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# Embedding method for the scattering phase in strongly correlated quantum dots

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**Abstract.** The embedding method for the calculation of the conductance through interacting systems connected to single channel leads is generalized to obtain the full complex transmission amplitude that completely characterizes the effective scattering matrix of the system at the Fermi energy. We calculate the transmission amplitude as a function of the gate potential for simple diamond-shaped lattice models of quantum dots with nearest neighbor interactions. In our simple models we do not generally observe an interaction dependent change in the number of zeroes or phase lapses that depend only on the symmetry properties of the underlying lattice. Strong correlations separate and reduce the widths of the resonant peaks while preserving the qualitative properties of the scattering phase.

## 1. Introduction

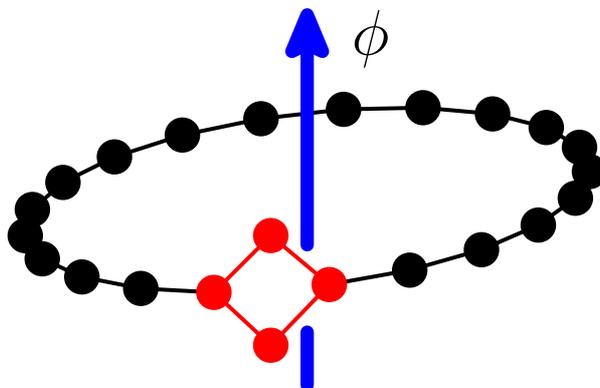
The scattering theory of quantum transport has demonstrated to be a very successful tool for the study and design of novel nanodevices. The information about the transport properties of a quantum mechanical system is encoded in the complex transmission amplitude. The conductance of such a system in a two-terminal configuration can be obtained from the trace of the transmission matrix  $t$  multiplied by its Hermitian conjugate  $t^\dagger$  according to the Landauer-Büttiker formula  $G = (e^2/h)\text{Tr}(t t^\dagger)$  [1, 2]. In a single channel device the conductance reduces

to the modulus of the transmission probability  $T$  through the channel and is easily measurable in experiments. On the other hand, even if the phases of the scattering matrix of a mesoscopic system show an interesting behaviour [3], their direct measurement cannot be done. One possible way to extract the phase of the transmission coefficient is to embed the system to measure in one of the arms of an Aharonov-Bohm (AB) interferometer. Under the appropriate conditions it was shown to be possible to extract the transmission phase from the phase of the conductance oscillations of the whole structure as a function of the magnetic flux threading the AB ring [4, 5].

The pioneering AB experiments that tried to measure the transmission phase of transport through a quantum dot in the Coulomb Blockade (CB) regime used a two-terminal geometry. In this situation, Onsager relations due to conservation of current and time-reversal symmetry imply that the conductance must be an even function of the magnetic flux and the measured transmission phase can only be 0 or  $\pi$  [2, 6]. Later experiments used a six-terminal configuration with additional leads that modify the reciprocity relations for transmission coefficients [5]. The obtained transmission phase presented a smooth increase of  $\pi$  as a function of a gate potential in the dot  $V_g$  when a resonance peak was crossed while a sudden jump of  $-\pi$  (a phase lapse following the experimentalists convention) was observed between resonances. In a more recent experiment the transition between this regime, termed universal regime, and a mesoscopic regime where phase lapses and resonances occur randomly with respect to each other was studied as a function of the number of electrons in the dot [7].

These observations have given rise to a whole body of literature devoted to study the transmission phase of quantum dots. Many theoretical works [8, 9, 10, 11, 12] were concerned with explaining the experimental results, in particular, the universal regime for the transmission phase which is at odds with the standard models for CB phenomena. Recent progress in that direction was obtained studying the statistical properties of the parity correlations between successive wave functions in ballistic chaotic quantum dots [13]. Another interesting path followed for the analysis of the experimental results is a many-body extension of the constant-interaction model (CIM) for the description of resonances in quantum dots [14, 15, 16]. In the regime where the distance between resonances is much smaller than the width of the resonant levels Karrasch *et al.* found that the number of phase lapses could be increased by moderate interactions. Although the results obtained do not seem to be generic, it has a lot of theoretical and practical interest to understand the effects of strong correlations on the behavior of the full complex scattering quantities that characterize the electronic transport properties.

