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Inferring untrained complex dynamics of delay systems using a physics-informed reservoir

Mirko Goldmann*[1], Claudio R. Mirasso, Ingo Fischer, Miguel C. Soriano

1. Instituto de Física Interdisciplinar y Sistemas Complejos, IFISC (CSIC-UIB), Campus Universitat Illes Balears, E-07122 Palma de Mallorca, Spain
*mirko@ifisc.uib-csic.es

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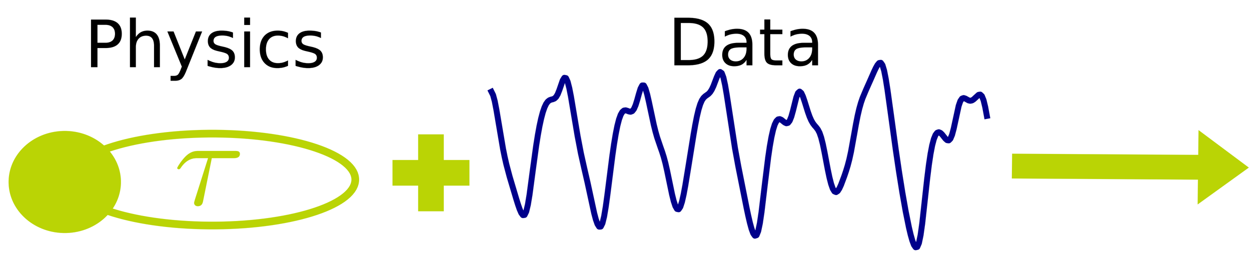
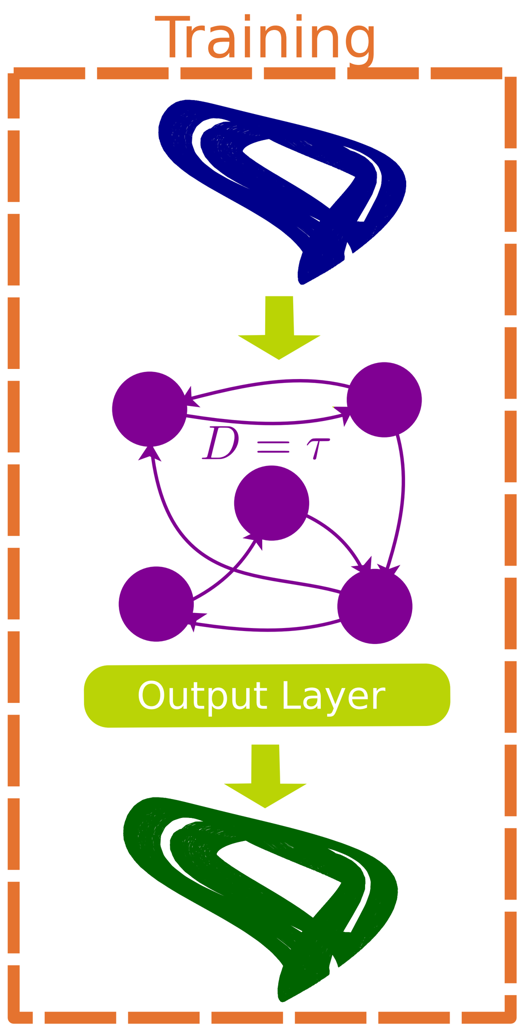
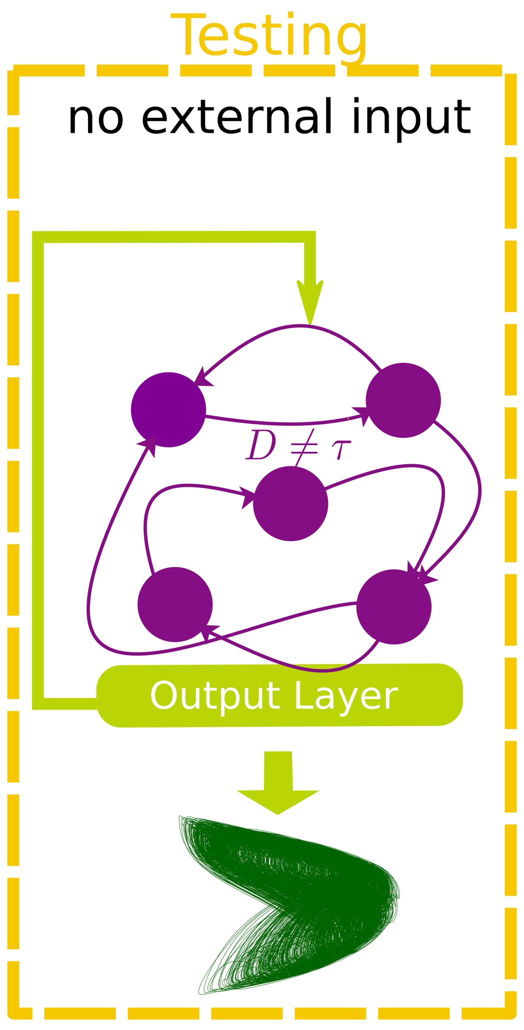
Abstract





