ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

NUCLEAR
PHYSICS B

Nuclear Physics B 663 [FS] (2003) 605–621

www.elsevier.com/locate/npe

Large N effects and renormalization of the long-range Coulomb interaction in carbon nanotubes

S. Bellucci ^a, J. González ^b, P. Onorato ^{a,c}

^a INFN, Laboratori Nazionali di Frascati, C.P. 13, 00044 Frascati, Italy

^b Instituto de Estructura de la Materia, Consejo Superior de Investigaciones Científicas, Serrano 1223, 28006 Madrid, Spain

^c Dipartimento di Scienze Fisiche, Università degli Studi di Napoli “Federico II”, Via Cintia, I-80126 Napoli, Italy

Received 6 February 2003; accepted 2 May 2003

Abstract

We develop a dimensional regularization approach to deal with the low-energy effects of the long-range Coulomb interaction in 1D electron systems. The method allows us to avoid the infrared singularities arising from the long-range Coulomb interaction at $D = 1$, providing at the same time insight about the fixed-points of the theory. We show that the effect of increasing the number N of subbands at the Fermi level is opposite to that of approaching the bare Coulomb interaction in the limit $D \rightarrow 1$. Then, we devise a double scaling limit, in which the large N effects are able to tame the singularities due to the long-range interaction. Thus, regular expressions can be obtained for all observables right at $D = 1$, bearing also a dependence on the doping level of the system. Our results imply a variation with N in the value of the exponent for the tunneling density of states, which is in fair agreement with that observed in different transport experiments involving carbon nanotubes. As the doping level is increased in nanotubes of large radius and multi-walled nanotubes, we predict a significant reduction of order $N^{-1/2}$ in the critical exponent of the tunneling density of states.

© 2003 Elsevier B.V. All rights reserved.

E-mail addresses: bellucci@lnf.infn.it (S. Bellucci), imtg64@pinar2.csic.es (J. González), pasquale.onorato@libero.it (P. Onorato).

1. Introduction

In the last decade carbon-based devices have been largely studied as a possible new frontier of microelectronics. The limits of further miniaturization (predicted by Moore's law) have increased the research toward the development of molecular electronics and new efforts of scientists have been stimulated by the progress in carbon technology, in order to build a carbon based microelectronics [1,2]. Carbon nanotubes are the bricks of this new building and recent experiments have revealed that they are also excellent systems for the investigations of electronic transport in one dimension. It has been shown that the single-walled nanotubes can have semiconducting or metallic behavior, depending on the helical arrangement of the carbon rings around the tubule [3]. The metallic nanotubes have all a typical band structure at low energies, in which two different subbands cross at opposite Fermi points (in the undoped system).

It is well known that the so-called Luttinger liquid behavior describes the regime with absence of electron quasiparticles that is characteristic of one-dimensional (1D) electron systems with dominant repulsive interactions [4–6]. In the case of the undoped metallic nanotubes, the fact of having four low-energy linear branches at the Fermi level introduces a number of different scattering channels, depending on the location of the electron modes near the Fermi points [7]. It has been shown, however, that processes which change the chirality of the modes, as well as processes with large momentum-transfer, are largely subdominant with respect to those between currents of like chirality [8,9]. It has been concluded, therefore, that the metallic carbon nanotubes should fall into the Luttinger liquid universality class [9,10]. Supporting that expectation, characteristic experimental signatures such as the power-law behavior of the tunneling density of states have been measured in ropes [11] as well as in individual single-walled nanotubes [12].

From the theoretical point of view, a relevant question is the determination of the effects of the long-range Coulomb interaction present in carbon nanotubes. In most part of the analyses carried out for such systems, the electron–electron interaction is taken actually as short-range, on the assumption that the finite length of the experimental samples has to impose in practice an infrared cutoff on the interaction. It is known, however, that the Coulomb interaction is not screened in one spatial dimension [13,14], and that it remains long-range even after coupling a parallel array of 1D systems—although the strength of the interaction can be consequently reduced in this case [15,16].

The issue of considering the effects of the long-range Coulomb interaction is significant, as they have been shown to lead in general to unconventional electronic properties [17]. Such effects have been shown to be responsible for a strong attenuation of the quasiparticle weight in graphite [18]. The long-range Coulomb potential $V(|\mathbf{r}|) \sim 1/|\mathbf{r}|$ yields the main electron interaction in carbon nanotubes as well as in the two-dimensional (2D) layers of graphite, which have a vanishing density of states at the Fermi level. An important confirmation of the marginal Fermi liquid behavior in such 2D layers comes from the experimental measurements in graphite of a quasiparticle decay rate linear in energy [19]. Owing to the singular nature of the Coulomb interaction, the imaginary part of the electron self-energy in the 2D layer behaves like $g^2\omega$ at weak g coupling [20]. On the other hand, the effective coupling scales at low energy like $g \sim 1/\log(\omega)$. This fact explains that

the quasiparticle weight is not driven to zero by its logarithmic renormalization, being corrected instead by terms of order $g^2 \log(\omega) \sim 1/\log(\omega)$ [21].

The experiments on tunneling into the bulk of multi-walled nanotubes (MWNTs) have shown a power-law behavior of the tunneling density of states, with an exponent $\alpha \sim 0.3$ [22]. This is close to the values measured in single-walled nanotubes [11,12], but it is much larger than expected in a wire with a large number N of subbands, where there should be a significant reduction in the strength of the electron–electron interaction. There have been attempts to confront this puzzle under the assumption of a disorder-enhanced renormalization of the interaction [23]. However, the exponents predicted in this treatment display a typical $1/N$ reduction that make them to fall short with respect to the experimental estimate [22]. The enhanced suppression of tunneling in the MWNTs supports therefore the view that the long-range character of the Coulomb interaction has to affect the system in a sizeable way.

