Looking for the exotic $X_0(2866)$ and its $J^P = 1^+$ partner in the $\bar{B}^0 \to D^{(*)+}K^-K^{(*)0}$ reactions

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We propose two reactions, $\bar{B}^0 \to K^0 D^+ K^-$ and $\bar{B}^0 \to K^{*0} D^{*+} K^-$, which have been already measured at Belle, to look into the $J^P = 0^+$, $X_0(2866)$ state and a 1^+ partner of molecular $D^* \bar{K}^*$ nature by looking at the D^+K^- and $D^{*+}K^-$ invariant mass distributions, respectively. Very clear peaks over the background are predicted and the branching ratios for the production of these states are evaluated to facilitate the task of determining the needed statistics for their observation. We conclude that with the upgrade of Belle II clear peaks should be seen in both reactions for the two resonances discussed.

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I. INTRODUCTION

In the paper [1] the Belle collaboration reported on the $\bar{B}^0 \to D^{(*)+}K^-K^{(*)0}$ decays, giving a list of eight reactions for which the branching ratios were provided. In some of the reactions, (i) $\bar{B}^0 \to D^+K^-K^{*0}$, (ii) $\bar{B}^0 \to D^{*+}K^-K^{*0}$, (iii) $\bar{B}^0 \to D^+K^-K^0$, (iv) $\bar{B}^0 \to D^{*+}K^-K^0$, one finds pairs [$D^+K^-$ in (i) and (ii), $D^{*+}K^-$ in (iii) and (iv)] that contain open charm and strangeness with c and s quarks. Should these pairs result from the decay of a physical state, it would be genuinely exotic since it cannot come from a $q\bar{q}$ conventional meson. The chosen pairs could correspond to states with isospin $I = 0$, while the other four cases of [1] would correspond to $D^{(*)+}\bar{K}$ states with isospin $I = 1$. The limited statistics prevented the authors from getting $D^{(*)+}K^-$ mass distributions, while the accumulation of K^-K^{*0} events from four reactions allowed them to get a K^-K^{*0} mass distribution that evidenced the $B \to D^{(*)+}a_1(1260)$ decay with $a_1(1260) \to K^-K^{*0}$. Yet, the abundant literature on tetraquark states from the very beginning of the quark model [2–12] (see Refs. [13–18] for reviews on more recent works) would have made it advisable to look at the $D^{(*)+}\bar{K}$ mass distributions in search of possible peaks corresponding to exotic states.

Recently, the answer to this question was provided by the LHCb collaboration [19,20] with the finding of the $X_0(2866)$ and $X_1(2900)$ states in the $B^+ \to D^+D^-K^+$ decay by looking at the D^-K^+ invariant mass distribution. In the charge conjugate reaction $B^- \to D^+D^-K^-$ one would find the peaks in the D^+K^- invariant mass distribution. Interestingly, the existence of a $I = 0, J^P = 0^+$ molecular state of $D^*\bar{K}^*$ nature, decaying to $D\bar{K}$, had been predicted in [21], with a mass of 2848 MeV and a width between 23–59 MeV, which is very close to the data of the $X_0(2866)$ with mass 2866 ± 7 MeV and width 57.2 ± 12.9 MeV. An update of that work in regard to the LHCb results is presented in [22].

The findings of Refs. [19,20] prompted many works offering an explanation for the $X_0(2866)$ as a tetraquark state [23–26] or a molecular $D^*\bar{K}^*$ state [27–32]. The sum rules studies [33–37] have also contributed its share to the discussion, some of them proposing a molecular structure [35–37]. Other studies suggest a structure coming from a triangle singularity [38] or cusps and analytical properties of triangle diagrams [39,40]. A triangle mechanism is also suggested in [41], and in [42] a detailed quark model calculation is shown to disfavor the compact tetraquark picture.

In Ref. [21], apart from the $J^P = 0^+$ state, two other states with $J^P = 1^+, 2^+$ also in $I = 0$ were found. In the update of [22], where the free parameters of the model were adjusted to experiment [19] for the $X_0(2866)$, the masses, widths, and couplings to the $D^*\bar{K}^*$ channel were evaluated, which are shown in Table I.

