gamma_5 u | 0 \rangle &\equiv -i\sqrt{2} f_\pi p^\mu, \\ \langle K^-(p) | \bar{s} \gamma^\mu \gamma_5 u | 0 \rangle &\equiv -i\sqrt{2} f_K p^\mu. \end{aligned} \quad (9.2)$$

The corresponding τ decay widths can then be predicted rather accurately [Eqs. (2.7) and (2.8)]. As shown in Table 3, these predictions are in good agreement with the measured values, and provide a quite precise test of charged-current universality.

Alternatively, the measured ratio between the $\tau^- \rightarrow \nu_\tau K^-$ and $\tau^- \rightarrow \nu_\tau \pi^-$ decay widths can be used to obtain a value for $\tan^2 \theta_C (f_K/f_\pi)^2$:

$$\left| \frac{V_{us}}{V_{ud}} \right|^2 \left(\frac{f_K}{f_\pi} \right)^2 = \left(\frac{m_\tau^2 - m_\pi^2}{m_\tau^2 - m_K^2} \right)^2 \frac{\Gamma(\tau^- \rightarrow \nu_\tau K^-)}{\Gamma(\tau^- \rightarrow \nu_\tau \pi^-)} \frac{1 + \delta R_{\tau/\pi}}{1 + \delta R_{\tau/K}} = (7.2 \pm 0.3) \times 10^{-2}. \quad (9.3)$$

This number is consistent with (but less precise than) the result $(7.67 \pm 0.06) \times 10^{-2}$ obtained from¹⁹ $\Gamma(K^- \rightarrow \mu^- \bar{\nu}_\mu)/\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$.

For the Cabibbo-allowed modes with $J^P = 1^-$, the matrix element of the vector charged current can also be obtained, through an isospin rotation, from the isovector part of the

e^+e^- annihilation cross-section into hadrons, which measures the hadronic matrix element of the $I = 1$ component of the electromagnetic current,

$$\langle H^0 | (\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d) | 0 \rangle. \quad (9.4)$$

The $\tau \rightarrow \nu_\tau V^-$ decay width is then expressed as an integral over the corresponding e^+e^- cross-section^{18,126}:

$$R_{\tau \rightarrow V} \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau V^-)}{\Gamma_{\tau \rightarrow e}} = \frac{3 \cos^2 \theta_C}{2\pi\alpha^2 m_\tau^8} S_{EW} \int_0^{m_\tau^2} ds (m_\tau^2 - s)^2 (m_\tau^2 + 2s) s \sigma_{e^+e^- \rightarrow V^0}^{I=1}(s), \quad (9.5)$$

where the factor $S_{EW} = 1.0194$ contains the renormalization-group improved electroweak correction at the leading logarithm approximation.²⁰ Using the available $e^+e^- \rightarrow$ hadrons data, one can then predict the τ decay widths for these modes.^{127,128,129,130,131}

Table 13: $R_{\tau \rightarrow V}$ from τ -decay^{17,19} and e^+e^- data.¹³¹

V^-	$R_{\tau \rightarrow V} \equiv \Gamma(\tau^- \rightarrow \nu_\tau V^-) / \Gamma_{\tau \rightarrow e}$	
	$\tau^- \rightarrow \nu_\tau V^-$	$e^+e^- \rightarrow V^0$
$\pi^- \pi^0$	1.413 ± 0.012	1.360 ± 0.043
$2\pi^- \pi^+ \pi^0$	0.239 ± 0.005	0.239 ± 0.031
$\pi^- 3\pi^0$	0.064 ± 0.008	0.059 ± 0.006
$\pi^- \omega$	0.108 ± 0.004	0.098 ± 0.011
$3\pi^- 2\pi^+ \pi^0$	0.0012 ± 0.0003	≥ 0.0010
$(6\pi)^-$	—	≥ 0.0052
$\pi^- \pi^0 \eta$	0.0101 ± 0.0013	0.0072 ± 0.0011
$K^- K^0$	0.0089 ± 0.0013	0.0062 ± 0.0016
$\pi^- \phi$	< 0.002	< 0.0006

The most recent results¹³¹ are compared with the τ -decay measurements in Table 13. The agreement is quite good. Moreover, the experimental precision of the τ -decay data is already better than the e^+e^- one.

The exclusive τ decays into final hadronic states with $J^P = 1^+$, or Cabibbo suppressed modes with $J^P = 1^-$, cannot be predicted with the same degree of confidence. We can only make model-dependent estimates¹³² with an accuracy which depends on our ability to handle the strong interactions at low energies. That just indicates that the decay of the τ lepton is providing new experimental hadronic information. Owing to their semileptonic character, the hadronic τ decay data are a unique and extremely useful tool to learn about the couplings of the low-lying mesons to the weak currents.

