Supersymmetry and Finite Radiative Electroweak Breaking from an Extra Dimension *

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Abstract

A five dimensional $N = 1$ supersymmetric theory compactified on the orbifold $S^1/\mathbb{Z}_2$ is constructed. Gauge fields and $SU(2)_L$ singlets propagate in the bulk ($U$-states) while $SU(2)_L$ doublets are localized at an orbifold fixed point brane ($T$-states). Zero bulk modes and localized states constitute the MSSM and massive modes are arranged into $N = 2$ supermultiplets. Superpotential interactions on the brane are of the type $UTT$. Supersymmetry is broken in the bulk by a Scherk-Schwarz mechanism using the $U(1)_R$ global $R$-symmetry. A radiative finite electroweak breaking is triggered by the top-quark/squark multiplet $T$ propagating in the bulk. The compactification radius $R$ is fixed by the minimization conditions and constrained to be $1/R \lesssim 10 - 15$ TeV. It is also constrained by precision electroweak measurements to be $1/R \gtrsim 4$ TeV. The pattern of supersymmetric mass spectrum is well defined. In particular, the lightest supersymmetric particle is the sneutrino and the next to lightest supersymmetric particle the charged slepton, with a squared-mass difference $\sim M_Z^2$. The theory couplings, gauge and Yukawa, remain perturbative up to scales $E$ given, at one-loop, by $ER \lesssim 30 - 40$. Finally, LEP searches on the MSSM Higgs sector imply an absolute lower bound on the SM-like Higgs mass, around 145 GeV in the one-loop approximation.

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1 Introduction

The Higgs boson is, for the time being, the only missing ingredient of the Standard Model (SM) of electroweak and strong interactions and, by far, the most intriguing one. While it is related to the origin of gauge boson and fermion masses, the mechanism of electroweak breaking is intimately related to the so-called hierarchy problem which has given rise to the (minimal) supersymmetric extension (MSSM) of the SM. In particular the radiative corrections to the squared Higgs mass in the SM have a quadratic sensitivity to the cutoff of the theory, $\Lambda_s$, which destabilizes the Higgs mass towards the region where the SM is no longer reliable \cite{1}. This behaviour is softened in the MSSM where the sensitivity to the SM cutoff is only logarithmic and can therefore be interpreted as the renormalization group running from the scale $\Lambda_s$ to the weak scale \cite{2}. Actually, one of the great successes of the MSSM is that the squared Higgs mass term can be driven by radiative corrections generated by the top Yukawa coupling from positive values at the scale $\Lambda_s$ to negative values at the weak scale thus triggering radiative electroweak breaking \cite{3}. Still the MSSM shows some (logarithmic) sensitivity to the cutoff scale $\Lambda_s$, whose value controls the total evolution of the squared Higgs mass.

The sensitivity of the squared Higgs mass term on the cutoff $\Lambda_s$ through radiative corrections can still be softened if the MSSM, and in particular the top/stop sector, is living in the bulk of an extra dimensional space of size $\mathcal{O}(\text{TeV}^{-1})$ \cite{4}. In that case the radiative corrections to the squared Higgs mass term are not sensitive at all to $\Lambda_s$. In fact they are finite, controlled by the inverse radius $1/R$ of the compactified extra dimensions \cite{1} and with a sign which depends on the spin of the bulk particle circulating in the loop \cite{5,6,7}. This observation gave rise to proposing the top/stop (hyper)multiplet living in the bulk \cite{7} as the source of a finite electroweak radiative breaking \cite{3}, while some explicit examples along that direction have been recently proposed in the frameworks of string \cite{8} and field theory in higher dimensions \cite{9,10}.

Another issue which is highly related to the hierarchy problem and electroweak breaking is that of supersymmetry breaking. The scale of supersymmetry breaking must not be hierarchically different from the weak scale since, on the one hand, we do not want to re-create the hierarchy problem, and on the other hand, it should trigger electroweak breaking. Because, in the finite radiative electroweak breaking, the weak scale is provided (apart from loop factors) by the inverse radius $1/R$ of the compactified extra dimensions, that is the expected order of magnitude for the scale of supersymmetry breaking. Although there are several mechanisms in the literature which can provide the correct order of magnitude for supersymmetry breaking, the one that naturally leads to supersymmetry breaking size of order $1/R$ is the Scherk-Schwarz (SS) mechanism \cite{11}-\cite{14}. In fact both recent examples of finite radiative electroweak breaking \cite{9,10} use, among other mechanisms, a variant of the SS-mechanism based on a discrete symmetry of the supersymmetric theory, the $R$-parity.

In this paper we will analyze a very simple five dimensional (5D) model where finite electroweak breaking is triggered by the top/stop multiplet living in the bulk of the extra

\footnote{This very well known fact in ordinary field theory at finite temperature $T$, i.e. compactified on the circle of inverse radius $T$, is at the origin of the so-called thermal (Debye) masses.}

\footnote{See footnote 11 in Ref. \cite{4}.}
dimension and supersymmetry is broken by a SS-mechanism based on a continuous symmetry of the 5D supersymmetric theory, $SU(2)_R$. So, unlike Refs. \cite{9,10} supersymmetry breaking is controlled by a continuous parameter, and the supersymmetric limit is continuously attainable. In this sense, and although the theoretical setup of our 5D theory is rather different from those presented in Refs. \cite{9,10}, our results can be considered in some aspects as more general than theirs.

The outline of this paper is as follows. In section 2 we will present the model and the mechanism of supersymmetry breaking. Finite radiative electroweak breaking will be analyzed in section 3 and the Higgs sector and supersymmetric spectrum will be presented in sections 4 and 5, respectively. In section 6 a discussion on unification and non-perturbation scales will be done and some comments concerning the relation of our paper with Refs. \cite{9,10} will be made. Finally in section 7 we will present our conclusions and comparison with recent related works.

2 The 5D MSSM and supersymmetry breaking

In this section we will describe a 5D $N = 1$ model whose massless modes constitute the usual four dimensional (4D) $N = 1$ MSSM, where supersymmetry breaking is a bulk phenomenon induced by the SS-mechanism, and with finite radiative electroweak breaking triggered by the presence of a bulk top/stop hypermultiplet.

