Origin of Zenneck-like waves excited by optical nanoantennas in non-plasmonic transition metals: supplement

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The scattering properties of metallic optical antennas are typically examined through the lens of their plasmonic resonances. However, non-plasmonic transition metals also sustain surface waves in the visible. We experimentally investigate in this work the far-field diffraction properties of apertured optical antennas milled on non-plasmonic W films and compare the results with plasmonic references in Ag and Au. The polarization-dependent diffraction patterns and the leakage signal emerging from apertured antennas in both kinds of metals are recorded and analyzed. This thorough comparison with surface plasmon waves reveals that surface waves are launched on W and that they have the common abilities to confine the visible light at metal-dielectric interfaces offering the possibility to tailor the far-field emission. The results have been analyzed through theoretical models accounting for the propagation of a long range surface mode launched by subwavelength apertures, that is scattered in free space by the antenna. This surface mode on W can be qualitatively described as an analogy in the visible of the Zenneck wave in the radio regime. The nature of the new surface waves have been elucidated from a careful analysis of the asymptotic expansion of the electromagnetic propagators, which provides a convenient representation for explaining the Zenneck-like character of the excited waves and opens new ways to fundamental studies of surface waves at the nanoscale beyond plasmonics.

1. EXPERIMENTAL DIFFRACTION PATTERNS AND TRANSMISSION SPECTRA

A. Diffraction regimes of single holes

Figure S1 shows the patterns of single holes in W film illuminated by the s- and p-polarized light, respectively. It is interesting to see that the diffraction patterns of SHs in W film show the similar three different regimes when the SH diameter $d < 2 \lambda$ as for the situation of SH in Ag film [1], here, $\lambda$ is the incident wavelength. In particular, $I_S(\theta) > I_P(\theta)$ for the SH with the diameter $d = 1.1 \mu m \ (\lambda < d < 2 \lambda)$, $I_S(\theta) = I_P(\theta)$ for the hole $d = 0.6 \mu m \ (d \sim \lambda)$ and $I_S(\theta) < I_P(\theta)$ for the hole $d = 0.2 \mu m \ (d < \lambda)$. The normalized diffraction patterns of SHs in W and Ag films do not show obvious difference and display the same polarization dependence.

Fig. S1. Polarized diffraction patterns of single holes of various diameters $d$ milled through a W film with illumination under s- and p-polarization, (a) $d = 1.1 \mu m$, (b) $d = 0.6 \mu m$, (c) $d = 0.2 \mu m$, respectively. The diffraction patterns for the same hole diameters through Ag film are also given.

B. Fano model for the single slit

The experimental data in k-space for the single slit shown in Fig. 2(d) of the main text can be reproduced with a fairly good agreement by the following Fano-like analytical expression for the

\[
I(\theta) = \frac{I_0}{1 + \left(\frac{\theta}{\theta_f}\right)^2} \cos^2(\theta)
\]
intensity in reciprocal space [2]:

\[ I(k) \approx t_0 + A e^{i\Phi} \frac{k_{SW}''}{(k - k_{SW}') + ik_{SW}''} \]

with \( t_0 \) the angular distribution of the direct transmission through the slit, and \( A \) a free parameter.

C. Enhanced transmission and visible light beaming W from bull’s eyes

Bull’s eyes (BEs) are milled in a W film with the periodical concentric grooves. When they are illuminated by white light with the pattern on the input side, the spectra show enhanced transmission. Figure S2(a) shows the transmission spectra for different groove periodicities \( p \). The resonant peaks are enhanced compared to that of the corresponding single hole (SH), and resonant wavelength is slightly smaller than the periodicity \( P \) of bull’s eye. Figure S2(b) show the polarized diffraction patterns from the SH and BE in W film. The bull’s eye is chosen with the resonant transmission peak at the incident wavelength. One can see that the BE show enhanced beaming for both the s- and p-polarization. Then enhancement factor is 6 with respect to that of SHs, and the angular distribution is as narrow as 5 degree for the p-polarization.

