Fertigation in furrows and level furrow systems I: model description and numerical tests

J. Burguete¹, N. Zapata², P. García-Navarro³, M. Maïkaka⁴, E. Playán⁵, and J. Murillo⁶

¹: *Researcher, Dept. Suelo y Agua, Estación Experimental de Aula Dei, CSIC.*

P.O. Box. 202, 50080 Zaragoza, Spain. E-mail: jburguete@eead.csic.es

²: *Researcher, Dept. Suelo y Agua, Estación Experimental de Aula Dei, CSIC.*

P.O. Box. 202, 50080 Zaragoza, Spain. E-mail: vzapata@eead.csic.es

³: *Professor, Dept. Fluid Mechanics, Centro Politécnico Superior, University of Zaragoza.*

María de Luna 3, 50018 Zaragoza, Spain. E-mail: pigar@unizar.es

⁴: *Engineering, SERS Consulting, Paseo Rosales 34, 50008 Zaragoza. Spain.*

E-mail: TIEMAGO@terra.es

⁵: *Researcher, Dept. Suelo y Agua, Estación Experimental de Aula Dei, CSIC.*

P.O. Box. 202, 50080 Zaragoza, Spain. E-mail: playan@eead.csic.es

⁶: *Assistant Professor, Dept. Fluid Mechanics, Centro Politécnico Superior, University of*

Zaragoza. María de Luna 3, 50018 Zaragoza, Spain. E-mail: Javier.Murillo@unizar.es

Abstract

The simulation of fertigation in furrows and level furrow systems faces a number of problems resulting in relevant restrictions to its widespread application. In this paper, a simulation model is proposed that addresses some of these problems by: 1) implementing an infiltration model that adjusts to the variations in wetted perimeter; 2) using a friction model that adjusts to different flows and which uses an absolute roughness parameter; 3) adopting an equation for the estimation of the longitudinal diffusion coefficient; and 4) implementing a second order TVD numerical scheme and specific treatments for the boundary conditions and the junctions. The properties of the proposed model were demonstrated using three

numerical tests focusing on the numerical scheme and the treatments. The application of the model to the simulation of furrows and furrow systems is presented in a companion paper, in which the usefulness of the innovative aspects of the proposed model is demonstrated.

Keywords: Infiltration, Furrow irrigation, Surface irrigation, Shallow water, Flow simulation, Numerical models.

INTRODUCTION

The numerical simulation of hydrodynamics is a common technique for irrigation analysis, from conveyance networks to on-farm systems. Surface irrigation systems are characterized by their operational simplicity and their complicated analysis and design. The numerical analysis of surface irrigation systems started in the 1970s, aiming at optimizing design and management by maximizing the insight obtained from resource consuming field experiments. Furrow fertigation is a popular surface irrigation system, characterized by one-dimensional flow and wetted perimeter dependent infiltration.

Fertigation (the application of fertilizers dissolved in irrigation water) is a common agricultural technique. It is not only agronomically suited to many crops, but it also constitutes the preferred technical solution to the fertilization of field crops developing tall canopies. For these cases, fertigation is much easier to implement and manage in sprinkler irrigation than in surface irrigation. However, surface irrigation farmers resort to fertigation for a number of crops and in a number of areas of the world. In fact, surface fertigation can result in a reduction of labour, energy, use of machinery and soil compaction as compared to the conventional application of fertilizers.

It was not until the end of the twentieth century that basin and border fertigation was addressed through experimentation and numerical analysis (Boldt et al. 1994; Playán and Faci 1997). These authors applied advective models to the results of surface irrigation simulation and to the identification of optimum fertilizer application practices. Field experiments permitted to explore the conditions of irrigation performance and fertilizer application resulting in adequate estimations of fertilizer distribution uniformity and application efficiency.

García-Navarro et al. (2000) presented a hydrodynamic model of basin and border fertigation, using a McCormack numerical scheme. The model was calibrated and validated using experiments on pervious and impervious borders. Impervious experiments were designed to eliminate the uncertainties derived from infiltration estimation. A diffusion coefficient was introduced in the formulation and estimated via calibration to experimental results. Solute transport in furrows represents an additional challenge due to the complexity of furrow infiltration. Abbasi et al. (2003a) reported the results of a detailed experiment revealing the 2D features of furrow fertigation in what refers to water and solute infiltration. Abbasi et al. (2003b) presented a Crank-Nicholson fertigation model including a routine for the estimation of the diffusion coefficient using a model initially derived for solute transport in soils (Bear 1972). Sabillón and Merkley (2004) presented an advective implicit model simulating furrow fertigation, and identified guidelines for optimum fertilizer application. A split-operator hydrodynamic simulation model for border and basin fertigation was proposed and evaluated by Zerihun et al. (2005a, 2005b), using the same approach for the diffusion coefficient proposed by Bear (1972). These authors coupled overland hydraulics to the HYDRUS-1D model (Simunek et al. 1998) for subsurface flow, and reported adequate agreement between observed and simulated solute distribution. Adamsen et al. (2005) reported a series of field experiments using bromide as a tracer. These authors identified strategies aiming at developing fertigation rules for their experimental conditions. Finally, Strelkoff et al. (2006) reported the extension of the surface irrigation model SRFR to simulate fertigation using an advective scheme.

The review of the literature shows that furrow irrigation has been simulated using a variety of approaches, with the hydrodynamic approach being the most common now a days. Following this approach, the well-known Saint Venant equations are typically solved in combination with two additional empirical equations representing the physical processes of roughness and infiltration. The characterization of roughness requires estimation of the Gauckler-Manning number, a parameter which is often described as dependent on soil surface

conditions, but which also depends on the irrigation discharge. Infiltration estimation is not an easy task even in flat geometry surface irrigation systems such as borders and basins. Numerical parameter estimation techniques have often been applied to this problem, and the resulting parameters only represent the soil surface in the particular experimental conditions. In furrow irrigation systems, infiltration additionally depends on furrow geometry and on wetted perimeter, therefore increasing the number of model parameters and making the use of models a more complicated task. Characterizing furrow infiltration therefore stands as a relevant obstacle to simulation. Although the complexity of furrow infiltration has been analysed and modelled using a number of approaches (Walker and Skogerboe 1987), practical applications have not been abundant due to the requirements on experimental data. When it comes to simulating the transport of neutrally buoyant solutes, furrow models fluctuate between the simplicity of the advective models and the complexity of advective-diffusive models. This complexity is not restricted to the programming effort, but also extends to the identification of the longitudinal diffusion coefficient. Even when predictive equations have been used to estimate the diffusion coefficient, the sensitivity of model results to this parameter has often been analysed in an attempt to derive better parameter estimates.

