Gravitational waves and quantum gravity

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Can GWs probe quantum gravity?

Production of GWs [Yunes, Yagi, Pretorius 2016]

Propagation of GWs

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■ **Modified Kerr black holes**

EMRIs kludge waveform [Canizares et al. 2012]

■ **Primordial blue-tilted spectra**

Stochastic GW background [G.C., Kuroyanagi 2021]

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Propagation of GWs

■ **Modified dispersion relations** [Ellis et al. 2015 Arzano, G.C. 2016]

Propagation speed

■ **Waveform phase** [Mirshekari et al. 2012]

■ **Luminosity distance** [Belgacem et al. 2019; G.C. et al. 2019a,b]

■ **Primordial blue-tilted spectra**

Stochastic GW background [G.C., Kuroyanagi 2021]

Modified dispersion relations

Modified dispersion relation for the graviton ($\Delta v = |\mathrm{d}\omega/\mathrm{d}k - 1|$ group velocity) [Arzano & G.C. PRD 2016]

$$\omega^2 = k^2 \left(1 \pm \frac{k^n}{M^n}\right) + O\left(k^{n+3}\right) \qquad \Rightarrow \qquad M \simeq \frac{\omega}{\Delta v^{\frac{1}{n}}}$$

GW150914: $\omega \approx 630\,\mathrm{Hz} \approx 10^{-13}$ eV, $|\Delta v| < 4 \times 10^{-20}$ [Abbott et al. 2016].

To get $M > 10\,\mathrm{TeV}$,

$$0 < n < 0.76\,.$$

This range is typical of field theories on multifractal geometries [G.C. 2012-2017], where $n = 1 - d_\mathrm{H}/4$ is related to the UV Hausdorff dimension of spacetime. For the typical $d_\mathrm{H} = 2$,

$$M > 10^{17}\,\mathrm{GeV}\,, \qquad n = 0.5$$

Luminosity distance

GW luminosity distance:

$$h \propto \frac{1}{d_L^{\mathrm{GW}}} \overset{\text{in GR}}{=} \frac{1}{d_L^{\mathrm{EM}}}$$

Known example (LIGO-Virgo/Fermi): BNS GW170817 / GRB170817A [Abbott et al. 2017]

$$\frac{d_L^{\mathrm{GW}}}{d_L^{\mathrm{EM}}} = 1 \pm |\gamma - 1| \left(\frac{d_L^{\mathrm{EM}}}{\ell_*}\right)^{\gamma-1}$$

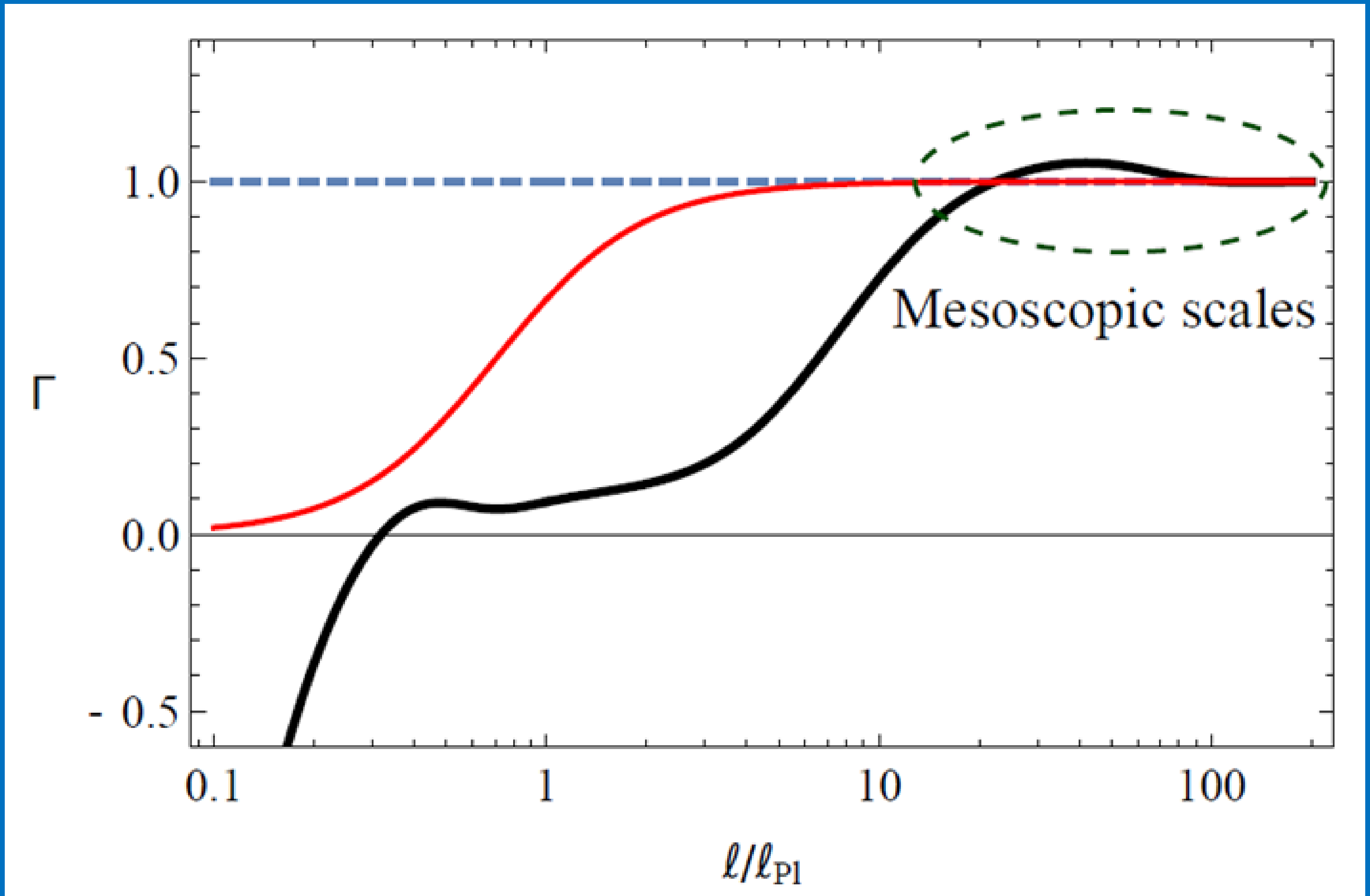
G.C. et al., PLB 2019; JCAP 2019

Detectable QG effect if $\gamma \gtrsim 1$, even when $\ell_* = O(\ell_{\mathrm{Pl}})$

Luminosity distance

$$S = \frac{1}{2}\int \mathrm{d}\varrho(x)\, h_{ij}\mathcal{K}(\Box)\, h^{ij}, \qquad [h_{ij}] = \frac{d_\mathrm{H} - [\mathcal{K}]}{2} =: \mathbf{\Gamma}$$

$$h_{ij} = \frac{\kappa \mathcal{F}_{ij}(t-r)}{(r^2)^{\frac{\Gamma}{2}}} \sim \frac{1}{r^\Gamma} \to \frac{1}{(d_L^{\mathrm{EM}})^\Gamma}$$



G.C. et al., PLB 2019;
JCAP 2019

LQG/spin foams/GFT states?