For reasons of parity and angular momentum conservation, the 0^+ state only decays to $D\bar{K}$ while the 1^+ state decays to $D^*\bar{K}$.

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TABLE I. Properties of the $D^*\bar{K}^*$ states from Ref. [22] accounting for $D\bar{K}$ and $D^*\bar{K}$ decays.

$I[J^P]$	M [MeV]	Γ [MeV]	Coupled channels	$g_{R,D^*\bar{K}^*}$ [MeV]	State
$0[2^+]$	2775	38	$D^*\bar{K}^*$	16536	?
$0[1^+]$	2861	20	$D^*\bar{K}^*$	12056	?
$0[0^+]$	2866	57	$D^*\bar{K}^*$	11276	$X_0(2866)$

The purpose of the present work is to investigate whether by looking at the $\bar{B}^0 \to D^{(*)+}K^-K^{(*)0}$ reactions one can observe clear peaks in the $D^{(*)+}K^-$ spectrum. The reaction is similar to the $B^- \to D^+D^-K^-$ one studied in [19,20]. The $K^{(*)0}$ in the Belle reactions would play the role of the D^- in the LHCb one. The study is stimulated by the success found in [43], fairly reproducing the $D\bar{K}$ peak versus the background of [19] in the study of the $B^- \to D^+D^-K^-$ reaction. This success was used in [43] to suggest the $\bar{B}^0 \to D^{*+}D^{*0}K^-$ decay in order to investigate the 1^+ state of Table I by looking at the $D^{*+}K^-$ mass distribution. It was found that the 1^+ state generated a peak in the distribution with a strength about seven times bigger than the background at the peak of the 1^+ contribution. Based on these findings, we propose here to study the $\bar{B}^0 \to D^{*+}K^-K^{*0}$ and $\bar{B}^0 \to D^+K^-K^0$ reactions. The reason to choose these two reactions from the eight reactions of Belle [1] is that, both in the signal for the exotic states as in the background, the amplitudes can proceed in the s wave, which is assumed to be dominant as usual, and one can correlate the background and the signal for the production of the 0^+ and 1^+ states.

II. FORMALISM AND RESULTS

A. Production of the 1^+ state in $\bar{B}^0 \to D^{*+}K^{*-}K^{*0} \to D^{*+}K^-K^{*0}$

The $D^*\bar{K}^*$ 1^+ state can only decay in $D^*\bar{K}$. Thus, we choose the $\bar{B}^0 \to D^{*+}K^-K^{*0}$ reaction and look at the $D^{*+}K^-$ mass distribution. The signal, however, will come from the $\bar{B}^0 \to D^{*+}K^{*-}K^{*0}$ reaction, after which the final state interaction of $D^{*+}K^{*-}$ will give the 1^+ state (R_1) and posterior decay into $D^{*+}K^-$. The primary step proceeds via external emission as depicted in Fig. 1. The $\bar{u}d$ component after the W^- vertex is hadronized with an $s\bar{s}$ component to give rise to $K^{*-}K^{*0}$ and the $c\bar{d}$ gives the D^{*+}. One has three vectors and one can write the s-wave component of the transition matrix matching the angular momentum of the $\bar{B}^0$ as

$$\tilde{t} = \mathcal{C}\boldsymbol{\epsilon}^{(1)} \cdot (\boldsymbol{\epsilon}^{(2)} \times \boldsymbol{\epsilon}^{(2)}) = \mathcal{C}\epsilon_{ijk}\epsilon_i^{(1)}\epsilon_j^{(2)}\epsilon_k^{(3)}, \qquad (1)$$

where the indices 1,2,3 apply to the K^{*0}, D^{*0}, and K^{*-}, respectively. We observe how the spins of the particles 2 and 3 combine to $J = 1$. The next step is to consider the $D^{*+}K^{*-}$ interaction. With our phase convention $(D^{*+}, -D^{*0})$, $(\bar{D}^{*0}, D^{*-})$, (K^{*+}, K^{*0}), $(\bar{K}^{*0}, -K^{*-})$, the $I = 0$ $D^*\bar{K}^*$ state is written as

$$|D^*\bar{K}^*; I = 0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}K^{*-} + D^{*0}\bar{K}^{*0}). \qquad (2)$$

The final state interaction of $D^{*+}K^{*-}$ to produce the R_1 state is taken into account as shown diagrammatically in Fig. 2.

