Anticipating climatic tipping points and regime shifts

Part 2: Classical and network methods of anticipation

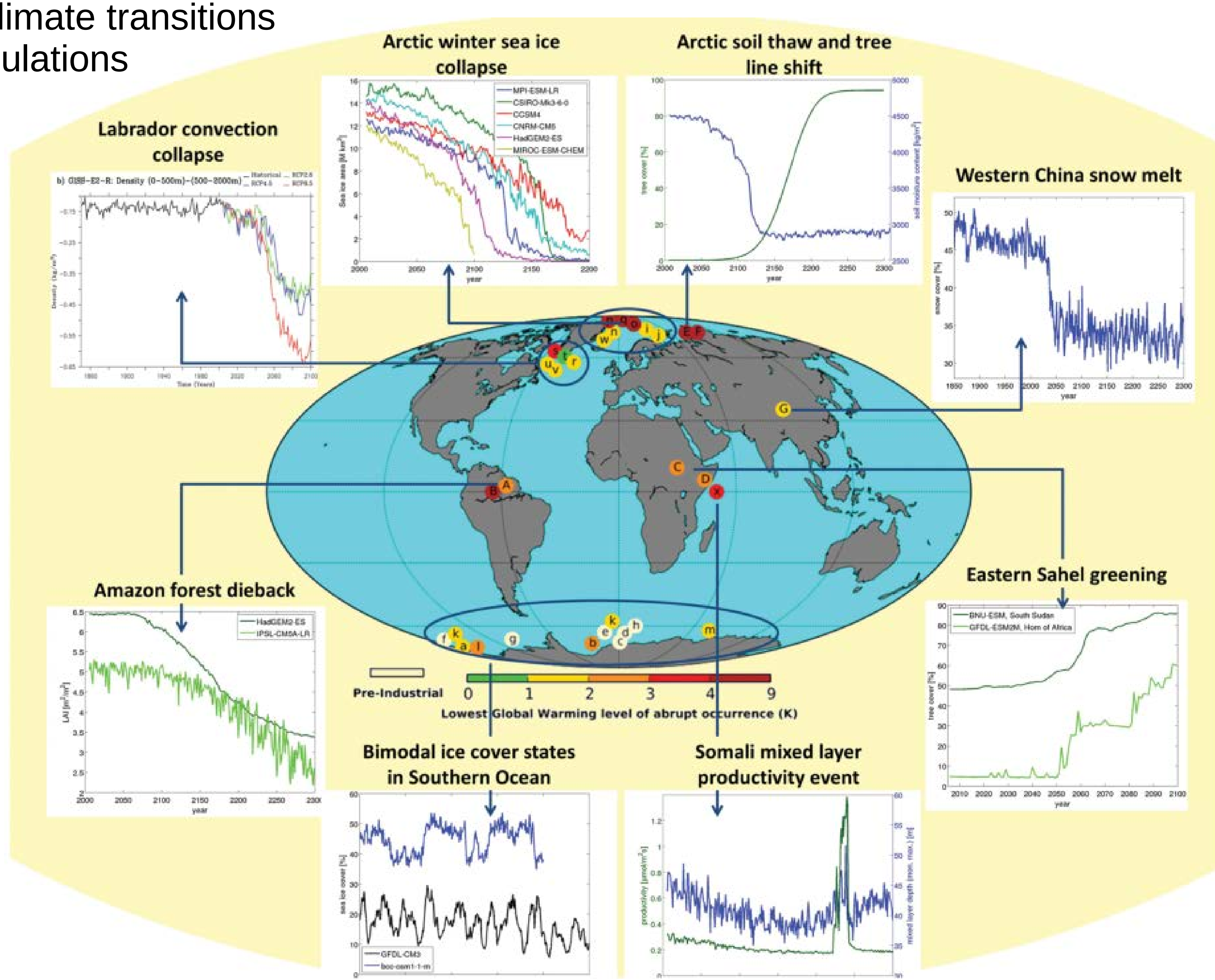
Emilio Hernández-García
IFISC (CSIC-UIB)
Palma de Mallorca, Spain

CAFE
Climate Advanced Forecasting of sub-seasonal Extremes

IFISC

CAFE first workshop, September 2020

Abrupt climate transitions from simulations



From Bathiany et al., Beyond Bifurcation: using complex models to understand and predict abrupt climate change. Dynamics and Statistics of the Climate System 1, dzw004 (2016)