New model of dark energy

- Luminosity distance of standard sirens are **sensitive to dark energy**

Nonanalytic nonlocality: **G.C., CQG, arXiv:2102.03363, 2106.15430**

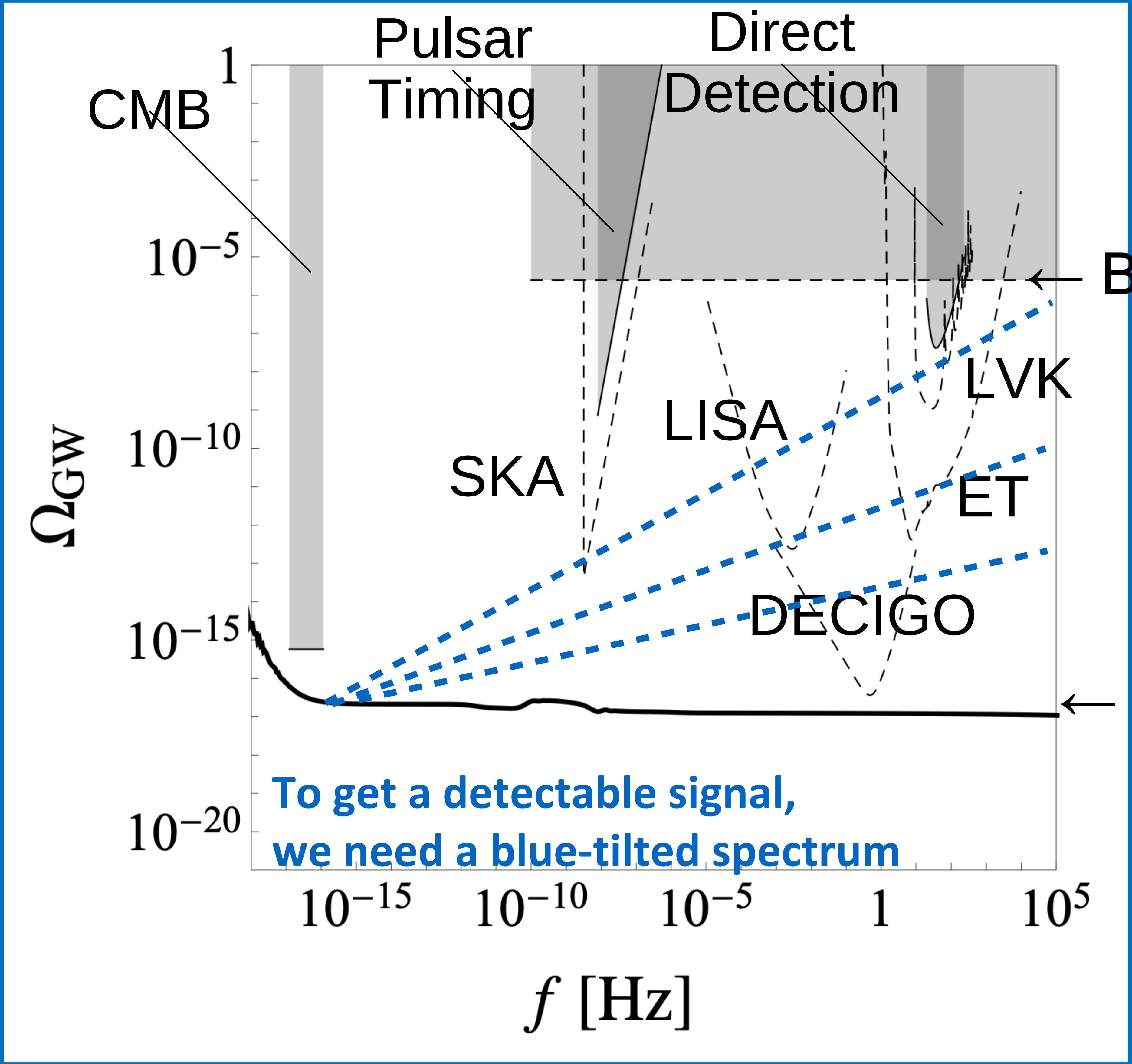
$$S = \frac{1}{2\kappa^2}\int d^D x\,\sqrt{|g|}\,\left[R - 2\Lambda + \ell_*^2 G_{\mu\nu}(-\ell_*^2\Box)^{\gamma-2}\,R^{\mu\nu}\right]$$

$$(-\Box)^{\gamma-1} = \frac{1}{\Gamma(1-\gamma)}\int_0^{+\infty} d\tau\,\tau^{-\gamma}e^{\tau\Box}$$