We also need the vertex $R_1D^*\bar{K}^*$, which incorporates the spin projection generator D^i

$$\tilde{g}_i = g_i V^{(i)}, \qquad (3)$$

with $V^{(i)}$ given by [44]

$$V^{(0)} = \frac{1}{3}\epsilon_l^{(2)}\epsilon_l^{(3)}\delta_{ij},$$
$$V^{(1)} = \frac{1}{2}(\epsilon_i^{(2)}\epsilon_j^{(3)} - \epsilon_j^{(2)}\epsilon_i^{(3)}),$$
$$V^{(2)} = \frac{1}{2}(\epsilon_i^{(2)}\epsilon_j^{(3)} + \epsilon_j^{(2)}\epsilon_i^{(3)}) - \frac{1}{3}\epsilon_l^{(2)}\epsilon_l^{(3)}\delta_{ij}. \qquad (4)$$

Considering the $\tilde{t}$ matrix of Eqs. (1) and (3) for $V^{(1)}$, $V^{(1)} = \frac{1}{2}(\epsilon_{i'}^{(2)}\epsilon_{j'}^{(3)} - \epsilon_{j'}^{(2)}\epsilon_{i'}^{(3)})$, and that in the loop function we sum over the spin polarization $\sum_{\rm pol}\epsilon_i^{(r)}\epsilon_j^{(r)} = \delta_{ij}$ ($r = 2$, 3), we obtain

$$t = \mathcal{C}\epsilon^{(1)}\epsilon_{ii'j'}G_{D^*\bar{K}^*}(M_{\rm inv}(D^*\bar{K}^*))\frac{-1}{\sqrt{2}}g_{R,D^*\bar{K}^*},$$

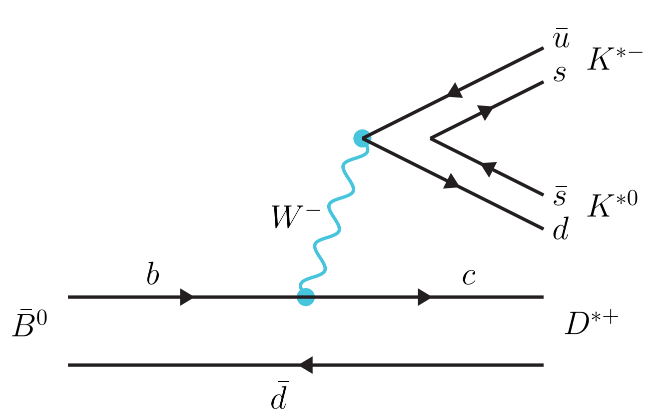

FIG. 1. Diagrammatic decay of $\bar{B}^0 \to D^{*+}K^{*-}K^{*0}$ at the quark level.

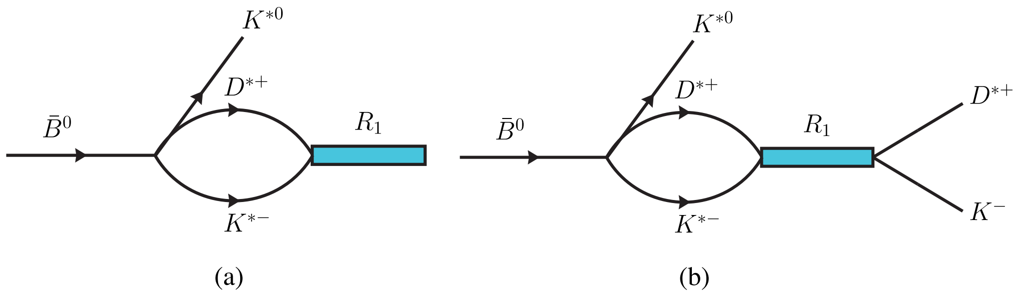

FIG. 2. (a) Rescattering of $D^{*+}K^{*-}$ to give the resonance R_1; (b) further decay of R_1 into $D^{*+}K^-$.