2. SCATTERING CROSS SECTION

A. Single hole

Under the fundamental model approximation, the scattering cross section of a single hole of radius \( R \) reads [1],

\[ \sigma_{0h}(\theta, \phi) = \sigma_0 \left[ I_{SH}(\theta, p) \cos^2(\phi - \phi_0) + I_{SH}(\theta, s) \sin^2(\phi - \phi_0) \right], \]

where

\[ I_{SH}(\theta, p) = \frac{|1 + z_s|^2 \cos^2 \theta \left[ k_\lambda R \sin \theta \right]^2}{|\cos \theta + z_s|^2} \left( 1 - k_\lambda^2 R^2 \frac{\sin^2 \theta}{\nu_p^2} \right)^2, \]

\[ I_{SH}(\theta, s) = \frac{|1 + z_s|^2 \cos^2 \theta \left[ k_\lambda R \sin \theta \right]^2}{1 + z_s \cos \theta} \left( 1 - k_\lambda^2 R^2 \frac{\sin^2 \theta}{\nu_s^2} \right)^2, \]

are the normalized angular distribution for p and s polarization of the radiative modes, respectively. The first term in both \( I_{SH}(\theta, p) \) and \( I_{SH}(\theta, s) \) gives the normalized intensity of an
infinitesimal hole for each polarization, while the second term deals with the geometry of the hole. The normalization factor,

$$ c_0 = \frac{k_\lambda^2 R^2}{4\pi (u_0^2 - 1)} \frac{|E'_{11}|^2}{|1 + |z_0|^2|}, $$

controls the total transmittance but does not affect the radiation pattern. $|E'_{11}|$ is the amplitude of the fundamental TE$_{11}$ at the hole opening. The metal properties are characterized by the surface impedance $z_0 = 1/\sqrt{\varepsilon_m}$ where $\varepsilon_m$ is the dielectric constant of the metal. This expression for $z_0$ is only valid under the approximation of surface impedance boundary conditions at the horizontal metal/air interface [3]. Notice that $u_0 \approx 1.84$ is the first root of the Bessel function $I_1(u_0) = 0$. A further discussion of the physical meaning of these terms can be found in Ref. [1].

The scattering cross section for a perfect electric conductor (PEC) is obtained when the metal impedance tends to zero,

$$ I_{PEC}^{SH}(\theta, p) = \frac{4k_\lambda^2 (k_\lambda R \sin \theta)}{k_\lambda^2 R^2 \sin^2 \theta}, $$

$$ I_{PEC}^{SH}(\theta, s) = \frac{4k_\lambda^2 (k_\lambda R \sin \theta)}{\left(1 - \frac{k_\lambda^2 R^2 \sin^2 \theta}{u_0^2}\right)^2} \cos^2 \theta, $$

B. Concentric hole and groove

The theoretical description of the angular radiation pattern can be performed in the framework of the coupled mode method (CMM) [3, 4] for this approach provides a simple analytical expression for the scattering cross section $\sigma(\theta, \phi)$ of the hole-groove system. Within this approach, we first compute the amplitude of the electric field at the output side of both hole ($E'_h$) and groove ($E'_g$) and, having them, the EM fields in the far field region are calculated as a second step using the angular spectrum representation [5]. Finally, the scattering cross section along the direction determined by the polar angle ($\theta$) and the azimuthal angle ($\phi$) can be straightforwardly computed from its definition as the power flux per solid angle, $\sigma(\theta, \phi) = dP(\theta, \phi)/d\Omega$. The general procedure is outlined in Ref. [1]. $\sigma(\theta, \phi)$ can be reduced to a simple analytical expression assuming that only the fundamental TE$_{11}$ mode is excited in both hole and groove, which is a good approximation for subwavelength defects. Using the fundamental mode approximation, $\sigma(\theta, \phi)$ yields

$$ \sigma(\theta, \phi) = c_0 \left[ I(\theta, p) \cos^2(\phi - \phi_0) + I(\theta, s) \sin^2(\phi - \phi_0) \right], $$

where the normalized angular distributions for both p and s polarization of the radiative modes,

$$ I(\theta, p) = I_{SH}(\theta, p) I_{SH}(\theta, p), $$

$$ I(\theta, s) = I_{SH}(\theta, s) I_{SH}(\theta, s), $$

are factorized into two terms, one that accounts for the power that is diffracted by the hole ($I_{SH}(\theta, \delta)$, with $\delta = p, s$) and other that results from the interference of the radiation coming form hole and groove ($I_{HG}(\theta, \delta)$). For the sake of convenience, the field amplitudes at the output side of both hole and groove are expressed as a function of the amplitude of a single (isolated) hole $E'_0$. The diffractive terms $I_{SH}(\theta, p)$ and $I_{SH}(\theta, s)$ for the central hole of radius $R$ are the quantities plotted in Fig. 1 of the main text. $c_0$, $I_{SH}(\theta, p)$ and $I_{SH}(\theta, s)$ are given in Sec. A. The interference terms,