The analysis of previous efforts suggests that three aspects of furrow fertigation simulation seem to require further attention: infiltration, roughness and fertilizer dispersion. Furrow fertigation is an active field of research in which simplified advective models are used because of the difficulties related to introducing additional simulation parameters and performing additional computations. While this may be an adequate choice in many cases, particular furrow configurations and experimental conditions require an adequate treatment of fertilizer hydrodynamic dispersion. There is a need for numerical models of furrow fertigation using a few, physically based parameters which can be either measured or estimated from experimental measures.

In the last decades, a particular type of furrow irrigation systems has become very popular among farmers in certain areas of the world: level furrows (Walker and Skogerboe 1987). In

this system, a zero-slope field with one inflow point is furrowed at the beginning of the season. Water builds up at the upstream distribution channel as it starts flowing down the irrigation furrows and recirculating through the downstream distribution channel once water advances to the downstream end of some irrigation furrows (Playán et al. 2004). Level furrows are characterized by requiring very little labour and by a high potential application efficiency. Irrigation simulation in level furrow systems was reported by García-Navarro et al. (2004).

In this work, a coupled model of water flow and solute transport is presented for the simulation of surface fertigation in furrows and level furrow systems. Particular attention is paid to the following aspects:

- the infiltration model in furrow geometry, incorporating the model proposed by Maïkaka (2004);
- the friction term, implementing the recent developments by Burguete et al. (2007c) aiming at introducing an absolute roughness parameter;
- the model proposed by Rutherford (1994) to describe the chemical diffusion coefficient; and
- the numerical techniques used for the solution of the governing set of equations.

An experimental field study was specifically designed to validate the proposed model and is presented in a companion paper, together with additional model applications.

GOVERNING EQUATIONS

Shallow-water model

The one-dimensional system formed by the cross sectional averaged liquid and solute mass conservation, momentum balance in main stream direction, infiltration and solute transport can be expressed in conservative form as:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{I} + \mathbf{S}^c + \frac{\partial \mathbf{D}}{\partial x} \quad (1)$$

117 where $\mathbf{U}$ is the vector of conserved variables, $\mathbf{F}$ the flux vector, $\mathbf{S}^c$ the source term vector, $\mathbf{I}$
 118 the infiltration vector and $\mathbf{D}$ stands for diffusion:

$$\mathbf{U} = \begin{pmatrix} A \\ Q \\ As \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} Q \\ gI_1 + \frac{Q^2}{A} \\ Qs \end{pmatrix}, \quad \mathbf{S}^c = \begin{pmatrix} 0 \\ g[I_2 + A(S_0 - S_f)] \\ 0 \end{pmatrix}, \quad \mathbf{I} = \begin{pmatrix} -Pi \\ 0 \\ -Pis \end{pmatrix},$$

$$\mathbf{D} = \begin{pmatrix} 0 \\ 0 \\ K_x A \frac{\partial s}{\partial x} \end{pmatrix} \quad (2)$$

119
 120 with A the wetted cross sectional area, Q the discharge, s the cross sectional average solute
 121 concentration, g the gravity constant, S_0 the longitudinal bottom slope, S_f the longitudinal
 122 friction slope, K_x the diffusion coefficient, i the infiltration rate, P the cross sectional wetted
 123 perimeter. I_1 and I_2 represent pressure forces:

$$I_1 = \int_0^H (H - z'') w dz'', \quad I_2 = \int_0^H (H - z'') \frac{\partial w}{\partial x} dz'' \quad (3)$$

124 with H the water depth and w the cross section width (see Figure 1 for the system of
 125 reference). The furrows are modelled as pervious prismatic channels of trapezoidal cross
 126 section as represented in Figure 2. In this case, the pressure integrals become:

$$I_1 = \frac{B_0 H^2}{2} + \frac{S H^3}{3}, \quad I_2 = 0 \quad (4)$$

127 with B_0 the base width and S the tangent of the angle between the furrow walls and the
 128 vertical direction. The set of equations is completed with the laws for infiltrated volume of
 129 water and solute:

$$\frac{\partial \alpha}{\partial t} = Pi, \quad \frac{\partial \phi}{\partial t} = Pis \quad (5)$$

130 with α the volume of water infiltrated per unit length of furrow and ϕ the mass of solute
 131 infiltrated per unit length of the furrow.

132 The system of equations (1) can be expressed in non-conservative form taking into ac-
 133 count:

$$\frac{d\mathbf{F}(x, \mathbf{U})}{dx} = \frac{\partial \mathbf{F}}{\partial x} + \mathbf{J} \frac{\partial \mathbf{U}}{\partial x}, \quad \mathbf{J} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} = \begin{pmatrix} 0 & 1 & 0 \\ c^2 - u^2 & 2u & 0 \\ -us & s & u \end{pmatrix} \quad (6)$$

134 where $\mathbf{J}$ is the flux Jacobian, $u = \frac{Q}{A}$ is the cross sectional average velocity, $c = \sqrt{\frac{gA}{B}}$ is the
 135 velocity of the infinitesimal waves and B is the cross section top width. Inserting in (1):

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{J} \frac{\partial \mathbf{U}}{\partial x} = \mathbf{I} + \mathbf{S}^{nc} + \frac{\partial \mathbf{D}}{\partial x} \quad (7)$$

136 with $\mathbf{S}^{nc}$ the non-conservative source term:

$$\mathbf{S}^{nc} = \mathbf{S}^c - \frac{\partial \mathbf{F}}{\partial x} = \begin{pmatrix} 0 \\ c^2 \frac{\partial A}{\partial x} - gA \left(\frac{\partial z_s}{\partial x} + S_f \right) \\ 0 \end{pmatrix} \quad (8)$$

137 where z_s is the water surface level. The Jacobian matrix can be made diagonal:

$$\mathbf{J} = \mathbf{P} \mathbf{\Lambda} \mathbf{P}^{-1}, \quad \mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ \lambda_1 & \lambda_2 & 0 \\ s & s & 1 \end{pmatrix}, \quad \mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \quad (9)$$

138 with $\mathbf{\Lambda}$ the eigenvalues diagonal matrix, $\mathbf{P}$ the diagonalizer matrix and λ_i the Jacobian
 139 eigenvalues corresponding to the propagation characteristic celerities:

$$\lambda_1 = u + c, \quad \lambda_2 = u - c, \quad \lambda_3 = u \quad (10)$$

140 Furrow infiltration model

141 One of the most widely used empirical models in surface irrigation is the Kostiakov model
 142 relating the infiltration depth Z to the opportunity time τ :

$$Z = K\tau^a \quad (11)$$

143 where K is the Kostiakov constant and a is the Kostiakov exponent, both empirical param-
 144 eters depend on soil type, soil water and compaction. From (11), the expression for the
 145 infiltration rate can be derived:

$$i = \frac{dZ}{dt} = Ka\tau^{a-1} \quad (12)$$

146 Working out τ from (11) and inserting it in (12) it can be re-expressed in terms of the
 147 infiltration depth (Maikaka 2004):

$$i = Ka \left(\frac{Z}{K} \right)^{\frac{a-1}{a}} \quad (13)$$

148 For long infiltration events, the Kostiakov model does not predict the correct infiltration
 149 rate. In these cases, it is necessary to introduce the saturated infiltration long-term rate i_c
 150 (Walker and Skogerboe 1987). Then, the Kostiakov-Lewis model is obtained:

$$i = i_c + Ka\tau^{a-1} \approx i_c + Ka \left(\frac{Z}{K} \right)^{\frac{a-1}{a}} \quad (14)$$