In a recent paper [24], we developed an analytic continuation in the number D of dimensions to accomplish the renormalization of the long-range Coulomb interaction at $D \rightarrow 1$. This limit turns out to be highly singular, since the marginal Fermi liquid behavior characteristic of the 2D graphene sheet is established as soon as one departs from $D = 1$. However, the attenuation of the electron quasiparticles is increasingly strong as $D \rightarrow 1$, leading to an effective power-law behavior of observables like the density of states. In this way, we were able to predict a lower bound of the corresponding exponent that turned out to be very close to the value measured in experimental observations of the tunneling conductance for MWNTs [22].

In this paper, we focus on the singular behavior that arises at the limit $D \rightarrow 1$ and introduce the effect of the number of subbands that contribute to the low-energy properties of carbon nanotubes. This issue is relevant for the investigation of the nanotubes of large radius that are present in the MWNTs, which are usually doped and may have a large number of subbands crossing the Fermi level [25]. From a technical point of view, it becomes suitable the implementation of a large- N approach in the renormalization of the electron system. Increasing the number of subbands has the effect opposite to approaching the bare Coulomb interaction in the limit $D \rightarrow 1$. We will see that it is possible to take a kind of double scaling limit, $N \rightarrow \infty$ and $D \rightarrow 1$, in which the large- N effects are able to tame the singularities related to the long-range interaction. Thus, regular expressions can be obtained for all observables right at $D = 1$, bearing also a dependence on the doping level of the nanotube system.

The property of the double scaling limit, of rendering finite the effects of the renormalized long-range interaction, can be explained by the onset of some sort of dynamical screening, in the limit of an infinite number of subbands. Of course, in the real carbon nanotubes of a multi-walled sample, the number of subbands at the Fermi level may be large but finite, and the singularity of the Coulomb potential is not precisely reached due to the finite size of the system. However, the double scaling limit describes the universality class capturing the physics of the doped multi-walled samples, as any other 1D approach to these systems is bound to bear the divergence of the long-range interaction in the low-energy limit.

In this paper we deal with a renormalization group (RG) approach, in order to obtain the low-energy behavior of carbon nanotubes with long-range Coulomb interaction and a large

number of subbands at the Fermi level. This approach is well-suited to the description of the scaling properties of observables like the quasiparticle weight or the density of states near the Fermi level. We will see that the renormalization introduces a factor of $1/\sqrt{N}$ in the critical exponent for those quantities. Our results can be compared with those from the experiments reported in Ref. [22], where measurements of the tunneling conductance have been carried out in doped MWNTs, with a number of subbands at the Fermi level $N \approx 10$ (in the outer layer). For such values of N , and taking into account the long-range Coulomb interaction, the critical exponents that we obtain match well the values estimated from the experiments.

2. Band structure of graphene and carbon nanotubes

We describe the band structure of carbon nanotubes by the technique of projecting the band dispersion of a 2D graphite layer into the 1D longitudinal dimension of the nanotube. The 2D band dispersion of graphene can be found in Ref. [26]. It consists of an upper and a lower branch that only touch at the corners of the hexagonal Brillouin zone. Thus, when the system is at half-filling, the metallic properties derive from a pair of inequivalent Fermi points, around which there is conical dispersion for the modes of the graphene sheet.

The 2D layers in graphite have a honeycomb structure with a simple hexagonal Bravais lattice and two carbon atoms in each primitive cell. We can start from the wavefunction associated to the electronic density, where we introduce two variational parameters, corresponding to two orbitals localized around the two carbon atoms in the primitive cell. Then, by introducing the wavefunctions in the energy functional for the lattice Hamiltonian, we can solve the variational problem and obtain the following band dispersion, shown in Fig. 1(a) [26]:

$$E(\mathbf{k}) = \pm \gamma \sqrt{1 + 4 \cos^2\left(\frac{\sqrt{3}}{2}k_x\right) + 4 \cos\left(\frac{\sqrt{3}}{2}k_x\right) \cos\left(\frac{3}{2}k_y\right)}. \quad (2.1)$$

Since the basis of the honeycomb lattice contains two atoms, there are two sublattices and two degenerate Bloch states at each Fermi point. If we choose the Bloch functions separately on each sublattice such that they vanish on the other, then we can expand the electron operator in terms of the Bloch waves

$$\Psi_\sigma(x, y) \sim \sum_{p\alpha} \exp(-i\alpha\mathbf{K} \cdot \mathbf{r}) F_{p\alpha\sigma}(x, y), \quad (2.2)$$

where $\alpha = \pm$ labels the Fermi point, $\mathbf{r} = (x, y)$ lives on the sublattice $p = \pm$ under consideration and $F_{p\alpha\sigma}(x, y)$ denote slowly varying operators. Thus, we can conclude that the low-energy excitations of the honeycomb lattice at half-filling are described by an effective theory of two 2D Dirac spinors [26].

Starting from the graphene band structure (2.1), after introducing periodic boundary conditions due to the cylindrical geometry of the tube, we obtain the energy bands of a

Fig. 1. (a) Energy levels in a graphite sheet: the cusps appear at the six corners of the first Brillouin zone. (b) Low-energy band structure of metallic carbon nanotubes with different radius. The energy is measured in units of the hopping parameter and the momentum in units of the inverse lattice spacing. As shown in (c), the radius is connected to the value of N_b in a simple way: $R = \sqrt{3}a/(2\pi \sin(\pi/N_b)) \approx N_b\sqrt{3}a/(2\pi)$, where a denotes the honeycomb lattice constant ($a/\sqrt{3} = 1.42 \text{ \AA}$).

carbon nanotube

$$E_m(k) = \pm \gamma \sqrt{1 - 4 \cos\left(\frac{\pi m}{N_b}\right) \cos\left(\frac{\sqrt{3}k}{2}\right) + 4 \cos^2\left(\frac{\sqrt{3}k}{2}\right)}, \quad (2.3)$$

where N_b is the number of periods of the hexagonal lattice around the compact dimension (in the y direction) of the cylinder. So, at half-filling, metallic nanotubes have two Fermi points corresponding to large momenta $\pm K_F$ (see Fig. 1(b)). Now the low-energy expansion (2.2) transforms correspondingly and the electron operator is written as [27]

$$\Psi_\sigma(x, y) = \sum_{p\alpha} (2\pi R)^{-1/2} \exp(-i\alpha \mathbf{K} \cdot \mathbf{r}) \psi_{p\alpha\sigma}(x) \quad (2.4)$$

which introduces 1D fermion operators $\psi_{p\alpha\sigma}(x)$ depending only on the longitudinal coordinate x .