The 5D space-time is compactified on $\mathcal{M}_4 \times S^1/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ parity is acting on the fifth coordinate as $x_5 \rightarrow -x_5$. The orbifold $S^1/\mathbb{Z}_2$ has two fixed points at $x_5 = 0, \pi R$ and the compactified space has two 3-branes located at the fixed points of the orbifold. In this way fields in the theory can be of two types: those living in the 5D bulk, similar to untwisted states in the heterotic string language ($U$-states), and those living on the branes localized at the fixed points of the orbifold, similar to the heterotic string twisted states ($T$-states). We will assume for simplicity that $T$-states are localized at the $x_5 = 0$ fixed point. While $U$-states feel the fifth dimension, i.e. their wave function depend on $x_5$, $T$-states do not.

Vector fields live in the bulk and they are in $N = 2$ vector multiplets in the adjoint representation of the gauge group $SU(3) \times SU(2)_L \times U(1)_Y$, $V = (A_\mu, \lambda_1; \Phi, \lambda_2)$. Matter fields in the bulk are arranged in $N = 2$ hypermultiplets, $H = (\tilde{\psi}_R, \psi_R; \tilde{\psi}_L, \psi_L)$. $\mathbb{Z}_2$, the parity in the fifth dimension, has an appropriate lifting to spinor and $SU(2)_R$ indices, such that we can decompose $V$ and $H$ into even, $(A_\mu, \lambda_1)$ and $(\tilde{\psi}_R, \psi_R)$, and odd, $(\Phi, \lambda_2)$ and $(\tilde{\psi}_L, \psi_L)$, $N = 1$ superfields. After the $\mathbb{Z}_2$ projection the only surviving zero modes are in $N = 1$ vector and chiral multiplets. If the chiral multiplet $(\tilde{\psi}_R, \psi_R)$ is not in a real representation of the gauge group, a multiplet of opposite chirality localized in the 4D boundary $(\tilde{\psi}_L, \psi_L)$ can be introduced to cancel anomalies. In order to do that $\mathbb{Z}_2$ must have a further action on the boundary under which all boundary states are odd. This action can be defined as $(-1)^{\varepsilon_i}$ for the chiral multiplet $X_i$, such that $\varepsilon_i = 1$ (0) for $X_i$.

\footnote{Where $i = 1, 2$ transform as $SU(2)_R$ indices and the complex scalar $\Phi$ is defined as, $\Phi \equiv \Sigma + i A_5$.}

\footnote{In fact, $(\tilde{\psi}_R, \psi_L)$ transforms as a doublet under $SU(2)_R$.}

\footnote{Of course the above lifting on hypermultiplets is arbitrary and we could equally well consider $(\tilde{\psi}_R, \psi_R)$ as odd and $(\tilde{\psi}_L, \psi_L)$ as even.}
living in the brane (bulk), which creates a selection rule for superpotential interactions on the brane.

The MSSM is then made up of zero modes of fields living in the 5D bulk and chiral $N = 1$ multiplets in the 4D brane, at localized points of the bulk. A superpotential interaction can only exist at the brane of the type $UTT$ or $UUU$ to satisfy the orbifold selection rule, although the latter, $UUU$, are expected to have Yukawa couplings which are suppressed with respect to those in $UTT$ by a factor $(R\Lambda_s)^{-1}$, and corresponding to the fact that localized couplings of states propagating in the bulk must be (volume) suppressed.

To allow for the MSSM superpotential on the brane,

\[ W = [h_U Q H_2 U + h_D Q H_1 D + h_E L H_1 E + \mu H_2 H_1] \delta(x_5) \]  

(2.1)

and to be consistent with the orbifold selection rule and with a top/stop hypermultiplet propagating in the bulk, to trigger a finite electroweak radiative breaking, the only solution is that the $SU(2)_L$ singlets, $U, D, E, H$ are localized on the brane and transform as chiral $N = 1$ multiplets.

Since both Higgs fields are on the brane, the $\mu$-parameter does not arise through compactification and, although it is an allowed term in the superpotential, one has to consider it as an effective parameter and rely on its generation as a result of the integration of the massive states of the underlying (supergravity or string) theory. In this sense the situation is no better than in the usual MSSM, except for the fact that the cutoff of the theory, $\Lambda_s$, the scale at which the structure of the underlying theory should be considered, is at most two orders of magnitude larger that the scale of supersymmetry breaking and therefore a modest suppression should be sufficient for phenomenological purposes.

After compactification on $S^1/\mathbb{Z}_2$ the zero mode of the vector multiplet $V^{(0)}$ is just the 4D MSSM $N = 1$ vector multiplet $(A^{(0)}_\mu, \chi^{(0)}_1)$, while the massive modes are the massive 4D $N = 2$ vector multiplets $(A_\mu^{(n)}, \chi^{(n)}_1, \chi^{(n)}_2, \Sigma^{(n)})$, with a mass $n/R$, where $\chi^{(n)}_1, \chi^{(n)}_2$ are Majorana spinors. Similarly the zero mode of matter hypermultiplets $\Phi^{(0)}$ is the 4D $N = 1$ chiral multiplet $(\psi^{(0)}_R, \psi^{(0)}_L)$, while the massive modes are 4D $N = 2$ hypermultiplets, $(\tilde{\psi}^{(n)}_R, \tilde{\psi}^{(n)}_L, \psi^{(n)}_R, \psi^{(n)}_L)$, with a mass $n/R$, where $\psi^{(n)}_R$ is a Dirac spinor with components $(\psi^{(n)}_R, \psi^{(n)}_L)$. Finally, the chiral fields which are localized on the brane are massless, except for the supersymmetric mass term $\mu$ introduced in the superpotential (2.1) that gives a common mass to Higgs bosons and higgsinos.

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6. Here we assume that all three generations propagate in the same way. Otherwise they can produce an acute flavor problem and trigger strong $CP$ violation, which translate into strong bounds on the scale $1/R$.

