A 3D analysis of the acoustic radiation force in microfluidic channel with rectangular geometry

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ARTICLE INFO

Article history:

Received 2 June 2020

Received in revised form 18 November 2020

Accepted 22 December 2020

Available online 28 December 2020

Keywords:

Standing waves

Ultrasonic

Acoustic radiation force

Optimized Forest–Ruth algorithm

ABSTRACT

Particles or cells in suspension and exposed to ultrasonic waves experience an acoustic radiation force (F_R) which, under certain conditions, drives them toward positions of acoustic equilibrium. In this paper, we present a three-dimensional model of the particle motions within the acoustic field generated by ultrasonic standing waves. This model allows a theoretical study of the three-dimensional F_R induced by a standing acoustic wave in a microfluidic chamber with rectangular geometry on micrometer-sized spherical particles. The approach models the agglomeration process and the behavior of particle clusters in the acoustic field. To achieve this, expressions for the 3D F_R are obtained as the time-average of a gradient of the acoustic potential established within the chamber with two different sets of boundary conditions. The particle motion under the action of this force was analyzed assuming a non-viscous fluid and a particle size much smaller than the acoustic wavelength. The 3D force expressions were used in a simulation employing an optimized Forest–Ruth algorithm to derive the dynamics of N spherical particles. This work provides novel results that predict some particle motion toward chamber or channel walls and the formation of pearl-chain aggregates within channels. These particle movements and the aggregate formation process were observed experimentally in an acoustic device built to assess the validity of the theoretical predictions.

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1. Introduction

Particles and cells in suspension exposed to ultrasonic waves experience a hydrodynamic force F_R which under certain conditions drives them toward positions of acoustic equilibrium where they agglomerate [1]. The steady-state hydrodynamic force is due to the nonlinear resultant of the interaction between the acoustic wave incident on each particle and the wave scattered by the particle. The theoretical basis of this mechanism has been understood since the first half of the 20th century [2], when the expression for this force was first derived for incompressible spherical particles in ideal fluids, and later extended to compressible particles [3]. From these studies, Gor'kov derived a general expression for the radiation force in terms of the gradient of the acoustic potential [1]. Doinikov subsequently developed a general expression for F_R that includes the fluid viscosity and heat [4], which was used to calculate the F_R acting on rigid

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microspheres [5] and compressible spherical particles [6] exposed to standing waves. These studies led to the conclusion that corrections to the radiation force due to the fluid viscosity are negligible if the thickness of the viscous boundary layer is much smaller than the size of the surrounded particle [7]. Efforts have since then continued to improve and refine the theoretical models.

The theoretical treatment of $\mathbf{F}_R$ in one dimension has largely been verified experimentally by reports of separation using acoustically induced cell enrichment within capillaries for applications such as blood plasmapheresis in rectangular capillaries and microfluidic channels [8–10], selective cell enrichment by separating lipids from erythrocytes in flowing blood samples [11], and isolation of tumor cells from peripheral blood samples [12,13] at acoustic frequencies of a few MHz. This label-free technology is based on the action of $\mathbf{F}_R$ to separate or manipulate populations, exploiting differences in size, density and compressibility between particles or cells in suspension [14]. Furthermore, the effects of this force on cellular properties can be used in other applications such as indirect characterization of mechanical properties of cells [15–17].

Several studies of the radiation force and its effects on particles and suspensions have been reported in the literature during recent decades. Analysis has been made of the force generated by acoustic waves confined to chambers or channels with different geometries [18]. Many theoretical studies of 3D standing waves have reported numerical solutions of the governing equations obtained using the COMSOL Multiphysics® finite element software [19–21]. It has also been shown that the acoustophoretic particle motion can be modeled using numerical solutions of the equations of motion with code that includes iterative methods such as a fourth-order Runge–Kutta [22–24]. Such software solutions decrease calculation time, facilitate study and accurate prediction of the simultaneous acoustophoretic motion of many particles or cells, and can be used to model dynamics of the aggregation processes in resonant chambers and to optimize separation throughput in different applications.

This paper presents a theoretical study of the 3D acoustic radiation force $\mathbf{F}_R$ that acts on N particles of spherical geometry in a rectangular resonant chamber, using custom developed software. 3D expressions of the radiation force $\mathbf{F}_R$ were derived from the acoustic potential in the chamber for two alternate boundary conditions in the x direction, namely rigid and free walls, which result in different expressions for $\mathbf{F}_R$ and consequent differential effects on the particle dynamics. These expressions of the $\mathbf{F}_R$ and the 3D physical model are described in Section 2 of the paper.