A complete theoretical understanding of transport through systems with strong correlations is generally missing, however. The problem can be formulated using the Keldysh approach [17, 18]. However, the calculations using Keldysh Green functions for interacting systems can usually only be performed within some approximation [19]. There are also many numerical methods in the literature but none are without problems. The more generally applicable ones rely on extensive real time simulations, see [20] and references therein. However, the computing power needed for the application of these methods limits their practical implementations for strongly correlated systems. A more recent approach is based on the exact diagonalization of the full but isolated, many-body system, and accordingly treats tunneling perturbatively [21]. It obviously is restricted to small systems. Other types of approaches rely on the calculation of an equilibrium quantity that can be related to the conductance. Although these approaches are usually limited to the linear response regime they can be extremely useful to understand the effects of strong correlations on electronic transport [22, 23, 24, 25, 26, 27, 28]. The purpose of this work is to present one of these methods, the embedding method, in a self-contained manner. This numerical method in combination with Density Matrix Renormalization Group (DMRG) allows the computation of the transmission amplitude of a system with strong correlations attached to one-dimensional leads. As an illustration, we apply the method to simple lattice models of small quantum dots.



**Figure 1.** Sketch of the system considered within the embedding approach: The interacting region of length  $L_s$  (red) is embedded in a one-dimensional ring formed by a non-interacting lead (black) of length  $L_L$ . The ring is threaded by an Aharonov-Bohm flux  $\phi$ . The interacting system we consider consists of four sites in a diamond-shaped configuration with nearest-neighbor interaction.

## 2. Embedding method for the transmission through a quantum dot

During the last decade, the embedding method has been used quite successfully to calculate the transmission of electrons through simple lattice models of strongly correlated systems [22, 23, 24, 25, 26, 27]. The system under consideration is embedded in a non-interacting ring. The ground state properties of the combined system pierced by a magnetic flux can be related to the scattering properties of the embedded system. In this section, we show the main steps of the derivation of the method. We follow closely the approach that allows to extract the modulus of the transmission amplitude presented in Appendix A of Ref. [29]. Moreover, we extend the method to calculate also the phase of the transmission amplitude. We will limit ourselves to the case of a single channel in the leads which is the simplest one although generalizations of the method to more than one channel have been reported [30]. The derivation is only rigorously valid for non-interacting impurities. However it was shown numerically that an interacting system in the limit of infinite lead length can be described by an effective scattering matrix whose matrix elements are accessible via the embedding method [29].

We can write the quantization condition of the single-particle states of the ring in terms of the transfer matrix of the non-interacting part (the lead) and the transfer matrix of the scatterer (see the sketch in Fig. 1) as

$$\det(I - M_L M_S) = 0, \quad (1)$$

where  $M_S$  and  $M_L$  are the transfer matrices of the system and the lead, respectively. In the presence of time-reversal symmetry, the transfer matrix of a one-dimensional scatterer can be expressed in terms of three independent angles  $\hat{\alpha}$ ,  $\theta$  and  $\varphi$ :

$$\begin{aligned} M_S &= \begin{pmatrix} 1/t^* & r^*/t^* \\ r/t & 1/t \end{pmatrix} \\ &= \frac{1}{\sin \varphi} \begin{pmatrix} e^{i\hat{\alpha}}/\sin \theta & -i \cot \theta + \cos \varphi \\ i \cot \theta + \cos \varphi & e^{-i\hat{\alpha}}/\sin \theta \end{pmatrix}, \end{aligned} \quad (2)$$