7. There is technically speaking another possibility: that matter doublets $Q, L$ are living in the bulk and matter singlets and Higgs doublets $U, D, E, H_2, H_1$ are localized on the brane. This possibility looks less natural and will not be explicitly considered in this paper, although it leads to results very similar to those that will be found. It also leads to a drawback in the supersymmetric spectrum concerning the lightest supersymmetric particle, as we will comment later on.

8. Another possibility, that has been recently pointed out in Ref. 10, is having a singlet field $S$ in the bulk acquiring a vacuum expectation value (VEV) by radiative corrections induced by another field, in a similar way to the one by which the top/stop sector makes the Higgs field to acquire a VEV in this paper. Of course this situation requires enlarging the MSSM to the NMSSM and the Higgs sector gets mixed with the singlet states.
Supersymmetry breaking was performed in Refs. [14, 6, 7] using the SS mechanism based on the subgroup $U(1)_R$ which survives after the orbifold action $S^1/\mathbb{Z}_2$. Using the $R$-symmetry $U(1)_R$ with parameter $\omega$ to impose different boundary conditions for bosons and fermions inside the 5D $N = 1$ multiplets, one obtains for the $n$-th Kaluza-Klein (KK) mode of gauge bosons and chiral fermions living in the bulk the compactification mass $n/R$, while for the $n$-th mode of gauginos, $\lambda^{(n)}$, and supersymmetric partners of chiral fermions living in the bulk, $\tilde\psi_R^{(n)}, \tilde\psi_L^{(n)}$ the mass $(n + \omega)/R$. Notice that for the particular case $\omega = 1/2$ the gauginos $\lambda^{(n)}$ and $\lambda^{-(n+1)}$, $n > 0$, are degenerate in mass and constitute a Dirac fermion. A detailed discussion was done in Ref. [14] for the case of gauginos. For matter scalars in hypermultiplets, $(\tilde\psi_R, \tilde\psi_L)$, that transform under the subgroup $U(1)_R$ we can write the SS boundary conditions as,

$$
\begin{pmatrix}
\tilde\psi_R \\
\tilde\psi_L
\end{pmatrix} = \begin{bmatrix}
\cos \omega x_5/R & -\sin \omega x_5/R \\
\sin \omega x_5/R & \cos \omega x_5/R
\end{bmatrix} \begin{pmatrix}
\varphi_R \\
\varphi_L
\end{pmatrix} \tag{2.2}
$$

where $\varphi_R$ ($\varphi_L$) are even (odd) periodic functions $\varphi_{R,L}(x_5) = \varphi_{R,L}(x_5 + 2\pi R)$. Making a Fourier expansion along the $x_5$ direction with coefficients $\varphi_{R,L}^{(n)}$ we can write:

$$
\begin{align*}
\tilde\psi_R &= \sum_{n=-\infty}^{\infty} \cos \left(\omega + n\right) x_5 \frac{R}{R} \tilde\psi_R^{(n)} \\
\tilde\psi_L &= \sum_{n=-\infty}^{\infty} \sin \left(\omega + n\right) x_5 \frac{R}{R} \tilde\psi_L^{(n)} \tag{2.3}
\end{align*}
$$

where

$$
\begin{align*}
\tilde\psi_R^{(n)} &\equiv \tilde\psi_L^{(-n)} = \frac{1}{2} \left( \varphi_R^{(n)} - \varphi_L^{(n)} \right), \quad n \geq 0 \\
\tilde\psi_L^{(n)} &\equiv \tilde\psi_R^{(-n)} = \frac{1}{2} \left( \varphi_R^{(n)} + \varphi_L^{(n)} \right), \quad n \geq 0 \tag{2.4}
\end{align*}
$$

are the mass eigenstates modes.

In this way the MSSM states, made out of bulk zero modes and localized states, acquire tree-level masses: gauge bosons, right-handed fermions and left-handed chiral multiplets localized on the brane are massless; gauginos and right-handed sfermions are massive, with masses $\omega/R$; Higgs bosons and higgsinos, localized on the brane, are massive with a common supersymmetric mass $\mu$.

Since the Higgs bosons have a positive squared mass $\mu^2$, electroweak symmetry is preserved at the tree-level. However, we will see in the next section how this tree-level mass pattern, along with the Yukawa couplings contained in the superpotential (2.1), and in particular the top Yukawa coupling $h_t$, will be able to break radiatively the electroweak symmetry and provide a well defined spectrum for the Higgs masses and the masses of the supersymmetric partners localized on the brane.
3 Radiative electroweak breaking

The Higgs potential along the direction of the neutral components of the fields $H_2 = h_2 + i \chi_2$ and $H_1 = h_1 + i \chi_1$ can be written as,

$$V(H_1, H_2) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 \cdot H_2 + h.c.) + \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2$$

$$+ \lambda_t |H_2|^4 + \lambda_b |H_1|^4$$

(3.1)

where the supersymmetric tree-level relations $m_1 = m_2 = \mu$ and $m_3^2 = 0$, $\lambda_t = \lambda_b = 0$ hold. These relations are spoiled by radiative corrections which provide contributions to all the above parameters. These corrections are driven by the $SU(2)_L \times U(1)_Y$ gauge couplings $g$ and $g'$, and by the top and bottom Yukawa couplings, defined as:

$$h_t = \frac{m_t}{v} \sqrt{\frac{1 + t^2}{t^2_\beta}}, \quad h_b = \frac{m_b}{v} \sqrt{\frac{1 + t^2}{t^2_\beta}}$$

(3.2)

where $t_\beta = \tan \beta = v_2/v_1$, $v_i = \langle H_i \rangle$ are the vacuum expectation values of the Higgs fields, $v = \sqrt{v_1^2 + v_2^2} = 174.1$ GeV, and $m_t$ and $m_b$ are the top and bottom running masses. Notice that $h_b$ can become important only for large values of $t_\beta$, as those that will be found by minimization of the one-loop effective potential. We will consider the leading radiative corrections: in particular $g^2$-corrections to the quadratic terms which are zero at the tree-level ($m_3^2$) and $h_{t,b}^4$ corrections to the quartic terms which are $O(g^2)$ at the tree-level. For this reason we will not consider any radiative term as $(H_1 \cdot H_2)^2$ in Eq. (3.1) and neglect $g'^2$-radiative corrections in the numerical analysis.