For later use, we note that Eq. (2.3) implies a simple dependence of the number of subbands that are found below the energy $E = \gamma\varepsilon$

$$n_s(\varepsilon) = \frac{N_b}{\pi} \arcsin(\sqrt{1 - \varepsilon^2}) \approx \frac{N_b}{\pi} \left(\varepsilon + \frac{\varepsilon^3}{6} + \dots \right).$$

At low energies, we can substitute the function $n_s(\varepsilon)$ with the first term of its Taylor expansion without a large disagreement.

3. Dimensional regularization near $D = 1$

In a previous paper [24], we have developed an analytic continuation in the number of dimensions in order to regularize the infrared singularity of the long-range Coulomb interaction at $D = 1$ [28]. Our aim was to find the effective interaction between the low-energy modes of metallic nanotubes, which have linear branches crossing at the Fermi level. For this purpose, we have dealt with the analytic continuation at general dimension D of the linear dispersion around each Fermi point. We start then with the Hamiltonian

$$H = v_F \sum_{\alpha\sigma} \int_0^\Lambda dp |\mathbf{p}|^{D-1} \int \frac{d\Omega}{(2\pi)^D} \psi_{\alpha\sigma}^+(\mathbf{p}) \boldsymbol{\sigma} \cdot \mathbf{p} \psi_{\alpha\sigma}(\mathbf{p}) + e^2 \int_0^\Lambda dp |\mathbf{p}|^{D-1} \int \frac{d\Omega}{(2\pi)^D} \rho(\mathbf{p}) \frac{c(D)}{|\mathbf{p}|^{D-1}} \rho(-\mathbf{p}), \quad (3.5)$$

where the σ_i matrices are defined formally by $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$. Here $\rho(\mathbf{p})$ are density operators made of the electron modes $\psi_{\alpha\sigma}(\mathbf{p})$, and $c(D)/|\mathbf{p}|^{D-1}$ corresponds to the Fourier transform of the Coulomb potential in dimension D . Its usual logarithmic dependence on $|\mathbf{p}|$ at $D = 1$ is obtained by taking the 1D limit with $c(D) = \Gamma((D-1)/2)/(2\sqrt{\pi})^{3-D}$.

A self-consistent solution of the low-energy effective theory has been found in [24] by determining the fixed-points of the RG transformations implemented by the reduction of the cutoff Λ . A phenomenological solution of the model was firstly obtained [14], carrying a dependence on the transverse scale needed to define the 1D logarithmic potential, which led to scale-dependent critical exponents and prevented a proper scaling behavior of the model [14,29]. The dimensional regularization approach of Ref. [24], which we follow here, overcomes the problem of introducing such an external parameter.

In the present paper, we incorporate the effect of having a number N of subbands crossing the Fermi level, which multiply consequently the number of electron fields and terms in the Hamiltonian (3.5). Actually, as long as the long-range Coulomb potential is strongly peaked at $\mathbf{p} = 0$, we will focus on the dominant interactions that take place between currents in which the electron modes remain in the same linear branch.

Thus, the one-loop polarizability $\Pi(\mathbf{k}, \omega_k)$ is given by the sum of particle-hole contributions within each branch. For a given subband, this leads in one spatial dimension to the well-known result [5]

$$\Pi(\mathbf{k}, \omega_k)|_{D=1} = \frac{2}{\pi} \frac{v_F \mathbf{k}^2}{v_F^2 \mathbf{k}^2 - \omega_k^2}. \quad (3.6)$$

For a number N of different subbands, we take the analytic continuation to general dimension D [30]

$$\Pi(\mathbf{k}, \omega_k) = 2Nb(D) \frac{v_F^{2-D} \mathbf{k}^2}{|v_F^2 \mathbf{k}^2 - \omega_k^2|^{(3-D)/2}}, \quad (3.7)$$

where $b(D) = \frac{2}{\sqrt{\pi}} \frac{\Gamma((D+1)/2)^2 \Gamma((3-D)/2)}{(2\sqrt{\pi})^D \Gamma(D+1)}$. In the case of carbon nanotubes we have $N = 2$, with the spin degeneracy already taken into account in (3.7). In the case of a thick nanotube with a large number of subbands, the doping modifies the number of subbands to be taken into account near the Fermi level. Moreover, it is also conceivable that, in the process of renormalization, N may be given effectively by a variable number of subbands depending on the energy cutoff, if the relevant energy scale of the problem is large enough, as we discuss Appendix A.

In the large- N limit we are interested in, the electron self-energy $\Sigma(\mathbf{k}, \omega_k)$ is obtained by dressing the bare Coulomb interaction with the polarization effects given by (3.7). We have then, at the dominant level,

$$\begin{aligned} \Sigma(\mathbf{k}, \omega_k) = & -e^2 \int_0^\Lambda dp |\mathbf{p}|^{D-1} \int \frac{d\Omega}{(2\pi)^D} \int \frac{d\omega_p}{2\pi} G_0(\mathbf{k} - \mathbf{p}, \omega_k - \omega_p) \\ & \times \frac{-i}{\frac{|\mathbf{p}|^{D-1}}{c(D)} + e^2 \Pi(\mathbf{p}, \omega_p)}, \end{aligned} \quad (3.8)$$

where G_0 is the bare electron propagator.

