GEOID COMPUTATION IN THE
NORTH-EAST ATLANTIC (AÇORES-PORUGAL)

J.C. Catalão
Departamento de Matemática.
Faculdade de Ciências. Universidade de Lisboa
1200 Lisboa. Portugal

M.J. Sevilla
Instituto de Astronomía y Geodesia
Facultad de Ciencias Matemáticas. Universidad Complutense
28040 Madrid. Spain

ABSTRACT. In this paper we present the gravimetric geoid computation results in the North-East Atlantic region (Açores-Portugal) as an extension of the work done by the Spanish Group participant in the GEOMED project. Have been processed 48028 gravity points and have been rejected 1263 ones. Geoid computation has been determined with free-air anomalies, OSU91A geopotential model and residual terrain effects in three blocks of 8°x 8° covering all area bounded by latitude between 36° and 42° and longitude -31° to -9°. The prediction was made using least squares collocation and the results are presented as a contour map.

1. INTRODUCTION

In the frame of GEOMED the analysis and validation of gravity data is a main task of the Spanish Group. Till now we have analyzed and validated the gravimetric data of the Mediterranean Sea, provided by Thesaloniki, BGI and DMA and from Portugal (Sevilla et al. 1991, 1992a, 1992c; Sevilla, 1993) and computed a precise gravimetric geoid in this area. In order to compare geoid in closed and open neighbouring seas we present the geoid computation in the north-east Atlantic Ocean.

The main objective is the geographic extend of the geoid from the Mediterranean till Atlantic crossing the Iberian Peninsula. This work can provide interesting answers to
the interrelations between closed and open seas marine geoids and land geoids. The first step of this work was the analysis and validation of the gravimetric data in the Açores-Portugal region (Sevilla and Catalão); then, the corresponding gravimetric geoid computation including the Iberian Peninsula, and finally we intent to use altimetric satellite data to get more information in this interesting region.

In this paper we summarize the results of the gravimetric geoid computation in the North-East Atlantic region ( Açores-Portugal). The selected geographic area included Açores and Madeira islands and Portugal. This region is characterized by an active geodynamic activity which comprises the three plate junction in Açores, the Atlantic rift and the well known Goringe bank in the south west of Portugal.

There are various methods for geoid computation such as Stokes integration or least squares collocation (LSC). Although the LSC is time consuming, and requires pre-processing, it offers the possibility of combining data from heterogeneous sources related to the gravity field. As our main objective is the precise geoid computation, which we known must integrate different sources of information (ex. gravity, satellite altimetric data), we decided to use the LSC method as the central step of the well known remove-restore methodology. In using this methodology we split the geoid computation in three steps: first remove the effect of the geopotencial model and of the residual terrain model (RTM), second computation of the residual geoid by LSC and finally to restore the geoid undulation implied by the geopotencial model and the RTM reduction.

2. DATA

The following data types of data are available in the North-East Atlantic : Potential coefficients, sea-gravity data and bathymetric data.
Gravimetric Data

The available gravimetry data bank has been provided by the Bureau Gravimetrique International (BGI). The covering data zone has the limits \(35^\circ < \phi < 43^\circ, -32^\circ < \lambda < -8^\circ\) with 48028 gravity points all of them corresponding of marine data.

The measured gravity \(g\) is referred to IGSN71 system and the theoretical gravity \(g\) to GRS67. The reference ellipsoid to which point coordinates are calculated is unknown. The points are irregularly distributed. There was 343 duplicate points in the BGI data.

A datum transformation is needed in order to standardize the data by referring all of them to Geodetic Reference System GRS80 (Moritz 1984) for coordinates, theoretical gravities and gradients, and to the International Gravity Standardization Net IGSN71 Datum for the observed gravities. The results of the transformations constitute the gravimetry data bank that will be used in the following.

The application of least squares collocation method requires a homogeneous and isotropic gravity field. We do not know if the gravity field in this area is or not homogeneous. For this reason it was decided to divide the whole area in three blocks, the splitting into three zones seems to correspond as well to geophysical and geological reasons. Moreover, the large size of the area, the large amount of data and the computer storage limitations confirm this decision. A classification in \(8^\circ \times 8^\circ\) non-overlapping zones was made in order to statistically analyze the data. Figure 1 shows the zone number and number of point gravity anomalies in the area used for validation and geoid computation.