All radiative corrections to the potential parameters in (3.1) will depend on $1/R$ and $\omega$. In particular the one-loop radiative corrections to any scalar localized on the brane were computed in Ref. [7] where the corresponding diagrams were identified. A simple application to the Higgs mass terms $m_1^2$ and $m_2^2$ yields:

$$m_2^2 = \mu^2 - \frac{6h_t^2 - 3g^2 \Delta(\omega)}{32\pi^4} \frac{R^2}{R^2}$$

$$m_1^2 = \mu^2 - \frac{6h_b^2 - 3g^2 \Delta(\omega)}{32\pi^4} \frac{R^2}{R^2}$$

(3.3)

where the function $\Delta$ is defined as

$$\Delta(\omega) = 2\zeta(3) - [Li_3(r) + Li_3(1/r)],$$

(3.4)

$r = \exp(2\pi i \omega)$, and $Li_n(x) = \sum_{k=1}^{\infty} x^k/k^n$ is the polylogarithm function of order $n$.

The mass term $m_3^2$ in (3.1) is generated by the one-loop diagram exchanging KK-modes of gauginos, $\lambda^{(n)}$ and localized higgsinos, $\tilde{H}_{1,2}$, as shown in Fig. 1.

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9See Figs. 2 and 3 of Ref. [7].
Figure 1: One-loop diagrams contributing to $m_3^2$.

The resulting contribution is given by,

$$m_3^2 = \mu \frac{3 g^2}{512 \pi^2} \frac{\omega}{R} \left[ i Li_2(r) - i Li_2(1/r) \right]$$

Notice that, for the particular case $\omega = 1/2$ ($r = -1$), $m_3^2 = 0$, reflecting the fact that the gauginos $\lambda^{(n)}$ are, in that case, Dirac fermions, as it was already mentioned.

The quartic couplings $\lambda_{t,b}$ are generated at the one-loop level by loop diagrams exchanging KK-modes, $\tilde{t}_R^{(n)}$ and $\tilde{b}_R^{(n)}$, and localized modes, $\tilde{t}_L$ and $\tilde{b}_L$. The diagrams contributing to $\lambda_t$ are shown in Fig. 2. The contribution to $\lambda_b$ being similar, just changing $H_2 \rightarrow H_1$ and $t \rightarrow b$.

Figure 2: One-loop diagrams contributing to $\lambda_t$.

Notice that in Fig. 2 we are sometimes propagating the modes $\tilde{t}_R^{(n)}$. They correspond, in our notation, to the odd modes of the hypermultiplets $(\tilde{t}_R^{(n)}, t_R^{(n)}, \tilde{t}_L^{(n)}, t_L^{(n)})$ that can couple to the brane through $\partial_5$ couplings, and should not be confused with the localized
multiplets \((\tilde{t}_L, t_L)\). The resulting expression is,

\[
\lambda_t = \frac{3h_t^4}{8\pi^2} \left\{ -1 + \log 2\pi R M_Z \right. \\
- \frac{1}{4 (r^2 - 1)} \left[ (r^2 - 1) (\log(1 - r) + \log(1 - 1/r)) + (1 + r^2) (Li_2(1/r) - Li_2(r)) \right] \right\}
\]

and a similar expression for \(\lambda_b\) just changing \(h_t \to h_b\).

Finally we have as free parameters, \(1/R, \omega, \mu\) and \(t_\beta\). Two of them will be fixed by the minimization conditions \(V_{h_1}' = V_{h_2}' = 0\) which read as

\[
3 \left( 1 + \frac{1}{t_\beta^2} \right) \frac{2h_t^2 - g^2 \Delta(\omega)}{32 \pi^4 R^2} = \mu^2 + \frac{1}{2} M_Z^2 + 2\lambda_t v^2 + \left( \mu^2 - m_A^2 - \frac{1}{2} M_Z^2 \right) \frac{1}{t_\beta^2}
\]

\[
\frac{2h_t^2 - g^2}{2h_b^2 - g^2} = \frac{\mu^2 + \frac{1}{2} M_Z^2 + 2\lambda_t v^2 + (\mu^2 - m_A^2 - \frac{1}{2} M_Z^2) / t_\beta^2}{\mu^2 - m_A^2 - \frac{1}{2} M_Z^2 + (\mu^2 - \frac{1}{2} M_Z^2 + 2\lambda_b v^2) / t_\beta^2},
\]

where

\[
m_A^2 = -m^2_3 \frac{1 + t_\beta^2}{t_\beta}
\]

is the mass of the CP-odd Higgs once the minimization conditions \(3.7\) have been used.

In fact we have chosen to select \(1/R\) and \(t_\beta\) as functions of the other variables. The corresponding plots are shown in Fig. 3.

![Figure 3: Plots of 1/R, in TeV (left panel) and t_\beta (right panel), as functions of \(\omega\) and \(\mu\), in TeV, as given from the minimization conditions (3.7).](image)

We can see from Fig. 3 that the minimization conditions impose a solution with large \(\tan \beta\) \((t_\beta \simeq 35 - 40)\) and values of the compactification scale going from a few TeV to \(\sim 10 - 15\) TeV, depending on the values of \(\omega\) and \(\mu\).

SM precision measurements settle bounds on electroweak observables which, for higher dimensional models with gauge fields living in the bulk of the extra dimension, translate on

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lower bounds on the compactification scale $1/R$. The model we are studying was analyzed in Ref. [17], where a very general class of models was considered. We found that, for large values of $t_\beta$, the lower bound on $1/R$ is $\sim 4$ TeV which is in the ballpark provided by Fig. 3.