The two boundary conditions used in these theoretical predictions of 3D particle agglomeration correspond to different materials used in fabrication of the chambers, which can be relevant for the development of new low-cost devices. Rigid walls correspond to chambers made of rigid materials, such as glass or silicon, typically used in conventional BAW (bulk acoustic wave) microresonators [25,26]. These devices use walls with acoustic impedance far exceeding those of the particle suspension, as is required for the establishment of a standing wave within the liquid phase between the walls. Free wall boundary conditions along the x direction, on the other hand, correspond to acoustically soft materials, with acoustic impedance as low as that of the liquid sample. This work represents a first approach to a theoretical study of suspensions using polymeric chips with densities close to 1 g/cm³ and sound speeds relatively close to 1500 m/s. The use of polymeric chip structures such as PDMS, SU-8 and PMMA has been reported since 2010 [13,24,27].

The present work analyzed simultaneous behaviors of N micrometer-sized spherical particles in water exposed to 3D standing waves within a microfluidic chamber of a rectangular geometry. These expressions were included in a numerical study described in Section 3 of the paper, for which C++ code along with an optimized Forest–Ruth algorithm [28] was used to obtain the dynamics of N spherical particles of much smaller size than the acoustic wavelength. Different particle trajectories and velocities were calculated by solving the equations of motion and taking into account gravitational and viscous drag forces.

This 3D model of the acoustophoretic motion of N particles may also be of use to study the rotation and/or precession of the particle or cell aggregates around pressure nodes, as has been reported in red blood cells, and which has been proposed as an effect of acoustic streaming [29]. Finally, since this algorithm allows modeling of inter-particle contact, this work may ultimately permit study of the formation of and interaction between particle stacked lines, phenomena whose study requires algorithms that take into account both contact and acoustic interactions between particles [30].

Theoretical and computational studies are described respectively in Sections 2 and 3 of this paper, with the experiment setup presented in Section 4. The results and discussion are found in Section 5, and the conclusions are presented in Section 6.

2. Theory

2.1. 3D $\mathbf{F}_R$ at different boundary conditions

When 3D acoustic standing waves are established inside a resonating chamber of rectangular geometry can be generated a harmonic pressure field, given by

$$p_1(\mathbf{r}, t) = p_1(\mathbf{r}) e^{-i\omega t} \quad (1)$$

where the spatial part of the acoustic pressure in a rectangular chamber with rigid walls in y and z directions can be described for rigid and free x walls, respectively, as

$$p_1(\mathbf{r}) = p_a \cos(k_x x) \cos(k_y y) \cos(k_z z) \quad (2)$$

and

$$p_1(\mathbf{r}) = p_a \sin(k_x x) \cos(k_y y) \cos(k_z z), \quad (3)$$

for an incident wave with a pressure amplitude p_a , angular frequency ω , and acoustic wave numbers $k_x = n_x \pi / \ell_x$, $k_y = n_y \pi / \ell_y$ and $k_z = n_z \pi / \ell_z$ that correspond to resonances established along x , y and z axes respectively, with $k^2 = k_x^2 + k_y^2 + k_z^2$. So in a chamber with dimensions ℓ_x , ℓ_y and ℓ_z , the integers n_x , n_y and n_z define the resonance frequency as

$$f_{n_x, n_y, n_z} = \frac{c_f}{2} \sqrt{\frac{n_x^2}{\ell_x^2} + \frac{n_y^2}{\ell_y^2} + \frac{n_z^2}{\ell_z^2}}. \quad (4)$$

A nonlinear interaction between the incident wave and the wave scattered by each single particle generates a hydrodynamic force referred to as the Acoustic Radiation Force $\mathbf{F}_R$, whose expression is derived from the gradient of the acoustic potential within the resonant chamber in which the 3D standing wave is established [1], i.e.

$$\mathbf{F}_R = \langle \mathbf{F}_R \rangle = -\nabla U_{ac}, \quad (5)$$

with the acoustic potential U_{ac} given as

$$U_{ac} = V_p \left[\frac{f_1}{2\rho_f c_f^2} \langle p_1^2 \rangle - \frac{3f_2 \rho_f}{4} \langle v_1^2 \rangle \right]. \quad (6)$$

In this equation V_p is the particle volume; ρ_f , the fluid density; c_f , the sound speed in the fluid; $\langle p_1^2 \rangle$ and $\langle v_1^2 \rangle$, the time average of the square of the pressure and velocity respectively; and f_1 and f_2 , the compressibility and density factors respectively, defined as [1]

$$f_1 = 1 - \frac{1}{\gamma \beta^2}, \quad f_2 = \frac{2\gamma - 2}{2\gamma + 1}. \quad (7)$$