Geopotential Model

The first step of the statistical analysis is the geopotential models comparison in the area; for this we have made a calculation of model anomalies with three geopotential models: IFE88E2 (Basic et al., 1990), OSU89B (Rapp and Pavlis, 1990) and OSU91A1F (Rapp et al., 1991); then a comparison with the observed anomalies is
made to establish the model that better fits the gravity field in the area. In view of the results we concluded that in all cases the model that best fits the gravity field in the North-East Atlantic region is the OSU91A1F one. We know that the rejected points in validation have not influence on this conclusion (Sevilla and Catalão). The OSU91A global model, complete up to degree and order 360, was subtracted from the free-air anomalies to get reduced anomalies.

**Bathymetric Data**

For regional and global scale computations there are global digital terrain models available with different resolutions. To model and remove the residual terrain effect we used the ETOPO5U mean heights and depths which gives the topography and bathymetry of the Earth on a regular grid of 5′x5′ (figure 2). The reference grid was computed from the ETOPO5U with a moving average of 30′x30′, accordingly with the order of the geopotential model. The residual terrain computation was preformed with TC program (R. Forsberg) which gives the gravimetric terrain component that we must remove from the original gravimetric data. This way we get the complete reduced gravity. The statistics of the remove procedure are presented in Table 1.

**Table 1. Statistics of the Residual Gravity anomaly**

<table>
<thead>
<tr>
<th></th>
<th>Numb. of data</th>
<th>Average (mgal)</th>
<th>St. Dev. (mgal)</th>
<th>Min (mgal)</th>
<th>Max (mgal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta g_{\text{obs}})</td>
<td>48028</td>
<td>16.091</td>
<td>47.280</td>
<td>-128.83</td>
<td>365.28</td>
</tr>
<tr>
<td>(\Delta g_{\text{obs}} - \Delta g_{\text{OSU91A}})</td>
<td>47543</td>
<td>-1.337</td>
<td>26.231</td>
<td>-117.02</td>
<td>211.74</td>
</tr>
<tr>
<td>(\Delta g_{\text{obs}} - \Delta g_{\text{OSU91A}} - \Delta g_{\text{RTM}})</td>
<td>47543</td>
<td>-1.805</td>
<td>19.766</td>
<td>-98.15</td>
<td>224.27</td>
</tr>
</tbody>
</table>

\(\Delta g_{\text{obs}}\) : observed gravity anomaly  
\(\Delta g_{\text{OSU91A}}\) : gravity anomaly from OSU91A
\( \Delta g_{\text{RTM}} \) : contribution of RTM (using ETOPO5U)

**Residual Terrain Model parameters**

- Reference grid : 30' x 30'
- Detailed grid : 5' x 5'
- Outer grid : 10' x 10'
- Radius \( R_1 = 30 \text{ Km} \) and \( R_2 = 200 \text{ Km} \)

### 3. COVARIANCE MODEL

After removing the short and long wavelength of the gravity field by subtracting the RTM effects and the OSU91A model, we computed the residual anomalies empirical covariance function for each zone, taking as sample points the closest ones to the centres of a 5' x 5' grid. This empirical covariance function has been obtained by using the program EMPCOV (Tscherning) (see fig. 3).

The following step was to fit the covariance model to the empirical covariance function using the least squares iterative inversion technique programmed by Knudsen (1987). The covariance model used was the following:

\[
C(P,Q) = \sum_{i=1}^{m} \sigma_i^e \left( \frac{R^2}{r \cdot r'} \right)^{n+1} P_n(\cos \psi) + \sum_{i=m+1}^{\infty} \sigma_i \left( \frac{R_B^2}{r \cdot r'} \right)^{n+1} P_n(\cos \psi)
\]

where \( \sigma_i^e \) are the error degree variances related to the potential coefficients set, \( R \) is the mean earth radius, \( r \) and \( r' \) are the radial distances of \( P \) and \( Q \), \( R_B \) is the radius of a Bjerhammar sphere, \( P_n \) are the Legendre polynomials, \( \psi \) is the spherical distance between \( P \) and \( Q \), and \( \sigma_i = A(i-1)/(i-2)(i+24) \). The parameters to fit are : a positive constant \( A \), the radius of the Bjerhammar sphere \( R_B \), and as we use the assumption of no error in the geopotential model coefficients development, we estimate the order of the local covariance function \( m \), i.e. the number of terms to be deleted, this order \( m \) is selected by fitting the first zero point of the empirical covariance function (Arabelos
el al. 1987).