where the two components correspond to right and left moving particles while  $r$  and  $t$  are the reflection and transmission amplitudes, respectively. The transmission amplitude is given by  $t = e^{i\hat{\alpha}} \sin \theta \sin \varphi = |t| \exp(i\alpha)$ . This relation between the transmission phase  $\alpha$  and the scattering phase  $\hat{\alpha}$  was at the origin of some misunderstandings about the interpretation of the

experimentally observed phase lapses until the issue was clarified by Taniguchi and Büttiker [31].  $\alpha$  is the relevant experimental quantity and is not well defined when there is a zero of the transmission, resulting in a phase lapse whenever the transmission vanishes changing sign as a function of some parameter.  $\hat{\alpha}$  is the relevant phase for the observance of the Friedel sum rule and only presents trivial  $2\pi$  jumps.

The transfer matrix of a lead of length  $L_L$  for a state with wave number  $k \geq 0$  reads

$$M_L = \exp(i\Phi) \begin{pmatrix} \exp(ikL_L) & 0 \\ 0 & \exp(-ikL_L) \end{pmatrix}, \quad (3)$$

taking into account that the flux threading the ring can be included by twisting the boundary condition of the system by a gauge transformation. Note that the scatterer embedded in the ring is not affected by this artificial flux such that we always work at zero magnetic field in the system of interest.

Inserting the transfer matrices (2) and (3), the eigenvalue condition (1) yields

$$\cos(\Phi) = \frac{1}{|t(k)|} \cos(kL + \delta\hat{\alpha}(k)). \quad (4)$$

The phase shift  $\delta\hat{\alpha} = \hat{\alpha} - kL_S$  is the phase of the scattering region relative to a perfect lead of the same length  $L_S$ . The solution of (4) yields the quantized momenta  $k$  of the energy eigenstates in the lead.

Since both  $t$  and  $\delta\hat{\alpha}$  are functions of  $k$ , it is in general impossible to obtain an analytic solution of (4). However, one can study the asymptotic limit of large  $L$ , which was done by Gogolin and Prokof'ev in their study of the persistent current of a one-dimensional ring with a defect [32]. We use a general dispersion relation  $\epsilon(k)$  in the lead that allows us to discuss continuum and tight-binding models at the same time.

The eigenvalue condition (4) can be rewritten as

$$k = k_n^0 + \frac{1}{L} f_{\pm}(k, \Phi). \quad (5)$$

Here,  $k_n^0 = 2\pi n/L$  with  $n \geq 0$  denotes the eigenvalues in the case of perfect transmission with  $|t| = 1$  and  $\delta\hat{\alpha} = 0$ . Following the notation of Ref. [32], we introduce the function

$$f_{\pm}(k, \Phi) = \pm \text{Arccos}(|t(k)| \cos \Phi) - \delta\hat{\alpha}(k). \quad (6)$$

Arccos denotes the principal branch of the inverse cosine function that takes values in the interval  $[0, \pi]$ . As  $k$  should be positive,  $f_{-}(k, \Phi)$  cannot be used for the case  $n = 0$ . The splitting of the solutions of (5) corresponding to “+” and “-” cannot exceed the spacing  $2\pi/L$  between the  $k_n^0$ , provided that  $\delta\hat{\alpha}(k)$  is smooth on this scale. This is the case in the limit  $L \rightarrow \infty$  and ensures that the order of the solutions with respect to energy is given by  $n$ .

Iterating (5) and expanding  $f_{\pm}$  for large systems, we obtain the expansion

$$k_n^{\pm} = k_n^0 + \frac{1}{L} f_{\pm}(k_n^0, \Phi) + \frac{1}{L^2} f_{\pm}(k_n^0, \Phi) \left( \frac{\partial f_{\pm}(k, \Phi)}{\partial k} \right)_{k=k_n^0} + O\left(\frac{1}{L^3}\right) \quad (7)$$

for the solutions of (5) in powers of  $1/L$ . Such an expansion is problematic in the vicinity of resonances where there is a rapid variation of the function  $f$  with  $k$ . In that case, the expansion is valid only for sufficiently large  $L$ .