The coefficient γ is the ratio of densities between particle and fluid, and β , the ratio of the sound velocity between particle c_p and fluid c_f [31]. Assuming an ideal fluid without fluid viscosity, the pressure field and acoustic velocity are related to the expression [1]

$$\mathbf{v}_1 = -\frac{1}{\omega \rho_f} \nabla p_1(\mathbf{r}). \quad (8)$$

The time averaged pressure and velocity squared are substituted in Eq. (6) for the acoustic potential U_{ac} , and the spatial derivatives are calculated in Eq. (5). The acoustically induced radiation force on a particle in a resonant chamber with either rigid or free x walls respectively is as follows:

$$\begin{aligned} \mathbf{F}_R = & 2k_x \phi U_0 \sin(2k_x x) \left[\frac{1}{4} + \frac{1}{4} \cos(2k_y y) \cos(2k_z z) + A_1 \cos(2k_y y) + A_2 \cos(2k_z z) - B_1 \right] \hat{\mathbf{i}} \\ & + 2k_y \phi U_0 \sin(2k_y y) \left[\frac{1}{4} + \frac{1}{4} \cos(2k_x x) \cos(2k_z z) + A_1 \cos(2k_x x) + A_3 \cos(2k_z z) - B_2 \right] \hat{\mathbf{j}} \\ & + 2k_z \phi U_0 \sin(2k_z z) \left[\frac{1}{4} + \frac{1}{4} \cos(2k_x x) \cos(2k_y y) + A_2 \cos(2k_x x) + A_3 \cos(2k_y y) - B_3 \right] \hat{\mathbf{k}} \end{aligned} \quad (9a)$$

or

$$\begin{aligned} \mathbf{F}_R = & -2k_x \phi U_0 \sin(2k_x x) \left[\frac{1}{4} + \frac{1}{4} \cos(2k_y y) \cos(2k_z z) + A_1 \cos(2k_y y) + A_2 \cos(2k_z z) - B_1 \right] \hat{\mathbf{i}} \\ & + 2k_y \phi U_0 \sin(2k_y y) \left[\frac{1}{4} - \frac{1}{4} \cos(2k_x x) \cos(2k_z z) - A_1 \cos(2k_x x) + A_3 \cos(2k_z z) - B_2 \right] \hat{\mathbf{j}} \\ & + 2k_z \phi U_0 \sin(2k_z z) \left[\frac{1}{4} - \frac{1}{4} \cos(2k_x x) \cos(2k_y y) - A_2 \cos(2k_x x) + A_3 \cos(2k_y y) - B_3 \right] \hat{\mathbf{k}}. \end{aligned} \quad (9b)$$

where

$$\begin{aligned} A_1 &= \frac{1}{4} - \frac{\phi' k_z^2}{2k^2}, \quad A_2 = \frac{1}{4} - \frac{\phi' k_y^2}{2k^2}, \quad A_3 = \frac{1}{4} - \frac{\phi' k_x^2}{2k^2}, \quad B_1 = \frac{\phi'}{2k^2} \{k_y^2 + k_z^2\}, \\ B_2 &= \frac{\phi'}{2k^2} \{k_x^2 + k_z^2\}, \quad B_3 = \frac{\phi'}{2k^2} \{k_x^2 + k_y^2\}, \end{aligned}$$

U_o is the maximal acoustic potential, which for a spherical particle with radius R_p is

$$U_o = \frac{2\pi}{3} R_p^3 E_{ac} \quad (10)$$

and E_{ac} is the acoustic energy density

$$E_{ac} = \frac{p_a^2}{4\rho_f c_f^2}. \quad (11)$$

In Eqs. (9)a and (9)b, ϕ and ϕ' are two nondimensional factors defined from the dispersion coefficients f_1 and f_2 as:

$$\phi = f_1 + \frac{3}{2}f_2, \quad \phi' = \frac{1}{\frac{2f_1}{3f_2} + 1}. \quad (12)$$

Inserting the coefficients of dispersion Eq. (7) into the expression for ϕ , one obtains

$$\phi = \frac{5\gamma - 2}{2\gamma + 1} - \frac{1}{\gamma\beta^2}. \quad (13)$$

This coefficient is referred to as the Acoustic Contrast Factor; its sign determines the direction of $\mathbf{F}_R$ toward the nodes of acoustic pressure (if $\phi > 0$) or toward the pressure antinodes (if $\phi < 0$), which are equilibrium positions where particles agglomerate.