151 In furrows, the amount of water infiltrated per unit time and furrow length is proportional
 152 to the wetted perimeter. Therefore, using opportunity time as the only independent variable
 153 in furrow infiltration such as in (11) or (12) is not correct, since infiltration will depend
 154 not only on the time during which water has been infiltrating but also on the surface water
 155 present in the furrow itself. Some authors (Playán et al. 2004) tried to modify expression
 156 (11) by introducing a dependence on the discharge, however, the results obtained with that
 157 model are contradictory in cases of transient inlet discharge. We believe that (13) can be

actually much more representative of the real event since it contains the dependence of i on Z , including the effects of opportunity time and the time evolution of wetted perimeter.

Our aim is to be able to move from (14) to an equivalent form valid in furrows trying to preserve the dimensionality and physical meaning of the Kostiakov-Lewis parameters. For that purpose, Z will be replaced by α divided by the furrow spacing D so that, in furrows:

$$i = i_c + Ka \left(\frac{\alpha}{DK} \right)^{\frac{a-1}{a}} \quad (15)$$

and this infiltration rate will be considered uniform all along the wetted perimeter P , in a form that permits to model the time variation of infiltrated area as (Maïkaka 2004):

$$\frac{d\alpha}{dt} = Pi = P \left[i_c + Ka \left(\frac{\alpha}{DK} \right)^{\frac{a-1}{a}} \right], \quad \frac{d\phi}{dt} = s \frac{d\alpha}{dt} = sP \left[i_c + Ka \left(\frac{\alpha}{DK} \right)^{\frac{a-1}{a}} \right] \quad (16)$$

Friction model

The friction slope is widely modelled by means of the Gauckler-Manning law (Gauckler 1867; Manning 1890):

$$S_f = \frac{n^2 Q |Q| P^{4/3}}{A^{10/3}} \quad (17)$$

For a furrow of trapezoidal cross section:

$$S_f = \frac{n^2 Q |Q| (B_0 + 2H\sqrt{1+S^2})^{4/3}}{(B_0 + 2SH)^{10/3}} \quad (18)$$

A more recent model (Burguete et al. 2007c), that showed a better performance in cases of high relative roughness, assumes that the velocity profile can be fit by means of a power function in the roughness upper zone, being negligible in the lower zone:

$$v_x = u_l \left(\frac{z - z_b - z'}{l} \right)^b, \text{ if } z \geq l + z_b + z' \quad (19)$$

where b is a fitting exponent and u_l is the water velocity at a vertical distance l of the

bed. This model also assumes that the bed roughness irregularities are of average size l . Neglecting the lateral exchanges of momentum, the velocity distribution that minimizes the friction energy losses can be obtained (Burguete et al. 2007c; Burguete et al. 2007a):

$$S_f = \frac{|Q|Q}{g \left[\int \frac{1}{(b+1)\sqrt{\epsilon}} \left(\frac{h^{b+(3/2)}}{l^b} - l\sqrt{h} \right) dy \right]^2} =$$

$$= \frac{\epsilon(b+1)^2 l^{2b} |Q|Q}{g \left\{ B_0 \left(H^{b+(3/2)} - \sqrt{H} l^{1+b} \right) + 2S \left[\frac{H^{b+(5/2)} - l^{b+(5/2)}}{b+(5/2)} - \frac{2}{3} l^{1+b} (H^{3/2} - l^{3/2}) \right] \right\}^2} \quad (20)$$

where ϵ is a dimensionless parameter of aerodynamical resistance depending only, in turbulent flows, on the roughness shape. This friction law is only valid for $H > l$. If $H < l$ a zero velocity condition is imposed for numerical stabilization of the advance over a dry bed. Figure 3 shows a comparison of the friction slopes estimated by the Gauckler-Manning and the proposed model for a typical furrow assuming a uniform flow velocity. The predicted values are similar for high water depth values. However, the proposed model provides higher values than the Manning model for low water depths.

Solute dispersion model

The diffusion coefficient contains all the information related to molecular or viscous diffusion, turbulent diffusion and dispersion derived from the averaging process. The model proposed by Rutherford (1994) will be used for practical applications:

$$K_x = 10\sqrt{gPA|S_f|} \quad (21)$$

NUMERICAL MODEL

The numerical scheme used in this paper is based on a previous study developed to demonstrate its suitability for the coupled simulation of the flux and transport equations (Burguete et al. 2007b). In order to incorporate the infiltration process to these equations, a four step algorithm is applied:

1. In the first step, the flow equations and the advective part of the transport equation are discretized with the explicit scheme, and the diffusion term is discretized implicitly:

$$\mathbf{U}_i^a = \mathbf{U}_i^n + \Delta t \left[\left(\mathbf{S}^c - \frac{\partial \mathbf{F}}{\partial x} \right)_i^n + \left(\frac{\partial \mathbf{D}}{\partial x} \right)_i^a \right] \quad (22)$$

2. In a second step infiltration is discretized as follows:

$$\mathbf{U}_i^b = \mathbf{U}_i^a + \Delta t \mathbf{I}_i^a \quad (23)$$

3. In a third step, the source terms are added with an implicit discretization:

$$\mathbf{U}_i^c = \mathbf{U}_i^b + \theta \Delta t (\mathbf{S}_i^c - \mathbf{S}_i^b) \quad (24)$$

where $\theta \in [0, 1]$ is the parameter controlling the degree of implicitness of the source term. We shall use $\theta = 0.5$ in all model runs.

4. Finally, the boundary conditions are applied at the inlet, outlet and furrow confluences (characteristic of level furrow systems) to obtain the conserved variable in the next step $\mathbf{U}_i^{n+1}$.