4 The Higgs mass spectrum

The neutral Higgs sector has one CP-odd and two CP-even scalar bosons. The mass of the CP-odd Higgs boson was already given in Eq. (3.8). The squared mass matrix for neutral CP-even scalar bosons is given by,

$$M_0^2 = \begin{pmatrix} m_A^2 s_\beta^2 + (M_Z^2 + 4 \lambda_b v^2) c_\beta^2 & (m_A^2 + M_Z^2) s_\beta c_\beta \\ (m_A^2 + M_Z^2) s_\beta c_\beta & m_A^2 c_\beta^2 + (M_Z^2 + 4 \lambda_t v^2) s_\beta^2 \end{pmatrix}$$

(4.1)

In the large $t_\beta$-limit the two eigenvalues are:

$$M_H^2 = m_A^2, \quad M_h^2 = M_Z^2 + 4 \lambda_t v^2,$$

(4.2)

where $h$ is the SM-like Higgs and $H$ the Higgs with non-SM couplings, while the mass of the charged Higgs $H^\pm$ is given by

$$M_{H^\pm}^2 = m_A^2 + M_W^2$$

(4.3)

The masses of the neutral CP-even and charged Higgs bosons are shown in Fig. 4.

Figure 4: Plots of $M_{h,H}$, in GeV (left panel) and $M_{H^\pm}$, in GeV (right panel), as functions of $\omega$ and $\mu$, in TeV.

In the left panel of Fig. 4, $M_h$ is the flat surface while $M_H$ corresponds to the steepest one. The crossing is characteristics of the large $t_\beta$ solution. For large values of $\mu$, $M_h$ is the lightest (SM-like) Higgs mass. Notice that the mass $M_h$ is not controlled by the compactification scale $1/R$, but only by the weak scale $v$. This is a reflection of a similar behaviour in the MSSM where the lightest SM-like Higgs mass is not controlled by the supersymmetry breaking scale. On the other hand, for small values of $\mu$, $M_H$ corresponds to the lightest Higgs mass. Its mass is controlled by the compactification scale.
To make contact with Refs. [9, 10] we can fix $\omega = 1/2$. In that case the $m_2^3$-term is not generated, as we said previously, and $m_A = 0$, unless we introduce an additional term like $\sim (\lambda/\Lambda_s)^2$ in the superpotential, giving rise to an $m_3^2$-term as $\sim \lambda \mu v^2/\Lambda_s$. In that case the Higgs spectrum depends on the parameters $\mu$ and $\lambda$, with some sensitivity on the cutoff $\Lambda_s$. We recover the results of Ref. [9] in the limit $\lambda, \mu \to 0$, in which case we obtain $M_h \simeq 128$ GeV, in agreement with the result in [9]. However, in general, for $\omega \neq n/2$ the PQ invariance is broken by the gaugino masses and there is no need for the $\lambda$-term in the superpotential.

Finally we will comment on the constraints imposed on our model from the Higgs searches at LEP. Preliminary results of last year run show a lower limit on the SM Higgs mass about 113 GeV. On the other hand, an excess of candidates for the process $e^+e^- \to Z^* \to Zh$ has been reported by the ALEPH and L3 Collaborations for center-of-mass energies $\sqrt{s} > 206$ GeV, for a SM-Higgs with a mass around $M_h \simeq 115$ GeV [19], which decays predominantly into $bb$. Now the question is whether this mass can be accommodated in our model. A quick glance at Fig. 4 (left panel) shows that such low masses should be described by what we name as $H$ eigenstate, with a mass $m_H \simeq m_A$. In the large $t_\beta$ region, the state $H$ is predominantly $H_1^0$, with unconventional couplings to gauge bosons and fermions, as opposed to the state $h$, the SM-like Higgs boson, with SM-like couplings to gauge bosons and fermions, and with a mass entirely controlled by the electroweak breaking parameter. However in this region we have $M_h > M_H$. In this way the coupling $ZZH_1 = (ZH_1)^{SM}c_\beta$ is strongly suppressed as $\sim 1/t_\beta$ and so does the $H$ direct production. In other words a SM-like Higgs with a mass $\sim 115$ GeV cannot be accommodated by our present model [10].

What are then the limits imposed to our model by the bounds on Higgs searches at LEP? For large $t_\beta$ and moderate values of the pseudoscalar mass $m_A$, such that $m_A = M_H$, LEP searches on the MSSM Higgs sector, based on the process $e^+e^- \to HA$, settle a lower bound on $m_A$ as $m_A \gtrsim 95$ GeV. This constraint translates into a lower limit $\mu \gtrsim \mu(\omega)$ on the $\mu$-parameter that is shown in Fig. 5 (left panel) where the shadowed region is excluded. In particular an absolute lower bound on $\mu$ around 350 GeV can be read off the plot. The corresponding lower bound on the SM-like Higgs mass, $M_h$, is shown in Fig. 5 (right panel) where again the shadowed region is the excluded one. From this plot we can see that the absolute lower limit for the SM-Higgs mass (in the one-loop approximation) is $\sim 145$ GeV. This mass is probably too heavy to be discovered at Tevatron and should await till the LHC collider.

In summary we see that the Higgs sector of this model is very constrained and will be probed in the next generation of colliders (LHC). It predicts, as any MSSM with large $t_\beta$, an almost degeneracy between the masses of one of the CP-even and the CP-odd states. LEP bounds on the MSSM Higgs sector set an absolute lower bound on the SM-like Higgs mass, around 145 GeV, and the $\mu$-parameter, around 350 GeV, which provides the higgssino masses. The model is very predictive and will be fully tested at LHC.

\[\text{We thank J.R. Espinosa and C. Wagner for pointing out this to us.}\]
Figure 5: Lower limit on $\mu$ from the LEP bound $m_A \gtrsim 95$ GeV (left panel), and the corresponding bound for $M_h$ (right panel).