where $g_{R_1,D^*\bar{K}^*}$ is the coupling of the resonance R_1 to the $(I=0)$ $D^*\bar{K}^*$ state and $G_{D^*\bar{K}^*}$ is the loop function of the $D^*, \bar{K}^*$ integrating the product of the propagators of the two particles. We use dimensional regularization for this loop with $\alpha = -1.474$ for a chosen $\mu = 1500$ MeV as was needed in [22] to obtain the right mass of the $X_0(2866)$ state.

The sum over the final vector polarization in $\sum |t|^2$ is implemented by summing $\sum |t|^2$ over the indices i', j' for the implicit VV components of R_1 and over the index i to sum over the K^{*0} polarization. We find

$$\sum_{\text{pol}} |t|^2 = 3\mathcal{C}^2 |g_{R_1,D^*\bar{K}^*}|^2 |G_{D^*\bar{K}^*}(M_{\text{inv}}(D^*\bar{K}^*))|^2.$$

The next step is to consider the decay of R_1 into $D^{*+}K^-$ as depicted in Fig. 2(b). This leads to a t' matrix containing the coupling of R_1 to $D^{*+}K^-$. This is accomplished by an effective coupling $g_{R_1,D^*\bar{K}}$ to the $D^*\bar{K}^*, I=0$ state, such that the coupling to $D^{*+}K^-$ is $\frac{-1}{\sqrt{2}} g_{R_1,D^*\bar{K}}$. To get the $g_{R_1,D^*\bar{K}}$ coupling we use the R_1 decay width via

$$\Gamma_{R_1} = \frac{1}{8\pi} \frac{1}{M_{R_1}^2} |g_{R_1,D^*\bar{K}}|^2 q_{\bar{K}},$$
$$q_{\bar{K}} = \frac{\lambda^{1/2}(M_{R_1}^2, m_{D^*}^2, m_{\bar{K}}^2)}{2M_{R_1}}, \tag{5}$$

taking the value of Γ_{R_1} from Table I. Hence

$$\sum_{\text{pol}} |t'|^2 = \frac{6}{4} \mathcal{C}^2 |g_{R_1,D^*\bar{K}^*}|^2 |G_{D^*\bar{K}^*}(M_{\text{inv}})|^2 \times |g_{R_1,D^*\bar{K}}|^2 \left| \frac{1}{M_{\text{inv}}^2(R_1) - M_{R_1}^2 + iM_{R_1}\Gamma_{R_1}} \right|^2. \tag{6}$$

The invariant mass distribution is then given by

$$\frac{d\Gamma}{dM_{\text{inv}}(D^{*+}K^-)} = \frac{1}{(2\pi)^3} \frac{1}{4M_{\bar{B}^0}^2} p_{\bar{K}^{*0}} \tilde{p}_{K^-} \sum |t'|^2, \tag{7}$$

where

$$p_{\bar{K}^{*0}} = \frac{\lambda^{1/2}(M_{\bar{B}^0}^2, m_{\bar{K}^{*0}}^2, M_{\text{inv}}^2(D^{*+}K^-))}{2M_{\bar{B}^0}},$$
$$\tilde{p}_{K^-} = \frac{\lambda^{1/2}(M_{\text{inv}}^2(D^{*+}K^-), m_{D^*}^2, m_{\bar{K}}^2)}{2M_{\text{inv}}(D^{*+}K^-)}. \tag{8}$$