Each component of Eqs. (9)a and (9)b show dependence on x , y and z and illustrate the fully 3D nature of $\mathbf{F}_R$. These expressions reduce to the form derived by Gor'kov [1] for rigid and free boundary conditions, respectively, in the limiting case of 1D resonance. A similar expression for acoustic force in three dimensions was found by Barmatz and Collas during the 1980s for the rigid x walls boundary condition [32,33]. The present work is the first to show the expression for the free x walls boundary condition.

2.2. Equations of motion

In addition to the $\mathbf{F}_R$ given by the Eqs. (9)a and (9)b, our model 3D includes other forces that affect particle motions, including:

The viscous drag force. exerted by the fluid on the particles during their motion toward the pressure node locations or acoustic equilibrium zones, given by:

$$\mathbf{F}_d = -6\pi\eta R_p (\mathbf{u} - \mathbf{v}), \quad (14)$$

where η is the dynamic fluid viscosity, $\mathbf{u}$ is the velocity acquired by the particle during its motion and $\mathbf{v}$ is the fluid velocity.

The buoyancy force. is the net force due to the difference between particle weight and the weight of the fluid displaced by the particle, along the z -direction of the chamber:

$$\mathbf{F}_b = \frac{4}{3}\pi r_o^3 (\rho_p - \rho_f) \mathbf{g}, \quad (15)$$

where $\mathbf{g} = -9.8\hat{k} \text{ m/s}^2$; and ρ_p and ρ_f are the particle and fluid density, respectively.

The Hertzian contact stress. established between the pairs of particles by their contact:

$$\mathbf{F}_{ij} = K\delta^n \hat{n}, \quad (16)$$

where the subscripts i and j represent the i 'th and j 'th particles in contact; K is the stiffness coefficient; δ is the relative penetration between the particles; n is a nonlinear exponential dependent on the elastic properties of the particles, their geometries and contact shapes [34]; and $\hat{n}$ is the unit normal vector. These forces are generated mainly at the pressure nodes or antinodes, where particle aggregates are formed depending on their acoustic contrast factor.

Th 3D equation of motion for particle i with mass m_i exposed to these forces in Eqs. (14), (15), (9) and (16)a or (9)b is:

$$\mathbf{f} = \mathbf{F}_R + \mathbf{F}_d + \mathbf{F}_b + \mathbf{F}_{ij} = m_i \mathbf{a}, \quad (17)$$

where $\mathbf{a}$ is the acceleration and $\mathbf{f}$ is the net force exerted on particle i . Eq. (17) was solved using an optimized Forest–Ruth algorithm. Some of the results from a numerical solution for N particles are presented below.

Table 1
Physical parameters [31,35].

Parameter	Symbol	Value
Density of particle	ρ_p	1050 kg/m ³
Density of water	ρ_f	998 kg/m ³
Water viscosity at 20°C	η	1×10^{-3} kg/(m·s)
Sound velocity in water	c_f	1483 m/s
Sound velocity in the particles	c_p	1700 m/s
Ratio of velocities	β	1.58
Relative density	γ	1.05
Young's Modulus of polystyrene	E	3300 MPa
Poisson's coefficient of polystyrene	ν	0.35

3. Numerical simulation

The initial ($t = 0$) conditions of position vector $\mathbf{r}$ and velocity vector $\mathbf{u}$ for each particle i were assumed to be:

$$\begin{aligned} u_x(0) &= u_y(0) = u_z(0) = 0, \\ 0 &< x(0) < \ell_x, \\ 0 &< y(0) < \ell_y, \\ 0 &< z(0) < \ell_z. \end{aligned} \quad (18)$$

The N particles were spaced at regular spatial intervals in x , y and z over the interior volume of the chamber so that the dynamics could be sampled over the entire volume. The system of differential equations described above was numerically solved for position x , y and z and velocity components u_x , u_y and u_z for each particle i .

This system was solved over time t . Use of the Forest–Ruth algorithm required that the step size for t was to be much shorter than the acoustic period T . The time step $h = T/10$ was used, as tests with smaller steps showed that the chosen value of h results in only minor errors ($< 0.01\%$) in the results, and therefore allows an accurate description of the time dependence of the N particle velocities and positions. Results for $\mathbf{r}$ and $\mathbf{u}$ were therefore calculated over the time steps between t and $t + h$.