5 The supersymmetric spectrum

The supersymmetric mass spectrum is fully determined by the mechanism of electroweak and supersymmetry breaking that we have described in previous sections. All soft breaking mass terms of zero modes propagating in the bulk of the fifth dimension are equal, and given by the SS supersymmetry breaking mechanism,

$$M_1 = M_2 = M_3 = m_{U_i} = m_{D_i} = m_{E_i} = A_{abc} = \frac{\omega}{R}$$ (5.1)

where $i = 1, 2, 3$ labels the generation number and $A_{abc}$ denotes generically all soft couplings corresponding to trilinear terms in the superpotential. For example, in the case of the top quark coupling in the superpotential, $h_t H_2 \tilde{Q} U_R$, $A_t$ comes from the term in the 5D Lagrangian

$$L_5 = -h_t H_2 \tilde{Q} \left( \partial_5 \tilde{U}_L \right) \delta(x_5) + h.c.$$ (5.2)

giving rise, after dimensional reduction and SS breaking, to the 4D Lagrangian

$$L_4 = - \sum_{n=-\infty}^{\infty} h_t \frac{n + \omega}{R} H_2 \tilde{Q} \tilde{U}_L^{(n)} + h.c.$$ (5.3)

in the notation of Eqs. (2.3) and (2.4). The term $n = 0$ of (5.3) gives rise to $A_t$.

Concerning now the $N = 1$ sector localized on the brane, it does not receive any tree-level mass, except for the higgsinos (and Higgs bosons) that get a tree-level mass $\mu$. However the scalar supersymmetric partners of left-handed quarks and leptons do receive a radiative mass from the bulk fields, where supersymmetry is broken, mediated by gauge and Yukawa interactions. These contributions were computed in full generality in Ref. [7] and we will just quote here the corresponding result applied to the present model.

Squarks receive the main radiative contribution from the gluon/gluino sector proportional to the QCD gauge coupling $g_3$. For the first and second generation squarks $\tilde{q}$ we
can neglect all other gauge and Yukawa couplings and write,

$$m_{\tilde{q}}^2 \simeq \frac{8}{9} \frac{g_3^2}{h_t^2 - g^2/2} \left( \mu^2 + \frac{1}{2} M_Z^2 \right)$$

Plugging numbers in Eq. (5.4) we can see that it yields $m_{\tilde{q}} \simeq 1.4 \mu$.

For the third generation squark doublet $\tilde{Q} = (\tilde{t}_L, \tilde{b}_L)$ there is, on top of the gauge contribution of (5.4), a negative contribution from $h_{t,b}$ Yukawa couplings. Using again the minimization conditions we can write,

$$m_{\tilde{Q}}^2 \simeq \left( 1 - \frac{3(h_t^2 + h_b^2)}{8 g_3^2} \right) m_{\tilde{q}}^2$$

which gives the numerical rough estimate $m_{\tilde{Q}} \simeq \mu$.

The mixing can be neglected for all states except for the third generation of up-type squarks for which the two mass eigenvalues can be approximately written as,

$$M_{\tilde{t}_2}^2 \simeq \left( \omega \frac{R}{R} \right)^2 + 2 m_t^2 + \frac{m_{\tilde{Q}}^2}{(\omega \frac{R}{R})^2 - m_{\tilde{Q}}^2} m_t^2$$

$$M_{\tilde{t}_1}^2 \simeq m_{\tilde{Q}}^2 - \frac{m_{\tilde{Q}}^2}{(\omega \frac{R}{R})^2 - m_{\tilde{Q}}^2} m_t^2$$

For the supersymmetric partners of lepton doublets $\tilde{L} = (\tilde{\nu}_L, \tilde{\ell}_L)$ (the three generations) we obtain, in the same way, the soft squared mass value,

$$m_{\tilde{L}}^2 \simeq \frac{9 \alpha_2}{16 \alpha_3} m_{\tilde{q}}^2$$

which gives the numerical estimate $m_{\tilde{L}} \simeq 0.55 \mu$. In this way, and neglecting the mixing, the mass of charged sleptons can be approximated by,

$$M_{\tilde{\ell}_L}^2 \simeq m_{\tilde{L}}^2 + m_{\tilde{\ell}}^2 + M_Z^2 \left( \frac{1}{2} - s_W^2 \right) \frac{t_\beta^2 - 1}{t_\beta^2 + 1}$$

$$M_{\tilde{\ell}_R}^2 \simeq \left( \omega \frac{R}{R} \right)^2 + m_{\tilde{L}}^2 + M_Z^2 s_W^2 \frac{t_\beta^2 - 1}{t_\beta^2 + 1}$$

while the mass of the sneutrinos

$$M_{\tilde{\nu}_L}^2 \simeq m_{\tilde{L}}^2 - \frac{1}{2} M_Z^2 \frac{t_\beta^2 - 1}{t_\beta^2 + 1}$$

In this way the $\tilde{\nu}_L$ turns out to be the lightest supersymmetric particle (LSP) while the slepton $\tilde{\ell}_L$ is the next to lightest supersymmetric particle (NLSP). Their masses satisfy the approximate relation

$$M_{\tilde{\ell}_L}^2 - M_{\tilde{\nu}_L}^2 \simeq M_Z^2$$

Since s-neutrinos are neutral particles any kind of cosmological problems associated with the existence of the LSP is automatically avoided in this model.\footnote{The other possibility pointed out in footnote 7, where Q, L live in the bulk, and U, D, E, H_{1,2} are localized on the brane, would yield the charged slepton, $\tilde{\ell}_R$, as the LSP, with a mass $m_{\tilde{\ell}_R} = 9 \alpha_1 m_3^2 / 20 \alpha_3 \simeq 0.35 \mu$, while the sneutrino $\tilde{\nu}_L$ is heavy, with a mass $M_{\tilde{\nu}_L} \simeq \omega / R$.}
6 Unification and non-perturbativity scales