We now calculate the ground state energy of the system as a function of the flux to order  $1/L^2$ . Using (7), the expansion of the one-particle energies in powers of  $1/L$  can be shown to be

$$\begin{aligned} \epsilon(k_n^\pm) = & \epsilon(k_n^0) + \frac{1}{L} \left( \frac{\partial \epsilon}{\partial k} f_\pm(k, \Phi) \right)_{k=k_n^0} \\ & + \frac{1}{2L^2} \frac{\partial}{\partial k} \left( \frac{\partial \epsilon}{\partial k} f_\pm^2(k, \Phi) \right)_{k=k_n^0} + O\left(\frac{1}{L^3}\right). \end{aligned} \quad (8)$$

We consider the simplest case of an odd number of particles in the ring. The leading order results for even number of particles give rise to the same final expression for the calculation of the transmission modulus [29]. For an odd number of particles  $N$  in the ring, all occupied states  $n$  come in pairs ( $[n,-]$  and  $[n,+]$ ), except for the one corresponding to  $n = 0$ . The total ground state energy is

$$\begin{aligned} E_0^{\text{odd}}(\Phi) = & \epsilon(k_0^+) + \sum_{n=1}^{n_F} [\epsilon(k_n^+) + \epsilon(k_n^-)] \\ = & \epsilon(0) + \frac{1}{2L^2} \left( \frac{\partial^2 \epsilon}{\partial k^2} [\text{Arccos}(|t| \cos \Phi) - \delta \hat{\alpha}]^2 \right)_{k=0} \\ & + \sum_{n=1}^{n_F} \left\{ 2\epsilon(k_n^0) - \frac{2}{L} \left( \frac{\partial \epsilon}{\partial k} \right)_{k=k_n^0} \delta \hat{\alpha}(k_n^0) \right. \\ & \left. + \frac{1}{L^2} \frac{\partial}{\partial k} \left( \frac{\partial \epsilon}{\partial k} [\text{Arccos}^2(|t| \cos \Phi) + \delta \hat{\alpha}^2] \right)_{k=k_n^0} \right\} + O\left(\frac{1}{L^3}\right). \end{aligned} \quad (9)$$

The sum runs up to  $n_F = (N - 1)/2$ . We have assumed  $(\partial \epsilon / \partial k)_{k=0} = 0$  and kept all terms which can give rise to contributions up to order  $1/L$ . The first term in the sum is the ground state energy in the absence of scattering. It is proportional to  $L$  when  $N$  is of order  $L$ . The second term represents the energy change due to the scattering potential and is of order 1. From this second term, we can obtain the scattering phase shift by changing the number of particles as it is discussed below. The third term is the leading flux-dependent correction which allows to obtain the modulus of the transmission.

We start by considering the leading order terms for the calculation of the modulus of the transmission (see App. A of Ref. [29] for next order corrections and for the case of an even number of particles). Converting the sums over  $n$  into integrals, these flux-dependent contributions can be expressed as

$$\begin{aligned} & \frac{1}{2\pi L} \int_{\pi/L}^{k_F + \pi/L} dk \frac{\partial}{\partial k} \left( \frac{\partial \epsilon}{\partial k} \text{Arccos}^2(|t| \cos \Phi) \right) \\ & = \frac{\hbar v_F}{2\pi L} \text{Arccos}^2(|t(k_F)| \cos(\Phi)) + O\left(\frac{1}{L^2}\right). \end{aligned} \quad (10)$$

Here,  $k_F = 2\pi n_F / L$  is the Fermi wave number and  $v_F = (\partial \epsilon / \hbar \partial k)_{k=k_F}$  is the Fermi velocity. The leading flux-dependent term of the ground state energy is

$$E_0^{\text{odd}(1)}(\Phi) = \frac{\hbar v_F}{2\pi L} \text{Arccos}^2(|t(k_F)| \cos(\Phi)), \quad (11)$$

where the superscript indicates the order of the result in  $1/L$ . In order to calculate the transmission modulus we need the leading order of the charge stiffness

$$\begin{aligned} D^{(1)} &= -\frac{L}{2} \left( E_0^{\text{odd}(1)}(0) - E_0^{\text{odd}(1)}(\pi) \right) \\ &= \frac{\hbar v_F}{2} \left[ \frac{\pi}{2} - \text{Arccos}(|t(k_F)|) \right]. \end{aligned} \quad (12)$$