The results of the adjustment for the covariance functions have been obtained by using the program COVFIT (Knudsen, 1987). (See Table 2)

Table 2. Empirical and Fitted Covariances

<table>
<thead>
<tr>
<th>Zone Number</th>
<th>Number Points</th>
<th>Average (mgal)</th>
<th>Variance (mgal²)</th>
<th>Signal (mgal²)</th>
<th>Noise (mgal²)</th>
<th>Order</th>
<th>Rₜ - Rₑ</th>
<th>Correl. length (min)</th>
<th>First zero length (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1682</td>
<td>-1.13</td>
<td>403.4</td>
<td>401.99</td>
<td>0.173</td>
<td>225</td>
<td>-1.682</td>
<td>6.79</td>
<td>23.96</td>
</tr>
<tr>
<td>2</td>
<td>1460</td>
<td>-2.41</td>
<td>225.0</td>
<td>231.61</td>
<td>0.010</td>
<td>198</td>
<td>-0.967</td>
<td>5.70</td>
<td>27.22</td>
</tr>
<tr>
<td>3</td>
<td>2468</td>
<td>-3.40</td>
<td>469.4</td>
<td>463.18</td>
<td>0.104</td>
<td>198</td>
<td>-2.292</td>
<td>8.35</td>
<td>27.16</td>
</tr>
</tbody>
</table>

The choice of covariance functions for validation purposes is easy to make since the collocation is used only to predict homogeneous quantities of the gravity field, i.e. gravity anomalies in a small area. With the previous results we have some covariance function parameters in each zone. By inspecting these parameters (number of points, correlation length, scale factor etc.) we can conclude if the size of the zone, the mesh-side of the regular grid and the relation signal/noise are accepted or not; in the last case the zone is again analyzed with different initial conditions until to obtain accepted results or to mark the zone as no relevant in order to obtain results by this method.

For prediction purposes and to avoid harmonicity problems, we have taken the same Bjerharmar radius in all three zones. The selected radius was the maximum of the three, which is Rₜ - Rₑ = -0.967.
4. Geoid Computation

The remove-restore methodology splits the geoid undulation (N) into three components:

\[ N = N_1 + N_2 + N_3 \]

in which \( N_1 \) represents the contribution of the OSU91A geopotential model, \( N_3 \) the residual terrain effect contribution, computed with the bathymetric data, and \( N_2 \) corresponds to the residual anomaly field contribution, computed by LSC.

In order to obtain the medium wavelength component \( N_2 \) we used GEOCOL program (Tscherning 1985). This program makes use of the following formulas (Moritz, 1980)

\[ N_2 = N_{col} = C_N \Delta g \ C^{-1} \Delta g_{res} \]

in which \( C_N \Delta g \) is the covariance between geoid undulation and completely reduced anomalies, and \( C \) is the sum of the autocovariance matrix with the error covariance.

In the computation of the local geoids for each zone, we used data within 8° × 8° area surrounding to 1° × 1° overlapping area in order to eliminate inconsistencies of the local geoids along the boundaries of the adjacent blocks. We computed the geoid heights on a grid in a zone of latitude between 36° and 42° and longitude between -31° and -9° with mesh size 5°.

The restore statistics are summarized in table 3 while the plot of the estimated geoid are represented in fig 4.

| \( N_{Col} + N_{OSU91A} \) | 23570 | 52.4 | 4.689 | 41.98 | 62.47 |
| \( N_{RTM} \) | 23570 | 0.001 | 0.337 | -1.19 | 2.83 |

Table 3. Estimated Gravimetric Geoid statistics
5. Conclusions

The datum transformations, the OSU91A model anomalies calculation, the 8°x 8° block subdivision and the statistical parameters estimation have been done for the North-East Atlantic ( Açores-Portugal) area; the estimation and analysis of the covariance functions have also been done.

The least squares prediction has been applied to compute the gravimetric geoid in the region. This new geoid will be useful in comparisons between closed seas (Mediterranean) and open seas (Atlantic) results, in a future work.

ACKNOWLEDGEMENTS

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References


Fig 1. Data sub-division on $8^\circ \times 8^\circ$
Fig. 2. ETOPO5U Terrain model contoured at 400 m.
Fig 3. Empirical (solid line) and Fitted (dashed line) Covariance function of completed reduced anomalies
Fig. 4. The estimated Gravimetric Geoid