- Derived from first principles of multifractal geometry.
- Unitary if **0< γ <1**, power-counting renormalizable if **γ >2**, 1-loop renormalizable if **γ >1, ≠ 3/2, 2**.
- Reproduces IR nonlocal models for $\gamma \to 0,1$ [Barvinsky 2005-2021, Maggiore et al. 2014-2020, Cusin, Ferreira, Foffa, Maroto, Nersisyan, …]

$$\mathcal{L} = R + c_0 R\Box^{-n_0}R + c_2 R_{\mu\nu}\Box^{-n_2}R^{\mu\nu}$$

Gravitational waves from inflation



$$\Omega_{GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k}$$

Starobinsky inflation: $r \sim 10^{-3}$

→ not detectable by interferometers

→ cannot be realized by standard slow-roll inflation

Starobinsky inflation

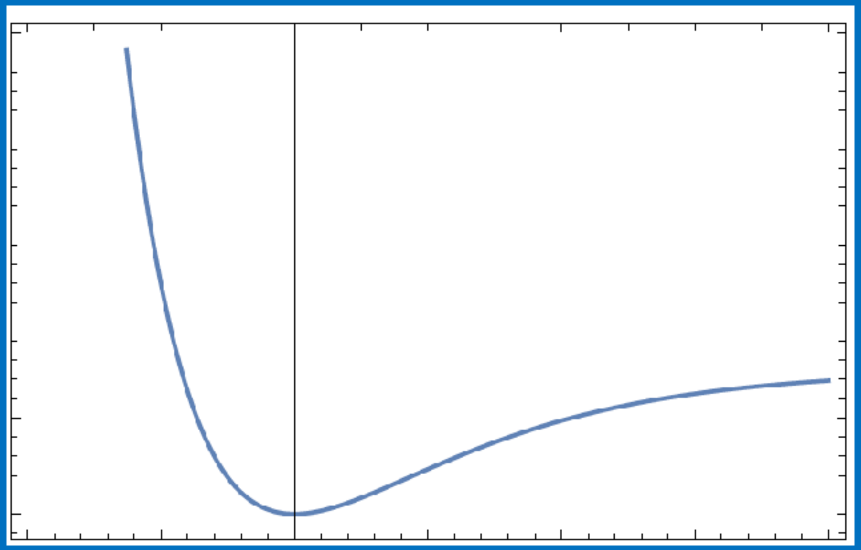
Action

Starobinsky PLB (1980)

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{|g|} \left[R + \frac{R^2}{6M_*^2} \right]$$

curvature-squared correction to the Einstein–Hilbert action

V(ϕ)

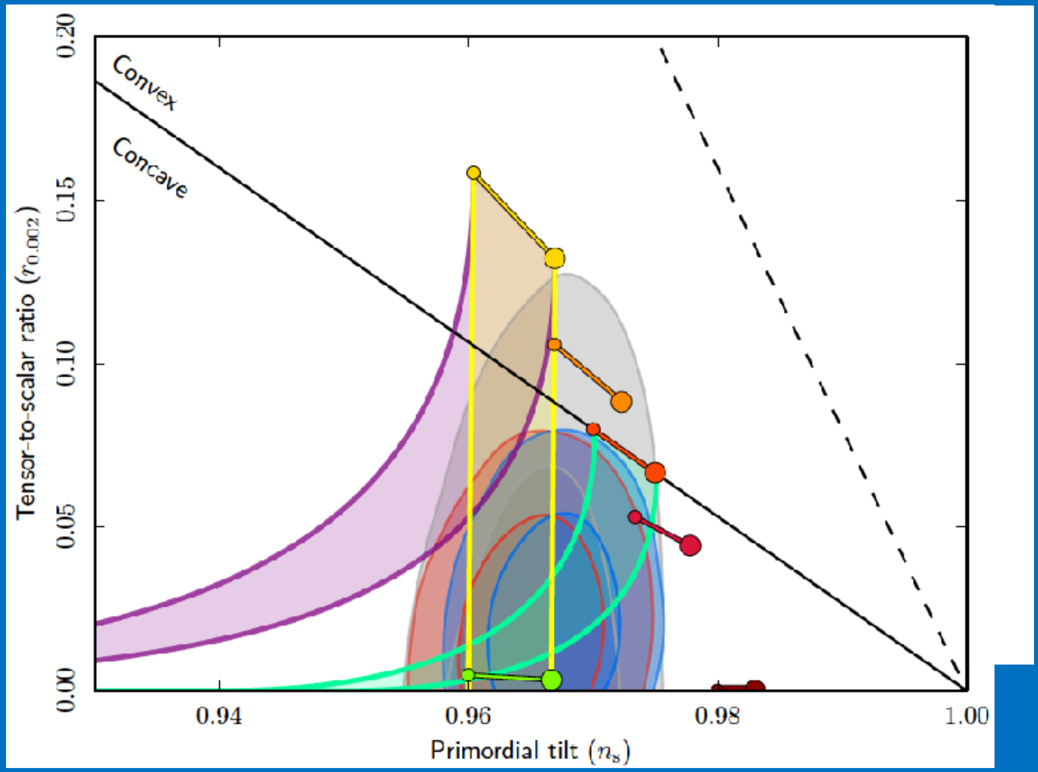


ϕ

Einstein frame

$$S = \int d^4x \sqrt{|g|} \left[\frac{\hat{R}}{2\kappa^2} - \frac{1}{2} \hat{\partial}_\mu \phi \hat{\partial}^\mu \phi - V(\phi) \right]$$

$$V(\phi) = \frac{3M_*^2}{4\kappa^2} \left(1 - e^{-\sqrt{\frac{2}{3}}\kappa\phi} \right)^2$$