The following system of equations for the positions and velocities correspond to the optimized Forest–Ruth algorithm [28] that was used to solve numerically the motion equations of the N particles at time t

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{r}(t) + \xi h \mathbf{u}(t), \\ \mathbf{u}_1 &= \mathbf{u}(t) + (1 - 2\lambda) \frac{h \mathbf{f}[\mathbf{r}_1]}{2m}, \\ \mathbf{r}_2 &= \mathbf{r}_1 + \chi h \mathbf{u}_1, \\ \mathbf{u}_2 &= \mathbf{u}_1 + \lambda \frac{h \mathbf{f}[\mathbf{r}_2]}{m}, \\ \mathbf{r}_3 &= \mathbf{r}_2 + (1 - 2(\chi + \xi)) h \mathbf{u}_2, \\ \mathbf{u}_3 &= \mathbf{u}_2 + \lambda \frac{h \mathbf{f}[\mathbf{r}_3]}{m}, \\ \mathbf{r}_4 &= \mathbf{r}_3 + \chi h \mathbf{u}_3, \\ \mathbf{u}(t+h) &= \mathbf{u}_3 + (1 - 2\lambda) \frac{h \mathbf{f}[\mathbf{r}_4]}{2m}, \\ \mathbf{r}(t+h) &= \mathbf{r}_4 + \xi h \mathbf{u}(t+h), \end{aligned} \quad (19)$$

Where $\mathbf{f}[\mathbf{r}]$ is the net force given by Eq. (17) and the constants ξ , λ and χ are specified in Omelyan [28] as

$$\begin{aligned} \xi &= +0.1786178958448091, \\ \lambda &= -0.2123418310626054, \\ \chi &= -0.06626458266981849, \end{aligned} \quad (20)$$

Table 1 shows values for the physical parameters of the fluid and the particles.

The C++ code was validated in a test case, by solving the equation of motion, Eq. (17), for a 5 μm diameter particle moving along the y direction under the action of a one dimensional acoustic force, obtained from Eqs. (9), together with the viscous drag force (14). The particle trajectory $y(t)$ was calculated for a particle with initial position $y(0) = 10 \mu\text{m}$, assuming acoustic energy density $E_{ac} = 6 \text{ J/m}^3$ at frequency $f = 3.7 \text{ MHz}$ and chamber width $\ell_y = 200 \mu\text{m}$. The pressure mode position is located at the midpoint along the y direction of the channel, shown with blue dots in Fig. 1. Fig. 1a

Fig. 1. (a) Particle y position vs. time and (b) particle velocity vs. position, both due to action of a 1D F_R at frequency $f = 3.7$ MHz. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

shows the numerically integrated trajectory (green line) and the analytical results of the Eq. (9) ([22,31]; (black line):

$$y(t) = \frac{1}{k_y} \arctan \left[\tan(k_y y(0)) \exp \left(\frac{4\phi}{9\eta} (k_y R_p)^2 E_{ac} t \right) \right]. \quad (21)$$

The numerically integrated behavior agrees well with the analytical solution.

The numerical and analytical solutions for the particle velocity in the y direction have been also compared in the one-dimensional case [36]:

$$u_y(y) = \frac{2\phi k_y R_p^2 E_{ac}}{9\eta} \sin(2k_y y). \quad (22)$$

The numerical results are shown as a green line in Fig. 1b, and analytical results of Eq. (22), by a black line. Both agree that the particle velocity is minimal near the wall at $y = 0$, increases continuously to a maximum at a distance $y = \lambda/8 = 50 \mu\text{m}$, and decreases to 0 at the pressure node at $y = 100 \mu\text{m}$ where $p_1 = 0$ (dotted blue line in Fig. 1b).

Note that the analytical expressions for the particle position and velocity, Eqs. (21) and (22), can be derived assuming equilibrium between the 1D radiation force F_R and the drag force F_d , but cannot be obtained analytically using the 3D radiation force, the buoyancy force, and the Hertzian contact force in Eq. (17).

4. Acoustic setup and experiments

A device was constructed to allow visualization of the response of particles in suspension to ultrasonic standing waves within a glass capillary tube of square cross section. The channel enclosure had cross section $200 \mu\text{m} \times 200 \mu\text{m}$ with length 4 mm. A piezoelectric transducer of 4 MHz (Pz26, Meggitt A/S, Kvistgaard, Denmark, $4 \text{ mm} \times 13 \text{ mm} \times 0.5 \text{ mm}$) was fixed to the capillary underneath. Silicone tubing was attached at the capillary ends, providing a microfluidic channel with one inlet and one outlet, Fig. 2.

The piezoelectric transducer was driven by a waveform generator (AFG3102C, Tektronix Inc., Beaverton, OR, USA) set to 10 V peak-to-peak with a frequency of 3.7 MHz. Image acquisition was performed using a BX51M reflection microscope (Olympus Corp., Tokyo, Japan) with an Olympus SC100 digital camera connected to a computer running Olympus Stream Basic 1.9 software. Aggregation processes in polystyrene particle suspensions were imaged during activation of the ultrasonic field in the capillary.