At general D , the self-energy (3.8) shows a logarithmic dependence on the cutoff at small frequency ω_k and small momentum $\mathbf{k}$. This is the signature of the renormalization of the electron field scale and the Fermi velocity. In the low-energy theory, with high-energy modes integrated out, the electron propagator becomes

$$\begin{aligned} \frac{1}{G} = & \frac{1}{G_0} - \Sigma \\ \approx & Z^{-1}(\omega_k - v_F \boldsymbol{\sigma} \cdot \mathbf{k}) - Z^{-1} \frac{f(D)}{2N} \sum_{n=0}^{\infty} (-1)^n g^{n+1} \\ & \times \left(\frac{n(3-D)}{n(3-D)+2} \omega_k + \left(1 - \frac{2}{D} \frac{n(3-D)+1}{n(3-D)+2} \right) v_F \boldsymbol{\sigma} \cdot \mathbf{k} \right) h_n(D) \log(\Lambda), \end{aligned} \quad (3.9)$$

where $g = 2Nb(D)c(D)e^2/v_F$, $f(D) = \frac{1}{2^D \pi^{(D+1)/2} \Gamma(D/2) b(D)}$, $h_n(D) = \frac{\Gamma(n(3-D)/2 + 1/2)}{\Gamma(n(3-D)/2 + 1)}$. The quantity $Z^{1/2}$ represents the scale of the bare electron field compared to that of the renormalized electron field for which G is computed.

The renormalized propagator G must be cutoff-independent, as it leads to observable quantities in the quantum theory. This condition is enforced by fixing the dependence of the effective parameters Z and v_F on Λ as more states are integrated out from high-energy

shells. We get the differential RG equations for the effective parameters g and Z

$$\Lambda \frac{d}{d\Lambda} \log Z(\Lambda) = -\frac{f(D)}{2N} \sum_{n=0}^{\infty} (-1)^n g^{n+1} \frac{n(3-D)}{n(3-D)+2} h_n(D) \equiv -\gamma(g), \quad (3.10)$$

$$\Lambda \frac{d}{d\Lambda} g(\Lambda) = \frac{f(D)}{2N} \frac{2(D-1)}{D} g^2 \sum_{n=0}^{\infty} (-g)^n \left(\frac{(3-D)n+1}{(3-D)n+2} \right) h_n(D) \equiv -\beta(g). \quad (3.11)$$

The right-hand side of Eq. (3.11) is a monotonous increasing function of g , for any dimension between 1 and 2. At $D = 2$, the right-hand sides of these equations can be summed up, and they coincide for $N = 2$ with the functions obtained in the case of the undoped graphene sheet [21]. For $D = 1$ the function $\beta(g)$ vanishes, so that the 1D model has formally a line of fixed-points as it happens in the case of short-range interaction. However, in the present model the variable g is sent to strong coupling in the limit $D \rightarrow 1$, and it remains to be checked the behavior of the RG flow in that regime.

In order to approach the limit $D \rightarrow 1$, we have to look for the asymptotic dependence on D of the functions appearing in the RG equations. We will see that this dependence appears as $D - 1$ and $D - 3$ factors, revealing that these are the two critical dimensions, corresponding to a marginal and a renormalizable theory, respectively.

Starting with the function $\beta(g)$, we need to carry out the sum at the right-hand side of Eq. (3.11). This is given in terms of the hypergeometric special functions

$$B_D(g) = \sum_{n=0}^{\infty} (-g)^n \left(\frac{(3-D)n+1}{(3-D)n+2} \right) h_n(D). \quad (3.12)$$

Since we need the β function near $D = 1$, we can approximate $B_D(g)$ with the simple function $B_1(g)$

$$B_1(g) = \frac{\sqrt{\pi}}{g} \left(1 - \frac{1}{\sqrt{1+g}} \right). \quad (3.13)$$

To first order in $D - 1$, we get then

$$\beta(g) \approx -\frac{f(D)}{2N} \frac{2(D-1)}{D} \sqrt{\pi} g \left(1 - \frac{1}{\sqrt{1+g}} \right). \quad (3.14)$$

As long as the coupling g depends implicitly on the dimension D , we also need to know its asymptotic behavior near $D = 1$. In practice, we can translate the analysis to the initial value of the coupling, $g_0(D) = 2Nb(D)c(D)e^2/v_F$. Carrying out an expansion near $D = 1$, we obtain

$$\frac{1}{g_0(D)} \approx \frac{4\pi^2 v_F}{2Ne^2} (D-1) \left[\frac{D}{2} - \frac{\zeta(3)}{4} (D-1)^3 \right], \quad (3.15)$$

where $\zeta(s) = \sum_{k=1}^{\infty} k^{-s}$ is the Riemann zeta function. So we can use in our calculations the simple expression

$$g_0(D) \approx 2N \frac{e^2}{4\pi^2 v_F} \frac{2}{(D-1)D}. \quad (3.16)$$

Finally, it can be seen that the right-hand side of Eq. (3.10) does not behave as a simple power law. This is due to the fact that the terms of the series vanish too slowly when n increases. The γ function can be expressed in the form

$$\gamma(g) = \frac{f(D)}{2N} \frac{3-D}{2} g T_D(g), \quad (3.17)$$

where the series $T_D(g)$ is available on tables. For $D = 1$ it has the simple expression

$$T_1(g) = \sqrt{\pi} \frac{2\sqrt{g+1} - g - 2}{g\sqrt{g+1}}. \quad (3.18)$$

The scaling of the electron wave function near $D = 1$ is therefore given by

$$\gamma(g) \approx \frac{f(D)}{2N} \sqrt{\pi} \left(2 - \sqrt{1+g} - \frac{1}{\sqrt{1+g}} \right). \quad (3.19)$$

This coincides formally with the anomalous dimension that is found at $D = 1$ in the exact solution of the Luttinger model, what provides an independent check of our RG approach to the 1D system.

