Fermionic backreaction in hybrid LQC

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Motivation

- Traditional QFT requires of renormalization (UV issues).
- Can be traced to the assumption of classical spacetime backgrounds: could quantum gravity change the picture?
- Hybrid scheme for cosmological models:
    - ⭑ QG representation for homogeneous d.o.f (LQC).
    - ⭑ Fock representation for inhomogeneities.

Motivation

- Traditional QFT requires of renormalization (UV issues).
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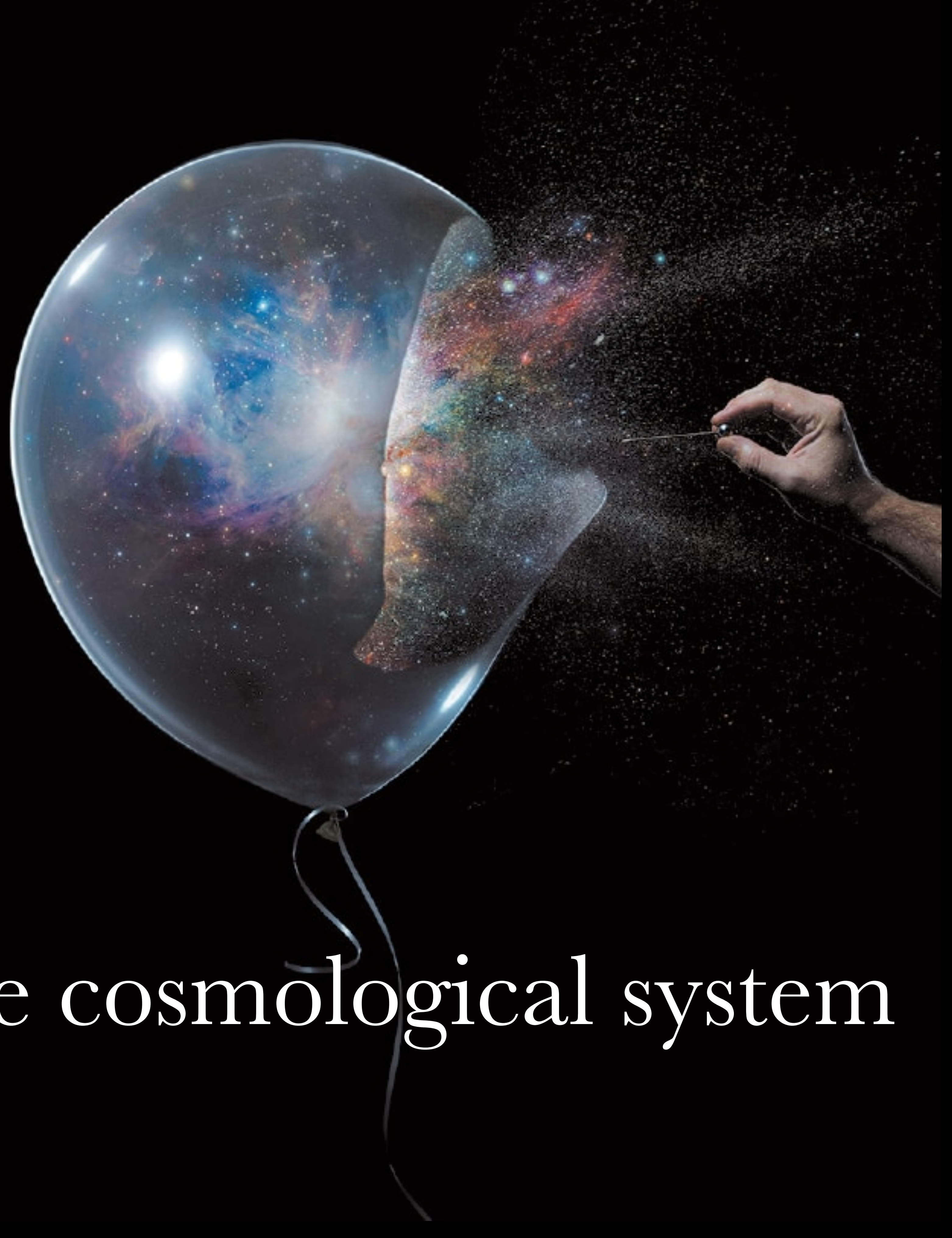
- Hybrid scheme for cosmological models.
- Freedom in performing canonical transformation that assign different dynamical roles to the homogeneous sector and the (matter and gravitational) inhomogeneities.
- Can this freedom allow for a well-defined quantum field theory for the inhomogeneities? ➡ analysis of quantum backreaction.
- First step in justifying that this is possible: fermionic perturbations in inflationary cosmology.