9.1. Chiral Dynamics

At low momentum transfer, the coupling of any number of π 's, K 's and η 's to the V-A current can be rigorously calculated with Chiral Perturbation Theory techniques.^{133,134,135,136}

In the absence of quark masses the QCD Lagrangian splits into two independent chirality (left/right) sectors, with their own quark flavour symmetries. With three light quarks (u , d , s), the QCD Lagrangian is then approximately invariant under chiral $SU(3)_L \otimes SU(3)_R$ rotations in flavour space. The vacuum is however not symmetric under the chiral group. Thus, chiral symmetry breaks down to the usual eightfold-way $SU(3)_V$, generating the appearance of eight Goldstone bosons in the hadronic spectrum, which can be identified with the lightest pseudoscalar octet; their small masses being generated by the quark mass matrix, which explicitly breaks chiral symmetry. The Goldstone nature of the pseudoscalar octet implies strong constraints on their low-energy interactions, which can be worked out through an expansion in powers of momenta over the chiral symmetry-breaking scale.^{133,134,135,136}

In the low-energy effective chiral realization of QCD, the vector and axial-vector currents take the form^{132,135}:

$$\begin{aligned} V_\mu &= -i \left(\Phi \overleftrightarrow{\partial}_\mu \Phi \right) + \mathcal{O}(\Phi^4) + \mathcal{O}(p^3) - \frac{iN_C}{6\sqrt{2}\pi^2 f_\pi^3} \varepsilon_{\mu\nu\alpha\beta} \left\{ \partial^\nu \Phi \partial^\alpha \Phi \partial^\beta \Phi + \mathcal{O}(\Phi^5) \right\}, \\ A_\mu &= -\sqrt{2}f_\pi \partial_\mu \Phi + \frac{\sqrt{2}}{3f_\pi} \left[\Phi, \left(\Phi \overleftrightarrow{\partial}_\mu \Phi \right) \right] + \mathcal{O}(\Phi^5) + \mathcal{O}(p^3) \\ &\quad + \frac{N_C}{12\pi^2 f_\pi^4} \varepsilon_{\mu\nu\alpha\beta} \left\{ \partial^\nu \Phi \partial^\alpha \Phi \left(\Phi \overleftrightarrow{\partial}^\beta \Phi \right) + \mathcal{O}(\Phi^6) \right\}, \end{aligned} \quad (9.6)$$

where the odd-parity pieces, proportional to the Levi-Civita pseudotensor, are generated by the Wess-Zumino-Witten term of the chiral Lagrangian,^{137,138} which incorporates the non-abelian chiral anomaly of QCD. The 3×3 matrix

$$\Phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}. \quad (9.7)$$

parametrizes the pseudoscalar octet fields. Thus, at lowest order in momenta, the couplings of the Goldstones to the weak current can be calculated in a straightforward way.

The one-loop corrections are known^{134,135,136,139} for the lowest-multiplicity states (π , K , 2π , $K\bar{K}$, $K\pi$, 3π). Moreover, a two-loop calculation for the 2π decay mode is already available.¹³⁹ Therefore, exclusive hadronic τ decay data at low values of q^2 could be compared with rigorous QCD predictions. There are also well-grounded theoretical results (based on a $1/M_\rho$ expansion) for decays such as $\tau^- \rightarrow \nu_\tau(\rho\pi)^-$, $\nu_\tau(K^*\pi)^-$, $\nu_\tau(\omega\pi)^-$, but only in the kinematical configuration where the pion is soft.¹⁴⁰

τ decays involve, however, high values of momentum transfer where the chiral symmetry predictions no longer apply. Since the relevant hadronic dynamics is governed by the non-perturbative regime of QCD, we are unable at present to make first-principle calculations for exclusive decays. Nevertheless, one can still construct reasonable models, taking into account the low-energy chiral theorems. The simplest prescription^{132,141,142,143} consists in extrapolating the chiral predictions to higher values of q^2 , by suitable final-state-interaction enhancements which take into account the resonance structures present in each channel in a phenomenological way. This can be done weighting the contribution of a given set of pseudoscalars, with definite quantum numbers, with an appropriate resonance form factor

such as

$$F_R(s) = \frac{M_R^2}{M_R^2 - s - iM_R\Gamma_R(s)}, \quad (9.8)$$

where M_R (Γ_R) denote the mass (width) of the resonance R . The requirement that the chiral predictions must be recovered below the resonance region fixes the normalization of those form factors to be one at zero invariant mass.

The extrapolation of the low-energy chiral theorems provides a useful description of the τ data in terms of a few resonance parameters. Therefore, it has been extensively used^{128,132,141,142,143,144,145} to analyze the main τ decay modes, and has been incorporated into the TAUOLA Monte Carlo library.¹⁴⁶ However, the model is too naive to be considered as an actual implementation of the QCD dynamics. Quite often, the numerical predictions could be drastically changed by varying some free parameter or modifying the form-factor ansatz. Not surprisingly, some predictions fail badly to reproduce the experimental data whenever a new resonance structure shows up.¹⁴⁷

The addition of resonance form factors to the chiral low-energy amplitudes does not guarantee that the chiral symmetry constraints on the resonance couplings have been correctly implemented. The proper way of including higher-mass states into the effective chiral theory was developed in Refs. 148. Using these techniques, a refined calculation of the rare decay $\tau^- \rightarrow \nu_\tau \eta \pi^-$ has been given recently.¹⁴⁹ A systematic analysis of τ decay amplitudes within this framework is in progress.¹⁵⁰

Tau decays offer a very good laboratory to improve our present understanding of the low-energy QCD dynamics. The general form factors characterizing the non-perturbative hadronic decay amplitudes can be experimentally extracted from the Dalitz-plot distributions of the final hadrons.¹⁵¹ An exhaustive analysis of τ decay modes would provide a very valuable data basis to confront with theoretical models.