Particle suspensions were prepared by diluting $100 \mu\text{l}$ of Polybead® Microspheres $4.5 \mu\text{m}$ or $15.7 \mu\text{m}$ (Polysciences, Inc., Warrington, PA, USA) in 1 ml of HPLC-grade water (DW099A, Promega Corp., Madison, WI, USA) used to eliminate ion-mediated electrical interactions between particles.

5. Results and discussion

The present study includes an analysis of the particle dynamics exposed to a 3D ultrasonic standing wave at a frequency $f = 3.7$ MHz in each of two resonating chambers, (1) a cubic geometry with side $\ell = 200 \mu\text{m}$ excited at its fundamental resonance mode ($n_x = n_y = n_z = 1$) and (2) a rectangular channel with the same cross section as the cube in (1) with double the length ($\ell_x = 400 \mu\text{m}$ and $\ell_y = \ell_z = 200 \mu\text{m}$). The particle motions within the resonant chambers was visualized with free software Gnuplot 5.0 [37].

Fig. 2. Bulk acoustic wave device built with a rectangular glass capillary tube.

5.1. Cubic chamber

The dynamics of $N = 64$ spherical polystyrene particles of diameter $15 \mu\text{m}$ exposed to a 3D acoustic radiation force $\mathbf{F}_R$ were determined by the numerical solving the equations of motion of Eq. (17) for an incident wave with pressure amplitude of 0.4 MPa (energy density $E_{ac} = 18.2 \text{ J/m}^3$) over a time interval of 5 s . The particles were assumed initially located at equidistant positions over a cubic chamber of dimensions $\ell_x = \ell_y = \ell_z = 200 \mu\text{m}$, in stagnant condition, i.e., with no flow, as shown in Figs. 3a and 4a.

5.1.1. Rigid x walls boundary condition

The numerical simulation assumed a pressure node at the center of the chamber along the x direction in the rigid-walled chamber, independent of E_{ac} , where particles were expected to cluster; see Fig. 3. During their motion the particles acquire different velocity amplitudes depending on their position history, as shown in supplementary simulation video 1.

The calculations predict specific movements of particles that depend on their initial location. Though the majority of particles collect at the pressure node position, some move toward regions on the chamber walls, identified in Fig. 3b on the (y, z) plane with black arrows. Yet other particles remain at rest at about $\lambda/8$, corresponding to positions of unstable equilibrium, shown labeled with numbers in Fig. 3b; these particles later move under the influence of the anisotropic gravitational force in the z direction to the walls and reach their final positions shown in Fig. 4c. The particle dynamics are shown in supplementary simulation videos 1 and 2 and in the 2D and 3D image sequences in Fig. 3, that illustrates how particle movement depends on initial position.

5.1.2. Free x walls boundary condition

The assumption of free walls in the x direction (at $x = 0$ and $x = \ell_x$) results in different pressure patterns inside the chamber, as shown in Fig. 4 for 64 particles that move with different velocities toward the walls (blue particles identified with black arrows in Fig. 4b). Most of these particles collect in the center area of the chamber walls to form six clusters, and some single particles collect in the corners, Fig. 4c. The model predicts that pairs of particles collect at four equidistant zones at $\lambda/8$ under unstable equilibrium, identified with black numerals in Fig. 4b, for a short duration until they are carried by the acoustic force to their equilibrium position on the lateral walls. This process can be observed in supplementary simulation videos 3 and 4. The downward movement of several particles due to the anisotropic gravitational force is observed with the free x walls boundary condition; these particles do not join the primary cluster at the chamber center.

5.2. Rectangular chamber

Numerical analysis was performed on the motion of 144 spherical polystyrene particles of $15 \mu\text{m}$ diameter exposed to a 3D $\mathbf{F}_R$ of Eqs. (9) at 18.2 J/m^3 in a rectangular chamber of length $\ell_x = 400 \mu\text{m}$, double of that of the cubic chamber, with cross section $\ell_y = \ell_z = 200 \mu\text{m}$. The particles were assumed initially in rest and distributed in a regularly spaced array within the chamber. The same acoustic frequency was assumed as with the cubic chamber, so $n_y = n_z = 1$ and $n_x = 2$.

5.2.1. Rigid x walls boundary condition

Fig. 5 shows the results for the 144 particles exposed to the 3D $\mathbf{F}_R$ in the channel assuming rigid boundary conditions for the x walls. Particles are predicted to acquire different velocity amplitudes and to form two distinct clusters, separated along the center of the x dimension, Fig. 5. Though the majority of particles aggregate in these central clusters, others are driven away and reach final positions beside the x walls. This is seen in the central region of the channel in $x = \lambda/2$, Fig. 5b, where the pressure antinode is located in the x direction. It should be highlighted that four aggregates are formed on the walls at the antinode as shown in Fig. 5c and in supplementary simulation video 5.