The cosmological system

The cosmology

- FLRW spacetime with flat & compact hypersurfaces.
- Minimally coupled homogeneous scalar field (inflaton).
- Minimally coupled inhomogeneous Dirac field, treated entirely as a perturbation.
- Scalar and tensor perturbations of the metric & inflaton.
- Expansion in spatial (Dirac or Laplace) eigenmodes.
- Canonical formalism: truncation of the action at quadratic order in <u>all</u> the perturbations.

The cosmology

- Hamiltonian, omitting scalar & tensor contributions:

$$N_0[H_{|0}(\alpha, \pi_\alpha, \phi, \pi_\phi) + H_D(\alpha)]$$

- At quadratic order, fermions do not contribute to the linearized Hamiltonian & diffeo constraints.

$\Downarrow$

Gauge-invariant perturbations

- Canonical set for homogeneous background+Dirac field:

$$\{(\alpha, \pi_\alpha), (\phi, \pi_\phi), \underbrace{(x_{\vec{k}}, \bar{x}_{\vec{k}}), (y_{\vec{k}}, \bar{y}_{\vec{k}})}\}$$

$\downarrow$ Quiral mode coefficients, $\vec{k} \in \mathbb{Z}^3$

Fermionic annihilation and creation-like variables

Fermionic variables

- Considering the system as a whole, freedom in:
  - ★ Dynamical separation of homogeneous geometry and fermionic d.o.f via canonical transformations.
  - ★ Choice of Fock vacuum for the fermionic perturbations, within the hybrid scheme.
- This ambiguity can be encoded in choices (that respect the symmetries of the fermionic Hamiltonian):

$$a_{\vec{k}}^{(x,y)} = f_1^k(\alpha,\pi_\alpha)\, x_{\vec{k}} + f_2^k(\alpha,\pi_\alpha)\, \bar{y}_{-\vec{k}-2\vec{\tau}},$$

$$\bar{b}_{\vec{k}}^{(x,y)} = g_1^k(\alpha,\pi_\alpha)\, x_{\vec{k}} + g_2^k(\alpha,\pi_\alpha)\, \bar{y}_{-\vec{k}-2\vec{\tau}}$$

Fermionic variables

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- For fixed background, these satisfy annihilation & creation-like Poisson algebra iff:

$$f_2^k = e^{iF_2^k}\sqrt{1 - \left|f_1^k\right|^2} \quad g_1^k = e^{iJ_k}\bar{f}_2^k, \quad g_2^k = -e^{iJ_k}\bar{f}_1^k$$

- The transformation can be completed to be canonical for the entire cosmology by introducing:

$$(\alpha, \pi_\alpha) \longrightarrow (\tilde{\alpha}, \tilde{\pi}_\alpha) = (\alpha + \Delta\tilde{\alpha}, \pi_\alpha + \Delta\tilde{\pi}_\alpha)$$

quadratic in fermionic perturbations

Fermionic variables

- We restrict the choices to those such that:
  - In QFTCS, the dynamics is unitarily implementable
  - Standard particle/antiparticle convention in the UV

Fermionic variables

- We restrict the choices to those such that, when $|\vec{k}| \to \infty$:

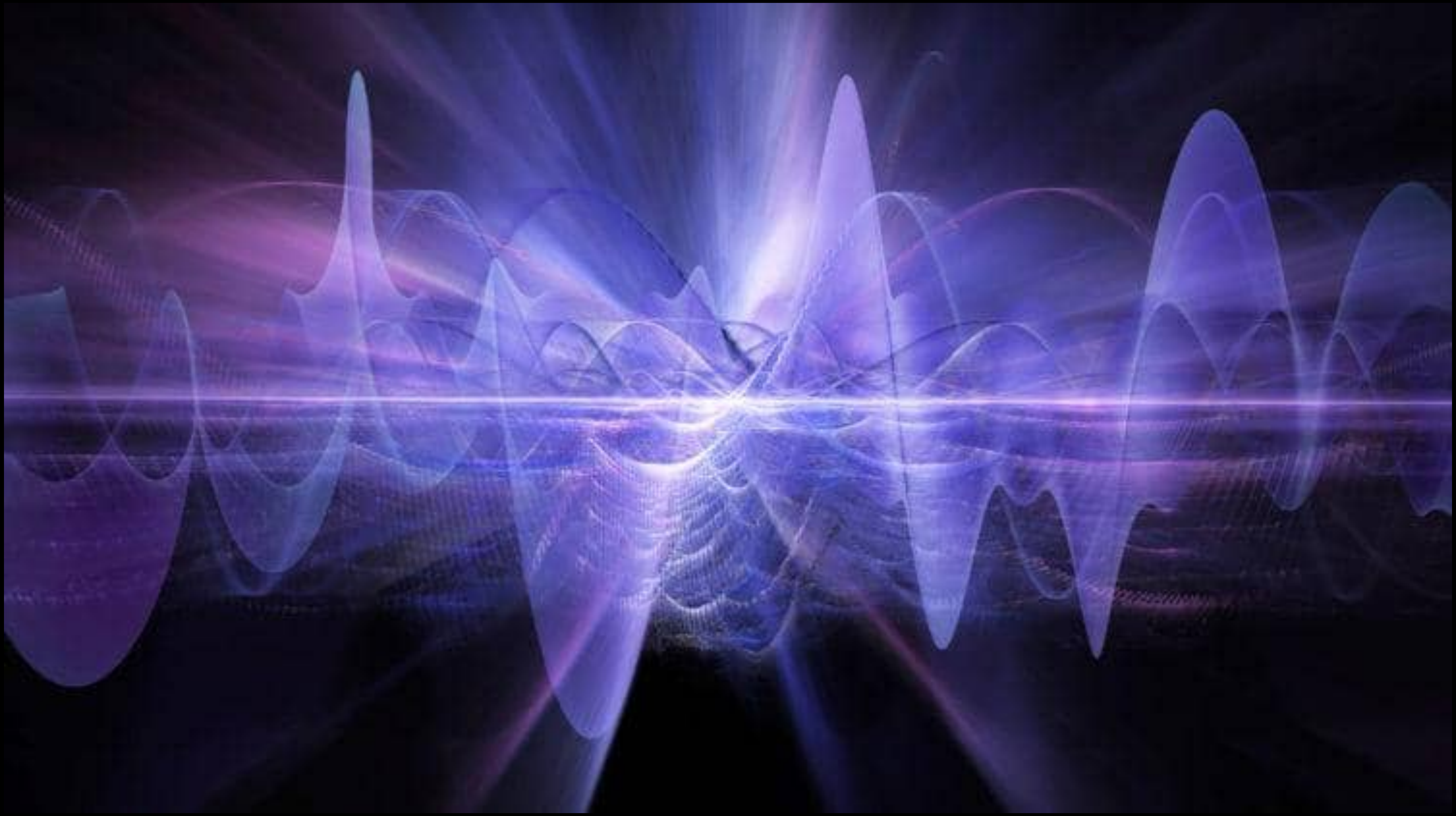
$$f_1^k = \sqrt{\frac{\xi_k - \omega_k}{2\xi_k}} + \frac{\tilde{M} e^{\alpha}}{2\omega_k}\left[e^{iF_2^k} - 1\right] + \theta_k, \qquad \sum_{\vec{k}} |\theta_k|^2 < \infty$$

$$\xi_k = \sqrt{\omega_k^2 + \tilde{M}^2 e^{2\alpha}} \qquad\qquad \omega_k \sim |\vec{k}|$$