10. QCD ANALYSIS OF THE TAU HADRONIC WIDTH

The inclusive character of the total τ hadronic width renders possible an accurate calculation of the ratio^{152,153,154,155,156,157,158,159,160,161} [(γ) represents additional photons or lepton pairs]

$$R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{ hadrons } (\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e (\gamma)]}, \quad (10.1)$$

using standard field theoretic methods.

The theoretical analysis of R_τ involves the two-point correlation functions

$$\Pi_{ij,V}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T(V_{ij}^\mu(x) V_{ij}^\nu(0)^\dagger) | 0 \rangle, \quad (10.2)$$

$$\Pi_{ij,A}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T(A_{ij}^\mu(x) A_{ij}^\nu(0)^\dagger) | 0 \rangle, \quad (10.3)$$

for the vector $V_{ij}^\mu = \bar{\psi}_j \gamma^\mu \psi_i$ and axial-vector $A_{ij}^\mu = \bar{\psi}_j \gamma^\mu \gamma_5 \psi_i$ colour-singlet quark currents ($i, j = u, d, s$). They have the Lorentz decompositions

$$\Pi_{ij,V/A}^{\mu\nu}(q) = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij,V/A}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,V/A}^{(0)}(q^2), \quad (10.4)$$

where the superscript ($J = 0, 1$) denotes the angular momentum in the hadronic rest frame.

The imaginary parts of the two–point functions $\Pi_{ij,V/A}^{(J)}(q^2)$ are proportional to the spectral functions for hadrons with the corresponding quantum numbers. The hadronic decay rate of the τ can be written as an integral of these spectral functions over the invariant mass s of the final–state hadrons:

$$R_\tau = 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im}\Pi^{(1)}(s) + \text{Im}\Pi^{(0)}(s) \right]. \quad (10.5)$$

The appropriate combinations of correlators are

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[\Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s) \right] + |V_{us}|^2 \left[\Pi_{us,V}^{(J)}(s) + \Pi_{us,A}^{(J)}(s) \right]. \quad (10.6)$$

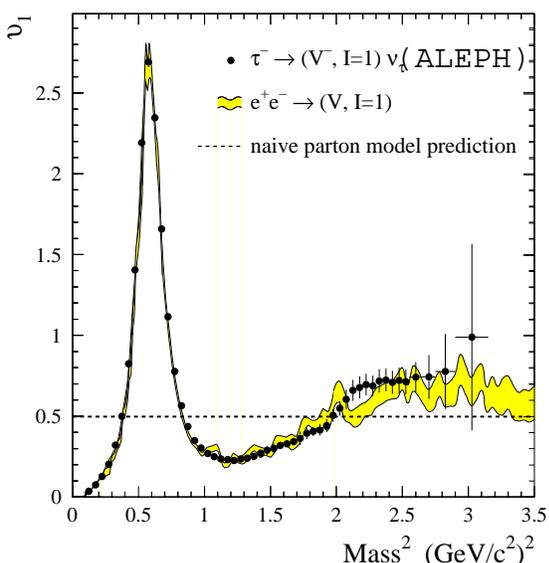


Figure 6: Measured vector–current spectral function¹⁶² [$v_1(s) \equiv 2\pi \text{Im}\Pi_{ud,V}^{(1)}(s)$]. The points correspond to τ decay data whereas the band has been obtained from e^+e^- . The dashed line represents the naive parton model prediction. (Taken from Ref. 162).

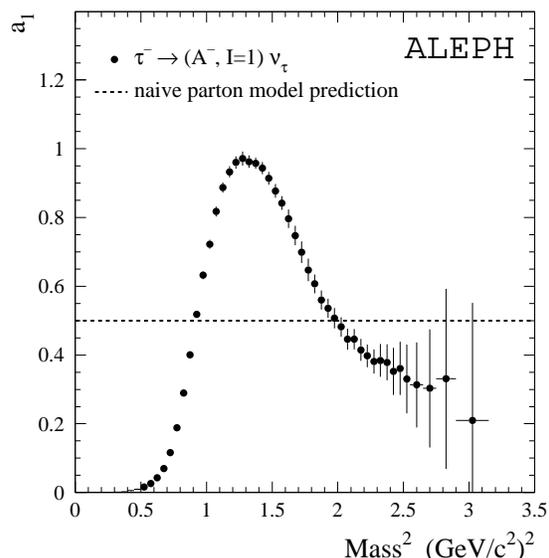


Figure 7: Measured axial–current spectral function¹⁶² [$a_1(s) \equiv 2\pi \text{Im}\Pi_{ud,A}^{(1)}(s)$]. The dashed line represents the naive parton model prediction. (Taken from Ref. 162).