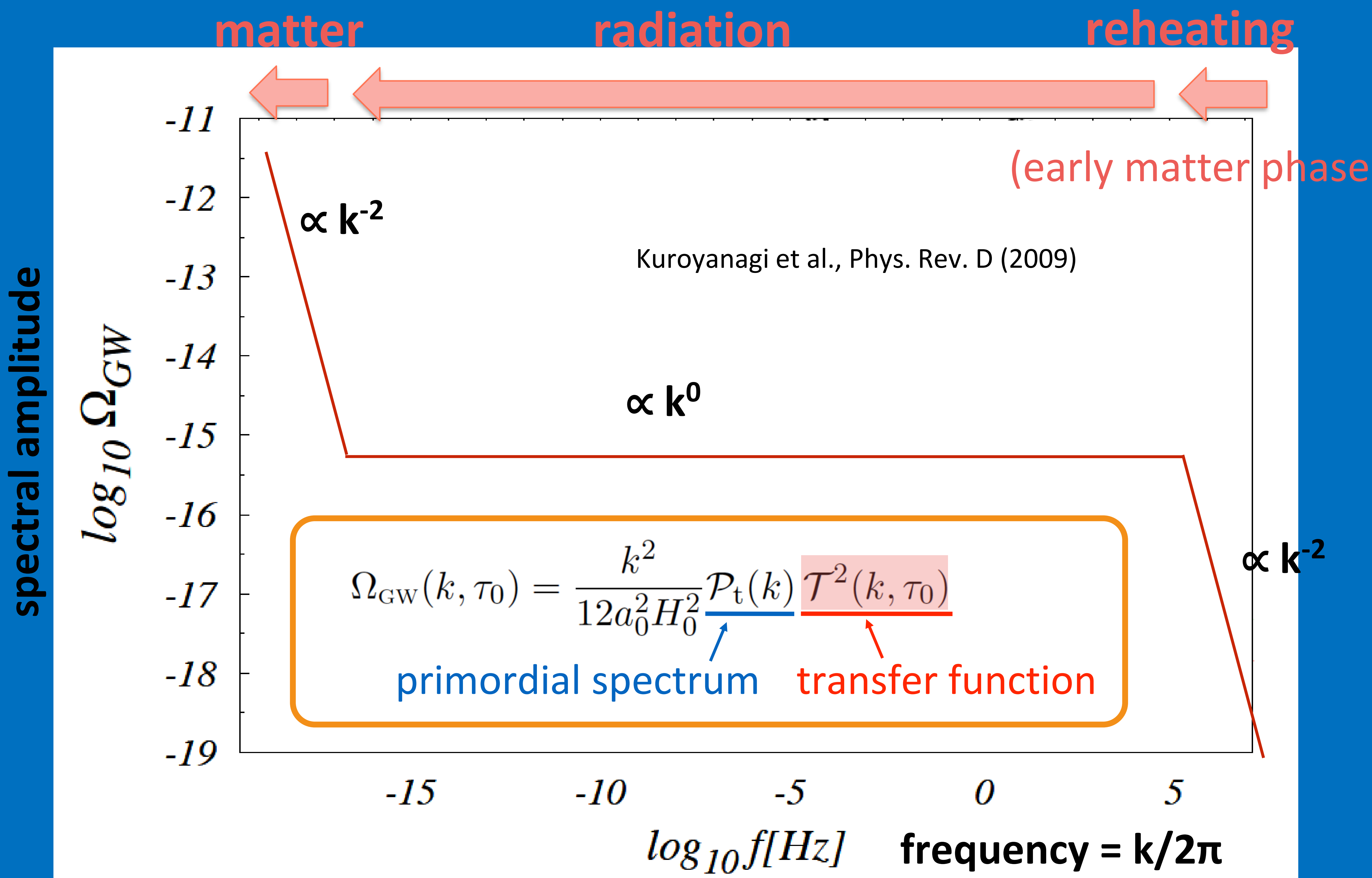
$n_s \simeq 0.967$

$r \simeq 3 \times 10^{-3}$

→ Good agreement with CMB observations

Spectral shape

Hubble expansion history affects the spectral shape



Theories beyond GR at inflationary energy scales can realize a blue-tilted spectrum

→ Models motivated by quantum gravity?

G.C., Kuroyanagi, JCAP (2021) [arXiv:2012.00170]

After discarding **many** models:

- **Nonlocal Starobinsky inflation**
- **Brandenberger–Ho noncommutative inflation**
- **Multifractional spacetimes**
- **String-gas cosmology**
- **New ekpyrotic scenario**

Nonlocal Starobinsky inflation

Action

A.S. Koshelev et al., JHEP 11 (2016) 067

$$S = \frac{1}{2\kappa^2}\int d^4x \sqrt{|g|}\,[R + R\,\gamma_S(\Box)R + C_{\mu\nu\rho\sigma}\gamma_C(\Box)C^{\mu\nu\rho\sigma}]$$

vanishes in a FLRW background

Embedded in quantum gravity

- Weyl tensor term is introduced to make the theory renormalizable
- Form factors are introduced to preserve unitarity (ghost freedom) and improve renormalizability

Briscese, Modesto, Tsujikawa PRD (2013); Koshelev, Modesto, Rachwal, Starobinsky JHEP (2016); Koshelev, Kumar, Starobinsky JHEP (2018); Koshelev, Kumar, Mazumdar, Starobinsky JHEP (2020)

Nonlocal Starobinsky inflation predicts a blue-tilted spectrum even up to n_t ~ 2 at the CMB scale?

Example

Tomboulis form factor

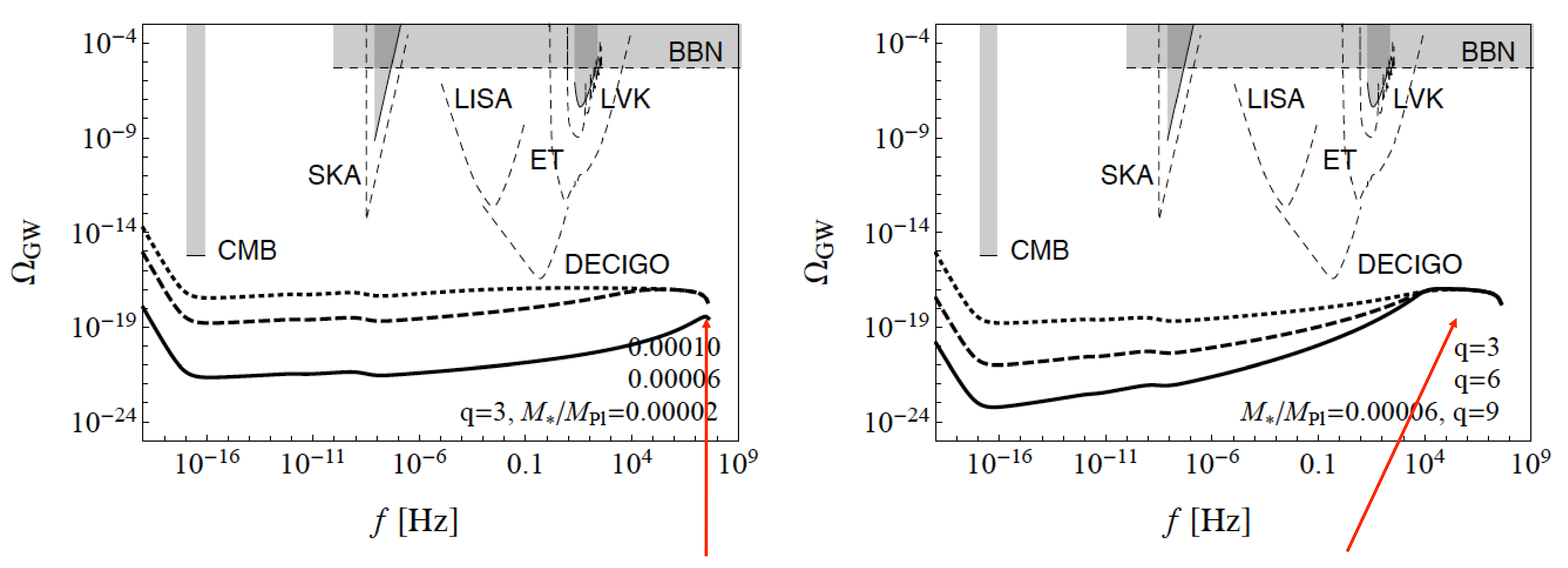
Tomboulis, hep-th/9702146
Modesto, PRD (2012)