- The fermionic Hamiltonian is a sum of modes over that have the asymptotic behavior, when $\omega_k \to \infty$:

$$\Big[\left(e^{-\tilde{\alpha}}\tilde{\xi}_k + 2h_D^k\right)\left(\bar{a}_{\vec{k}}^{(x,y)} a_{\vec{k}}^{(x,y)} + \bar{b}_{\vec{k}}^{(x,y)} b_{\vec{k}}^{(x,y)}\right) + 2h_J^k\left(\bar{b}_{\vec{k}}^{(x,y)} b_{\vec{k}}^{(x,y)}\right)$$
$$+ e^{i(J_k - F_2^k)} e^{-\tilde{\alpha}}\left(2\omega_k\bar{\theta}_k + \bar{h}_I^k\right) a_{\vec{k}}^{(x,y)} b_{\vec{k}}^{(x,y)} + \text{H.c.}\Big]$$

$$h_J^k = \mathcal{O}(1) \qquad h_D^k = \mathcal{O}(1) \qquad h_I^k = i\tilde{\pi}_\alpha \frac{\tilde{M} e^{-\tilde{\alpha}}}{2\omega_k} e^{iF_2^k} + \mathcal{O}(\text{Max}[\theta_k, \omega_k^{-2}]).$$



Quantum fermionic dynamics

Quantum dynamics

- Kinematical Hilbert space $\mathcal{H}^{\text{grav}}_{\text{LQC}} \otimes L^2(\mathbb{R}, d\phi) \otimes \mathcal{F}_D$.
- Ansatz for quantum states: $\Gamma(V,\phi)\psi_D(\mathcal{N}_D,\phi), \quad V \propto e^{3\tilde{\alpha}}$

$$-i\partial_\phi \Gamma(V,\phi) = \hat{\tilde{\mathcal{H}}}_0 \Gamma(V,\phi)$$

- Γ "almost-solutions" of the unperturbed cosmology.
- Imposition of the constraint & Born-Oppenheimer approx. (neglect $\partial_\phi^2 \psi_D$):

$$i\partial_\phi \psi_D(\mathcal{N}_D,\phi) = \frac{l_0 \langle \widehat{V^{2/3} e^{\tilde{\alpha}} \tilde{H}_D} \rangle_\Gamma - C_D^{(\Gamma)}(\phi)}{\langle \hat{\tilde{\mathcal{H}}}_0 \rangle_\Gamma} \psi_D(\mathcal{N}_D,\phi)$$

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Measures (mean) backreaction

Quantum dynamics

- Associated Heisenberg equations for fermionic operators:

$$d_{\eta_\Gamma} \hat{a}^{(x,y)}_{\vec{k}}(\eta,\eta_0) = -iF^{(\Gamma)}_k \hat{a}^{(x,y)}_{\vec{k}}(\eta,\eta_0) + G^{(\Gamma)}_k \hat{b}^{(x,y)\dagger}_{\vec{k}}(\eta,\eta_0),$$

$$d_{\eta_\Gamma} \hat{b}^{(x,y)\dagger}_{\vec{k}}(\eta,\eta_0) = i\left(F^{(\Gamma)}_k + \tilde{J}^{(\Gamma)}_k\right) \hat{b}^{(x,y)\dagger}_{\vec{k}}(\eta,\eta_0) - \bar{G}^{(\Gamma)}_k \hat{a}^{(x,y)}_{\vec{k}}(\eta,\eta_0)$$