We can separate the inclusive contributions associated with specific quark currents:

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}. \quad (10.7)$$

$R_{\tau,V}$ and $R_{\tau,A}$ correspond to the first two terms in (10.6), while $R_{\tau,S}$ contains the remaining Cabibbo–suppressed contributions. Non–strange hadronic decays of the τ are resolved experimentally into vector ($R_{\tau,V}$) and axial–vector ($R_{\tau,A}$) contributions according to whether the hadronic final state includes an even or odd number of pions. Strange decays ($R_{\tau,S}$) are of course identified by the presence of an odd number of kaons in the final state.

Since the hadronic spectral functions are sensitive to the non-perturbative effects of QCD that bind quarks into hadrons, the integrand in Eq. (10.5) cannot be calculated at present from QCD. Nevertheless the integral itself can be calculated systematically by exploiting the analytic properties of the correlators $\Pi^{(J)}(s)$. They are analytic functions of s except along the positive real s axis, where their imaginary parts have discontinuities. The integral (10.5) can therefore be expressed as a contour integral in the complex s plane running counter-clockwise around the circle $|s| = m_\tau^2$:

$$R_\tau = 6\pi i \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \Pi^{(0+1)}(s) - 2\frac{s}{m_\tau^2} \Pi^{(0)}(s) \right]. \quad (10.8)$$

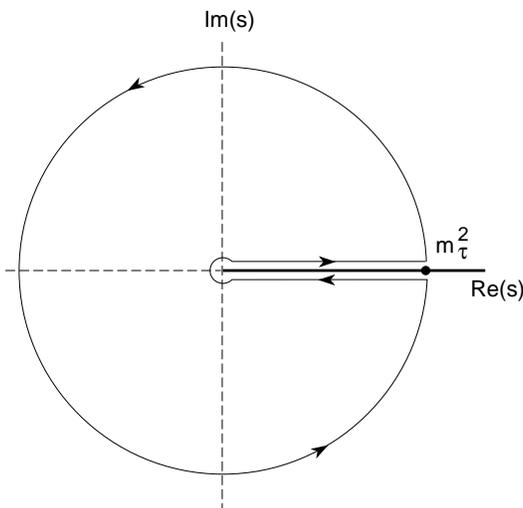


Figure 8: Integration contour in the complex s plane, used to obtain Eq. (10.8).

The advantage of expression (10.8) over (10.5) for R_τ is that it requires the correlators only for complex s of order m_τ^2 , which is significantly larger than the scale associated with non-perturbative effects in QCD. The short-distance Operator Product Expansion (OPE) can therefore be used to organize the perturbative and non-perturbative contributions to the correlators into a systematic expansion¹⁶³ in powers of $1/s$,

$$\Pi^{(J)}(s) = \sum_{D=2n} \sum_{\dim\mathcal{O}=D} \frac{\mathcal{C}^{(J)}(s, \mu) \langle \mathcal{O}(\mu) \rangle}{(-s)^{D/2}}, \quad (10.9)$$

where the inner sum is over local gauge-invariant scalar operators of dimension $D = 0, 2, 4, \dots$. The possible uncertainties associated with the use of the OPE near the time-like axis are negligible in this case, because the integrand in Eq. (10.8) includes a factor $(1 - s/m_\tau^2)^2$, which provides a double zero at $s = m_\tau^2$, effectively suppressing the contribution from the region near the branch cut. The parameter μ is an arbitrary factorization scale, which separates long-distance non-perturbative effects, which are absorbed into the vacuum matrix elements $\langle \mathcal{O}(\mu) \rangle$, from short-distance effects, which belong in the Wilson coefficients $\mathcal{C}^{(J)}(s, \mu)$. The $D = 0$ term (unit operator) corresponds to the pure perturbative

contributions, neglecting quark masses. The leading quark–mass corrections generate the $D = 2$ term. The first dynamical operators involving non-perturbative physics appear at $D = 4$. Inserting the functions (10.9) into (10.8) and evaluating the contour integral, R_τ can be expressed as an expansion in powers of $1/m_\tau^2$, with coefficients that depend only logarithmically on m_τ .

It is convenient to express the corrections to R_τ from dimension– D operators in terms of the fractional corrections $\delta_{ij,V/A}^{(D)}$ to the naive contribution from the current with quantum numbers ij, V or ij, A :

$$R_{\tau,V/A} = \frac{3}{2}|V_{ud}|^2 S_{EW} \left(1 + \delta'_{EW} + \delta^{(0)} + \sum_{D=2,4,\dots} \delta_{ud,V/A}^{(D)} \right), \quad (10.10)$$

$$R_{\tau,S} = 3|V_{us}|^2 S_{EW} \left(1 + \delta'_{EW} + \delta^{(0)} + \sum_{D=2,4,\dots} \delta_{us}^{(D)} \right), \quad (10.11)$$

where $\delta_{ij}^{(D)} = (\delta_{ij,V}^{(D)} + \delta_{ij,A}^{(D)})/2$ is the average of the vector and axial–vector corrections, and the factors²⁰

$$S_{EW} = \left(\frac{\alpha(m_b^2)}{\alpha(m_\tau^2)} \right)^{\frac{9}{19}} \left(\frac{\alpha(M_W^2)}{\alpha(m_b^2)} \right)^{\frac{9}{20}} \left(\frac{\alpha(M_Z^2)}{\alpha(M_W^2)} \right)^{\frac{36}{17}} = 1.0194 \quad (10.12)$$

and¹⁶⁴

$$\delta'_{EW} = \frac{5}{12} \frac{\alpha(m_\tau^2)}{\pi} \simeq 0.0010, \quad (10.13)$$

contain the known electroweak corrections.