4. RG scaling and low-energy density of states

Now we can rewrite the RG Eq. (3.11) near $D = 1$ as follows:

$$\frac{dg}{dx} = -\frac{f(D)}{2N} \frac{2(D-1)}{D} \sqrt{\pi} g \left(1 - \frac{1}{\sqrt{1+g}} \right) \quad (4.20)$$

where $x = -\log(\Lambda)$. At $D = 1$ we have formally from Eq. (4.20) a line of fixed-points, which cover all values of e^2/v_F . In spite of the fact that g goes to strong coupling as $D \rightarrow 1$, we can see that the behavior in Eq. (3.16) does not suffice to cancel the $D - 1$ factor at the right-hand side of Eq. (4.20). Therefore, we make sure in this way that the long-range Coulomb interaction is at a fixed-point of the RG at $D = 1$.

We can also rewrite the RG equation (3.10) as follows:

$$\frac{d \log(Z)}{dx} = \frac{f(D)}{2N} \frac{3-D}{2} T_D(g) g. \quad (4.21)$$

Near $D = 1$ we find a crossover to a behavior with a sharp reduction of the quasiparticle weight in the low-energy limit $\Lambda \rightarrow 0$. All that is displayed in Fig. 5 in Appendix A, where we have represented the electron field scale square Z . If D is above ≈ 1.2 , we have a clear signature of quasiparticles in the nonzero value of Z at low energies, whereas for lower values of D the picture cannot be distinguished from that of a vanishing quasiparticle weight.

Our target is to compare theoretical results with measurements of the tunneling density of states carried out in nanotubes of large radius [22]. The density of states $n(\varepsilon)$ computed at dimensions between 1 and 2 displays an effective power-law behavior which is given by

$$\begin{aligned} n(\varepsilon) &\approx 2N Z(\varepsilon) |\varepsilon|^{D-1} \Rightarrow \\ \log(n(\varepsilon)) &\approx \log(2N) + \log Z(\varepsilon) + (D-1) \log(|\varepsilon|). \end{aligned} \quad (4.22)$$

The last term in Eq. (4.22) comes from the analytic continuation of the conical dispersion, which gives rise at $D = 2$ to the well-known behavior of the graphite layer, $n(\varepsilon) \sim |\varepsilon|$.

In order to obtain the linear dependence of $\log(n(\varepsilon))$ on $\log(|\varepsilon|)$, we have to introduce the low-energy behavior of $Z(\varepsilon)$ in Eq. (4.22), by which

$$\log(n(\varepsilon)) \sim (\alpha_Z - (D - 1))x \equiv \alpha_D x. \quad (4.23)$$

If we want to keep the linear behavior of $\log(Z)$ in $x = -\log(|\varepsilon|)$, we have to make an approximation. We can solve the RG equation (3.10) near $x = 0$ with a simple first order expansion, $\log(Z) \sim \gamma(g_0)x$, where g_0 is the initial value of the coupling. Then we can analyze the dependence of α_Z on the dimension D and at large N starting from the expression

$$\alpha_Z \approx \frac{f(D)}{2N} \frac{3 - D}{2} T_D(g_0) g_0. \quad (4.24)$$

4.1. Analytic continuation near $D = 1$

Following Ref. [24], we can obtain a lower bound for the exponent of the density of states, for the purpose of comparison with experiments, by estimating the minimum of the absolute value of α_D , for dimensions ranging between $D = 1$ and $D = 2$. The evaluation can be carried out using Eq. (4.23), after plugging the expression of α_Z in (4.24), with its strong dependence on D .

In order to obtain an analytical expression for α_D , we can approximate $T_D(g)$, starting from $T_1(g)$, by using for $1 < D < 2$ the simple approximation $T_D(g) \approx \frac{D+1}{2} T_1(g)$. Thus, we can give a simple formula for α_D rewriting Eq. (4.23) in the form

$$\alpha_D \approx g_0 T_1(g_0) \frac{(3 - D)f(D)(D + 1)}{8N} - (D - 1). \quad (4.25)$$

We obtain a minimum value for $|\alpha_D|$ as a function of D when we introduce the expression of $g_0(D)$ obtained in the previous section. From Fig. 2 we can see that the maximum value for α_D (α_M) corresponds to a dimension between 1 and 1.2. If the number of subbands is increased, then the value of $|\alpha_M|$ decreases while the corresponding dimension approaches 1.

Our analytic computation allows to understand the interplay between the number of subbands N and the dimension D at which the minimum of the critical exponent $|\alpha_D|$ is obtained. We find that this is around $|\alpha_D| \approx 0.3$ for not too large values of N , which is in agreement with the values measured experimentally. However, the fact that the minima are obtained above $D = 1$ stresses once again the singular character of the 1D Coulomb interaction. The physical description can be clarified by noticing that the purely 1D limit can be approached by increasing the number N of subbands, as we propose below.

4.2. Double scaling approach

Here we exploit the dependence on the number of subbands N as a way of regularizing the divergent effects of the Coulomb interaction, when D approaches the critical dimension $D = 1$. As it is easily seen inspecting the α_Z function in Eq. (4.24), a large N can be

Fig. 2. In (a) we show α as a function of the dimension (Eq. (4.25)), and we choose the maximum value of this function between $D = 1$ and $D = 2$ as an estimate of the critical exponent. We find a value for α in the usual $N = 2$ undoped nanotube that reproduces the anomalous exponent measured experimentally ($\alpha \approx -0.3$), corresponding to a dimension for the crossover between 1.1 and 1.2. The value of α is compared to the term $\alpha_Z(D)$ (dashed line) dominant near the divergence $D \rightarrow 1$. As shown in (b), when the number of subbands increases, the absolute value of the maximum decreases, while the corresponding dimension approaches 1 (each curve corresponds to a different value of N).

combined with the divergence in $D - 1$ from the coupling constant g_0 in Eq. (3.16), so that both quantities regularize each other in a sort of double scaling limit. From the physical standpoint, this mechanism of regularization can be regarded as a kind of screening of the singular Coulomb interaction at $D = 1$.