$$\mathrm{H}_{\mathrm{Tom}}(z) := \frac{1}{2}\left\{\ln p^2(z) + \Gamma[0, p^2(z)] + \gamma_{\mathrm{E}}\right\}$$

$$p^2(z) = z^q$$

exact formula

$$\mathcal{P}_{\mathrm{t}} \simeq \frac{m^2}{2\pi^2 M_{\mathrm{Pl}}^2}(1-3\epsilon)\, e^{-\tilde{\mathrm{H}}_2(z_*)}$$



converges to standard Starobinsky inflation at high frequency

Instant reheating is assumed

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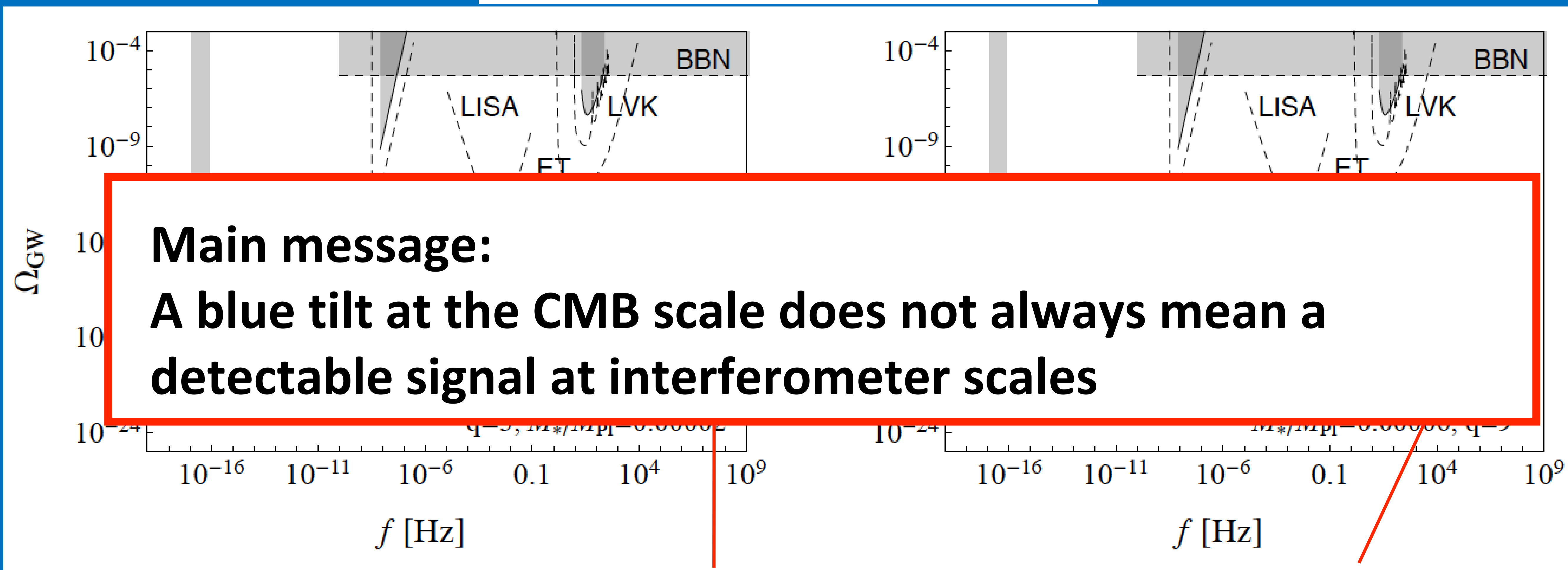


Main message:
A blue tilt at the CMB scale does not always mean a detectable signal at interferometer scales