In this section we will comment on the sensitivity of the model to the sector of the theory beyond the scale $1/R$. It is a well known fact [20] that, because of the non-renormalizability of the 5D theory, the gauge and Yukawa couplings run with a power law behaviour of the scale. This idea was on the basis of the so-called “accelerated” unification proposed in Ref. [21] and subsequently analyzed in different papers [22]-[29]. In particular, the gauge and Yukawa coupling one-loop renormalization for the model analyzed in this paper were studied in Refs. [26, 15] with one-loop $\beta$-functions:

\[
16\pi^2 \beta_{g_1} = \left( \frac{48}{5} e^t - 3 \right) g_1^3 \\
16\pi^2 \beta_{g_2} = \left( -4 e^t - 3 \right) g_2^3 \\
16\pi^2 \beta_{g_3} = -3 g_3^3 \\
16\pi^2 \beta_{h_t} = \left\{ e^t \left( 4h_t^2 + h_b^2 - 3g_2^2 - \frac{1}{3}g_1^2 - \frac{8}{3}g_3^2 \right) + 2h_t^2 - \frac{8}{15}g_1^2 - \frac{8}{3}g_3^2 \right\} h_t \\
16\pi^2 \beta_{h_b} = \left\{ e^t \left( 4h_b^2 + h_t^2 - 3g_2^2 - \frac{1}{3}g_1^2 - \frac{8}{3}g_3^2 \right) + 2h_b^2 - \frac{2}{15}g_1^2 - \frac{8}{3}g_3^2 \right\} h_b
\]

(6.1)

where $g_2 \equiv g$ and $g_1 \equiv \sqrt{5/3}g'$ are the $SU(2)_L \times U(1)_Y$ gauge couplings with hypercharge normalization $k_1 = 5/3$.

We have run the one-loop renormalization group equations (RGE) from $M_Z$ to scales $\mu > 1/R$ using, for $M_Z \leq \mu \leq M_{\text{SUSY}}$ the SM beta functions for gauge and Yukawa couplings $^{12}$, those of the MSSM for $M_{\text{SUSY}} \leq \mu \leq 1/R$, and those in Eq. (6.1) for scales $\mu > 1/R$. We have chosen for the plot $t \beta \simeq 37$ and $1/R \simeq 4$ TeV.

\[\text{Figure 6: Plot of the gauge couplings } \alpha_i, i = 1, 2, 3 \text{ (left panel), } i = t, b \text{ (right panel) as a function of exp}(t) \equiv R\mu.\]

The result is shown in Fig. 6 where we plot the gauge couplings $\alpha_i = g_i^2/4\pi$, $i = 1, 2, 3$ (solid curves in the left panel) and the Yukawa couplings $\alpha_{t,b} = h_{t,b}^2/4\pi$ (right panel). We

\[^{12}\text{To simplify the analysis we use here a common scale of supersymmetry breaking } M_{\text{SUSY}} \simeq 1 \text{ TeV.}\]
see that all couplings, except $\alpha_1$, are asymptotically free and, as a consequence, in the region $\mu R \lesssim 30$ shown in the plots the theory is weakly coupled. For $\mu R > 30$, $\alpha_1$ keeps on growing and for $\mu R \simeq 40$ the theory becomes strongly coupled.

We can compare here the above scales with those obtained in models [9, 10] where all SM particles are propagating in the bulk. There are two main differences:

- The first important difference concerns the running of the QCD gauge coupling, $g_3$. In our case there is no linear running in $\beta_{g_3}$ because only half of the SM $SU(3)$ triplets (the right-handed ones) are propagating in the bulk which leads to cancellation of the coefficient of the linear term in $\beta_{g_3}$. When also the left-handed triplets propagate in the bulk there is an extra contribution to the $SU(3)$ $\beta$-function as $16\pi^2 \Delta \beta_{g_3} = 6e^t g_3^2$ which makes $g_3$ to increase with the scale and become non-perturbative at $\mu_{NP} R \simeq 6 - 8$, depending on the value of $1/R$ [13]. However in our case we have seen that only $\alpha_1$ is non-asymptotically free and has a linear running, which translates into a much larger non-perturbative scale $\mu_{NP} R \simeq 40$.

- The second important difference concerns the running of Yukawa couplings. As a consequence of the superpotential structure $UTT$ the $\beta$-functions of the Yukawa couplings are governed by the anomalous dimensions of fields in the brane. These anomalous dimensions involve a single $U$-field propagating in the loop, which makes their scale dependence linear. On the other hand, theories with all matter fields propagating in the bulk rely on localized Yukawa couplings with a superpotential structure of the type $UUU$. In that case the one-loop anomalous dimensions involve two $U$-fields propagating in the loop, which makes the scale dependence Yukawa $\beta$-functions quadratic and worsens the perturbative behaviour of Yukawa couplings. In fact we have seen that, while in our case the top and bottom Yukawa couplings are one-loop asymptotically free, in theories with all SM fields in the bulk they become non-perturbative at scales $\mu_{NP} R \simeq 3 - 6$ [9, 10].

Finally we can see from the left plot of Fig. 6 that the theory does not unify. This issue was analyzed in Ref. [26] where it was shown that by adding two zero hypercharge triplets propagating in the bulk, and contributing to the $\beta$-functions $16\pi^2 \Delta \beta_{g_2} = 8e^t g_2^3$, the theory unifies at $\mu_{GUT} R \simeq 30$. The running of $\alpha_2$ is shown in Fig. 6 (left panel) in dashed where unification is explicit. Of course sensitivity with the scale is large and to draw firm conclusions on unification predictions we should control all threshold effect at the scale $\mu_{GUT}$ from the underlying (string) theory.

7 Conclusions

In this paper we have presented a 5D model with the fifth dimension compactified on the orbifold $S^1/Z_2$, where the parity $Z_2$ is defined by $x_5 \rightarrow -x_5$ and an appropriate lifting to spinor and $SU(2)_R$ indices. The gauge bosons are propagating in the bulk ($U$-states) while matter and Higgs fields can either propagate in the bulk or be localized on the 3-branes.

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13 Since $SU(3)$ is asymptotically free in the SM, the smaller $1/R$ the larger $\alpha_3(1/R)$ and the smaller $\mu_{NP} R$.

14 In the spirit of Refs. [30].
at the orbifold fixed points (T-states). The orbifold selection rules allow superpotential interactions on the branes of the type $UUU$ and $UTT$, where the former are suppressed as $(\Lambda_s R)^{-1}$ with respect to the latter. We then assume a superpotential of the form $UTT$.