We use the above method of regularization near the critical dimension $D = 1$, in order to give an alternative evaluation of the critical exponent α . In the limit of large N , the dimension where the crossover effectively takes place, corresponding to the maximum value of α_D , approaches $D = 1$. In this regime, the last term in Eq. (4.25) vanishes and we can use the value of α_Z as an estimate of α .

In the limit of large N and small values of $D - 1$, we have $T_1(g) \rightarrow -\sqrt{\pi/g}$, so that Eq. (4.24) becomes

$$\alpha_Z \approx -\frac{f(D)(3-D)(D+1)}{8N} \sqrt{\pi g_0}. \quad (4.26)$$

Taking into account the form of g_0 in Eq. (3.16), we see that α_Z vanishes at large N as $\sim 1/\sqrt{N}$, and it diverges at $D \rightarrow 1$ as $\sim 1/\sqrt{D-1}$.

Now we can use the regularization from the number of subbands, in order to access a physical regime that would be otherwise hidden in the usual treatment of the 1D Coulomb interaction. In other words, we take simultaneously the limits $N \rightarrow \infty$ and $D \rightarrow 1$ by keeping a finite renormalized value N_{ren} for the number of subbands, $N_{\text{ren}} \equiv N(D-1)$. We use then N_{ren} as a way of parameterizing the dependence of the critical exponent α on the doping level, which sets the number of subbands at the Fermi level in the carbon nanotubes. From Eq. (4.26) we obtain a simple expression for α as a function of the renormalized

Fig. 3. In this paper we propose also a different way to estimate the critical exponent, as the value of α_Z in the double limit $N \rightarrow \infty$ and $D \rightarrow 1$. A comparison between the results of the double scaling approach (dashed line) and those from the analytic continuation in D (full line) is shown in (a). The increase of α for a moderate number of subbands is shown in (b). We have taken in all cases a value of the coupling $e^2/(\pi^2 v_F) \approx 0.5$, which is appropriate for MWNTs, owing to the reduction in this value due to the interaction with the inner metallic cylinders [15]. We can conclude that, when N is very large, there is a good agreement between the results obtained with the two different approaches, i.e., double scaling and analytic continuation. The dashed portion of the curve in (b) agrees, in the one-mode limit (i.e., for $N = 0.5$), with the results of Ref. [24].

number of subbands

$$\alpha = -\frac{1}{4} \sqrt{\frac{e^2}{\pi^2 v_F N_{\text{ren}}}}. \quad (4.27)$$

The agreement between the two estimates of the critical exponent α by the double scaling limit and by analytic continuation near $D = 1$ is shown in Fig. 3(a). The dependence of α_Z on the number of subbands in the second approach can be actually borne out by numerical results. These are in good agreement with our analytical predictions, as we show in Fig. 4. The dependence of α_Z on the number of subbands is rather similar to $1/\sqrt{N}$, especially for dimensions near 1. Thus, from this overall picture we can predict that, by changing the doping level in carbon nanotubes of large radius, a sizeable variation of the anomalous electron dimension should be observed in the range $N \sim 2$ –100, with a consequent change in the power-law behavior of the tunneling density of states given by Eq. (4.27).

5. Discussion

In this paper we have described the appropriate framework to deal with the low-energy effects of the long-range Coulomb interaction in 1D electron systems. Our interest has been focused in the scaling behavior of quantities like the quasiparticle weight or the low-energy density of states, which can be compared directly with the results of transport experiments. For this purpose, we have developed a RG approach in a dimensionally regularized theory devised to interpolate between the 2D conical band dispersion of graphene and the 1D band

Fig. 4. On the left, we show the linear dependence of α_Z on the inverse square root of N , for different values of D . On the right, we observe a rather smooth behavior of the slope of the curves for each dimension in the first figure.

dispersion of the carbon nanotubes. The analytic continuation in the number of dimensions has allowed to avoid the infrared singularities that the long-range Coulomb interaction poses at $D = 1$, providing insight at the same time about the fixed-points and universality classes of the theory in the limit $D \rightarrow 1$.

Thus, we have seen that the 1D electron system with long-range Coulomb interaction is at a fixed-point of the RG flow, for arbitrary values of the interaction strength. However, the scaling behavior of observables like the quasiparticle weight is dictated by divergent critical exponents in the limit $D \rightarrow 1$. In order to make contact with the results of transport experiments on MWNTs, we have introduced the relevant dependence on the number N of subbands at the Fermi level. By increasing the value of N , it has been possible to approach the critical dimension $D = 1$ while producing finite, sensible estimates of the critical exponents in the carbon nanotubes. The double scaling limit, $N \rightarrow \infty$ and $D \rightarrow 1$, gives rise to a phase of the system that captures the physical properties measured in nanotubes of large radius, which use to be significantly doped. In this way, we have been able to describe a new universality class that arises in the large- N limit, in which the effect of the large number of subbands is to regularize the singular 1D Coulomb interaction. The new phase arises from a sort of 1D screening of the long-range interaction, giving rise to a modified Luttinger liquid picture in which the critical exponents depend on the doping level as well as on the strength of the interaction.