First step: flow and transport

This part is based on defining the vectors at the cell interfaces:

$$\mathbf{G}_{i+(1/2)}^n = \left(\mathbf{S}^c - \frac{\delta \mathbf{F}}{\delta x} \right)_{i+(1/2)}^n \quad (25)$$

using the notation $\delta f_{i+(1/2)} = f_{i+1} - f_i$ and $f_{i+(1/2)} = (f_{i+1} + f_i)/2$. It is important to note that in the part of the source term corresponding to the friction term, a numerical limitation

207 of the friction source term is performed (Burguete et al. 2007a):

$$(gAS_f)_{i+(1/2)}^n = \min \left[\frac{(gAS_f)_{i+1}^n + (gAS_f)_i^n}{2}, \frac{Q_{i+(1/2)}^n}{\Delta t} - \frac{\delta}{\delta x} \left(\frac{Q^2}{A} \right)_{i+(1/2)}^n - \left(gA \frac{\delta z_s}{\delta x} \right)_{i+(1/2)}^n \right] \quad (26)$$

208 Then, the numerical scheme is built by defining the upwind vectors as:

$$\mathbf{G}^\pm = \frac{1}{2}[\mathbf{1} \pm \mathbf{P} \text{sign}(\mathbf{\Lambda}) \mathbf{P}^{-1}] \mathbf{G}, \quad \mathbf{\Lambda}^\pm = \frac{1}{2}(\mathbf{\Lambda} \pm |\mathbf{\Lambda}|) \quad (27)$$

209 where the matrices $\mathbf{P}$ and $\mathbf{\Lambda}$ are based on Roe's averages:

$$\lambda_{i+(1/2)} = \frac{\sqrt{A_{i+1}}\lambda_{i+1} + \sqrt{A_i}\lambda_i}{\sqrt{A_{i+1}} + \sqrt{A_i}}, \quad s_{i+(1/2)} = \frac{\sqrt{A_{i+1}}s_{i+1} + \sqrt{A_i}s_i}{\sqrt{A_{i+1}} + \sqrt{A_i}} \quad (28)$$

210 The artificial viscosity coefficient defined is as (Burguete and García-Navarro 2004):

$$\nu_{i+(1/2)}^n = \max_k \begin{cases} \frac{1}{4} [\delta(\lambda_k) - 2|\lambda_k|]_{i+(1/2)}^n, & \text{if } (\lambda_k)_i^n < 0 \text{ and } (\lambda_k)_{i+1}^n > 0 \\ 0, & \text{otherwise} \end{cases} \quad (29)$$

211 the second order vectors as:

$$\mathbf{L}^\pm = \left(\mathbf{1} \mp \mathbf{\Lambda}^\pm \frac{\Delta t}{\delta x} \right) \mathbf{P}^{-1} \mathbf{G}^\pm \quad (30)$$

212 and the flux limiting matrices as:

$$\mathbf{\Psi}_{i+(1/2)}^\pm = \begin{pmatrix} \Psi \left(\frac{(\mathbf{L}^\pm)_1^1}{(\mathbf{L}^\pm)_1^1} \right) & 0 & 0 \\ 0 & \Psi \left(\frac{(\mathbf{L}^\pm)_2^2}{(\mathbf{L}^\pm)_2^2} \right) & 0 \\ 0 & 0 & \Psi \left(\frac{(\mathbf{L}^\pm)_3^3}{(\mathbf{L}^\pm)_3^3} \right) \end{pmatrix} \quad (31)$$

213 where $(\mathbf{L}^\pm)^k$ is the k component of the vector $\mathbf{L}^\pm$ and Ψ is the flux limiter function. A
 214 number of particular flux limiter functions have been defined in the literature (Hirsch 1990).

215 In this paper we will use the *Superbee* flux limiter:

$$\Psi(r) = \max[0, \min(1, 2r), \min(2, r)] \quad (32)$$

216 Then, the second order in space and time TVD scheme is written as (Burguete et al. 2007b):

$$\begin{aligned} \mathbf{U}_i^a = \mathbf{U}_i^n + \Delta t \left\{ \left(\mathbf{G}^+ - \nu \frac{\delta \mathbf{U}}{\delta x} \right)_{i-(1/2)}^n + \left(\mathbf{G}^- + \nu \frac{\delta \mathbf{U}}{\delta x} \right)_{i+(1/2)}^n - \frac{\mathbf{D}_{i+(1/2)}^{n+\theta} - \mathbf{D}_{i-(1/2)}^{n+\theta}}{\delta x} + \right. \\ \left. + \frac{1}{2} \left[\left(\mathbf{P}\Psi^+\mathbf{L}^+ \right)_{i-(1/2)}^n - \left(\mathbf{P}\Psi^+\mathbf{L}^+ \right)_{i-(3/2)}^n + \left(\mathbf{P}\Psi^-\mathbf{L}^- \right)_{i+(1/2)}^n - \left(\mathbf{P}\Psi^-\mathbf{L}^- \right)_{i+(3/2)}^n \right] \right\} \quad (33) \end{aligned}$$

218 Second step: infiltration

219 In a second step, the contribution of the infiltration term is incorporated. Since infil-
220 tration is produced at the flow layer in contact with the porous bed (characterized by null
221 velocity in viscous flows), there is no loss of momentum. In order to avoid numerical errors
222 in the form of negative water volumes:

$$\Delta \alpha_i^a = \min(A, \Delta t P i)_i^a, \quad \begin{pmatrix} A \\ Q \\ As \\ \alpha \\ \phi \end{pmatrix}_i^b = \begin{pmatrix} A \\ Q \\ As \\ \alpha \\ \phi \end{pmatrix}_i^a + \Delta \alpha_i^a \begin{pmatrix} -1 \\ 0 \\ -s \\ 1 \\ s \end{pmatrix}_i^a \quad (34)$$

223 Exact conservation of water volume and solute mass (to the limit of machine accuracy) is
224 produced in this step, since the following equation holds:

$$A_i^b + \alpha_i^b = A_i^a + \alpha_i^a, \quad (As)_i^b + \phi_i^b = (As)_i^a + \phi_i^a \quad (35)$$

225 **Third step: source terms**

226 In the third step, as mentioned in the context of (24), an implicit discretization of the
 227 source terms is applied. Taking into account that only the momentum equation contains
 228 source terms, the mass conservation and the solute transport equations are trivial in this
 229 step :

$$A_i^c = A_i^b, \quad (As)_i^c = (As)_i^b \quad (36)$$

230 The friction laws considered are singular, tending to infinity for small values of the water
 231 depth, which can introduce numerical instabilities in transient calculations. A threshold
 232 value for the depth H_{min} will be used in order to avoid those situations. Below that value,
 233 the discharge will be set to zero. We use:

- 234 • $H_{min} = 0.01m$ for the Manning friction model.
- 235 • $H_{min} = l$ for the power law velocity model.

236 otherwise, a friction factor $r = r(A) = S_f/(|Q|Q)$ depending only of A is defined for the
 237 considered friction models, leading to a simple second order equation for the water discharge.
 238 Therefore, discharge is evaluated according to:

$$Q_i^c = \begin{cases} 0; & (H_i^c \leq H_{min}) \\ Q_i^b + g\theta\Delta t\{[A(S_0 - r|Q|Q)]_i^c - [A(S_0 - r|Q|Q)]_i^b\}; & (H_i^c > H_{min}) \end{cases} \quad (37)$$

239 **Fourth step: boundary conditions**

240 *Inlet and outlet*

241 A correct numerical model for unsteady flow problems must be based not only on a
 242 numerical scheme with the required properties but also on an adequate procedure to discretize
 243 the boundary conditions. The theory of characteristics provides clear indications about the
 244 number of necessary external boundary conditions to define a well posed problem (Hirsch
 245 1990).