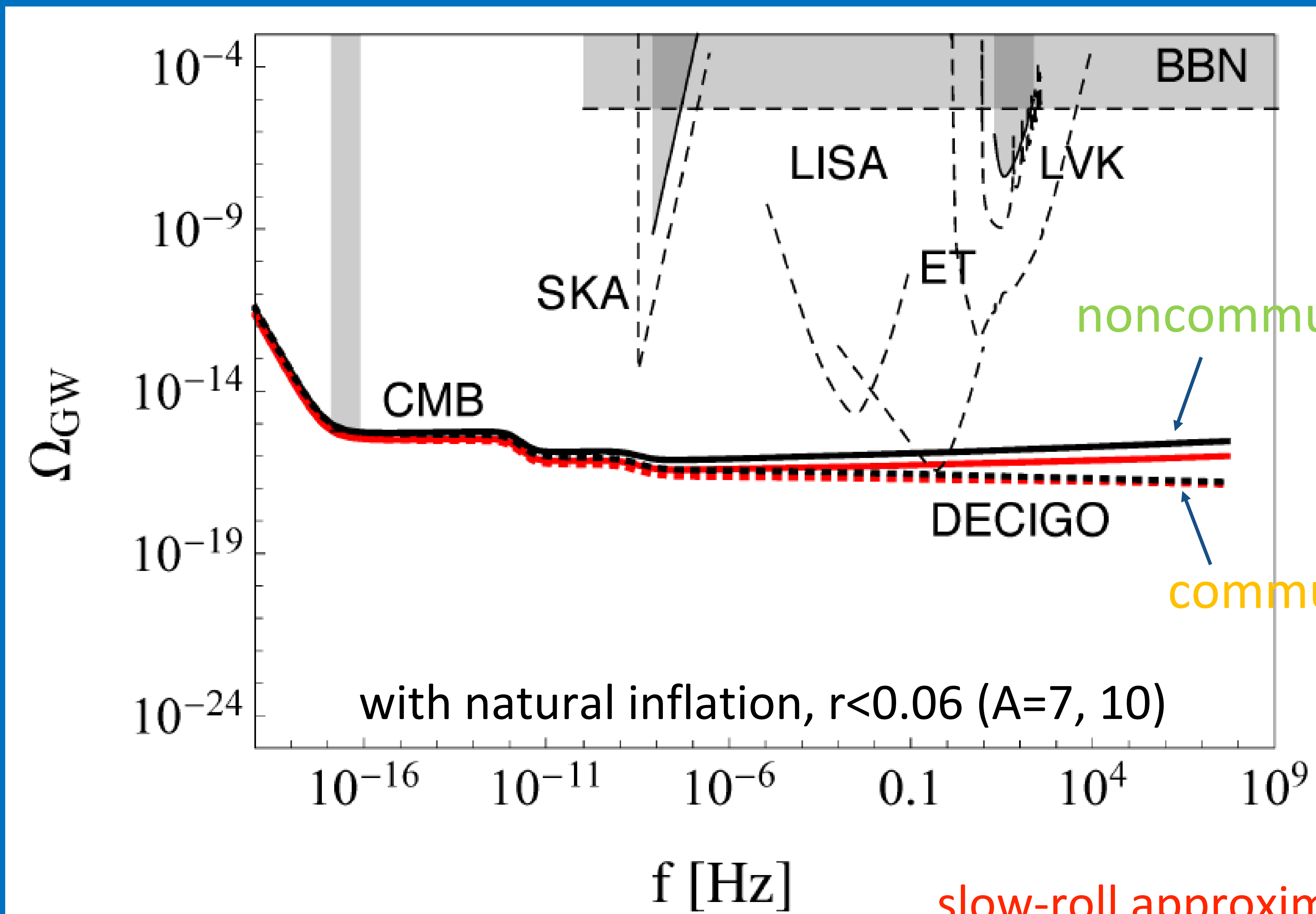
converges to standard Starobinsky inflation at high frequency