$$ I_{HG}(\theta, \delta) = \frac{I(\theta, \delta)}{I_{SH}(\theta, \delta)} = \left| \frac{E'_0}{E'_h} + \frac{E'_0}{E'_g} X_{SH}(\theta) \right|^2, \quad \text{(S2)} $$

with $\delta = p, s$, account for the normalized radiation patterns measured experimentally. The $X_{SH}$ function in Eq. (S2) is equal to the overlap of the mode in either the hole ($i = h$) or the groove ($i = g$) with the $\delta$-polarized radiation modes. Explicit expression of these functions are given in
Sec. C. The field amplitudes appearing in Eq. (S2),
\[
E'_h^{(g)} = G_{gg} - \Sigma_g - (G_{hh} - \Sigma_h)^2 - G_{hh}^{-1} (G_{hh} - \Sigma_h)G_{gg}^2,
\]
\[
E'_g^{(h)} = G_{hh} - \Sigma_h - (G_{hh} - \Sigma_h)^2 - G_{hh}^{-1} (G_{hh} - \Sigma_h)G_{gg}^2,
\]
are solution of the linear system of equations described in Sec. D. These quantities are independent of the radiated field and are function of geometrical and material parameters only. Their constitutive quantities account for the different scattering mechanisms. In fact, \( G_{gg} \) represents the coupling between the EM fields at the two sides of the hole, \( \Sigma_g \) takes account for the bouncing back and forth of the fields inside the hole, \( \Sigma_h \) is the equivalent expression for the groove,
\[
G_{ij} = \pi \sum_{l=p,s} \int_0^{\infty} \frac{Y_l k_l n_l}{1 + Y_l z_x} k_s dk_s,
\]
is the propagator, for which i, j = h, g, \( Y_3 \) is the admittance of the \( \delta \)-polarized radiation modes, \( z_x = 1/\sqrt{\epsilon_m} \) is the metal impedance under the approximation of surface impedance boundary conditions at the horizontal metal/air interface [3], and \( \epsilon_m \) is the dielectric constant of the metal. Explicit expression of these functions are reported in Sec. D. The propagator \( G_{hh} \), represents the reillumination of the groove by the hole \( (G_{hh} = G_{gh}) \). The EM radiation interchanged by hole and groove propagates along the metal surface. Notice that Eq. (S5) results from the projection of the real-space propagator into plane-wave modes (k-space) [4]. The modal amplitudes are also a function of self-interaction terms (due to the self-illumination) of both hole \( (G_{hh}) \) and groove \( (G_{gg}) \).

C. Overlap functions
The overlap functions \( X_{ik} \) of \( \delta = p, s \) radiation modes with the TE_{11} mode in either a hole \( (i = h) \) of radius \( R \) or a groove \( (i = g) \) with inner (outer) radius \( r_h \) \((r_g)\) read,
\[
X_{ph}(k) = \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{u^2 - 1}} f_1(k R),
\]
\[
X_{sh}(k) = R u_h^2 \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{u^2 - 1}} f_1(k R),
\]
\[
X_{pg}(k) = Z_1(u_g) f_1(k r_h) - Z_1(u_g) f_1(k r_g),
\]
\[
X_{gh}(k) = R u_g^2 \frac{c Z_1(u_g) f_1'(k r_g) - Z_1(u_g) f_1'(k r_g)}{u_g^2 - R^2},
\]
where \( f_1(x) \) is the first order Bessel function, \( f_1'(x) \) its derivative, \( k_{ij} = k \sin \theta_i \) and \( c = r_h/r_g \).
The approximate value \( u_g \approx 2/(c + 1) \) [6] for the solution of the secular equation for coaxial waveguide modes is used in the calculations.