- learning to replicate the chaotic dynamics of the training data set

changing the reservoir's topology



- inferring complex dynamics not seen in the training but corresponding to system trained for in a different dynamical regime

Physics-informed reservoir computing

- using prior knowledge about the task's physics to bias the training data, the reservoir's topology or loss function
- data-driven learning of the reservoir's output layer
  - ✓ high prediction accuracy
  - ✓ far-reaching inference
  - ✓ data efficient learning

Model

- Goal: predict the dynamics of high-dimensional chaotic delay systems
- Example: Mackey-Glass system

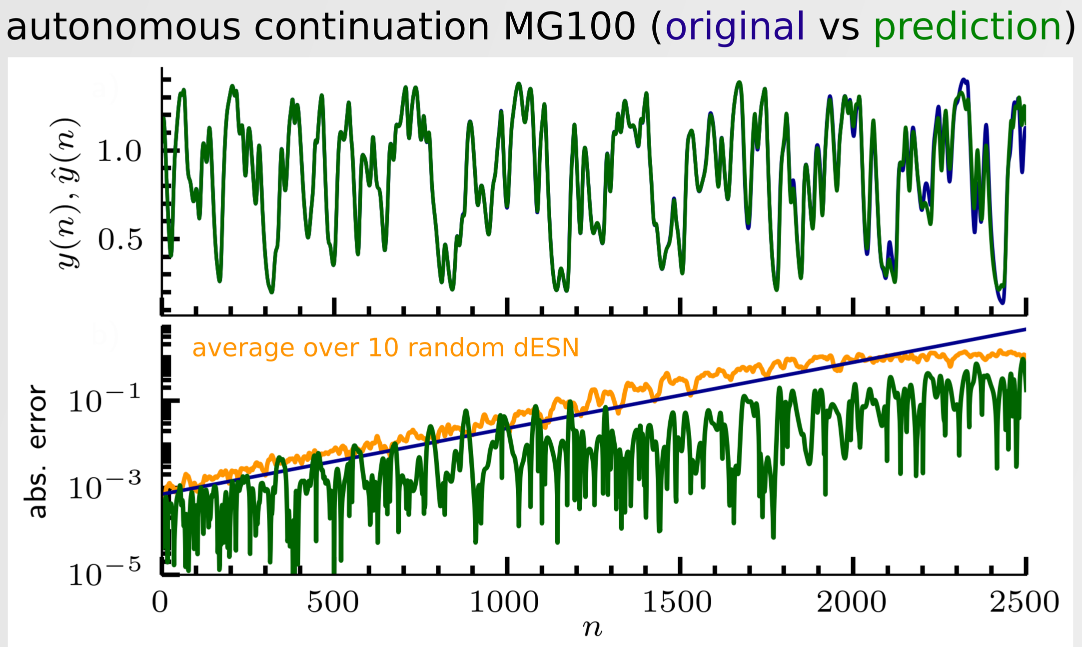
$$\dot{y}(t) = -0.1y(t) + \frac{0.2y(t-\tau)}{1-y^{10}(t-\tau)}$$