Instant reheating is assumed

Brandenberger–Ho noncommutative inflation

Brandenberger, Ho, PRD 2002

Time and space coordinates do not commute: $[\tilde{\tau}, x] = i/M^2$



noncommutative case

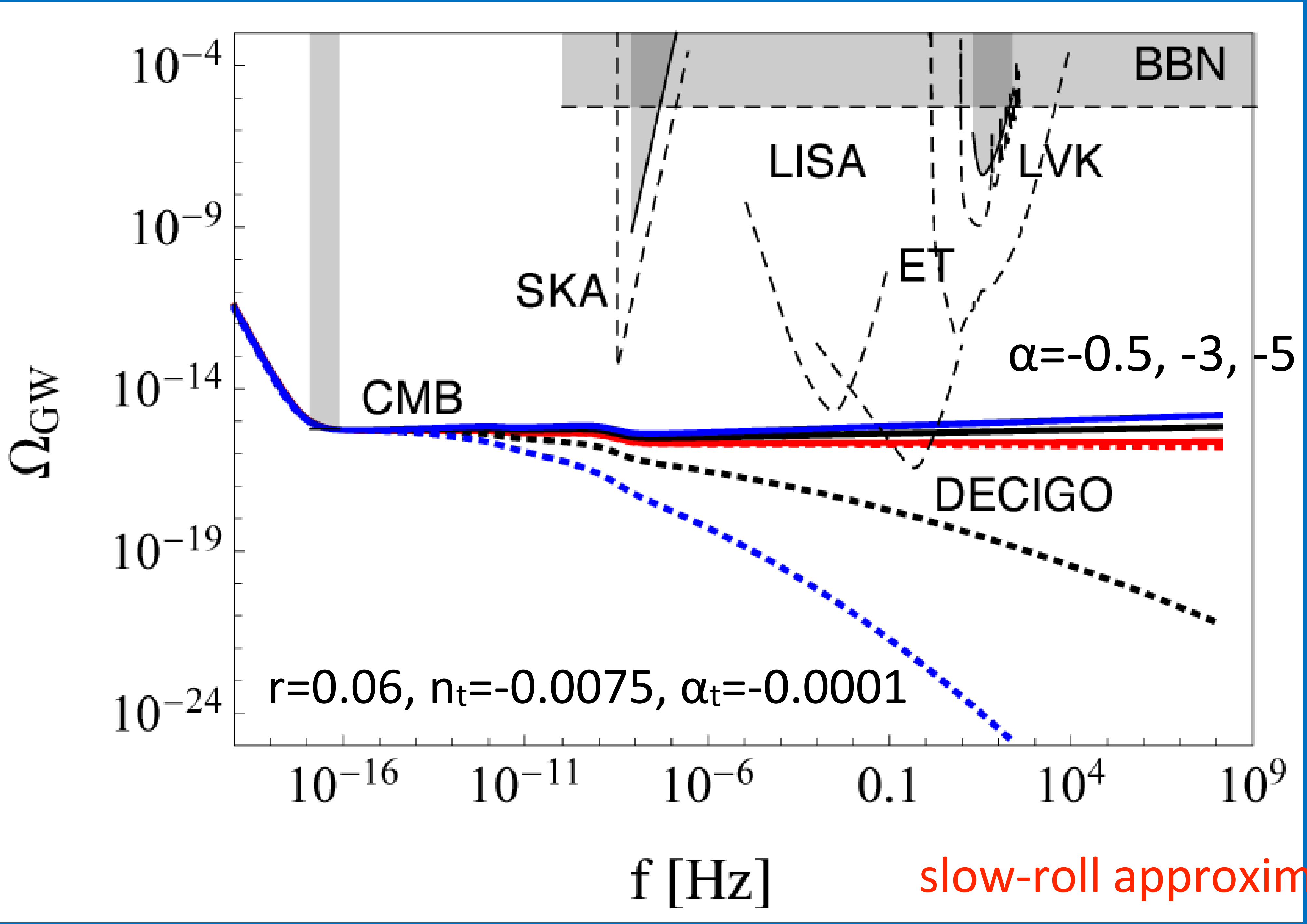
commutative case

slow-roll approximation is valid

Multifractional spacetimes

G.C., JHEP 2017; G.C., MPLA 2021

The dimension of spacetime changes with the probed scale



$d_S = d_H = \alpha_0 + 3\alpha$

$n_t \to \alpha n_t$

$\alpha_t \to \alpha^2 \alpha_t$

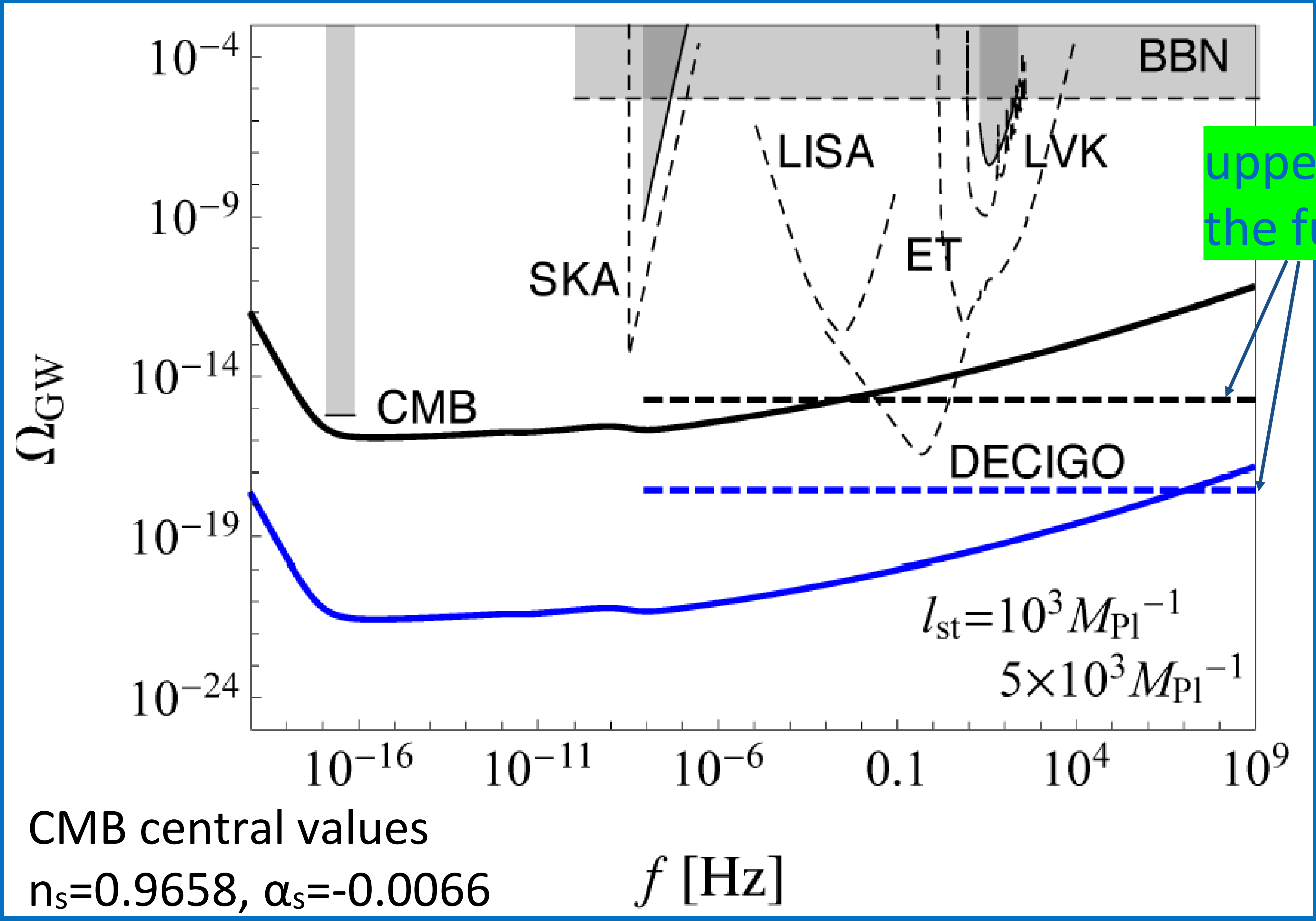
slow-roll approximation is valid

String-gas cosmology

Brandenberger et al., PRL 2007

produces both scalar and tensor primordial spectra
Via thermal mechanism alternative to inflation