Adding the three terms, the total ratio R_τ is

$$R_\tau = 3 \left(|V_{ud}|^2 + |V_{us}|^2 \right) S_{EW} \left\{ 1 + \delta'_{EW} + \delta^{(0)} + \sum_{D=2,4,\dots} \left(\cos^2 \theta_C \delta_{ud}^{(D)} + \sin^2 \theta_C \delta_{us}^{(D)} \right) \right\}, \quad (10.14)$$

where $\sin^2 \theta_C \equiv |V_{us}|^2 / (|V_{ud}|^2 + |V_{us}|^2)$.

10.1. Perturbative corrections

The dimension–0 contribution, $\delta^{(0)} = \delta_{ij,V/A}^{(0)}$, is the purely perturbative correction neglecting quark masses, which is the same for all the components of R_τ . It is given by^{152,153,154,155,156,157,158}.

$$\delta^{(0)} = \sum_{n=1} K_n A^{(n)}(\alpha_s) = \frac{\alpha_s(m_\tau^2)}{\pi} + 5.2023 \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^2 + 26.366 \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^3 + \mathcal{O}(\alpha_s^4). \quad (10.15)$$

The dynamical coefficients K_n regulate the perturbative expansion of the correlator $D(s) \equiv -s \frac{d}{ds} \Pi^{(0+1)}(s)$ in the massless–quark limit [$s\Pi^{(0)}(s) = 0$ for massless quarks]; they are known^{165,166,167} to $\mathcal{O}(\alpha_s^3)$: $K_1 = 1$; $K_2 = 1.63982$; $K_3(\overline{MS}) = 6.37101$. The kinematical effect of the contour integration is contained in the functions¹⁵⁶

$$A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left(\frac{\alpha_s(-s)}{\pi} \right)^n \left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right) = \left(\frac{\alpha_s(m_\tau^2)}{\pi} \right)^n + \mathcal{O}(\alpha_s^{n+1}), \quad (10.16)$$

which only depend on $\alpha_s(m_\tau^2)$. Owing to the long running of the strong coupling along the circle, the coefficients of the perturbative expansion of $\delta^{(0)}$ in powers of $\alpha_s(m_\tau^2)$ are larger than the direct K_n contributions. This running effect can be properly resummed to all orders in α_s by fully keeping¹⁵⁶ the known three-loop-level calculation of the integrals $A^{(n)}(\alpha_s)$.

Table 14: $\delta^{(0)}$ for different values of $\alpha_s(m_\tau^2)$.

$\alpha_s(m_\tau^2)$	$\delta^{(0)}$		$\Delta(\delta^{(0)})$
	$K_4 = 0$	$K_4 = 27.5$	
0.30	0.161	0.164	± 0.006
0.31	0.168	0.172	± 0.007
0.32	0.176	0.180	± 0.008
0.33	0.183	0.188	± 0.008
0.34	0.191	0.196	± 0.009
0.35	0.198	0.203	± 0.010
0.36	0.205	0.211	± 0.010
0.37	0.213	0.219	± 0.011
0.38	0.220	0.226	± 0.012
0.39	0.227	0.234	± 0.012
0.40	0.234	0.241	± 0.013

The dominant perturbative uncertainties come from the unknown higher-order coefficients $K_{n>3}$. The $\mathcal{O}(\alpha_s^4)$ contribution has been estimated¹⁶⁸ using scheme-invariant methods, namely the principle of minimal sensitivity¹⁶⁹ and the effective charge approach,¹⁷⁰ with the result¹⁶⁸:

$$K_4^{\text{est}} = 27.5. \quad (10.17)$$

This number is very close to the naïve guess¹⁵⁶ $K_4 \sim (K_3/K_2)K_3 \approx 25$. A similar estimate, $K_4^{\text{NNA}} = 24.8$, is obtained^{171,172,173,174,175,176,177} in the limit of a large number of quark flavours, using the so-called *naive non-abelianization* prescription¹⁷⁸ ($n_f \rightarrow 3\beta_1 = n_f - \frac{33}{2} = -\frac{27}{2}$). From a fit to the experimental τ data, the value $K_4^{\text{fit}} = 29 \pm 5$ has been also quoted.¹⁷⁹

Using the estimate (10.17), the $\mathcal{O}(\alpha_s^4)$ correction amounts to a 0.004 increase of $\delta^{(0)}$ for $\alpha_s(m_\tau^2) = 0.35$. The resulting perturbative contribution $\delta^{(0)}$ is given¹⁶¹ in Table 14 for different values of the strong coupling constant $\alpha_s(m_\tau^2)$. In order to be conservative, and to account for all possible sources of perturbative uncertainties, we have used^{157,158,161}

$$\Delta(\delta^{(0)}) = \pm 50 A^{(4)}(\alpha_s), \quad (10.18)$$

as an estimate of the theoretical error on $\delta^{(0)}$. Note that, for the relevant values of α_s , this is of the same size as $K_3 A^{(3)}(\alpha_s)$; thus, this error estimate is conservative enough to apply¹⁵⁸ in the worst possible scenario, where the onset of the asymptotic behaviour of the perturbative series were already reached for $n = 3, 4$.