D. Modal amplitudes
The modal amplitudes are solution of the linear system of equations [3]
\[
(G_{hh} - \Sigma_h) E_h - G_{eh} E_h' = I_h,
\]
\[
(G_{hh} - \Sigma_h) E_h - G_{eh} E_h' = 0,
\]
\[
(G_{gg} - \Sigma_g) E_g + G_{gh} E_g' = 0,
\]
assuming that only the fundamental \( a = TE_{11} \) mode is excited on each defect. The solution of this linear system of equations is given by Eqs. (S3,S4) of the main text. It can be compared with the solution of the simple system of two identical holes [7, 8]. Functions appearing in such expressions are the illumination term \( I = 2 Y_p X_{ph}(\theta_0) / (1 + z_x Y_p) \),
\[
G_{i} = 2 Y_x \left[ f_\alpha^+ 2 \Phi - f_\alpha^- 2 / \Phi \right]^{-1},
\]
which represents the coupling between the EM fields at the two sides of the hole,
\[ \Sigma_h = G_{iG} \left[ f^+ \Phi - f^- / \Phi \right] / 2, \]
that takes account for the bouncing back and forth of the fields inside the hole,
\[ \Sigma_g = Y_{iG} \left[ f^+ \Phi - f^- / \Phi \right] / \left[ f^+ f^- (\Phi - \Phi^{-1}) \right] \]
is the equivalent expression for the groove, and the propagators \( G_{iG} \) that is defined by Eq. (S5),
where \( i, j = h \) (hole), \( g \) (groove). In the above expressions, \( f^\pm = 1 \pm Y_s z \), with \( \delta = p, s, a \), \( Y_p = c k_s / k_a \) is the admittance of the \( p \) (s) plane-wave modes at either side of the metal and \( Y_s \) is the admittance of the waveguide mode in either the hole or the groove, \( \Phi = \exp(i n_{eff} k h) \), \( n_{eff} \) its effective index of the waveguide mode, \( k_\lambda = 2\pi / \lambda \) is the vacuum wavenumber, and \( h \) is the metal thickness.

### 3. Asymptotic Expansion of the Hole-Groove Propagator

This section briefly describes the asymptotic expansion of the integral for the hole-groove propagator \( G_{hg} \) following the steepest descent method of Refs. [9, 10]. The general framework is described in [11]. The propagator can be written as the difference of two integrals, \( G_{hg} = I(r_h) - I(r_a) \), where

\[
I(r) = \int_{-\infty}^{\infty} F(qk_\lambda r) e^{iqk_\lambda r} dq,
\]

\[
F(x) = e^{-3i\pi/4} k_s X_{pq}(qk_\lambda R)(r_s^2 - 1) x^{-1/2},
\]

where \( R \) is the central hole radius, and \( r_a \) and \( r_b \) are the inner and outer radii of the groove, respectively. The argument of (S6) is analytically prolonged to the complex plane. The integration contour \( C \) corresponds to the real axis in the complex \( q \)-plane. For the sake of convenience, the integration variable is transformed to polar coordinates, \( q = \sin \phi \). The integral (S6) has a pole at \( q_{\phi} = \cos^{-1}(-z_p^2) \), which is located near the saddle point at \( q_{\phi} = \pi / 2 \). The saddle point is obtained from the condition \( dq\phi / dq = 0 \). The imaginary part of \( q \) is \( q_{\phi} = q_{\phi} + i q_{\phi}'' \) is constant along the steepest descent path (SDP), \( q_{\phi}'(\phi) = q_{\phi}'(q_{\phi}) = i \), i.e. \( \cos(\phi' - \pi / 2) \cos(\phi'' = 1 \), where \( \phi = \phi' + i \phi'' \). We introduce the new variable \( s = \sqrt{1 - \sin(\phi')} \). The saddle point is at \( s = 0 \), while the pole is at \( s_p = s_{(p)} \). Values of \( q_{\phi}' \) and \( s_p \) are given in Table S1 for both Ag and W. The new SDP runs along the real \( s \) axis (\( s'' = 0 \)). The transformed integral reads,

\[
I = e^{ik_{\lambda} r} \int_{-\infty}^{\infty} \Phi(s)e^{-k_{\lambda} r^2} ds,
\]

where \( \Phi(s) = F(q(s)k_{\lambda} r) \cos(q(s) dq / ds). \)

When the deformed path crosses the pole, as for Ag, its residue, \( I_p = 2\pi C_{pq} e^{i q_{\phi} k_{\lambda} r} \), should be added to the resulting integral, where \( C_{pq} = -F(q_{p} k_{\lambda} r) / \tan \theta_p \) and \( \tan \theta_p = \text{Im}(q_{p}) / \text{Re}(q_{p}) \). By contrast, \( I_p = 0 \) for W.