This last result is independent of the parity of the number of particles. Then, the transmission modulus can be obtained as

$$|t(E_F)|^2 = \lim_{L \rightarrow \infty} \sin^2 \left( \frac{\pi D}{2D^0} \right), \quad (13)$$

where  $D^0$  is the charge stiffness of a clean ring without scatterer.

We now keep the flux fixed. Then, the lowest order change of the ground state energy (9) in  $1/L$  with respect to the change of the particle number  $N$  depends on  $\delta\hat{\alpha}$ . It is thus possible to express the scattering phase as

$$\delta\hat{\alpha}(k_F) = - \lim_{L \rightarrow \infty} \frac{L}{2} \left( \frac{E(N) - E(N-2) - 2\epsilon(k_{n_F}^{(0)})}{d\epsilon/dk|_{k=k_{n_F}^{(0)}}} \right) \quad (14)$$

in terms of the many-body energies at particle numbers  $N$  and  $N-2$  and the properties of the dispersion relation at the Fermi energy. In the special case of a tight binding chain with unit hopping and lattice spacing at half filling, this formula reduces to

$$\delta\hat{\alpha}(k_F) = - \lim_{L \rightarrow \infty} \frac{L}{4} [E(L/2+1) - E(L/2-1)]. \quad (15)$$

Eqs. 14 and 15 show that the scattering phase is related to the change in total ground state energy in large rings when the particle number is changed. It is therefore possible to use the embedding method to extract the scattering phase.

From the embedding method, we can thus obtain  $\hat{\alpha}$ . However, the quantity of interest from the experimental point of view is  $\alpha$  with its phase lapses coming from the different branches of the relation  $|t| \exp(i\alpha) = \sin \theta \sin \varphi \exp(i\hat{\alpha})$ . To achieve this goal, we need to access the sign of  $\varphi$ . For that purpose we go back to the general expression (2) of the transfer matrix and see that in order to have left-right symmetry we need  $r = r'$ , and therefore  $\sin \theta \cos \varphi = 0$ .

We have three possibilities: i)  $\sin \theta = 0$ , ii)  $\varphi = \pi/2$ , iii)  $\varphi = -\pi/2$ . Thus, in a left-right symmetric case we have two branches ( $\sin \varphi = 1$  and  $\sin \varphi = -1$ ) and we can pass from one to the other only when  $t = 0$ .

The embedding method allows to follow the branch-switching since the phase sensitivity is given by [29]

$$\Delta E = (E(\pi) - E(0)) = \frac{\hbar v_F}{L} \left[ \frac{\pi}{2} - \text{Arccos}(\sin \theta \sin \varphi) \right], \quad (16)$$

and therefore

$$\sin \theta \sin \varphi = \sin \left( \frac{\Delta E L}{\hbar v_F} \right) = \sin \left( \frac{\pi}{2} \frac{\Delta E}{|\Delta E^{(0)}|} \right), \quad (17)$$

where  $\Delta E^{(0)}$  is the phase sensitivity of a perfectly transmitting scatterer. Therefore, the sign changes of  $\Delta E$  determine the branch-switch of  $\varphi$  and thus the jumps in  $\alpha$ . One should also notice that there can also be transmission zeroes without phase lapses in cases when there is a zero of  $\Delta E$  without sign change. This possibility does occur for particular values of the parameters in some of the models we have analyzed.