$n_t \sim 1 - n_s$



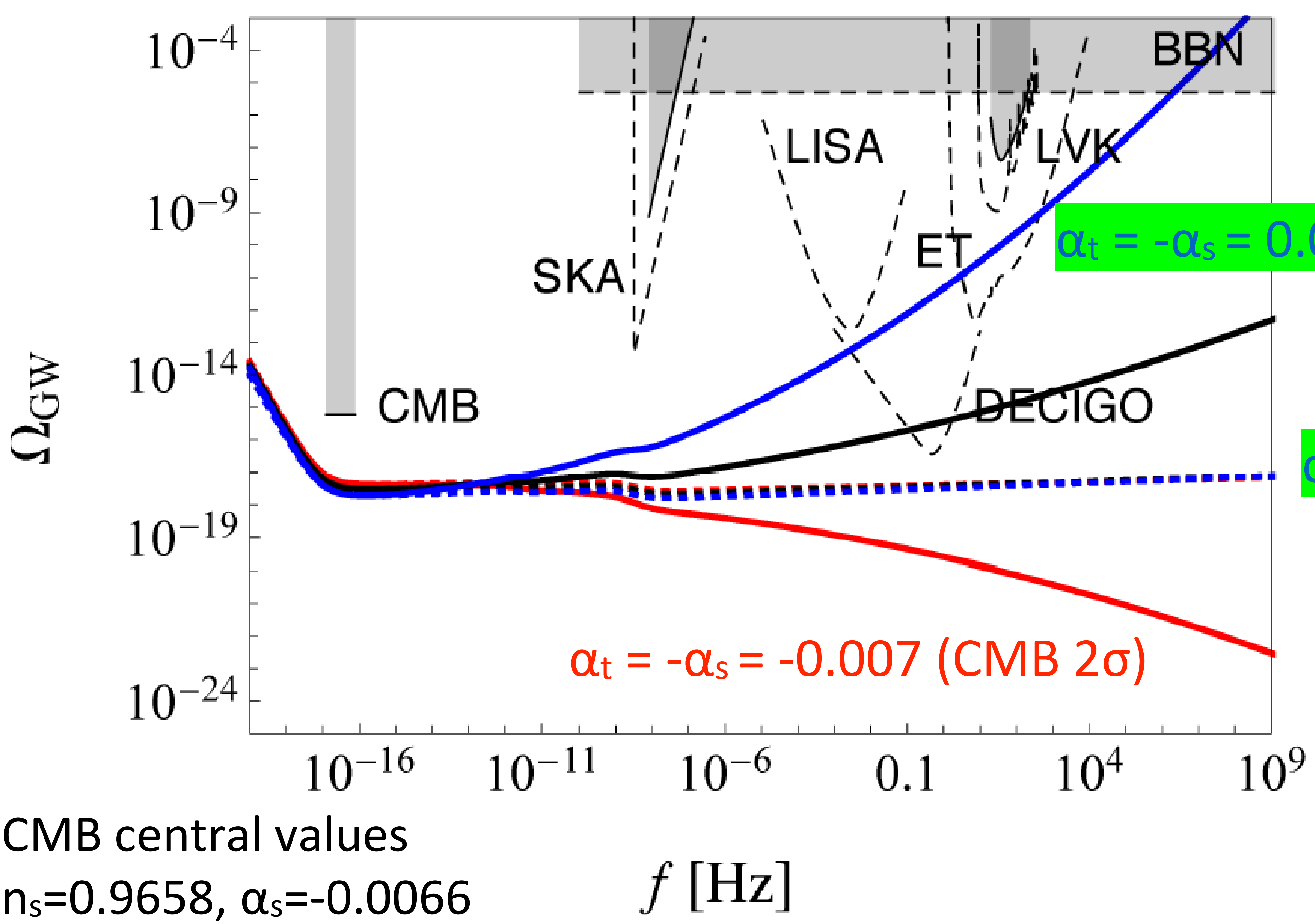
upper bound from the full expression

CMB central values
n_s=0.9658, α_s=-0.0066

New ekpyrotic scenario

Brandenberger, Wang, PRD 2020

contraction (ekpyrosis) + bounce + expansion
perturbations generated by brane collision (+ S-brane)



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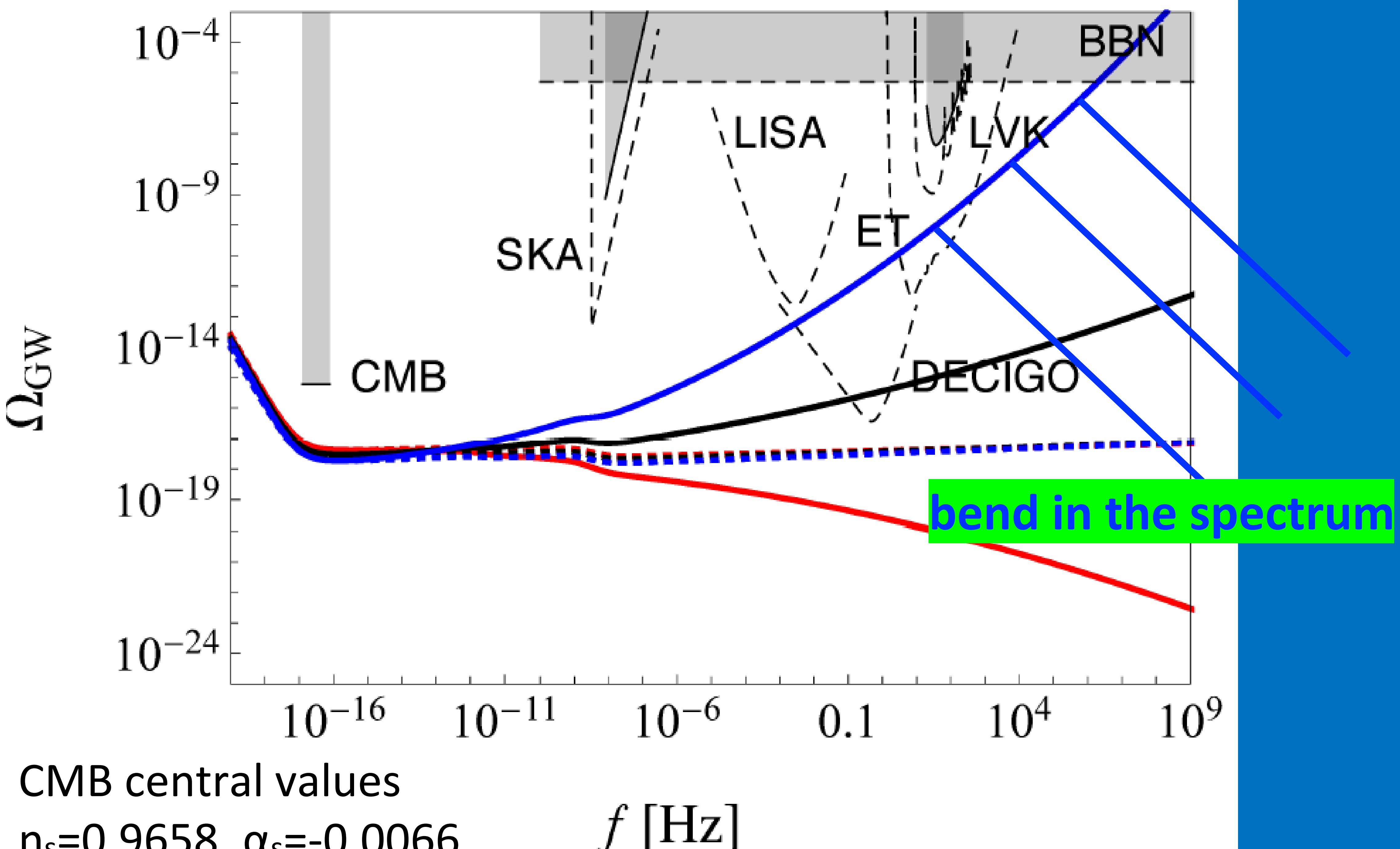

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Summary

GWs can become a powerful probe of QG in the very early and late-time Universe

- QG **dispersion relations** are tightly constrained
- Luminosity distance of standard sirens can yield important information on GW propagation in QG
- Some models of/motivated by QG predict a SGWB detectable by DECIGO or ET, but likely not by LISA

Thank you!