The integral along the SDP can be formally separated into resonant, \( C_{pq} (s - s_p)^{-1} \) and non-resonant, \( \Phi_0 \equiv \Phi - C_{pq} (s - s_p)^{-1} \), terms. Writing \( (s - s_p)^{-1} = s(s^2 - s_p^2)^{-1} + s_p(s^2 - s_p^2)^{-1} \), only the even function contributes to the integral of the resonant term for the integral with the odd argument vanishes. If \( \text{Re}(s_p^2) < 0 \),

\[
I_{\text{res}_{sp}} = s_p C_{pq} e^{i q_{\phi} k_{\lambda} r} \int_{-\infty}^{\infty} ds \int_{k_{\lambda} r}^{\infty} dze^{-z(s^2 - s_p^2)} = \sqrt{\pi} s_p C_{pq} e^{i q_{\phi} k_{\lambda} r} \int_{k_{\lambda} r}^{\infty} e^{-z^2} dze^{-z(s^2 - s_p^2)} = \sqrt{\pi} s_p C_{pq} e^{i q_{\phi} k_{\lambda} r} \int_{k_{\lambda} r}^{\infty} e^{-z^2} dze^{-z(s^2 - s_p^2)}.
\]

Making the change of variable \( x^2 = -zs_p^2 \), with \( x = \mp is_p \sqrt{z} \), the integral may be written as

\[
I_{\text{res}_{sp}} = \pm i \pi C_{pq} e^{i q_{\phi} k_{\lambda} r} \text{erfc}(z \mp is_p \sqrt{k_{\lambda} r}),
\]

in terms of the complementary error function, \( \text{erfc}(z) \equiv 2\pi^{-1/2} \int_0^{\infty} e^{-t^2} dt \). The real part of its argument must be positive to guarantee that \( \lim_{k_{\lambda} r \rightarrow \infty} \text{erfc}(z \mp is_p \sqrt{k_{\lambda} r}) = 0 \).

The non-singular term is expanded in Taylor series near the saddle point \( s = 0 \). The integral vanishes for odd power of \( s \). The second order approximation,

\[
e^{ik_{\lambda} r} \int_{-\infty}^{\infty} \Phi_0(s)e^{-k_{\lambda} r^2} ds = e^{ik_{\lambda} r} \int_{-\infty}^{\infty} \frac{1}{k_{\lambda} r} \left[ \Phi_0(k_{\lambda} r) + \frac{1}{4k_{\lambda} r} \frac{d^2 \Phi_0(k_{\lambda} r)}{ds^2} \right],
\]
contributes with two additional terms to the total integral,
\[ I_N = -\frac{\sqrt{2\pi F(k\lambda r)}}{z'^2} e^{i(k\lambda r-3\pi/4)}, \]
\[ I^{NR}_{sdp} = C_p\sqrt{\pi}\frac{e^{ik\lambda r}}{s_p(k\lambda r)^{1/2}} + C_p\sqrt{\pi}\frac{e^{ik\lambda r}}{2s_p(k\lambda r)^{3/2}}. \]

The final resulting integral is
\[ I = I_p + I^{res}_{sdp} + I_N + I^{NR}_{sdp}. \]  

The integral along the steepest descent path, \( I_{sdp} = I^{res}_{sdp} + I^{NR}_{sdp} \), decays faster than both \( I_p \) and \( I_N \) for large values of the distance. In fact, \( I_{sdp} \) vanishes for asymptotically large values of \( r \), as we can deduce from the series expansion of \( \text{erfc}(z) \) [12],
\[ I^{res}_{sdp} \approx -C_p\sqrt{\pi}\frac{e^{ik\lambda r}}{s_p} \left[ (k\lambda r)^{-1/2} + s_p^{-2}(k\lambda r)^{-3/2}/2 + O((k\lambda r)^{-5/2}) \right]. \]

Thus, \( I \approx I_p + I_N \) at large distances. SSP and Zenneck waves decay exponentially with distance \( \sim C_p(r)e^{-r/L_{sw}} \), where \( L_{sw} = (k_1\text{Im}(q_p))^{-1} \) is the wave propagation length), while Norton waves are algebraical functions \( \sim F(k\lambda r)(k\lambda r)^{-3/2} \). Therefore, the pole contribution dominates at moderate distances, while Norton waves are dominant at very large distances. Near the crossover \( (I_p = I_N) \) we have a strong interference of both kind of waves.