For a strictly one-dimensional system, the sign of  $\Delta E$  is fixed by Leggett's theorem [33, 34]. For an odd number of particles we are in the branch of positive  $\varphi$ ,  $t$  never vanishes, and there cannot be branch-switches. For a quasi-one dimensional scatterer this is no longer true, there can be parameter values where  $t$  vanishes and branch-switches appear. The embedding method for the phase of the transmission has been checked for a one-dimensional chain and its predictions compared with the Friedel sum rule. Its validity has been confirmed for interacting scatterers attached to reservoirs via non-interacting leads, where it describes the effective one-body scattering at the Fermi-energy [16, 29].

### 3. Numerical results for diamond-shaped quantum dots

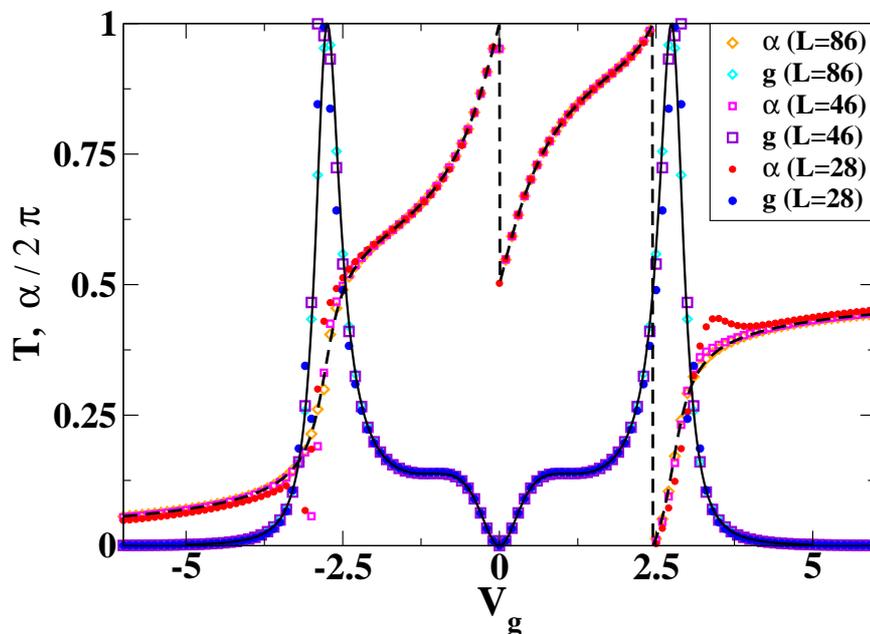
A one-dimensional scatterer does not exhibit zeroes of the transmission and thus no phase lapses are observed. We therefore study a minimal model of a scatterer in which transmission zeroes occur, and where phase lapses are expected. In this context, the diamond lattice quantum dots were introduced by Levy-Yeyati and Büttiker as a simple example to understand the physics behind the phase lapses of the transmission [35]. In the non-interacting case the form of the transmission as a function of the gate voltage  $V_g$  can be obtained analytically [35]. The results for the non-interacting case can serve as a guide for the convergence properties of the embedding method for quantum dot lattice models beyond one dimension.

The Hamiltonian of the system under study including interactions reads

$$\begin{aligned}
 H = & -t \left( c_1^\dagger c_2 + c_1^\dagger c_3 + c_2^\dagger c_4 + c_3^\dagger c_4 + \text{H.C.} \right) + \epsilon n_2 - \epsilon n_3 \\
 & + V_g \sum_{i=1}^4 n_i + U (\tilde{n}_1 \tilde{n}_2 + \tilde{n}_1 \tilde{n}_3 + \tilde{n}_2 \tilde{n}_4 + \tilde{n}_3 \tilde{n}_4), \quad (18)
 \end{aligned}$$

where  $\tilde{n}_i = n_i - 1/2$ . The nearest neighbor interaction is added in that form in order to enforce charge neutrality at half-filling. A sketch of the lattice used in the model is shown in Fig. 1. Site 1 is connected to the left lead and to sites 2 and 3 while site 4 is connected to the right lead and also to sites 2 and 3. The single particle energies of sites 2 and 3 are the same but with opposite signs while the single particle energies at the sites connected to the leads 1 and 4 are taken to be 0. This symmetry ensures that a zero of the transmission appears exactly at  $V_g = -2U$ .