We would like to compare this mass distribution with the one of the background for the same reaction, $\bar{B}^0 \to K^{*0}D^{*+}K^-$. The process proceeds with the same topology as in Fig. 1, changing K^{*-} by K^-. As shown in [45] the difference between pseudoscalar and vector production can be taken into account by means of Racah coefficients of the same order of magnitude, so approximately we can put for the $\bar{B}^0 \to K^{*0}D^{*+}K^-$ background

$$t = \mathcal{C}\boldsymbol{\epsilon}(K^*)\boldsymbol{\epsilon}(D^*)$$

with the same constant $\mathcal{C}$ as in Eq. (1), such that now the background distribution is given by

$$\frac{d\Gamma_{\text{bac}}}{dM_{\text{inv}}(D^{*+}K^-)} = \frac{1}{(2\pi)^3} \frac{1}{4M_{\bar{B}^0}^2} p_{K^{*0}} \tilde{p}_{\bar{K}} 3\mathcal{C}^2. \tag{9}$$

The assumption of taking the same constant $\mathcal{C}$ is supported by the results of [43], reproducing fairly well the signal versus the background of the LHCb experiment [19].

The results can be seen in Fig. 3. We can see a peak clearly sticking out of the background, as was also found in [43] with a different reaction. It is clear that even if there were uncertainties of a factor of two or three, the signal should be clearly seen.

In order to test the feasibility of the experiment, we integrate the mass distributions over the whole invariant mass range, for both, the resonance peak and the background. We find

$$\frac{\Gamma_{\text{peak}}}{\Gamma_{\text{back}}} = 0.125. \tag{10}$$

Next, we see from Fig. 2(d) of Ref. [1] that for $\bar{B}^0 \to K^{*0}D^{*+}K^-$ there are about 45 events reported in [1] at the $\bar{B}^0$ peak. This means we can expect about six events in the

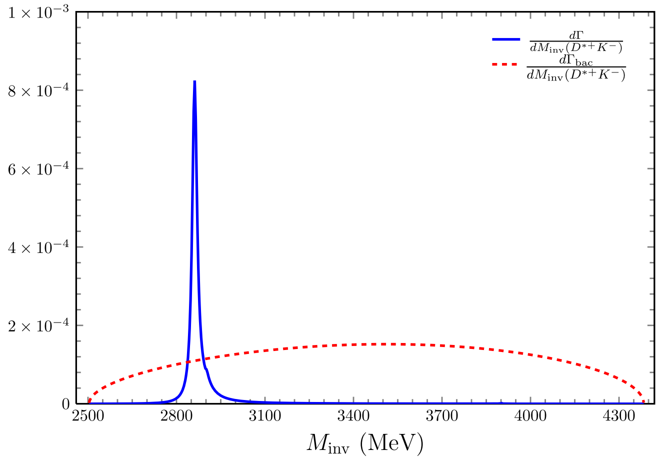

FIG. 3. $\frac{d\Gamma}{dM_{\rm inv}}$ for R_1 production and $\frac{d\Gamma_{\rm bac}}{dM_{\rm inv}}$ for background in the $\bar{B}^0 \to K^{*0}D^{*+}K^-$ reaction in global arbitrary units. $M_{\rm inv}$ is the $D^{*+}K^-$ invariant mass.

peak with the present setup but not enough to see a clean structure. Yet, with the Belle II prospects where there will be about 30 times more events than so far collected in *BABAR* and Belle [46], one could have 170 events, which is much more than sufficient to see clearly the peak, given the clear signal of $\bar{B}^0$ mesons seen with 45 events in [1]. Even accepting a rate three times smaller than estimated, there would be enough statistics to see clearly the peak.