246 In furrow irrigation, where the water flow is always subcritical, both a physical and
 247 a numerical boundary condition at the inlet and at the outlet are necessary. The most
 248 usual physical boundary condition at the inlet is a discharge hydrograph $Q_{in} = Q_{in}(t)$.
 249 At the outlet, it is common practice to use a rating curve of the type $Q_{out} = Q_{out}(H_{out})$.
 250 A closed outlet can be considered a particular case with $Q_{out} = 0$. For solute transport, a
 251 physical boundary condition at the inlet, usually a concentration input $s_{in}(t)$, and a numerical
 252 boundary condition at the outlet are required.

253 The method of global mass conservation (Burguete et al. 2002; Burguete et al. 2006) is
 254 based on enforcing the integral form of the mass conservation extended to all the computa-
 255 tional domain in combination with a conservative scheme for the interior points to generate
 256 the numerical boundary condition. In order to ensure the global mass conservation of the
 257 scheme, the numerically generated volume variation must be combined with the desired vol-
 258 ume variation and therefore the following corrections must be enforced over the wetted cross
 259 section at the inlet:

$$A_1^{n+1} = A_1^c + \frac{\int_{t^n}^{t^{n+1}} Q_{in}(t)dt - \Delta t Q_1^n}{\delta x}, \quad (As)_1^{n+1} = (As)_1^c + \frac{\int_{t^n}^{t^{n+1}} Q_{in}(t)s_{in}(t)dt - \Delta t (Qs)_1^n}{\delta x} \quad (38)$$

260 In order to ensure the correct formulation of the boundary we must enforce subcritical flow
 261 at the inlet in the following form:

$$Q_1^{n+1} = \min[Q_{in}(t^{n+1}), A_1^{n+1}c_1^{n+1}] \quad (39)$$

262 At the outlet, an exact estimation of the outflowing mass is impossible when using a rating
 263 curve as boundary condition. Hence the following approximations are used:

$$Q_N^{n+1} = \min[Q_{out}(H_N^{n+1}), A_N^{n+1}c_N^{n+1}], \quad A_N^{n+1} = A_N^c, \quad (As)_N^{n+1} = (As)_N^c \quad (40)$$

We will concentrate on furrow junctions of the “T” type, that is, involving only a main furrow and a perpendicular secondary furrow as in Figure 4. In this way, the momentum addition from the tributary furrow is in the normal direction to the main flow and viceversa.

The main hypothesis used to solve at the junction area is that the m main furrow grid cells involved at the junction (from j to $j + m$) as well as the secondary furrow grid cell involved (k) share a unique water surface level and a unique value of solute concentration. The total volume of water $V_{junction}^c$ and mass of solute $M_{junction}^c$ at the junction cells are therefore:

$$V_{junction}^c = A_k^c \delta x_k + \sum_{i=j}^{j+m} A_i^c \delta x_i, \quad M_{junction}^c = (As)_k^c \delta x_k + \sum_{i=j}^{j+m} (As)_i^c \delta x_i, \quad (41)$$

By requiring the conservation of water volume and the uniform surface water level z_s^{n+1} , a second order equation for this variable can be written:

$$V_{junction}^{n+1} = \{(B_0)_k + S_k[(z_s)^{n+1}_k - (z_b)_k]\}[(z_s)^{n+1}_k - (z_b)_k] \delta x_k + \sum_{i=j}^{j+m} \{(B_0)_i + S_i[(z_s)^{n+1}_i - (z_b)_i]\}[(z_s)^{n+1}_i - (z_b)_i] \delta x_i = V_{junction}^c \quad (42)$$

this formulation immediately leads to the values of A_i^{n+1} and A_k^{n+1} . On the other hand, the requirements of solute mass conservation and uniform concentration at the junction result in:

$$s_i^{n+1} = s_k^{n+1} = \frac{M_{junction}^c}{V_{junction}^c}, \quad \forall i \in [j, j + m] \quad (43)$$

Finally, momentum interchanges at the junction must be considered. We will assume that velocity is uniform in a cell, so that the momentum exchange is proportional to mass exchange. In fact, the furrow supplying mass to the confluence loses an amount of momentum in its longitudinal direction which is proportional to its loss of mass. However, since the confluence is perpendicular, the furrow supplies momentum in perpendicular fashion, with

no component in the longitudinal direction of the receiving furrow. Taking this effect into account, the following correction over the discharges at the junction grid cells is performed:

$$Q_i^{n+1} = \begin{cases} Q_i^c; & (A_i^c \leq A_i^{n+1}) \\ Q_i^c \frac{A_i^{n+1}}{A_i^c}; & (A_i^c > A_i^{n+1}) \end{cases}, \quad Q_k^{n+1} = \begin{cases} Q_k^c; & (A_k^c \leq A_k^{n+1}) \\ Q_k^c \frac{A_k^{n+1}}{A_k^c}; & (A_k^c > A_k^{n+1}) \end{cases} \quad (44)$$

TEST CASES AND APPLICATIONS

Test I: ideal dambreak with solute discontinuity

The ideal dambreak problem is one of the classical examples used as test case for unsteady shallow water flow simulations. The reason is that for flat and frictionless bottom, rectangular cross section and no diffusion, the problem defined by zero initial velocity and initial discontinuities in the water depth and solute concentration has an exact solution (Stoker 1957).

A rectangular channel 200m long and 1m wide has been considered with an initial depth ratio 1m : 0.1m and with an initial discontinuity in the concentration of 1kg/m³ : 0kg/m³ in the same location as the depth jump. A grid spacing of $\delta x = 2m$, $CFL = 0.9$ and $t = 20s$ was used for all simulations.

The plots in Figure 5 show the numerical solution for the water depth from the numerical scheme described in this work and for the classical McCormack scheme (García-Navarro and Savirón 1992) versus the exact solution for $t = 20s$. The numerical scheme used in this research clearly shows better performance than classical MacCormack.

Figure 6 shows the results for the solute concentration provided by the 2nd order TVD scheme using the discretization described in this work and the separate discretization (see (Burguete et al. 2007b) for different discretizations of the solute transport equation with this scheme), and the classical McCormack scheme. The proposed scheme yields best results when used in conjunction with the proposed discretization.