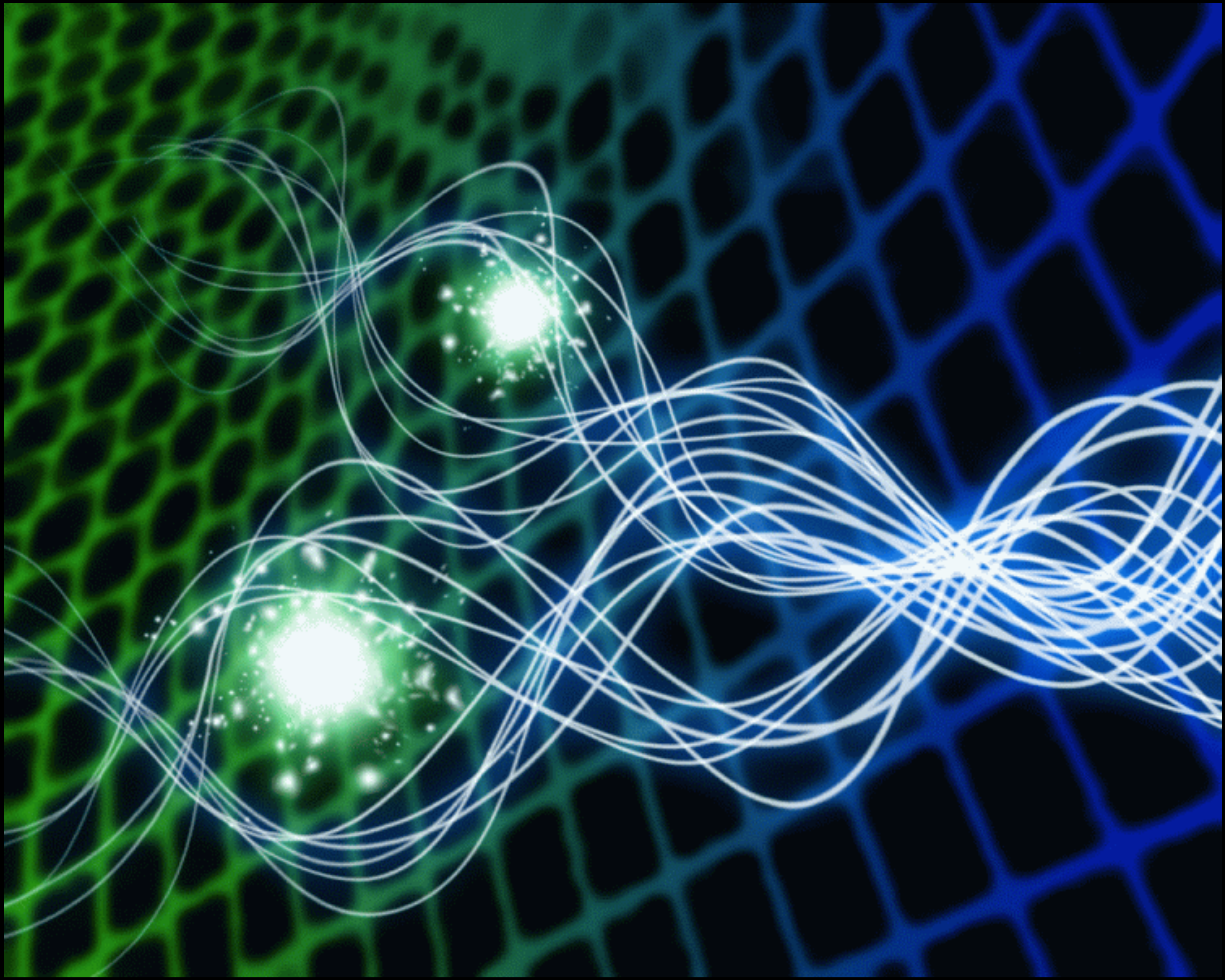
$$d\eta_\Gamma = \frac{l_0 \langle \hat{V}^{2/3}\rangle_\Gamma}{\langle \hat{\tilde{\mathcal{H}}}_0\rangle_\Gamma} d\phi$$

- By construction of the representation, known asymptotics of expectation values when $\omega_k \to \infty$:

$$\tilde{J}^{(\Gamma)}_k = \frac{\langle 2\widehat{e^{\tilde{\alpha}} V^{2/3}} h^k_J\rangle_\Gamma}{\langle \hat{V}^{2/3}\rangle_\Gamma} = \mathcal{O}(1),$$

$$F^{(\Gamma)}_k = \omega_k + \frac{\langle 2\widehat{e^{\tilde{\alpha}} V^{2/3} h}^k_D\rangle_\Gamma}{\langle \hat{V}^{2/3}\rangle_\Gamma} + \frac{M^2}{2l_0^2\omega_k}\frac{\langle \hat{V}^{4/3}\rangle_\Gamma}{\langle \hat{V}^{2/3}\rangle_\Gamma} + \mathcal{O}(\omega_k^{-3}),$$

$$G^{(\Gamma)}_k = \frac{2i\omega_k \langle \widehat{e^{i(F^k_2 - J_k)} V^{2/3}} \theta_k\rangle_\Gamma + i\langle \widehat{e^{i(F^k_2 - J_k)} V^{2/3}} h^k_I\rangle_\Gamma}{\langle \hat{V}^{2/3}\rangle_\Gamma}$$