Fig. 3. Particle movements generated by the 3D $\mathbf{F}_R$, Eq. (9)a, in the cubic chamber with rigid walls. (a) Initial motions due to the acoustic force; (b) cluster formation in the chamber center; (c) final particle positions.

Fig. 4. Particle movements generated by the 3D F_R , Eq. (9)b, in the cubic chamber with free walls in the x direction. (a) Initial motion due to the acoustic force; (b) formation of small clusters on the walls of cubic chamber; (c) final positions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 5. Particle movements generated by the 3D F_R , Eq. (9)a, in the rectangular chamber with rigid walls. (a) Initial motion due to the acoustic force; (b) formation of two clusters at the center of chamber; (c) final positions.

Similar particle motion was obtained in the (y, z) plane of the rectangular chamber with rigid walls, Fig. 5. Again, most particles collect along the pressure node due to the acoustic radiation force, Eq. (9)a, but some remain in specific positions of unstable equilibrium equidistant from the chamber center, labeled with black numerals in Fig. 5b. The lower half-volume of the channel contains more particles in aggregates, again due to gravity. It is important to note that clusters formed in the center of the rectangular chamber with rigid walls remain stable during the application of the acoustic field.

Fig. 6. Particle movements generated by the 3D F_R , Eq. (9)b, in the rectangular chamber with free walls in the x direction. (a) Initial motion due to the acoustic force; (b) formation of one cluster at the center of the chamber; (c) final positions.

5.2.2. Free x walls boundary conditions

The model predicts different particle motions for the case of free walls in the x direction. A single particle cluster is first formed at the chamber x center ($\lambda/2$), Fig. 6. Unlike with the cubic chamber, the rectangular chamber model predicts that a large number of particles reach their equilibrium positions at the lateral channel walls, Fig. 6b and c, in particular at $x = \lambda/4$ and $x = 3\lambda/4$ where two pressure antinodes are generated in the x direction, respectively. This aggregation process can be observed in supplementary simulation video 6.

Fig. 7. Pearl chain aggregate formed in the center of a rectangular chamber, with $\ell_x = 4$ mm, generated by 3D F_R with free walls, Eq. (9)b. (a) Initial particle positions; (b) cluster formation at the channel center; (c) final positions.

Fig. 6b and c also show the particle cluster formation in the (y, z) plane at $x = 200$ μm , especially at the center of the channel. As with rigid walls, free walls are also associated with four regions of unstable equilibrium where some particles remain for a short interval until they fall in the x direction due to gravity.

Fig. 8. Sequential images of the formation of a pearl-chain aggregate on the nodal plane located in $z = \lambda/4$ with polystyrene particles of diameter $4.5 \mu\text{m}$. The black arrows at time t_2 show the motion of two particles toward the channel walls, and the arrows at t_4 show the formation of small particle clusters.

Pearl chain aggregate

An interesting result was found with $3D \mathbf{F}_R$ in the case of free walls with $\ell_x \gg \ell_y, \ell_z$: the formation of pearl-chain aggregates in the center of the channel, as shown in Fig. 7. To simulate generation of the pearl-chain aggregate, 96 spherical polystyrene particles of $15 \mu\text{m}$ diameter were assumed distributed homogeneously inside a channel of length 10 wavelengths ($\ell_y = \ell_z = 200 \mu\text{m}$ and $\ell_x = 10\lambda = 4 \text{ mm}$) exposed to $3D \mathbf{F}_R$ at frequency 3.7 MHz and energy density 18.2 J/m^3 . Fig. 7 shows a sequence of images the formation process of the pearl-chain aggregate at time $t = 5 \text{ s}$ after the acoustic field is turned on. The model shows particle aggregation in small clusters in the center, Fig. 7b, but over time these small clusters bond to become a continuous aggregate shaped like a pearl chain as shown in Fig. 7b and c. Supplementary simulation video 7 show the aggregation process, where the final positions of some particles on the edges of the channel, beside the walls, are highlighted. These types of aggregates are frequently observed in experiments with microfluidic channels, as shown in Figs. 8 and 9.

It is of interest that during the formation of pearl-chain aggregates no aggregation at unstable equilibrium regions is observed in the (y, z) plane, Fig. 7. This prediction and that of particle motions toward channel walls were confirmed by experimental observations as described below.

The particles appeared randomly distributed before onset of activation of the acoustic field and were rapidly reordered by the action of the $3D$ radiation force. The particles located in the nodal plane were driven toward the central axis where small groups of particles aggregated in small local clusters, identified by black arrows at the time t_4 in Fig. 8. These small clusters gradually joined along the x direction to finally form the pearl-chain aggregate at the time t_6 .