The Zenneck pole of \( W \) is not crossed when the initial integration path is deformed into the SDP for this pole is in the lower Riemann sheet of the complex plane, so that we have that \( I_p \) does not contribute to \( I_N \) in this case (the pole is improper [13]). The other two expansion terms, \( I_N \) and \( I^{NR}_{sdp} \), remain the same. Interestingly, the asymptotic representation for \( \text{Ag} \) can still be applied to \( W \) thanks to the well-known property of the complementary error function, \( \text{erfc}(z) = 2 - \text{erfc}(z) \), that transforms \( I^{res}_{sdp}(-is_p\sqrt{k_1T}) \) into \( I_p + I_{sdp}^{res}(is_p\sqrt{k_1T}) \) [10, 11]. Both expressions are totally equivalent and can be used indistinctly for either \( \text{Ag} \) or \( W \).

Figures S3(a) and (b) show the three asymptotic terms for \( \text{Ag} \) and \( W \), respectively, as a function of the hole-groove distance \( a_1 \). We observed that Zenneck waves in \( W \) decays 10 times faster than SSP waves in \( \text{Ag} \) in accordance with the respective values of \( L_{sw} \), see Table S1. However, \( I_N \) is of the same order of magnitude for both metals, see inset of Fig. S3(b). The crossover distance is \( a_1 = 9.6L_{sw} \) for \( \text{Ag} \) (in agreement with [9]) and \( a_1 = 4.1L_{sw} \) for \( W \). \( I_N \) is the main contribution to the asymptotic expansion after the crossover for both metals. Below the crossover distance, SSP waves in \( \text{Ag} \) accurately describe the asymptotic expansion \( I \approx I_{sw} \), in remarkable contrast to Zenneck waves in \( W \) that largely differs from the total propagator at both small and large values of \( a_1 \).

The main text shows that QC waves, \( G_{QC} \), represent a small contribution to the total hole-groove propagator \( G_{hp} \) of \( \text{Ag} \) at moderate distances, which is accurately described by \( G_p \). However, when the standard representation (S7) is employed to \( W \), \( G_{hp} \) results from the interference of Zenneck and QC waves having both a large contribution to the total propagator, see Fig. S3(c). Moreover, \( G_p \) and \( G_{QC} \) have opposite phases. We observe in the inset of Fig. S3(c) that the total contribution of the two non-resonant terms, \( G^{NR} = G_N + G^{NR}_{sdp} \), is practically the same for both metals. The two curves are indistinguishable. Therefore, differences come from the resonant terms \( G_p \) and \( G_{hp}^{res} \).

The alternative representation presented in the main text overcomes the issues found in Figs. S3(b) and (c). We show in the main text that \( G_p/2 \) is a good approximation of the full propagator for small values of \( a_1 \), while \( G_{QC}^{res} \) represents a small contribution of \( G_{hp}^{res} \). Figure S3(d) shows that \( G_N \) remains the main contribution for large values of \( a_1 \) when the new representation is used.
Asymptotics
Full
SPP
Norton
SDP

(a)

(b)

Asymptotics
Full
Zenneck
Norton
SDP

(c)

(d)

Fig. S3. (Color online). Log plot of the asymptotic values of $|G_{\text{hg}}|$ vs. $a_1$ for (a) Ag and (b) W. The full expansion of $|G_{\text{hg}}|$ (S7) is compared with SPP or Zenneck waves, Norton waves, and SDP terms. Inset: $|G_N|$ for W and Ag. (c) Exact $\text{Re}[G_{\text{hg}}]$ for W vs. full asymptotic expansion (AE, Eq. S7), pole ($G_p$), and QC waves ($G_{\text{QC}}$) terms. Inset: non-resonant term $G_{\text{NR}}$ for W and Ag. (d) $|G_{\text{hg}}|$ of W for the alternative expansion (main text Eq. 8). $a_1$ is normalized by either $\lambda$ or $L_{sw}$ (Table S1).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$\Lambda_X$</th>
<th>W</th>
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<td>$\epsilon_m$</td>
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<td>4.7 +25.8 i</td>
</tr>
<tr>
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<td>0.150 -0.125 i</td>
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<td>skin depth (nm)</td>
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Table S1. Material parameters at $\lambda = 660$ nm.