There have been attempts^{173,174,175,176,177} to improve the perturbative prediction by performing an all-order summation a certain class of higher-order corrections (the so-called ultraviolet renormalon chains). This can be accomplished using exact large- n_f results and applying the *naive non-abelianization* prescription.¹⁷⁸ Unfortunately, the naive resummation turns out to be renormalization-scheme dependent beyond one loop.^{174,180} More recently, a renormalization-scheme-invariant summation has been presented.¹⁸¹ The final effect of the higher-order corrections (beyond K_4) turns out to be small.

10.2. Power corrections

The leading quark-mass corrections $\delta_{ij}^{(2)}$ are known^{155,182} to order α_s^2 . They are certainly tiny for the up and down quarks ($\delta_{ud}^{(2)} \sim -0.08\%$), but the correction from the strange quark mass is important for strange decays ($\delta_{us}^{(2)} \approx -20\%$). Nevertheless, because of the $\sin^2 \theta_C$ suppression, the effect on the total ratio R_τ is only $-(1.0 \pm 0.2)\%$.

The leading non-perturbative contributions can be shown to be suppressed by six powers of the τ mass,^{152,153,154,155} and are therefore very small. This fortunate fact is due to the phase-space factors in (10.8); their form is such that the leading $1/s^2$ corrections to $\Pi^{(1)}(s)$ do not survive the integration along the circle. Moreover, there is a large cancellation between the vector and axial-vector $D = 6$ contributions to the total hadronic width (the $D = 6$ operator with the largest Wilson coefficient contributes with opposite signs to the vector and axial-vector correlators, due to the γ_5 flip). Thus, the non-perturbative corrections to R_τ are smaller than the corresponding contributions to $R_{\tau,V/A}$.

The numerical size of the non-perturbative corrections can be determined from the invariant-mass distribution of the final hadrons in τ decay.¹³² Although the distributions themselves cannot be predicted at present, certain weighted integrals of the hadronic spectral functions can be calculated in the same way as R_τ . The analyticity properties of $\Pi_{ij,V/A}^{(J)}$ imply^{132,183}:

$$\int_0^{s_0} ds W(s) \text{Im} \Pi_{ij,V/A}^{(J)} = \frac{i}{2} \oint_{|s|=s_0} ds W(s) \Pi_{ij,V/A}^{(J)}, \quad (10.19)$$

with $W(s)$ an arbitrary weight function without singularities in the region $|s| \leq s_0$. Generally speaking, the accuracy of the theoretical predictions can be much worse than the one of R_τ , because non-perturbative effects are not necessarily suppressed. In fact, choosing an appropriate weight function, non-perturbative effects can even be made to dominate the final result. But this is precisely what makes these integrals interesting: they can be used to measure the parameters characterizing the non-perturbative dynamics.

To perform an experimental analysis, it is convenient to use moments of the directly measured invariant-mass distribution¹⁸³ ($k, l \geq 0$)

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}. \quad (10.20)$$

The factor $(1-s/s_0)^k$ supplements $(1-s/m_\tau^2)^2$ for $s_0 \neq m_\tau^2$, in order to squeeze the integrand at the crossing of the positive real axis and, therefore, improves the reliability of the OPE analysis; moreover, for $s_0 = m_\tau^2$ it reduces the contribution from the tail of the distribution,

which is badly defined experimentally. A combined fit of different $R_\tau^{kl}(s_0)$ moments results in experimental values for $\alpha_s(m_\tau^2)$ and for the coefficients of the inverse power corrections in the OPE. $R_\tau^{00}(m_\tau^2) = R_\tau$ uses the overall normalization of the hadronic distribution, while the ratios $D_\tau^{kl}(m_\tau^2) = R_\tau^{kl}(m_\tau^2)/R_\tau$ are based on the shape of the s distribution and are more dependent on non-perturbative effects.¹⁸³

The predicted suppression^{152,153,154,155} of the non-perturbative corrections has been confirmed by ALEPH^{162,184,185} and CLEO,¹⁸⁶ using the moments (0,0), (1,0), (1,1), (1,2) and (1,3). The most recent ALEPH analysis¹⁸⁷ gives:

$$\delta_{\text{NP}} \equiv \sum_{D \geq 4} \left(\cos^2 \theta_C \delta_{ud}^{(D)} + \sin^2 \theta_C \delta_{us}^{(D)} \right) = -(0.02 \pm 0.5)\%, \quad (10.21)$$

in agreement with previous estimates.¹⁵⁵

10.3. Phenomenology

The QCD prediction for R_τ is then completely dominated by the perturbative contribution $\delta^{(0)}$; non-perturbative effects being of the order of the perturbative uncertainties from uncalculated higher-order corrections.^{158,159,160,161} Furthermore, as shown in Table 14, the result turns out to be very sensitive to the value of $\alpha_s(m_\tau^2)$, allowing for an accurate determination of the fundamental QCD coupling.