In the non-interacting case with  $\epsilon = 2$  the conductance for this model has two symmetric resonant peaks at  $V_g = 2.75$  and  $V_g = -2.75$ . The zero of the transmission is associated with a phase lapse at  $V_g = 0$ . There are two more symmetric resonances around the transmission zero at  $V_g = -1.01$  and  $V_g = 1.01$  but corresponding to very badly coupled wave-functions so they hardly surpass the transmission background from the bigger resonances. We compare the exact results to calculations using the embedding method. For obtaining results from the embedding method an extrapolation to infinite lead lengths is needed. A finite size scaling analysis of the results is made using increasing lead lengths and linear or quadratic extrapolation formulas as explained in detail in Ref. [29]. This procedure is performed for the transmission modulus and for the transmission phase in exactly the same fashion. We show in Fig. 2 the values for  $g$  and  $\alpha$  obtained with total lengths of the system plus the auxiliary ring of  $L = 28, 46, 86$  and compare them with the exact result. The actual value of the length that we need to use in the extrapolation formulas (Eq. 12 and Eq. 15) is  $L - 1$  due to the geometrical form of the system (sites 2 and 3 are just to alternative ways for the electron to go through the system and count as a single site for the purpose of the extrapolation formulas). The results for all the three cases are very good around the zero of the transmission. There are small deviations close to the resonances for  $L = 28$  and  $L = 46$  while the results for  $L = 86$  coincide almost exactly with the exact results. The qualitative behavior is correctly reproduced even for the smallest ring size  $L = 28$ . The deviation from the exact results close to the resonances is characteristic of the



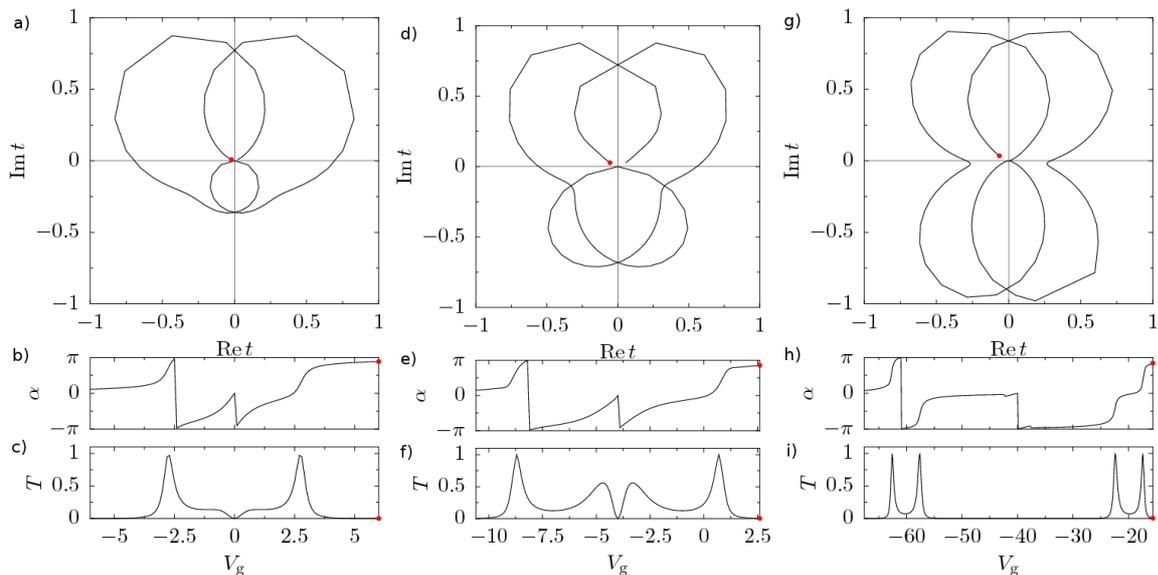
**Figure 2.** Transmission probability and transmission phase for the non-interacting diamond dot model with  $\epsilon = 2$  calculated using the embedding method. The solid lines represent the exact results.

method when full convergence in the length of the auxiliary ring has not been reached. Similar deviations are observed when interactions are switched on.