Test II: closed furrows with a confluence

In this section, the performance of the proposed numerical scheme is assessed for different treatments of the boundary conditions in a set of two furrows closed in their downstream ends and arranged in a “T” confluence. Figure 7 presents the geometry of the test case. Furrows have a trapezoidal section with dimensions $B_0 = 0.20m$, $S = 1$, $S_0 = 0$ and a depth of $0.4m$. Roughness is modelled using Gauckler-Manning equation (18) with $n = 0.03sm^{-1/3}$, solute diffusion follows Rutherford equation (21), and infiltration follows equation (16), with $K = 0.0015ms^{-a}$ and $a = 0.3$. These parameters are characteristic of a low infiltration clay soil. The furrow spacing, D , is $1m$. A constant inflow $Q_{in} = 0.01m^3/s$ is introduced in the domain with a solute concentration of $s_{in} = 1kg/m^3$. Although the problem does not have an analytical solution, the total water volume and solute mass follow:

$$V = Q_{in}t, \quad M = Q_{in}s_{in}t \quad (45)$$

These volumes and masses can be compared with the numerical results, which are computed as:

$$V^n = \sum_{i=1}^N (A + \alpha)_i^n \delta x, \quad M^n = \sum_{i=1}^N (As + \phi)_i^n \delta x \quad (46)$$

The respective conservation errors can be determined as:

$$E_V = 100 \frac{V^n - V}{V} \%, \quad E_M = 100 \frac{M^n - M}{M} \% \quad (47)$$

Figure 8 presents the longitudinal profiles of H as a function of the distance to the upstream inlet point, using the proposed numerical scheme and treatments of the boundary conditions and the confluence for (a) $t = 900s$ and (b) $t = 1500s$.

Table 1 presents a comparison of the mass errors at time $t = 1500s$ with the proposed treatments for the boundary conditions (global mass conservation) and the confluence (conservative junction). Results are also provided for other treatments of the boundary condi-

tions (local mass conservation (Jin and Fread 1997)) and a simple treatment of the confluence based on equalling the free water surface level and the solute concentration at the receiving furrow to the supplying furrow at the confluence. Different combinations of treatments result in large errors, while the combination of proposed treatments reduces the conservation errors to machine accuracy.

Test III: Confluence with experimental measurements

Qu (2005) and Ramamurthy et al. (2007) reported an experimental and numerical analysis of the flows resulting from a confluence in a laboratory channel. These papers include a detail flow analysis and simulations performed with a 3D numerical model. Since the channel walls were smooth, the Manning roughness model was used, with $n = 0.009sm^{-1/3}$.

In the practical simulation of a level-furrow system there is a large number of confluences between the conveyance channels and the irrigation furrows. Additionally, the computational mesh required to simulate these problems in reasonable time is often coarse. As a consequence, the computational time devoted to each confluence is limited, and the confluence should be simulated as just one cell in the conveyance channel and another cell in the irrigation furrow.

Test III was performed to assess if - despite its crude approach - the proposed two-cell confluence can produce a reasonable approximation of the flow partition. Figure 9 presents a scheme of the experimental device and the simulation mesh. A mesh with $\delta x = 0.61m$, in the range of typical furrow simulations, was designed. The mesh uses 14 cells for the main furrow and 4 for the secondary furrow, with the confluence involving just one cell in each furrow.

The paper by Qu (2005) reports measurements of discharge, flow depth and velocity for five different flow conditions obtained through modifications of the weirs installed at the downstream end of each furrow. However, the author did not report on the settings (elevations) of the regulating weirs. In order to overcome this difficulty, a critical flow law was implemented at the downstream end of each furrow using a range of weir settings.

353 The weir setting minimizing the error between measured and simulated measurements was
 354 adopted as representative of the experimental conditions. Let h_m^e be the flow depth, with
 355 $\overline{h_m^e}$ the average value and n_m^e the number of experimental measurements in the main furrow;
 356 h_m^s will be the simulated flow depth at the main furrow. In the secondary furrow these
 357 magnitudes will be denoted as h_s^e , $\overline{h_s^e}$, n_s^e and h_s^s , respectively. Additionally, Q_{in} will be
 358 the inflow discharge, Q_s^e and Q_s^s will be the experimental and simulated discharge at the
 359 secondary furrow, respectively. In these conditions, the error can be defined as:

$$E = \frac{|Q_s^e - Q_s^s|}{Q_{in}} + \frac{1}{\overline{h_m^e}} \sqrt{\frac{\sum_{i=1}^{n_m^e} [(h_m^e)_i - (h_m^s)_i]^2}{n_m^e - 1}} + \frac{1}{\overline{h_s^e}} \sqrt{\frac{\sum_{i=1}^{n_s^e} [(h_s^e)_i - (h_s^s)_i]^2}{n_s^e - 1}} \quad (48)$$

360 The model will be considered valid if under conditions of minimum error it can reproduce
 361 the experimental measurements in a reasonable fashion.

362 Table 2 presents the discharges measured at the inflow and at the secondary furrow for
 363 the five experimental flow conditions, together with the optimum weir settings resulting in
 364 minimum error according to (48). Figure 10 presents maps of the errors corresponding to the
 365 main and secondary weir settings in the five reported flow conditions. It can be concluded
 366 that the weir settings could be estimated in an accurate way. Figure 11 presents a scatter plot
 367 of experimental vs. optimum simulated discharge at the secondary channel for the different
 368 flow conditions. All five points are distributed along the 1 : 1 line, providing an additional
 369 indication of the accuracy in the estimation of the weir settings. Finally, Figure 12 presents
 370 the longitudinal profiles for flow depth (measured and simulated). The agreement between
 371 both sources of data suggests that the proposed simple method for hydraulic computations
 372 in confluences is accurate enough to be used in the simulation of a level furrow system.

373 CONCLUSIONS

374 A mathematical model including shallow water flow and solute transport has been pre-
 375 sented and solved using a second order TVD scheme. The model is adapted to furrow

fertigation and implements an infiltration equation that automatically adjusts to variations in the wetted perimeter, a roughness equation based on an absolute roughness parameter, and an equation for the estimation of the longitudinal diffusion parameter. The parameterization problem is therefore reduced to estimating infiltration in reference conditions, and estimating a physically based roughness parameter that will result in a flow-dependent roughness. The model also incorporates a specific treatment of the boundary conditions formulated to ensure global mass conservation at machine accuracy.

In order to extend the model to furrow networks, a simple and computationally efficient approach to the junction conditions, considered as internal boundaries, has been proposed. Three numerical tests have been used to assess the shock-capturing model properties for both water level and solute concentration front advance, and to evaluate the performance of the treatment of boundary conditions and junctions. The results of these tests have confirmed the adequacy of the model to address the problems of unsteady flows with solute transport in single channels and junctions in channels. In a companion paper the model is calibrated and validated using *ad hoc* furrow fertigation experiments, and is applied to the simulation of level furrow systems.