Unitarity & backreaction

Unitarity

- Solutions to the Heisenberg equations at time $\eta_\Gamma = \eta$, with initial conditions $\hat{a}_{\vec{k}}^{(x,y)}, \hat{b}_{\vec{k}}^{(x,y)\dagger}$:

$$\hat{a}_{\vec{k}}^{(x,y)}(\eta,\eta_0) = \alpha_k(\eta,\eta_0)\hat{a}_{\vec{k}}^{(x,y)} + \beta_k(\eta,\eta_0)\hat{b}_{\vec{k}}^{(x,y)\dagger}, \qquad |\alpha_k|^2 + |\beta_k|^2 = 1$$

$$\hat{b}_{\vec{k}}^{(x,y)\dagger}(\eta,\eta_0) = -e^{i\int_{\eta_0}^{\eta} d\eta_\Gamma\, \tilde{J}_k^{(\Gamma)}}\bar{\beta}_k(\eta,\eta_0)\hat{a}_{\vec{k}}^{(x,y)} + e^{i\int_{\eta_0}^{\eta} d\eta_\Gamma\, \tilde{J}_k^{(\Gamma)}}\bar{\alpha}_k(\eta,\eta_0)\hat{b}_{\vec{k}}^{(x,y)\dagger}$$

- Known asymptotics of Bogoliubov coefficients when $\omega_k \to \infty$:

$$\alpha_k(\eta,\eta_0) = e^{-i\omega_k(\eta-\eta_0)-i\Phi_k} + o(1), \qquad \Phi_k(\eta,\eta_0) = \mathcal{O}(1) \in \mathbb{R}$$

$$\beta_k(\eta,\eta_0) = \mathcal{O}(G_k^\Gamma \omega_k^{-1}) = \underline{\mathcal{O}\left(\mathrm{Max}[\omega_k^{-2}, \theta_k]\right)}$$

↓

Square summable:
Unitary evolution

Backreaction

- Explicit construction of the unitary operator that implements the dynamics, in terms of α_k, β_k :

$$\hat{U}_D = \hat{U}_B \hat{U}_L,$$

$\hat{U}_L \longrightarrow$ dominant phase of alpha coeffs.

$\hat{U}_B \longrightarrow$ rest of Bogoliubov transformation

- The evolved vacuum $\hat{U}_D|0\rangle$ can be seen to provide solutions to the Schrödinger equation, with:

$$C_D^{(\Gamma)}(\phi) = l_0 \langle \hat{V}^{2/3} \rangle_\Gamma \sum_{\vec{k},(x,y)} \left[\Im(G_k^{(\Gamma)} \bar{\Delta}_k) - d_{\eta_\Gamma} c_{\vec{k}}^{(x,y)} \right]$$

$c_{\vec{k}}^{(x,y)} \longrightarrow$ arbitrary part of the phase of $\hat{U}_D|0\rangle$

$$\Delta_k = e^{-i\omega_k(\eta-\eta_0) - i\Phi_k - i\int_{\eta_0}^{\eta} d\eta_\Gamma \tilde{J}_k^{(\Gamma)}} \beta_k$$ at dominant order $(\omega_k \to \infty)$

Backreaction

- In order to avoid any regularization by means of $c_{\vec{k}}^{(x,y)}$, or conditional sums, we impose that:

$$\Im(G_k^{(\Gamma)}\bar{\Delta}_k) \quad \text{is absolutely summable over } \vec{k} \in \mathbb{Z}^3$$

(independently of Γ)

- We know its dominant asymptotic order in terms of $G_k^{(\Gamma)}$.

- The backreaction is absolutely convergent iff:

$$\theta_k = -i\frac{\tilde{M}e^{-\alpha}}{4\omega_k^2}\pi_\alpha e^{iF_2^k} + \vartheta_k, \qquad \sum_{\vec{k}\in\mathbb{Z}^3} \omega_k|\vartheta_k|^2 < \infty$$

- Similar arguments show that the fermionic Hamiltonian is well-defined on the vacuum iff: $\displaystyle\sum_{\vec{k}\in\mathbb{Z}^3} \omega_k^2|\vartheta_k|^2 < \infty$

Conclusions & outlook

- Separation of the phase space in inflationary hybrid LQC so that the fermionic backreaction is well-defined.
- Leads to a very specific choice of annihilation and creation-like variables for the fermionic degrees of freedom:
    - ⋆ Dynamical characterization of the excitations.
    - ⋆ Restriction of the choices of vacua.
- Conditions can be strengthen so that the fermionic Hamiltonian is well-defined on the Fock vacuum.
- Results point to further possible restrictions via a diagonalization of the Hamiltonian on particle-states.