B. Production of the 0^+ state in $\bar{B}^0 \to K^0D^{*+}K^{*-} \to K^0D^+K^-$

Proceeding like in the former subsection, we would now compare the signal for the 0^+ state from the $\bar{B}^0 \to K^0D^{*+}K^{*-}$ with $D^{*+}K^{*-}$ interaction leading to the 0^+ state (R_0) and its decay into D^+K^-, and the background from the $\bar{B}^0 \to K^0D^+K^-$ reaction. We can proceed as before and for the $\bar{B}^0 \to K^0D^{*+}K^{*-}$ we assume a transition matrix

$$t = \mathcal{C}'\boldsymbol{\epsilon}(D^*)\boldsymbol{\epsilon}(K^*) \tag{11}$$

and similarly for the $\bar{B}^0 \to K^0D^+K^-$

$$t = \mathcal{C}' \tag{12}$$

with the same $\mathcal{C}'$, for both reactions, as we have done before. We shall come back to this assumption. Following the same steps as before, we obtain

$$\frac{d\Gamma'}{dM_{\rm inv}(D^+K^-)} = \frac{1}{(2\pi)^3}\frac{1}{4M_{\bar{B}^0}^2}p_{K^0}\tilde{p}_{K^-}\sum|t'|^2, \tag{13}$$

where

$$\sum|t'|^2 = \frac{3}{4}\mathcal{C}'^2|G_{D^*\bar{K}^*}(M_{\rm inv}(D^+K^-))|^2|g_{R_0,D^*\bar{K}^*}|^2 \times\left|\frac{1}{M_{\rm inv}^2(D^+K^-) - M_{R_0}^2 + iM_{R_0}\Gamma_{R_0}}\right|^2|g_{R_0,D\bar{K}}|^2, \tag{14}$$

with

$$p_{K^0} = \frac{\lambda^{1/2}(M_{\bar{B}^0}^2, m_{K^0}^2, M_{\rm inv}^2(D^+K^-))}{2M_{\bar{B}^0}},$$
$$\tilde{p}_{K^-} = \frac{\lambda^{1/2}(M_{\rm inv}^2(D^+K^-), m_{D^+}^2, m_{K^-}^2)}{2M_{\rm inv}(D^+K^-)}, \tag{15}$$

with $g_{R_0,D^*\bar{K}^*}$ given in Table I, and the effective $|g_{R_0,D\bar{K}}|^2$ coupling obtained from

$$\Gamma_{R_0} = \frac{1}{8\pi}\frac{1}{M_{R_0}^2}|g_{R_0,D\bar{K}}|^2 q_{\bar{K}},$$
$$q_{\bar{K}} = \frac{\lambda^{1/2}(M_{R_0}^2, m_{D^+}^2, m_{\bar{K}}^2)}{2M_{R_0}}. \tag{16}$$

For the background we find

$$\frac{d\Gamma'_{\rm bac}}{dM_{\rm inv}(D^+K^-)} = \frac{1}{(2\pi)^3}\frac{1}{4M_{\bar{B}^0}^2}p_{K^0}\tilde{p}_{K^-}\mathcal{C}'^2. \tag{17}$$

The results for these two distributions are shown in Fig. 4. We also see a signal that sticks out of the background clearly, as was shown in [43] for the $B^- \to D^+D^-K^-$ reaction in the production of the $X_0(2866)$.

As we have done before, we integrate over the range of the invariant mass the signal and background in Fig. 4 and we find

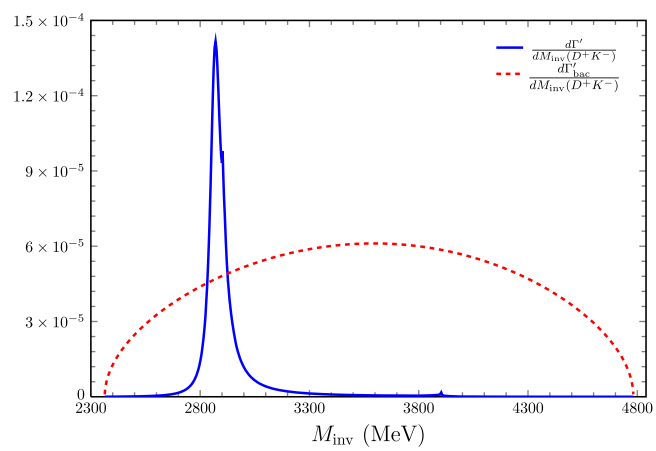

FIG. 4. $\frac{d\Gamma'}{dM_{\rm inv}}$ for R_0 production and $\frac{d\Gamma'_{\rm bac}}{dM_{\rm inv}}$ for background in the $\bar{B}^0 \to K^0D^+K^-$ reaction in global arbitrary units. $M_{\rm inv}$ is the D^+K^- invariant mass.