Radiative finite electroweak symmetry breaking is triggered by right-handed stops propagating in the bulk, where supersymmetry is broken by a Scherk-Schwarz mechanism that uses the $U(1)_R$ subgroup of the $SU(2)_R$ ($R$-symmetry) left unbroken by the compactification. The Scherk-Schwarz parameter $\omega$ is considered as a free parameter in this paper. To allow fermion masses and superpotential interactions on the brane, consistent with the orbifold selection rules, we must assume that $SU(2)_L$ singlets propagate in the bulk while $SU(2)_L$ doublets are localized on the 3-brane, e.g. at the fixed point $x_5 = 0$. In this way the 4D theory of zero modes and localized states constitute the MSSM ($N = 1$), while that of massive modes possesses $N = 2$ supersymmetry in four dimensions.

The Higgs sector is localized on the brane and then the $\mu$-parameter does not arise through compactification, although it is an allowed term in the superpotential. One has to consider it as an effective parameter from the underlying, supergravity or string, theory. In this sense the situation is better than in the MSSM since the cutoff $\Lambda_s$ is in the TeV range.

We have proven that electroweak breaking is induced by finite radiative corrections triggered by the top/stop sector, $\mu$ and $\omega$ remaining as the only free parameters. In fact, the compactification radius and $\tan \beta$ are fixed by the minimization procedure. While $\tan \beta$ is large, $\tan \beta \sim 40$, $1/R$ is in the range, $1 - 15$ TeV, depending on the values of $\mu$ and $\omega$. The lightest Higgs mass is similar to the MSSM one [31] for large $t_\beta$, in particular for large values of $1/R$ for which the heavy KK-modes can be integrated out. We have checked that their integration produces threshold effects that increase the lightest Higgs mass by an amount $\lesssim 5$ GeV so we expect that once we include the genuine MSSM two-loop corrections [32] the final value predicted for the model will not differ much from the MSSM one. On the other hand LEP searches on the MSSM Higgs sector imply (\omega dependent) lower bounds on the $\mu$ parameter and the mass of the SM-like Higgs, $M_h$. They translate into the absolute bounds $\mu \gtrsim 350$ GeV and $M_h \gtrsim 145$ GeV.

The supersymmetric mass spectrum has a well defined pattern. The heaviest states are right-handed sfermions, the gauginos and the gravitino, with a mass $\sim \omega/R$, and higgsinos, with a mass $\sim \mu$. The other supersymmetric partners (left-handed sfermions) receive radiative masses from the supersymmetry breaking in the bulk. The next to heaviest states are the left-handed squarks which receive radiative masses from the gluon/gluino sector. We have found that the lightest supersymmetric particles are the sneutrinos, and the next to lightest supersymmetric particles the charged sleptons, with a squared mass difference between them $\sim M_Z^2$.

The gauge and Yukawa couplings run linearly with the scale $\mu$ (a reflection of the non-renormalizability of the 5D theory). The theory remains perturbative for scales $\mu R \lesssim 40$ while $\alpha_1$ and $\alpha_3$ unify at $\mu R \approx 30$. This is a great difference with respect to recently proposed models, where all particles propagate in the bulk [9, 10] and that rely on superpotential interactions of the type $UUU$. In this case wave function renormalization involves one-loop diagrams with two bulk states propagating and vertices that do not conserve the KK-number. This translates into quadratic running for the Yukawa couplings and make the theory non-perturbative for $\mu R \approx 3 - 6$. Since all properties of the
theory rely on summing over the infinite tower of KK states, having such a low cutoff is a drawback which can make threshold effects from heavy KK-modes non-negligible.

Concerning the recent related works already mentioned, Ref. [9] uses a single Higgs hypermultiplet in the bulk and all matter fields propagating in the bulk. It breaks $N=1$ supersymmetry by a SS-mechanism using $R$-parity as the global symmetry, which amounts to choosing $\omega = 1/2$ as the SS-parameter. On the one hand, since they have a single Higgs field, they solve the $\mu$-problem by nullification. On the other hand, they have to rely on $UUU$ Yukawa couplings to give masses to quarks and leptons, which makes, as we noticed above, the theory non-perturbative for low scales. The prediction on the Higgs mass ($\sim 128$ GeV) is reached by our model for the particular case $\omega = 1/2$, $\mu = 0$, although this is a non-physical point for our theory. Finally the LSP in Ref. [9] is the stop, also making a big difference with respect to our theory which predicts the sneutrino as the LSP.

Concerning Ref. [10], its authors use two Higgs multiplets (either in the bulk or localized on the brane) and also all matter fields propagating in the bulk. Supersymmetry is broken by the SS-mechanism using $R$-parity, i.e. again $\omega = 1/2$. Since massive gauginos form Dirac fermions, they cannot generate radiatively the $m_3^2 H_1 \cdot H_2$ term in the potential and this model has to rely on non-renormalizable terms in the superpotential, as $\lambda (H_1 \cdot H_2)^2 / \Lambda_s$ that, along with the $\mu$-term, generate the $m_2^2$-parameter, which then gets a sensitivity to the cutoff scale, at the tree level. Also Yukawa couplings renormalize quadratically with the scale and so the theory becomes non-perturbative at low scales.

Let us finally conclude by emphasizing the beauty of the mechanism of electroweak breaking triggered by a top-quark propagating in the bulk of an extra dimension. In our opinion it constitutes a step forward in our understanding of electroweak breaking since the generated Higgs field instability at the origin is neither “imposed” by hand at tree-level nor sensitive to the Standard Model cutoff, in spite of its radiative origin. Of course the top-quark propagation in the bulk is not without phenomenological consequences, that should be worked out in future works and must be tested in future colliders. On the other hand, the pattern of supersymmetric mass spectrum should give a hint on the particular mechanism of supersymmetry breaking realized by the Nature.

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References


\textsuperscript{15}Actually Ref. [10] also proposes another, non-SS mechanism for supersymmetry breaking based on a strong supersymmetry breaking localized on a hidden brane that has little relation with that used in this paper and that we will not comment on here.