When  $U > 0$ , the ground state properties of the ring including the diamond-shaped quantum dot are calculated using the Density Matrix Renormalization Group (DMRG) algorithm [36]. We keep up to 650 states in order to get a good accuracy in the calculation of the charge stiffness of the ring for the larger sizes.

From very general arguments based on the Green's function approach to non-interacting transport it can be shown that a zero of the transmission appears between two consecutive resonant peaks depending on the relative symmetry properties of the wave functions corresponding to the resonances with respect to the positions of the leads [35]. If consecutive resonances have the same symmetry a zero appears between peaks and when they have opposite symmetry the zero is missing. The four resonances of the diamond quantum dot Hamiltonian Eq. 18 have even, odd, odd and even symmetry with respect to sites 1 and 4 so there is only one zero in the middle.

The effect of the interactions is to change the position of the zero of the conductance (due to the charge neutrality requirements) and the position of the resonance peaks while deforming their overall shape. The resonances get more isolated and symmetric. However, the phase lapse of  $\alpha$  when we go through the transmission zero is the same as in the non-interacting case. Within our model, The symmetries of the resonances are not modified by the many-body correlations that are induced through the increase of the interaction strength  $U$ . In the different panels of Fig. 3 we show the behavior of the complex transmission coefficient as a function of  $V_g$ . We compare the exact results for  $U = 0$  with embedding method results calculated with an extrapolation up to  $L = 76$  for the cases of  $U = 2$  and  $U = 20$ .



**Figure 3.** Results for the diamond quantum dot model with  $\epsilon = 2$  and different values of the interaction strength  $U$ . The left, middle, and right panels show the trajectory of the complex transmission amplitude (a, d, and g), transmission phase as a function of  $V_g$  (b, e, and h) and transmission probability  $T$  as a function of  $V_g$  (c, f, and i) for  $U = 0$ ,  $U = 2$ , and  $U = 20$ , respectively. The red dot marks the same  $V_g$  point in each of the three graphs with the same value  $U$  so it is easier to follow the trajectory of the transmission amplitude in the complex plane.

Due to the symmetry of the problem, the transmission zero is always placed between the two groups of resonances. As we increase the interaction the symmetric resonances close to the zero of transmission that are barely visible in the case  $U = 0$  fully develop into lorentzian peaks and a wide gap opens between them. This is due to the transition between extended wave functions for small  $U$  to spatially localized wave-functions for large  $U$ . However, there is no qualitative change in the behavior of the transmission phase as we increase the interaction  $U$ . The trajectories of the complex transmission amplitude as we vary  $V_g$  shown in the top panel of the figure show similar features. The mechanisms proposed for the change of the number of phase lapses in interacting models like population switching[14, 15, 37] are not at work in this simple geometrical model of a quantum dot precisely because the geometrical properties of the lattice dominate the symmetry properties of the wave functions even in the many-body case.

#### 4. Conclusions

The embedding method for the calculation of the transmission through a strongly correlated system connected to non-interacting leads can be successfully extended to calculate also the phase of the transmission amplitude. The phase of the effective transmission amplitude of a correlated system connected to non-interacting one-dimensional leads can be extracted from differences of the ground state energy of an auxiliary ring at different electron numbers. We have illustrated the application of this method for the calculation of the full complex transmission coefficient of a diamond-shaped quantum dot with nearest-neighbor interactions. The calculations of the ground state energy differences have been performed using the DMRG algorithm that allows for great accuracy in the models considered. For the small systems that we treated in the work, interactions modify substantially the shape of the resonances although the number and position of the zeros between resonances are not changed and depend only on

the symmetry properties of the quantum dot model. Larger structures which represent a harder computational challenge may present richer phenomena concerning the scattering phase. These and further issues will be explored in future work.

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