The framework that we have introduced is appropriate for the description of carbon nanotubes of large radius, in which the shift of the Fermi level leads to a sizeable variation of the number of subbands contributing to the low-energy electronic properties. Such a situation is realized in the MWNTs, which are concentric arrangements of carbon nanotubes with a diameter of ~ 10 nm and typically ~ 5 inner cylinders. The transport measurements carried out in the MWNTs reflect usually the electronic properties of the

outer layer, to which the electrodes are attached. In these systems the number of subbands at the Fermi level can be large, i.e., of the order of ~ 10 – 20 taking into account the electron spin, because of the electrochemical doping.

One of the most significant observations made in the MWNTs has been the power-law behavior of the tunneling conductance as a function of the temperature or the bias voltage. In the experimental measurements, the maximum energy scale reached in those variables has been always below 100 meV. This is of the order of the typical energy spacing between the different subbands, so that the transport measurements refer to the contribution of a fixed number of subbands at low energies. In the case that the experiments were carried out over a larger energy range, the physical description should be modified to allow for a variable number of subbands depending on the cutoff energy, producing the results that are shown in Appendix A.

The measurements carried out in the MWNTs have displayed a power-law behavior of the tunneling conductance, that gives a measure of the low-energy density of states, with exponents ranging from 0.24 to 0.37 [22]. These values are, on the average, below those measured in the single-walled nanotubes, which are typically about ≈ 0.35 [12]. We see that our results can account satisfactorily for this slight reduction in the critical exponent with the change of the nanotube thickness. We observe from Fig. 3(b) that, for a number of subbands $N = 2$, the value of $|\alpha|$ corresponding to an undoped single-walled nanotube is slightly below 0.3. This means that our choice for the interaction strength (with some reduction from the dielectric constant) has been appropriate. The important point is that, by varying the number N of subbands from 2 to 10, we obtain a shift in the value of the critical exponent which is in fair agreement, on the average, with that observed from the different transport experiments. This lends strong support to our picture of the renormalization of the long-range Coulomb interaction.

The main prediction that comes from our study is that there should be a significant reduction in the critical exponent of the tunneling density of states as the doping level is increased in nanotubes of large radius. We have seen that the presence of a large number of subbands at the Fermi level implies a reduction of $|\alpha|$, by a factor of the order of $N^{-1/2}$. It would be of considerable importance to test such a dependence in experiments carried out using samples with different amount of doping. That kind of effect should arise also as a consequence of considering wider compact nanotube structures, stressing in another way the significance of the geometry in the electronic properties of the nanotubes.

Acknowledgements

This work was partly supported by the Programma di Ricerca di Interesse Nazionale cofinanced by MIUR (Es. Fin. 2002) “Effetti di spin, interazione e proprietà di trasporto in sistemi elettronici fortemente interagenti a bassa dimensionalità”.

Appendix A. Number of subbands depending on cut-off energy

Next, we consider a number of subbands N_S depending on the cut-off. So, we introduce the energy dependent function $n_s(\varepsilon)$ which substitutes the usual n_s . We define a unit energy

as ε_p , so that we have $x_p = -\log(\varepsilon_p) = 0$. The corresponding number of subbands reads $N_s(\varepsilon_p) = N_0 + N_p$ ¹

$$\begin{aligned} N_s(\varepsilon) &= N_0 + N_p \frac{\varepsilon}{\varepsilon_p} \Rightarrow \\ N_s(x) &= N_0 + N_p e^{-x} \Rightarrow n_s(x) = 1 + n_p e^{-x}. \end{aligned} \quad (\text{A.1})$$

Here n_p represents the number of subbands, in a unit of energy normalized with respect to N_0 .

When n_s depends on the energy we obtain, by increasing the number of subbands, that α_Z vanishes as $n_s^{-1/2}$. However a stronger α_N contribution causes a growth of α (although its absolute value stays less than 1). The relevant correction in Eq. (4.23) is due to the direct contribution of the number of subbands depending on the energy. This is the α_N term which can strongly modify the α_D value. Near $x = 0$, we can obtain this term as a simple function of n_p , which represents the number of subbands, in a unit of energy normalized with respect to N_0

$$\alpha_N \approx -\frac{n_p}{1 + n_p}.$$

This term does not depend on the dimension, so it does not affect the search of the maximum in the α_D , rather it gives a simple shift in the global α value, which we have to compare with the experimental value of the parameter. Larger values of n_p give larger contributions (in absolute value) from this term which ranges between 0 and 1 (once again, in absolute value).

In the case of a linear dependence of the number of subbands on the energy, a large correction to the measurable α exponent can arise from the α_N contribution ($\alpha_N \sim 0-1$), while the effective parameter Z yields strong deviations from the behavior at $n = 4$, even when the crossover dimension approaches the fixed point $D = 1$.

Some interesting effects regarding the numerical calculation of Z arise, when the number of subbands depends on the cutoff energy (n_p is the density of the number of subbands in the energy unit). In this case a strong deviation from the usual behavior appears. The renormalization of the electron field scale Z , for a fixed number of subbands (4 subbands, i.e., $N = 2$), shows a drastic suppression of the electron field scale, which takes place over a variation of only two orders of magnitude in the energy scale. When the number of subbands increases, we observe that the scale of energies, and also the shape of the curve, are quite different. The energy scale goes towards lower and lower energies, while a large plateau appears in the curve $Z(-\log(E))$. A simple mechanism can explain this behavior. When the cutoff increases, many subbands have to be taken into account and the system, at any dimension greater than 1, is more similar to a 2-dimensional graphene sheet than to a 1-dimensional wire. Only when the energy goes below a small value (i.e., $x > x_T$) the system restores its Luttinger liquid behavior and the quasiparticle amplitude vanishes, following the usual behavior. This is what we can observe on the left of Fig. 5.

¹ We use a simple formula we obtained from the Taylor expansion of the $n_s(\varepsilon)$ function in Section 2. However, we have to implement a correction, because we know that, when E is set to 0, we have $N_s(0) = N_0 = 4$.