- dynamics rely on long history function
- ML Model: delayed echo state network

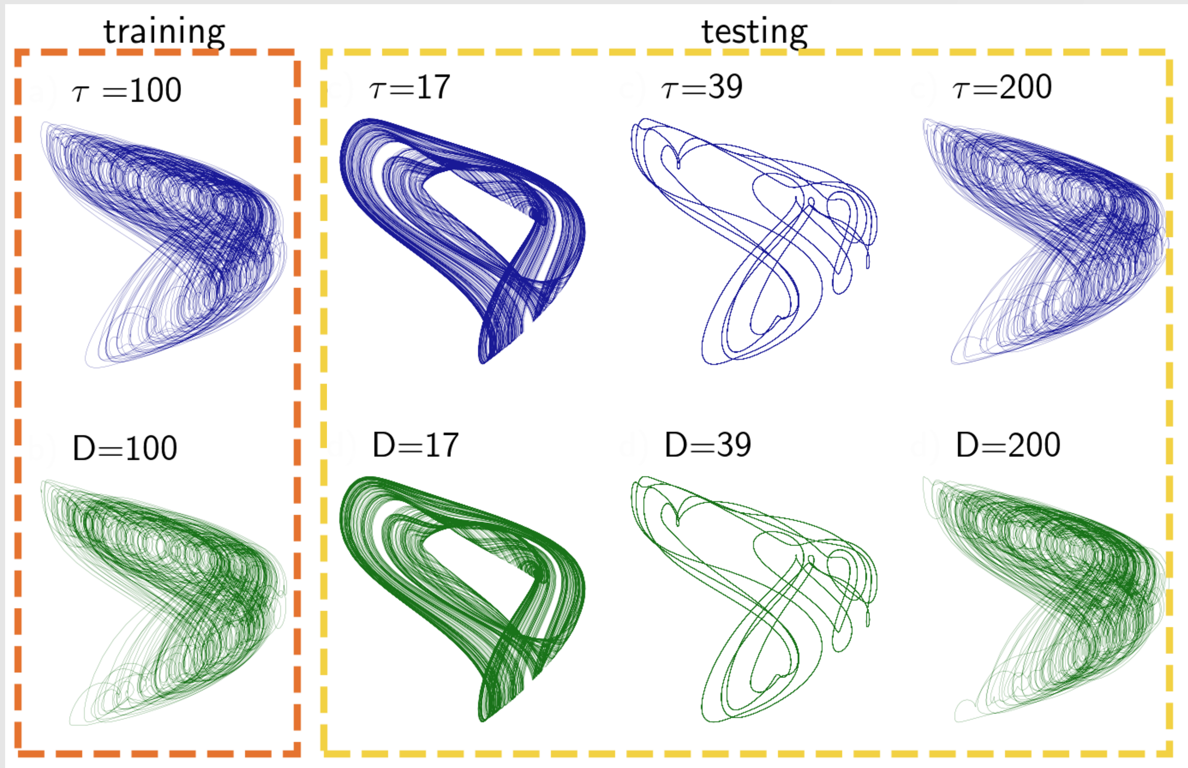
$$\vec{x}(n+1) = \alpha\vec{x}(n) + \\ + \beta\tanh(\mathbf{W}\vec{x}(n-D) + \gamma\mathbf{W}_{in}s(n) + \mathbf{W}_b)$$

- learn to predict time series one-step-ahead $\hat{y}(n) = W_{out}x(n)$
- feed back prediction as new input -> autonomous running reservoir

autonomous continuation MG100 (original vs prediction)