A. Quasi-cylindrical waves

It is convenient to recall that, in the opposite limit of very small distances, the propagator has been found to behave as if the metal were a perfect electric conductor (PEC) [14]. The PEC limit is obtained for a vanishing small metal impedance, i.e. by taking $z_s \to 0$ in Eq. (S5). In this case, all $q$ values contribute to the propagator, and not a single point as for Norton waves. A different point of view to describe the same behavior is that quasi-cylindrical (QC) waves are excited at the metal surface. QC waves (initially coined as creeping waves) were defined by Lalanne and Hugonin as the difference between the exact field and pole contribution, $G_{\text{QC}} = G_{\text{hg}} - G_p$ [15]. These authors pointed out that the QC waves oscillate with the free-space wavelength (as in a PEC) decaying faster than the pole contribution at distances of the order of a few wavelengths.

There is not a simple mathematical representation available for the QC wave. Nevertheless, in order to evaluate the contribution of QC waves, we can take advantage of the high numerical accuracy of the asymptotic representation beyond its range of validity (at large distances) to
compute the propagator at distances that are even a small fraction of the wavelength, provided that enough terms are used in the expansion [10]. This is, in fact, a well-known mathematical property of the asymptotic expansion exploited in several fields of physics. Figures 3(c) and (d) of the main text show a nice agreement between the exact Re$G_{hq}$ (black dashed line) and its asymptotic expansion (black solid line) for both Ag and W. Therefore, we can assure that the asymptotic expansion is a reliable mathematical representation of the propagators at the measured distances and, therefore, define the QC-wave contribution to $G_{hq}$ as $G_{QC} = G_N + G_{sdp}$ so that $G_{hq} = G_F + G_{QC}$. We can estimate in this way the relative contribution of $G_F$ and $G_{QC}$ to the the propagator at moderate values of the hole-groove distance.

**B. Derivation and analysis of Eq. (7)**

Eq. (7) of the main text is obtained from the original expression for $I^{res}_{sdp}$ (Eq. (6) of the main text),

$$I^{res}_{sdp}(r) = ±πiC_p(q_pk_I r)e^{ijkl} \text{erfc}(±ipr \sqrt{λr})$$

by choosing a contour in the complex plane for erfc($z$) $≡ 2π^{-1/2} \int_{z}^{∞} e^{-t^2} dt$ that is directed from $z$ via a straight line to the origin, and then along the real axis to $+∞$. Notice that the second integral is equal to 1. This contour is equivalent to write erfc($z$) in terms of the error function, erfc($z$) $≡ 1 − \text{erf}(z)$, where $\text{erf}(z) = ±2π^{-1/2} \int_{z}^{∞} e^{-t^2} dt$ [13]. As $\text{erf}(z)$ is an odd function of $z$, we have $\text{erf}(±z) = 1 ± \text{erf}(z)$. Thus, we have Eq. (7) of the main text,

$$I^{res}_{spd} = ±πiC_p e^{ijkl} \text{erf}(ipr \sqrt{λr}) = ±πiC_p e^{ijkl} \text{erf}(ipr \sqrt{λr})$$

We find that the second term is a surface wave with half the amplitude of the surface wave $I_p$ excited at the pole. On the other hand, the first term oscillates with the free-space wavelength, like Norton and QC waves. It can be deduced from the series expansion of the error function, $\text{erf}(z) = e^{−z^2} p(z)$, where $p(z) = 2π^{-1/2} \sum_{n=0}^{∞} (2n+1)! 1·3· ... · (2n+1)^{-1}$ [12]. The dependence on $q_p$ is removed from the exponential function in $e^{ijkl} \text{erf}(ipr \sqrt{λr})$ using the definition of the integration variable for the SDP, $s_p^2 = i − iq_p$. Notice that, in numerical calculations, the infinite series $p(z)$ is truncated at a given expansion order $n$, obtaining a polynomial $p_n(z)$ of order $n$. The polynomial $p_n(z)$ is a monotonic function in the parameter space considered in the paper. With $n > 5$ the error function is accurately represented in our calculations. Also notice that this decomposition into surface and free-space waves is still valid for asymptotically larger values of the argument if we use the asymptotic expansion of Eq. (S8).