NOTATION

A = cross-sectional wetted area;

a = Kostiakov infiltration exponent;

B = cross section top width;

B_0 = cross section base width;

c = velocity of the infinitesimal waves;

D = distance between furrows;

$\mathbf{D}$ = diffusion vector;

400 E_M = error of solute mass conservation;

401 E_V = error of water volume conservation;

402 $\mathbf{F}$ = conservative flux vector;

403 g = gravitational constant;

404 H = cross-sectional maximum water depth;

405 h = water depth;

406 H_{min} = minimum depth to allow water flowing;

407 $\mathbf{I}$ = infiltration vector;

408 i = infiltration rate;

409 I_1, I_2 = pressure force integrals;

410 $\mathbf{J}$ = conservative flux Jacobian,

411 K = Kostiakov infiltration parameter;

412 K_x = diffusion coefficient;

413 L = weir setting (elevation over the furrow base);

414 $\mathbf{L}$ = second order vector;

415 l = characteristic roughness length;

416 M = solute mass;

417 n = Gauckler-Manning roughness coefficient;

418 P = cross-sectional wetted perimeter;

419 $\mathbf{P}$ = Jacobian eigenvectors matrix;

420 Q = discharge;

421 Q_{in} = inlet hydrograph discharge;

422 Q_{out} = outlet rating curve of discharge;

423 r = friction factor;

424 S = furrow wall slope;

425 s = cross-sectional average solute concentration;

426 S_0 = longitudinal bottom slope;

427 $\mathbf{S}^c$ = conservative source term vector;

428 S_f = longitudinal friction slope;

429 s_{in} = inlet solute concentration input;

430 $\mathbf{S}^{nc}$ = non-conservative source term;

431 t = time;

432 $\mathbf{U}$ = conserved variable vector;

433 u = cross-sectional averaged velocity;

434 V = water volume;

435 v_x = longitudinal component of the velocity at any point of the cross section;

436 w = cross section width;

437 x = longitudinal coordinate;

438 y = transversal coordinate;

439 Z = cumulative infiltration length;

440 z = vertical coordinate;
 441 z' = vertical distance to bed level;
 442 z'' = vertical distance over the lowest point in the cross section;
 443 z_b = level of the lowest point in the cross section;
 444 z_s = water surface level;
 445 α = infiltrated cross section;
 446 Δ = temporal finite increment;
 447 δ = spatial finite increment;
 448 ϵ = friction coefficient;
 449 Λ = eigenvalues diagonal matrix;
 450 λ_i = Jacobian eigenvalues;
 451 ϕ = solute mass infiltrated per unit length of the furrow; and
 452 τ = opportunity time.

453 References

- 454 Abbasi, F., Feyen, J., Roth, R. L., Sheedy, M., and van Genuchten, M. T. (2003a). "Water
 455 flow and solute transport in furrow-irrigated fields." *Irrig. Sci.*, 22, 57–65.
 456 Abbasi, F., Simunek, J., van Genuchten, M. T., Feyen, J., Adamsen, F. J., Hunsaker, D. J.,
 457 Strelkoff, T., and Shouse, P. J. (2003b). "Overland water flow and solute transport: model
 458 development and field-data analysis." *ASCE J. Irrig. and Drainage Eng.*, 129(2), 71–81.
 459 Adamsen, F. J., Hunsaker, D. J., and Perea, H. (2005). "Border strip fertigation: effect of
 460 injection strategies on the distribution of bromide." *Trans. of the ASAE*, 48(2), 529–540.
 461 Bear, J. (1972). *Dynamics of fluids in porous media*. Elsevier Science, New York, USA.

- Boldt, A. L., Watts, D. G., Eisenhauer, D. E., and Schepers, J. S. (1994). "Simulation of water applied nitrogen distribution under surge irrigation." *Trans. of the ASAE*, 37(4), 1157–1165.
- Burguete, J. and García-Navarro, P. (2004). "Implicit schemes with large time steps for non-linear equations: application to river flow hydraulics." *Int. J. for Numerical Methods in Fluids*, 46(6), 607–636.
- Burguete, J., García-Navarro, P., and Aliod, R. (2002). "Numerical simulation of runoff from extreme rainfall events in a mountain water catchment." *Natural Hazards in Earth System Sci.*, 2, 1–9.
- Burguete, J., García-Navarro, P., and Murillo, J. (2006). "Numerical boundary conditions for globally mass conservative methods to solve the shallow-water equations and applied to river flow." *Int. J. for Numerical Methods in Fluids*, 51(6), 585–615.
- Burguete, J., García-Navarro, P., and Murillo, J. (2007a). "Friction term discretization and limitation to preserve stability and conservation in the 1d shallow-water model: application to unsteady irrigation and river flow." *Int. J. for Numerical Methods in Fluids*, in press.
- Burguete, J., García-Navarro, P., and Murillo, J. (2007b). "Preserving bounded and conservative solutions of transport in 1d shallow-water flow with upwind numerical schemes: application to fertigation and solute transport in rivers." *Int. J. for Numerical Methods in Fluids*, in press.
- Burguete, J., García-Navarro, P., Murillo, J., and García-Palacín, I. (2007c). "Analysis of the friction term in the one-dimensional shallow water model." *ASCE J. Hydraulic Eng.*, 133(9), 1048–1063.
- García-Navarro, P., Playán, E., and Zapata, N. (2000). "Solute transport modeling in overland flow applied to fertigation." *ASCE J. Irrig. and Drainage Eng.*, 126(1), 33–41.
- García-Navarro, P., Sánchez, A., Clavero, N., and Playán, E. (2004). "A simulation model for level furrows II: description, validation and application." *ASCE J. Irrig. and Drainage Eng.*, 130(2), 113–121.

489 García-Navarro, P. and Savirón, J. M. (1992). “McCormack’s method for the numerical
490 simulation of one-dimensional discontinuous unsteady open channel flow.” *J. Hydraulic*
491 *Res.*, 30(1), 95–105.

492 Gauckler, P. G. (1867). “Études théoriques et pratiques sur l’écoulement et le mouvement
493 des eaux.” Comptes Rendues de l’Académie des Sciences, Paris.

494 Hirsch, C. (1990). *Computational methods for inviscid and viscous flows: Numerical compu-*
495 *tation of internal and external flows.* John Wiley & Sons, New York.

496 Jin, M. and Fread, D. L. (1997). “Dynamic flood routing with explicit and implicit numerical
497 solution schemes.” *ASCE J. Hydraulic Eng.*, 123(3), 166–173.

498 Maïkaka, M. (2004). “Modelos numéricos unidimensionales en riego por superficie,” PhD
499 thesis, University of Zaragoza.

500 Manning, R. (1890). “On the flow of water in open channels and pipes.” Institution of Civil
501 Engineers of Ireland.

502 Playán, E. and Faci, J. M. (1997). “Border fertigation: field experiments and a simple
503 model.” *Irrig. Sci.*, 17, 163–171.

504 Playán, E., Rodríguez, J. A., and García-Navarro, P. (2004). “Simulation model for level
505 furrows. I: analysis of field experiments.” *ASCE J. Irrig. and Drainage Eng.*, 130(2), 106–
506 112.

