Equilibrium selection in coordination games

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1 Introduction

Cooperation among individuals is often crucial for the survival of the whole, be it economic system, small social group, or animal cohort. In many man-made systems proper regulations ensure that we end up in the optimal configuration. However, in many complex systems coordination emerges without any external supervision or regulation. It’s not always clear how the system evolves into cooperative state and which state will be selected, if more than one exists. Understanding, this phenomena is relevant in a range of fields, from economics and sociology to biology. The conventional approach to model such phenomena is by using evolutionary game theory. In this framework individuals play games using different strategies and receive payoffs. Based on the payoff and the strategy of their neighbors they update their strategy in order to maximize the profit.

2 Results

We study equilibrium selection under different update rules by investigating a spectrum of coordination games played in a population of agents [1]. Firstly, we look at their behavior in a single layer network of $N$ nodes, each with a degree $k$ (random regular graph). We run numerical simulations with agents initially using a random strategy for two games: the pure coordination game (PCG) with the payoff matrix:

\[
A \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\]

with two equivalent equilibria, and the general coordination game (GCG) with the payoff matrix:

\[
A \begin{pmatrix} 1 & S \\ T & 0 \end{pmatrix},
\]

for $T < 1$ and $S < 0$ with payoff- and risk-dominant equilibria [2–4]. To analyze the time evolution of the system, we update the state of nodes, i.e. the last payoff and the used strategy, in each time step until a stationary state or a frozen configuration is reached. We use the asynchronous update rule – in each time step a node, called a focal or active
node, is randomly selected to play the game with its neighbors and update its strategy based on its payoff and the applied update rule.

There are several update rules used in the field of evolutionary game theory [5, 6]. Three of the most common ones are: the replicator rule (RR), the best response (BR), and the unconditional imitation (UI). The replicator rule can be seen mostly in biological applications, whether to describe replication of genes, species, or individuals. The best response update rule is usually considered in economic literature, where the assumption of rational agents is typical. The unconditional imitation is popular in complexity science, as it resembles social imitation. Surprisingly, all three update rules can lead to substantially different outcomes, therefore the evolutionary game environment is defined by the update rule as much as by the payoff matrix. Therefore, in order to describe experimental data one must properly identify the appropriate update rule.

To measure the level of coordination we incorporate a parameter $\alpha \in [0, 1]$ – the fraction of nodes using the strategy A. For the PCG we observe a transition from a frozen disorder configuration to full coordination with increasing $k$ when using UI (Fig. 1a). In two other update rules the system coordinates for much sparser networks ($k > 3$). In the GCG again results depend on the degree for UI – the theoretical transition...
line $T = S + 1$ at which the risk-dominant strategy changes is reached only for a complete graph, while for smaller values of $k$ it it shifted towards lower $S$ (Fig. 1b). This novel result goes into opposite direction than previously studied behaviour for rings (circular city) [7]. For $T = -1$ the critical value $S_c$ approaches $-2$ as a power law with increasing $k$ for any size of the network (Fig. 1c).

Finally, we study coordination between two layers in a multiplex networks. On each layer a GCG with a different payoff matrix is played with such payoff values that each layer separately would coordinate on a different strategy. Both games, however, are chosen to be equally distant from the transition. Therefore, the setup is symmetrical in principle. We vary the node overlap $q$ (aka the degree of multiplexity) between the layers and analyze equilibrium selection. Overlapping nodes must be in the same state at both layers at all times [8]. Surprisingly, we observe symmetry breaking – in most cases coordination on the Pareto-optimal strategy is preferred with bigger values of the overlap (Fig. 1d).

All three studied update rules assume that a rational agent aims at increasing its payoff. Either by directly computing its payoff and choosing the bigger one like in the BR (therefore this rule requires players’ knowledge about the payoff matrix), or by imitating a more successful neighbour with a bigger payoff like in the RR and UI. However, the outcome can be very different between those update rules in coordination games, even for the simplest PCG. Additionally, the sensitivity to changes in the degree varies largely between the rules, with UI promoting Pareto-optimal outcome for sparser networks. A simple one-round two-player game is fully described by its payoff matrix. The conclusion from our simulations is that an evolutionary game is defined by the update rule as much as by the payoff matrix.

References