**4. FUNCTION D(Q) FOR A SINGLE SLIT**

The function $D(q)$ for a vary narrow single slit has been reported in Ref. [9],

$$D(q) = \frac{1}{2π} \frac{Z_m}{Z_0 + Z_m}$$

where $Z_m = q_{zm}/ε_m$ and $Z_0 = q_z$ are the impedance of the metal and the air, respectively, while $q_z = \sqrt{1−q^2}$, $q_{zm} = \sqrt{ε_m−q^2}$, $ε_m = ε_1+iε_2$ is the complex dielectric function of the metal, $ε_1$ is its real part, and $ε_2$ its imaginary part. $D(q)$ is characterized by a narrow peak centered at $q_{zp}$ for Au and by a broader peak with lower intensity, which is not centered at $q_{zm}$, for W, see Fig. 2(e) of the main text. In order to understand the origin of those peaks, the denominator of $D(q)$, i.e. the total impedance $Z_t = Z_0 + Z_m$, is plotted in Fig. S4. We observe that the local minimum in the absolute values of $Z_t$ occurs when its imaginary part is equal to zero.
Fig. S4. (Color online). Denominator of $D(q)$, $|Z_t|$, for (a) Ag and (b) W. Absolute values (black line), real parts (red line) and imaginary parts (blue line) are compared.

$Z_t$ is zero at the surface wave complex wavenumber $q_{sw} = \sqrt{\epsilon_m / (\epsilon_m + 1)}$. At the experimental wavelength, $\lambda = 785$ nm, we obtain $q_{sp} = 1.02 + 1.5 \times 10^{-3}i$ for Au and $q_{zw} = 0.996 + 2.0 \times 10^{-2}i$ for W. However, we are interested in the zeros of the imaginary part of $Z_t$ for real values of $q$, i.e. the situation considered in Fig. S4. Neglecting the small absorption of Au ($\epsilon_m = -23 + 1.5i$), we find immediately that $\text{Im}[Z_t] = 0$ for the real part of $q_{sp}$, i.e. for $q = \sqrt{1 / (\epsilon_1 + 1)} = 1.02$, at which the peak of $D(q)$ is centered in Fig. 2(e) of the main text. We cannot apply the same procedure to W due to its large absorption ($\epsilon_m = 4.3 + 23.9i$). It is convenient in this case to write explicit expressions for both real and imaginary parts of $q_{zm}$,

$$
-i q_{zm} = \sqrt{q^2 - \epsilon_1 - i\epsilon_2} = q_{zm1} + iq_{zm2},
$$

where

$$
q_{zm1} = \sqrt{\frac{q^2 - \epsilon_1 + \sqrt{(q^2 - \epsilon_1)^2 + \epsilon_2^2}}{2}},
$$

$$
q_{zm2} = -\sqrt{-q^2 + \epsilon_1 + \sqrt{(q^2 - \epsilon_1)^2 + \epsilon_2^2}}.
$$

The imaginary part of the total impedance becomes,

$$
\text{Im}[Z_t] = \sqrt{q^2 - 1} - \frac{\epsilon_1 q_{zm1} + \epsilon_2 q_{zm2}}{\epsilon_1^2 + \epsilon_2^2}.
$$

Taking into account that $\epsilon_2 \gg |q^2 - \epsilon_1|$, this expression can be simplified to

$$
\text{Im}[Z_t] \approx \sqrt{q^2 - 1} - \frac{1}{\sqrt{2} \epsilon_2}.
$$

It vanishes for

$$
q \approx \sqrt{1 + \frac{1}{2 \epsilon_2}}.
$$

We have $q = 1.01$ for W at $\lambda = 785$ nm, in agreement with the value at which $D(q)$ is centered in Fig. 2(e) of the main text. This value is larger than $\text{Re}[q_{zw} = 0.996]$. We have shown in this way that the peak is not centered at the Zenneck wavenumber. Therefore, Zenneck waves are not exited by the slit in W. However, other kind of waves can be excited at the small peak produces at $q > 1$. 


REFERENCES