507 Qu, J. (2005). “Three-dimensional turbulence modeling for free surface flows,” PhD thesis,
508 Concordia University.

509 Ramamurthy, A. S., Qu, J., and Vo, D. (2007). “Numerical and experimental study of
510 dividing open-channel flows.” *ASCE J. Hydraulic Eng.*, 133(10), 1135–1144.

511 Rutherford, J. C. (1994). *River mixing.* John Wiley & Sons.

512 Sabillón, G. N. and Merkley, G. P. (2004). “Fertigation guidelines for furrow irrigation.”
513 *Spanish Journal of Agricultural Research*, 2, 576–587.

514 Simunek, J., Sejna, M., and van Genuchten, M. T. (1998). “The HYDRUS-1D software pack-
515 age for simulating the movement of water, heat and multiple solutes in variably saturated

516 porous media, version 2.0.” United States Salinity Laboratory, USDA-ARS, Riverside,
517 California. USA.

518 Stoker, J. J. (1957). *Water waves*. Interscience, New York.

519 Strelkoff, T. S., Clemmens, A. J., and Perea-Estrada, H. (2006). “Calculation of non-reactive
520 chemical distribution in surface fertigation.” *Agric. Water Mgmt.*, 86(1–2), 93–101.

521 Walker, W. R. and Skogerboe, G. V. (1987). *Surface irrigation. Theory and practice*.
522 Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

523 Zerihun, D., Furman, A., Warrick, A. W., and Sanchez, C. A. (2005a). “Coupled surface-
524 subsurface solute transport model for irrigation borders and basins I: model development.”
525 *ASCE J. Irrig. and Drainage Eng.*, 131(5), 396–406.

526 Zerihun, D., Sanchez, C. A., Furman, A., and Warrick, A. W. (2005b). “Coupled surface-
527 subsurface solute transport model for irrigation borders and basins II: model evaluation.”
528 *ASCE J. Irrig. and Drainage Eng.*, 131(5), 407–419.

529 **List of Tables**

530	1	Errors in water volume and solute mass conservation in Test II at time $t =$	
531		1500s as solved with the proposed treatments for the boundary conditions	
532		(global mass conservation, GC) and the confluence (conservative junction,	
533		CJ). Results are also provided for other treatments of the boundary conditions	
534		(local mass conservation LC) and a simple treatment of the confluence (SC).	30
535	2	Discharges measured at the inflow and at the secondary furrow for the five ex-	
536		perimental flow conditions, together with the optimum weir settings (elevation	
537		over the furrow base, L) resulting in minimum simulation error.	31

538 **List of Figures**

539	1	Coordinate system in a cross section.	33
540	2	Trapezoidal furrow geometry	34

541	3	S_f versus H for a flow velocity $U = 1m/s$ and a typical furrow shape with	
542		$B_0 = 0.14m$, $S = 1.22$ and where $n = 0.035sm^{-1/3}$, $\epsilon = 0.04$, $b = 0.25$ and	
543		$l = 0.02m$	35
544	4	Discretization at a junction. The grid cells j to $j + m$ at the main furrow and	
545		the grid cell k at the secondary furrow are involved in the junction.	36
546	5	Ideal dambreak depth with McCormack and 2nd order TVD schemes.	37
547	6	Results of Test I, dambreak with discontinuous concentration, for $t = 20s$ with	
548		the MacCormack method, the proposed coupled 2nd order TVD, the separate	
549		2nd order TVD and the exact solution.	38
550	7	Plan view of the simulated system in Test II.	39
551	8	Longitudinal profiles of flow depth as a function of the distance to the up-	
552		stream inlet for Test II with the proposed numerical scheme and treatments	
553		for (a) $t = 900s$ and (b) $t = 1500s$	40
554	9	Test III: plan view of the experimental system and scheme of the mesh used	
555		for its simulation.	41
556	10	Map of simulation error as a function of the weir settings for the five flow	
557		conditions reported in Test III.	42
558	11	Scatter plot of experimental vs. simulated discharge at the secondary channel	
559		for the five flow conditions described in Test III.	43
560	12	Longitudinal flow depth at the main and secondary furrows, as experimentally	
561		observed and simulated, for the five flow conditions described in test III. . .	44

Table 1. Errors in water volume and solute mass conservation in Test II at time $t = 1500s$ as solved with the proposed treatments for the boundary conditions (global mass conservation, GC) and the confluence (conservative junction, CJ). Results are also provided for other treatments of the boundary conditions (local mass conservation LC) and a simple treatment of the confluence (SC).

Numerical treatments	$E_V(\%)$	$E_M(\%)$
LC+SJ	46.6	46.5
GC+SJ	45.9	45.9
LC+CJ	0.37	0.28
GC+CJ	$-2.0 \cdot 10^{-15}$	$-2.0 \cdot 10^{-15}$

Table 2. Discharges measured at the inflow and at the secondary furrow for the five experimental flow conditions, together with the optimum weir settings (elevation over the furrow base, L) resulting in minimum simulation error.

Case	Q (m^3/s)	Q (m^3/s)	L (m)		Error
	inlet	secondary	main	secondary	
IIIa	0.047	0.007	0.110	0.156	0.034
IIIb	0.046	0.014	0.098	0.108	0.031
IIIc	0.046	0.019	0.104	0.092	0.067
IIId	0.047	0.032	0.145	0.088	0.082
IIIe	0.046	0.038	0.180	0.102	0.078

Figure 2. Trapezoidal furrow geometry

Figure 3. S_f versus H for a flow velocity $U = 1m/s$ and a typical furrow shape with $B_0 = 0.14m$, $S = 1.22$ and where $n = 0.035sm^{-1/3}$, $\epsilon = 0.04$, $b = 0.25$ and $l = 0.02m$.

Figure 4. Discretization at a junction. The grid cells j to $j + m$ at the main furrow and the grid cell k at the secondary furrow are involved in the junction.

Figure 5. Ideal dambreak depth with McCormack and 2nd order TVD schemes.

Figure 6. Results of Test I, dambreak with discontinuous concentration, for $t = 20s$ with the MacCormack method, the proposed coupled 2nd order TVD, the separate 2nd order TVD and the exact solution.

Figure 7. Plan view of the simulated system in Test II.

Figure 8. Longitudinal profiles of flow depth as a function of the distance to the upstream inlet for Test II with the proposed numerical scheme and treatments for (a) $t = 900s$ and (b) $t = 1500s$.

Figure 9. Test III: plan view of the experimental system and scheme of the mesh used for its simulation.

Figure 10. Map of simulation error as a function of the weir settings for the five flow conditions reported in Test III.

Figure 11. Scatter plot of experimental vs. simulated discharge at the secondary channel for the five flow conditions described in Test III.

Figure 12. Longitudinal flow depth at the main and secondary furrows, as experimentally observed and simulated, for the five flow conditions described in test III.