During this process the formation of small clusters was observed, as well as motion of some particles toward the walls of the capillary. Fig. 8 and supplementary video 8 demonstrate the observations.

The same aggregation process was observed using particles of $15.7 \mu\text{m}$ diameter, Fig. 9, where the particles formed a pearl-chain at the nodal plane located at $z = \lambda/4$, Fig. 9a and c. A few particles were observed to reach an equilibrium position at the walls, as is indicated with the black squares in Fig. 9c.

An aggregate of particles was found at the bottom of the channel ($z = 0$), at one of the vertices, Fig. 9b. This experimental observation corroborates the calculated particle movement toward the vertices of the channel due to the $3D$ Acoustic Radiation Force, Fig. 7.

This method allows the study of time-dependent single particle behavior under the action of the acoustic field, and predicts formation of aggregates at the pressure node. Novel predictions include particles remaining at temporary rest in regions of unstable equilibrium, and their ultimate motion toward equilibrium positions at the channel walls. This shows more complex behavior than anticipated, i.e. the presence of other zones of acoustic equilibrium not described in previous work that modeled $\mathbf{F}_R$ in one dimension only. These new effects may therefore be related to the dependence of $\mathbf{F}_R$ on the

Fig. 9. Images of aggregate formation in a rectangular capillary with polystyrene particles of $15.7\ \mu\text{m}$ diameter. (a) Aggregate formed on the nodal plane; (b) aggregate formed on a channel bottom edge; (c) image of nodal plane where some particles are located on the walls.

three spatial dimensions, Eqs. (9). In particular, it should be noted that, in the rectangular chamber, the direction of $\mathbf{F}_R$ is toward the walls when the pressure antinodes coincide in the three spatial directions, which predicts formation of four aggregates on walls at $x = \lambda/2$ for rigid walls, Fig. 5, and at $x = \lambda/4$ and $x = 3\lambda/4$ for free walls, Fig. 6.

The current study considers two boundary conditions along the chamber length, rigid and free x walls. Novel predictions were made for these conditions, in particular in the case of channels with free x walls for $\ell_x \gg \ell_y, \ell_z$, where pearl chain particle aggregates were predicted to lie along the center of the channel, and that some particles reached their equilibrium positions at the vertices. These results from numerical simulation were corroborated by experiments with a microfluidic setup for study of particles exposed to a 3D standing wave. Good agreement between the present theoretical work and experimental results was obtained, as shown in Figs. 7–9.

The novel 3D model and computational approach may lend itself to the study of particle dynamics related to fluid viscosity, acoustic streaming, as well as inter-particle acoustic forces. Applications could include plasmapheresis of highly concentrated samples. In this case it would be necessary to consider the secondary acoustic radiation (Bjerknes) force between particles due to interactions of waves scattered by the suspended particles [38]. This force could be included in the present 3D model, along with simulation of the acoustic radiation force 3D and Hertzian contact force, and would allow the study of hundreds of particles subjected to the action of these forces.

6. Conclusions

This work presents a 3D model of the effects of a three dimensional radiation force generated by an acoustic standing wave in resonant cavities of rectangular and square geometry, with boundary conditions corresponding to either rigid or free walls. The analysis of particle dynamics under different boundary conditions is of particular importance with respect to the development of new low-cost acoustic devices, since this allows examination of the effect of building materials upon devices with respect to throughput in the separation or manipulation processes [24]. A numerical simulation in C++ was developed to predict the simultaneous dynamics of N particles in microfluidic channels using a novel computational approach with an optimized Forest–Ruth algorithm. This simulation provides a comprehensive understanding of the behavior of microparticles due to the 3D Acoustic Radiation Force in microfluidic devices where different acoustic equilibrium zones were identified. Finally, the simultaneous simulation-based study of the dynamics of and interactions between many particles allows study of aggregation processes, as was verified in the case of the pearl-chain aggregate.

CRedit authorship contribution statement

A. Vargas-Jiménez: Conceptualization, Methodology, Software, Formal analysis. **M. Camacho:** Writing - review & editing. **J.D. Muñoz:** Software. **I. González:** Methodology, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This research was supported and funded by the Institute of Physical Technologies and Information ITEFI-CSIC, España, through project COOPA20348, I-COOP+2018 and by the Administrative Department of Science, Technology and Innovation of Colombia, Colciencias, through project 222856 933541, conv. 569-2012. We thank Dr. Michael Delay, IDEX Health & Science (Semrock), for comments, suggestions, and assistance with the manuscript.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.wavemoti.2020.102701>.

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