The experimental value for R_τ can be obtained from the leptonic branching fractions or from the τ lifetime. The average of those determinations¹³

$$R_\tau = 3.649 \pm 0.014, \quad (10.22)$$

corresponds to

$$\alpha_s(m_\tau^2) = 0.35 \pm 0.02. \quad (10.23)$$

Once the running coupling constant $\alpha_s(s)$ is determined at the scale m_τ , it can be evolved to higher energies using the renormalization group. The size of its error bar scales roughly as α_s^2 , and it therefore shrinks as the scale increases. Thus a modest precision in the determination of α_s at low energies results in a very high precision in the coupling constant at high energies. After evolution up to the scale M_Z , the strong coupling constant in (10.23) decreases to

$$\alpha_s(M_Z^2) = 0.1217 \pm 0.0025, \quad (10.24)$$

in excellent agreement with the direct measurement^{188,189} at $\mu = M_Z$, $\alpha_s(M_Z^2) = 0.121 \pm 0.003$, and with a similar error bar. The comparison of these two determinations of α_s in two extreme energy regimes, m_τ and M_Z , provides a beautiful test of the predicted running of the QCD coupling.

With $\alpha_s(m_\tau^2)$ fixed to the value in Eq. (10.23), the same theoretical framework gives definite predictions^{155,158} for the semi-inclusive τ decay widths $R_{\tau,V}$, $R_{\tau,A}$ and $R_{\tau,S}$, in good agreement with the experimental measurements.^{162,190} The separate analysis of the vector and axial-vector contributions allows to investigate the associated non-perturbative corrections. Figure 9 shows¹⁶² the (preliminary) constraints on $\delta_V^{(6)}$ and $\delta_A^{(6)}$ obtained from the

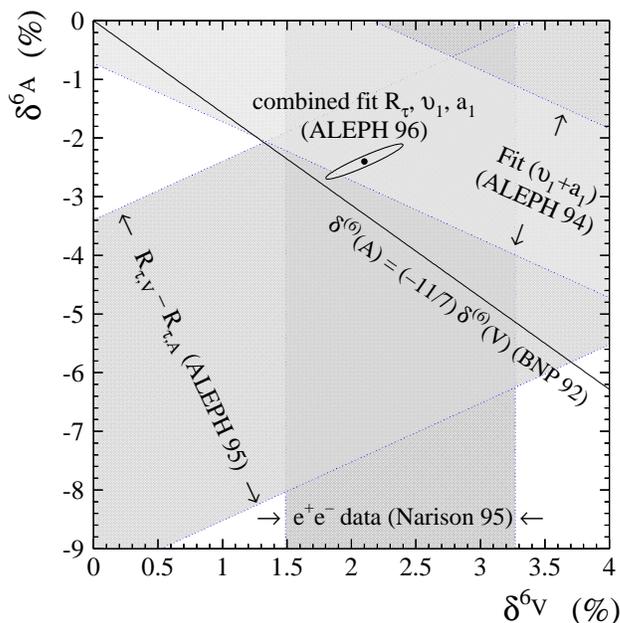


Figure 9: Constraints on $\delta_V^{(6)}$ and $\delta_A^{(6)}$ obtained from ALEPH data.¹⁶² The ellipse depicts the combined fit. All results are still preliminary. (Taken from Ref. 162).

most recent ALEPH analyses.^{184,162,191} A clear improvement over previous phenomenological determinations^{155,192} is apparent.

The Cabibbo-suppressed width $R_{\tau,S}$ is very sensitive to the value of the strange quark mass,¹⁵⁵ providing a direct and clean way of measuring m_s . A very preliminary value, $m_s(m_\tau^2) = (212_{-35}^{+30+1})$ MeV, has been already presented at the last τ workshop.¹⁹⁰

Using the measured invariant-mass distribution of the final hadrons, it is possible to evaluate the integral $R_\tau^{00}(s_0)$, with an arbitrary upper limit of integration $s_0 \leq m_\tau^2$. The experimental s_0 dependence agrees well with the theoretical predictions¹⁸³ up to rather low values¹⁹³ of s_0 (> 0.7 GeV²). Equivalently, from the measured^{185,186} $R_\tau^{00}(s_0)$ distribution one obtains $\alpha_s(s_0)$ as a function of the scale s_0 . As shown¹⁹³ in Figure 10, the result exhibits an impressive agreement with the running predicted at three-loop order by QCD. It is important to realize¹⁹³ that the theoretical prediction for $R_\tau^{00}(s_0)$ does not contain inverse powers of s_0 (as long as the s -dependence of the Wilson coefficients is ignored). The power corrections are suppressed by powers of $1/m_\tau^2$; thus, they do not drive a break-down of the OPE. This could explain the surprisingly good agreement with the data for $s_0 \lesssim 1$ GeV².