$$\frac{\Gamma'_{\mathrm{peak}}}{\Gamma'_{\mathrm{bac}}} = 0.124. \quad (18)$$

In Ref. [1] [see Fig. 2(f) of [1]] one finds about 30 events for $\bar{B}^0 \to D^+K^-K_s^0$ around the $\bar{B}^0$ peak. This means that one could expect around four events in the peak of the resonance with present statistics, which is clearly insufficient to determine the peak. With 30 times more statistics from the Belle II upgrade there would be about 110 events, more than sufficient to see clearly the peak.

As this point we would like to make some discussion. The $\bar{B}^0 \to K^0 D^{*+}K^{*-}$ can proceed with the topology of Fig. 1 changing K^{*0} by K^0 where the K^0 and K^{*-} are produced by hadronization of the $\bar{u}d$ component. Yet, the production of K^0K^- from the same vertex is suppressed, as discussed in [47]. Indeed, the vertex WPP is given by $W_\mu\langle[P,\partial_\mu P]\rangle$ in chiral theory [48,49], the s wave going as the difference in the energies of the two pseudoscalars for the WPP vertex, which vanishes in the W rest frame if the particles have the same mass. The argument does not hold if one produces a vector and a pseudoscalar, as it was the case for the signal of the 0^+ state. The argument given above is corroborated by the branching ratio of the $\bar{B}^0 \to D^+K^-K^0$, which is about one order of magnitude smaller than the one of $\bar{B}^0 \to D^{*+}K^-K^{*0}$ (see Table II of Ref. [1]). Certainly we could now have contributions from higher partial waves, but the argumentation given above, with the support of the small $\bar{B}^0 \to D^+K^-K^0$ branching ratio, would tell us that we can expect in practice a peak showing even stronger with respect to the background than what is shown in Fig. 4.

In order to make this argument more quantitative, we take from Table II of Ref. [1] the following branching ratios:

$$\mathcal{B}(\bar{B}^0 \to K^-D^{*+}K^{*0}) = (1.29 \pm 0.22 \pm 0.25)10^{-3}, \quad (19)$$

$$\mathcal{B}(\bar{B}^0 \to K^0D^+K^-) = (0.16 \pm 0.08 \pm 0.03)10^{-3}. \quad (20)$$

By analogy to Eq. (11) we would assume now for the $\bar{B}^0 \to K^0D^{*+}K^{*-}$ amplitude,

$$t = \tilde{C}'\boldsymbol{\epsilon}(D^*)\cdot\boldsymbol{\epsilon}(K^*), \quad (21)$$

and for $\bar{B}^0 \to K^0D^+K^-$ the amplitude of Eq. (12) with a different coupling,

$$t = \tilde{C}''. \quad (22)$$

The mass distribution for the case of Eq. (21) is given as

$$\frac{d\Gamma}{dM_{\mathrm{inv}}(D^{*+}K^{*0})} = \frac{1}{(2\pi)^3}\frac{1}{4M_{B_0}^2}p_{K^-}\tilde{p}_{K^{*0}}3\tilde{C}'^2, \quad (23)$$

with p_{K^-} being the K^- momentum in the $\bar{B}^0$ rest frame and $\tilde{p}_{K^{*0}}$ the momentum of the K^{*0} in the $D^{*+}K^{*0}$ rest frame. The mass distribution for the case of Eq. (22) is given by

$$\frac{d\Gamma}{dM_{\mathrm{inv}}(D^{+}K^{0})} = \frac{1}{(2\pi)^3}\frac{1}{4M_{B_0}^2}p_{K^-}\tilde{p}_{K^{0}}\tilde{C}''^2, \quad (24)$$

with the same meaning for the p_{K^-} and $\tilde{p}_{K^0}$ as before.