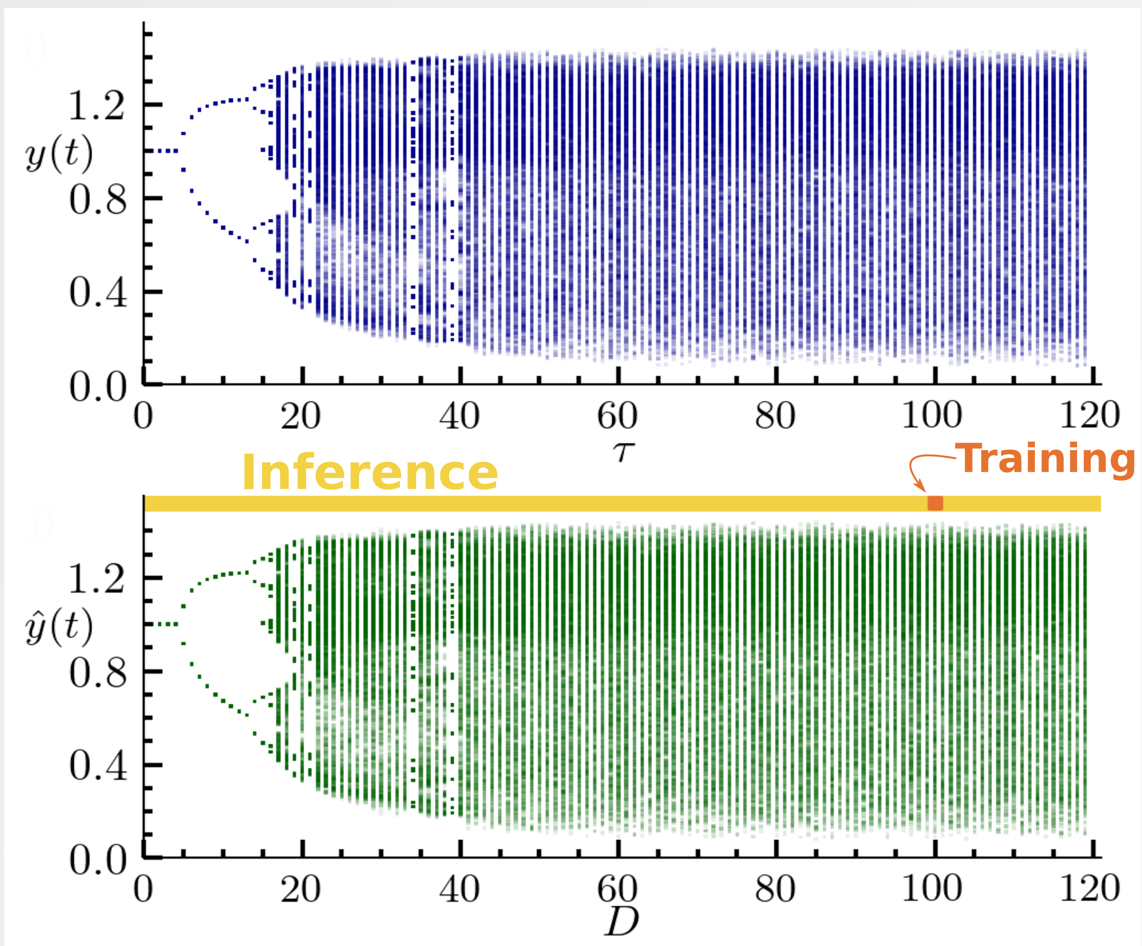
Infer Different Dynamical Regimes



- after replicating the training data, changing the delay of the dESN
- reservoir infers dynamics of the Mackey-Glass system with different delays

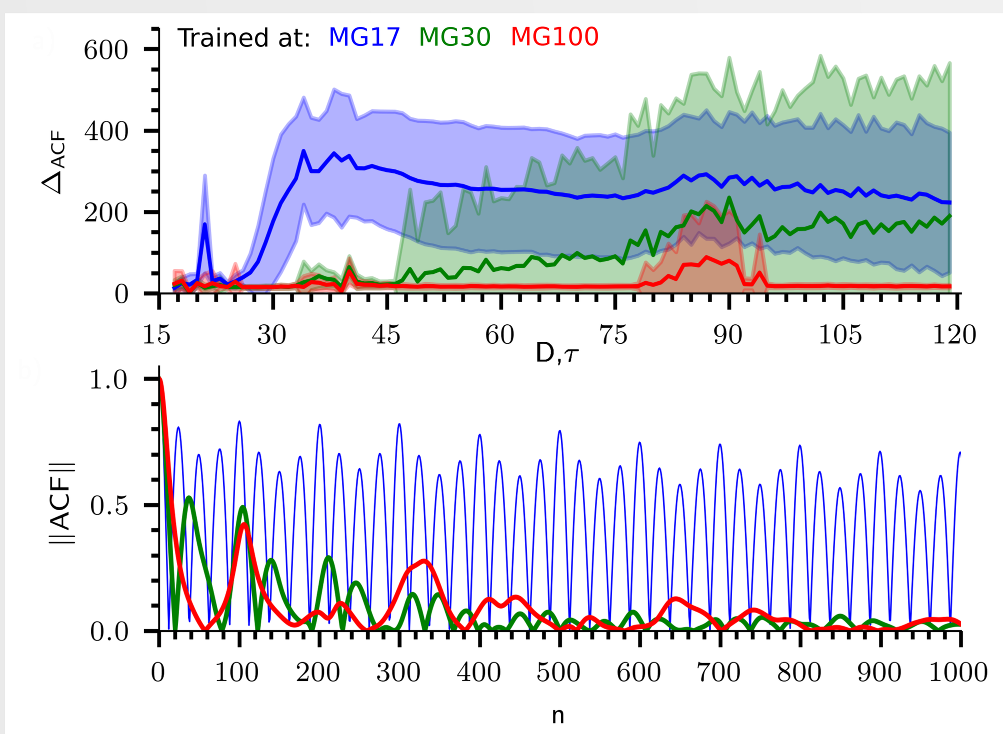
  - output layer left unchanged
  - no further training needed

- prediction reveals bifurcations towards:
  - limit cycles
  - fixed points
- trained reservoir can infer unseen multistabilties
- learning from a single example enables to infer the entire bifurcation diagram of the delay system



What are the limitations?

- more complexity in the training data improves the inference ability
- e.g. learning from a chaotic system with a long delay enables to predict dynamics of systems with respectively shorter delay
- learning in the long delay limit enables prediction of much longer delays up to τ=1000



Conclusion & Outlook

- training a physics-informed reservoir on data enables prediction of dynamical features not seen during the training
- building digital twins of real world systems to infer dynamics of regimes where data is not accesible
- possible extension to other dynamical system such as:
  - delay-coupled oscillators
  - spatio-temporal systems

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