A similar test was performed before¹³⁰ for $R_{\tau,V}$, using the vector spectral function measured in $e^+e^- \rightarrow$ hadrons, and varying the value of the τ mass. This allows to study the behaviour of the OPE at lower scales. The theoretical predictions for $R_{\tau,V}$ as function of m_τ^2 agree¹³⁰ well with the data for $m_\tau > 1.2$ GeV. Below this value, higher-order inverse power corrections become very important and eventually generate the expected break-down of the expansion in powers of $1/m_\tau^2$.

The theoretical analysis of the τ hadronic width has reached a very mature level. Many different sources of possible perturbative and non-perturbative contributions have been an-

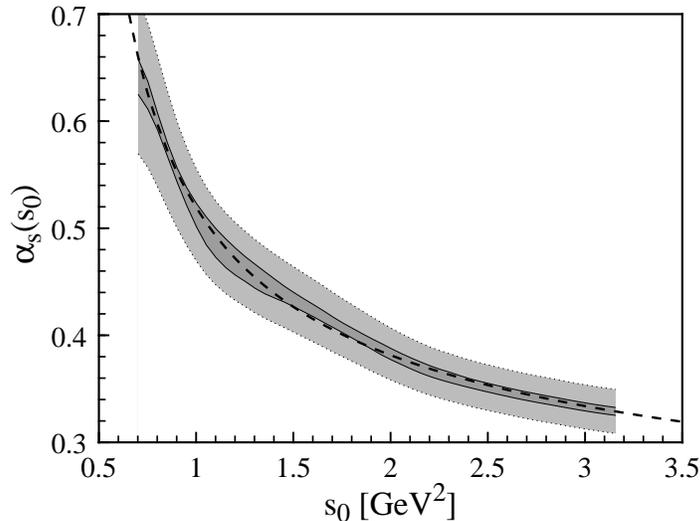


Figure 10: Values of $\alpha_s(s_0)$ extracted¹⁹³ from the $R_\tau^{00}(s_0)$ data.^{185,186} The dashed line shows the three-loop QCD prediction for the running coupling constant. (Taken from Ref. 193).

alyzed. A very detailed study of the associated uncertainties has been given in Ref. 158. The comparison of the theoretical predictions with the experimental data shows a successful and consistent picture. The resulting $\alpha_s(m_\tau^2)$ determination is in excellent agreement with (and more precise than) the measurements at the M_Z scale, providing clear evidence of the running of α_s . The QCD predictions are further confirmed by analyses of the semi-inclusive components of the τ hadronic width, $R_{\tau,V}$, $R_{\tau,A}$ and $R_{\tau,S}$, and the invariant-mass distribution of the final decay products.

In addition to provide beautiful tests of perturbative QCD, the hadronic spectral functions measured in τ decay contain valuable dynamical information on non-perturbative aspects of the strong interactions¹³² which could greatly enhance our present understanding of these phenomena. For instance, $R_{\tau,V} - R_{\tau,A}$ is a pure non-perturbative quantity; basic QCD properties force the associated invariant-mass distribution to obey a series of chiral sum rules,^{132,194,195,196,197,198,199} which have been recently tested with τ data.¹⁶² The measurement of the vector spectral function²⁰⁰ $\text{Im}\Pi_V(s)$ has also been used²⁰¹ to reduce the present uncertainties in fundamental QED quantities such as $\alpha(M_Z)$ and $(g_\mu^\gamma - 2)$.

11. SUMMARY

The flavour structure of the Standard Model is one of the main pending questions in our understanding of weak interactions. Although we do not know the reason of the observed family replication, we have learned experimentally that the number of Standard Model fermion generations is just three (and no more). Therefore, we must study as precisely as possible the few existing flavours to get some hints on the dynamics responsible for their observed structure.

The τ turns out to be an ideal laboratory to test the Standard Model. It is a lepton, which means clean physics, and moreover it is heavy enough to produce a large variety of

decay modes. Naïvely, one would expect the τ to be much more sensitive than the e or the μ to new physics related to the flavour and mass-generation problems.

QCD studies can also benefit a lot from the existence of this heavy lepton, able to decay into hadrons. Owing to their semileptonic character, the hadronic τ decays provide a powerful tool to investigate the low-energy effects of the strong interactions in rather simple conditions.

Our knowledge of the τ properties has been considerably improved during the last few years. Lepton universality has been tested to rather good accuracy, both in the charged and neutral current sectors. The Lorentz structure of the leptonic τ decays is certainly not determined, but begins to be experimentally explored. The quality of the hadronic τ decay data has made possible to perform quantitative QCD tests and determine the strong coupling constant very accurately. Searches for non-standard phenomena have been pushed to the limits that the existing data samples allow to investigate.

At present, all experimental results on the τ lepton are consistent with the Standard Model. There is, however, large room for improvements. Future τ experiments will probe the Standard Model to a much deeper level of sensitivity and will explore the frontier of its possible extensions.

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