By integrating Eqs. (23) and (24) and writing $B = \Gamma/\Gamma_{\mathrm{tot}}$, we find using Eqs. (19) and (20),

$$\frac{\tilde{C}'^2}{\Gamma_{\mathrm{tot}}} = 5.86\times10^{-3}\ \mathrm{MeV}^{-1}, \qquad \frac{\tilde{C}''^2}{\Gamma_{\mathrm{tot}}} = 1.42\times10^{-3}\ \mathrm{MeV}^{-1}, \quad (25)$$

which leads to $\tilde{C}'/\tilde{C}'' \simeq 2$. By assuming $\tilde{C}' = \tilde{C}''$ as we have done in Eqs. (11) and (12) we would be underestimating the signal for the resonance in about a factor of 4. This makes more quantitative the discussion made above, indicating that we should expect a fairly larger signal over the background than shown in Fig. 4.

In the LHCb case [19] a different reaction was used, the $B^+ \to D^+D^-K^+$, or analogously $B^- \to D^-D^+K^-$. Even if the reaction seems the same except for small changes as $\bar{B}^0 \to K^0D^+K^-$, replacing the D^- with K^0, the reactions are topologically different since the LHCb one, as well as the associated $B^- \to D^-D^{*+}K^{*-}$ reaction, proceeds via internal emission, and the argument discussed above is peculiar to the $W^\mu\langle[P,\partial_\mu P]\rangle$ vertex of external emission. In the LHCb reaction the formalism used here for the signal and background gave rise to a distribution in fair agreement with experiment. There is no analog reaction to the $B^- \to D^-D^+K^-$ that proceeds with the internal emission of the type $B \to DK\bar{K}$. The reaction that we have chosen to observe the 0^+, $X_0(2866)$ state, $\bar{B}^0 \to K^0D^+K^-$, stands as a good one, where the signal over background is expected to be even bigger than shown in Fig. 4.

III. CONCLUSIONS

We have chosen two reactions, already performed by the Belle collaboration [1], to observe the $0^+, 1^+$ states obtained from the $D^*\bar{K}^*$ interaction, where the 0^+ state is associated to the $X_0(2866)$ state. From the eight reactions of the type $\bar{B} \to D^{(*)}K^-K^{*0}$ of Ref. [1] we have selected two, the $\bar{B}^0 \to K^{*0}D^{*+}K^-$ and $\bar{B}^0 \to K^0D^+K^-$, in order to observe the 1^+ and 0^+ states, respectively. In the first case the signal of the 1^+ state stems from the original $\bar{B}^0 \to K^{*0}D^{*+}K^{*-}$ reaction, followed by $D^{*+}K^{*-}$ interaction to give the R_1 resonance, which decays posteriorly to $D^{*+}K^-$. In the second case, the signal for the 0^+ state comes from the $\bar{B}^0 \to K^0D^{*+}K^{*-}$ with the posterior $D^{*+}K^-$ interaction producing the 0^+ state, which decays lately into D^+K^-. We could relate the mass distributions of the signal and the background, finding very clear peaks for the 1^+ and 0^+ states. However, we have argued that in the case of the 0^+ state we expect the signal to be even more pronounced with respect to the background than what is calculated here because of the suppressed $\bar{B}^0 \to D^+ K^- K^0$ decay versus $\bar{B}^0 \to D^{*+} K^- K^{*0}$ decay at the tree level.

These reactions, already measured at Belle [1], would need somewhat more statistics to show the $D^{(*)+} K^-$ peaks clearly.

Based upon the number of events presently observed at Belle [1], we have estimated a few events for the peak of the resonances, but with a factor of 30 increase in the number of events expected in Belle II, the number of events in the peak would be fairly larger than 100, which is much more than sufficient to see the strength and shape of the peaks, corroborating the existence of the $X_0(2866)$ and observing its 1^+ partner predicted around 2861 MeV with around 20 MeV width.

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