Fig. 5. On the left, we plot $Z(-\log(E))$ for a different number of subbands, depending on the energy. On the right, we show the dependence of $\alpha_Z \approx \frac{d \log(Z)}{dx}$ on x .

All that is clear also on the right, where we show the dependence of $\alpha_Z \approx \frac{d \log(Z)}{dx}$ on x . Here the asymptotic behavior reflects the small n_p results, whereas the values for ‘high energies’ are quite similar to those of the usual 2D graphene.

All the phenomenology of a tube with a very large number of subbands below the cutoff energy is more similar to that of 2D graphene than to a 1D Luttinger liquid. This effect could be seen either as a smaller crossover dimension, or as the persistence of quasiparticle poles at relatively high energy. We can obtain the same physics if we imagine the radius of the tube going to infinity: the geometrical limit of the tube is the graphene layer, while the energy spacing between two subbands vanishes, so that an infinite number of subbands have to be taken into account, also for very small energies.

References

- [1] A. Bachtold, P. Hadley, T. Nakanishi, C. Dekker, *Science* 294 (2001) 1317.
- [2] P.L. McEuen, *Nature* 393 (1998) 15.
- [3] J.W. Mintmire, B.I. Dunlap, C.T. White, *Phys. Rev. Lett.* 68 (1992) 631;
N. Hamada, S. Sawada, A. Oshiyama, *Phys. Rev. Lett.* 68 (1992) 1579;
R. Saito, M. Fujita, G. Dresselhaus, M.S. Dresselhaus, *Appl. Phys. Lett.* 60 (1992) 2204.
- [4] V.J. Emery, in: J.T. Devreese, R.P. Evrard, V.E. Van Doren (Eds.), *Highly Conducting One-Dimensional Solids*, Plenum, New York, 1979.
- [5] J. Sólyom, *Adv. Phys.* 28 (1979) 201.
- [6] F.D.M. Haldane, *J. Phys. C* 14 (1981) 2585.
- [7] Yu.A. Krotov, D.-H. Lee, S.G. Louie, *Phys. Rev. Lett.* 78 (1997) 4245.
- [8] L. Balents, M.P.A. Fisher, *Phys. Rev. B* 55 (1997) R11973.
- [9] R. Egger, A.O. Gogolin, *Phys. Rev. Lett.* 79 (1997) 5082;
R. Egger, A.O. Gogolin, *Eur. Phys. J. B* 3 (1998) 281.
- [10] C. Kane, L. Balents, M.P.A. Fisher, *Phys. Rev. Lett.* 79 (1997) 5086.
- [11] M. Bockrath, D.H. Cobden, J. Lu, A.G. Rinzler, R.E. Smalley, L. Balents, P.L. McEuen, *Nature* 397 (1999) 598.

- [12] Z. Yao, H.W.Ch. Postma, L. Balents, C. Dekker, *Nature* 402 (1999) 273.
- [13] R. Egger, H. Grabert, *Phys. Rev. Lett.* 79 (1997) 3463.
- [14] S. Bellucci, J. González, *Eur. Phys. J. B* 18 (2000) 3.
- [15] R. Egger, *Phys. Rev. Lett.* 83 (1999) 5547.
- [16] J. González, *Phys. Rev. Lett.* 88 (2002) 076403.
- [17] P.-A. Bares, X.G. Wen, *Phys. Rev. B* 48 (1993) 8636;
A. Houghton, H.-J. Kwon, J.B. Marston, R. Shankar, *J. Phys. Condens. Matter* 6 (1994) 4909;
C. Castellani, C. Di Castro, A. Maccarone, *Phys. Rev. B* 55 (1997) 2676;
C. Castellani, S. Caprara, C. Di Castro, A. Maccarone, *Nucl. Phys. B* 594 (2001) 747.
- [18] J. González, F. Guinea, M.A.H. Vozmediano, *Nucl. Phys. B* 424 (1994) 595.
- [19] S. Xu, J. Cao, C.C. Miller, D.A. Mantell, R.J.D. Miller, Y. Gao, *Phys. Rev. Lett.* 76 (1996) 483.
- [20] J. González, F. Guinea, M.A.H. Vozmediano, *Phys. Rev. Lett.* 77 (1996) 3589.
- [21] J. González, F. Guinea, M.A.H. Vozmediano, *Phys. Rev. B* 59 (1999) R2474.
- [22] A. Bachtold, M. de Jonge, K. Grove-Rasmussen, P.L. McEuen, M. Buitelaar, C. Schonenberger, *Phys. Rev. Lett.* 87 (2001) 166801;
A. Bachtold, M. de Jonge, K. Grove-Rasmussen, P.L. McEuen, M. Buitelaar, C. Schonenberger, *cond-mat/0012262*.
- [23] R. Egger, A.O. Gogolin, *Phys. Rev. Lett.* 87 (2001) 066401.
- [24] S. Bellucci, J. González, *Phys. Rev. B* 64 (2001) 201106(R).
- [25] M. Krüger, M.R. Buitelaar, T. Nussbaumer, C. Schönenberger, L. Forró, *Appl. Phys. Lett.* 78 (2001) 1291.
- [26] J. González, F. Guinea, M.A.H. Vozmediano, *Nucl. Phys. B* 406 (1993) 771.
- [27] C.L. Kane, E.J. Mele, *Phys. Rev. Lett.* 78 (1997) 1932.
- [28] The method of analytic continuation in the number of dimensions has been proposed in the case of a short-range interaction by C. Castellani, C. Di Castro, W. Metzner, *Phys. Rev. Lett.* 72 (1994) 316.
- [29] See also: D.W. Wang, A.J. Millis, S. Das Sarma, *Phys. Rev. B* 64 (2001) 193307.
- [30] C. Itzykson, B. Zuber, *Quantum Field Theory*, McGraw–Hill, New York, 1980.