CLIMATIC PHENOMENA AT SHORTER TIME SCALES THAT CAN BENEFIT FROM A 'TIPPING-POINT PERSPECTIVE'

- El Niño phenomenon
  - Rodriguez-Mendez, Eguiluz, Hernandez-Garcia, Ramasco. *Percolation-based precursors of transitions in extended systems.* Scientific Reports 6, 29552 (2016)
  - Meng, Fan, Ashkenazy, Havlin. *Percolation framework to describe El Niño conditions.* Chaos 27, 035807 (2017)
- Monsoon onset
  - Stolbova, Surovyatkina, Bookhagen, Kurths (2016), *Tipping elements of the Indian monsoon: Prediction of onset and withdrawal*, *Geophys. Res. Lett.*, 43, 3982 (2016)
- Cyclones
  - Shraddha Gupta, Kurths, Pappenberger. *Study of Tropical Cyclones in the North Indian Ocean basin using Percolation in Climate Networks*, EGU General Assembly 2020, Online, 4–8 May 2020, EGU2020-5916, https://doi.org/10.5194/egusphere-egu2020-5916, 2020
- ???

A working scientific hypothesis is that abrupt climatic transitions are associated to dangerous and to explosive bifurcations

Since climatic abrupt transitions may have important impact on human activities (without time to slowly adapt, as with safe bifurcations), it is relevant to devise methods of early warning: is there something in the system behavior before the bifurcation telling us that we are close to a tipping point?

- Critical slowing-down
  - Slower recovery from perturbations
  - Increased autocorrelation
  - Increased variance
- Skewness, flickering, potential recovery
- Spatial indicators
  - Increased spatial variance
  - Increased correlation length
- Network indicators
  - Degree, clustering, …
  - Percolation-based methods

Dakos et al. *Slowing down as an early warning singal for abrupt climate change,* PNAS 105, 14308 (2008)

Thompson and Sieber, *Predicting Climate tipping as a noisy bifurcation: A review,* International Journal of Bifurcation and Chaos 21, 399 (2011).

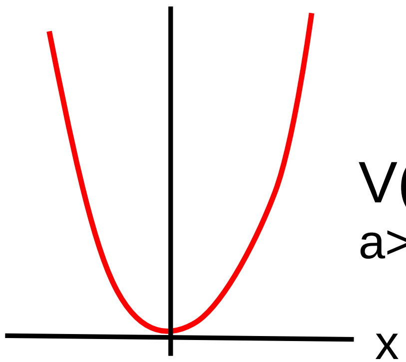
Lenton, *Early warning of climate tipping points.* Nature Clim. Change 1, 201 (2008)

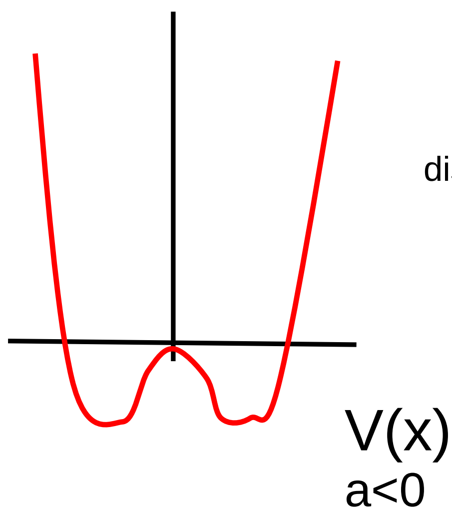
Dijkstra, Hernandez-Garcia, Masoller, Barreiro. *Networks in Climate.* Cambridge (2019).

van der Mheen et al. *Interaction network based early warning indicators for the Atlantic Meridional Overturning Circulation collapse.* Geophys. Res. Lett., 40, 2714 (2013).

Rodriguez-Mendez et al. *Percolation-based precursors of transitions in extended systems*. Scientific Reports 6, 29552 (2016).

Example (for a pitchfork bifurcation):





Let us consider overdamped motion in a potential:

$$\dot{x}(t) = -\frac{\partial}{\partial x} V(x)$$

with

$$V(x) = \frac{a}{2}x^2 + \frac{b}{4}x^4$$

i.e.:

$$\dot{x} = -ax - bx^3$$

close to $x \approx 0$ we have for both $a > 0$ and $a < 0$:

$$\dot{x} \approx -ax \ ,$$

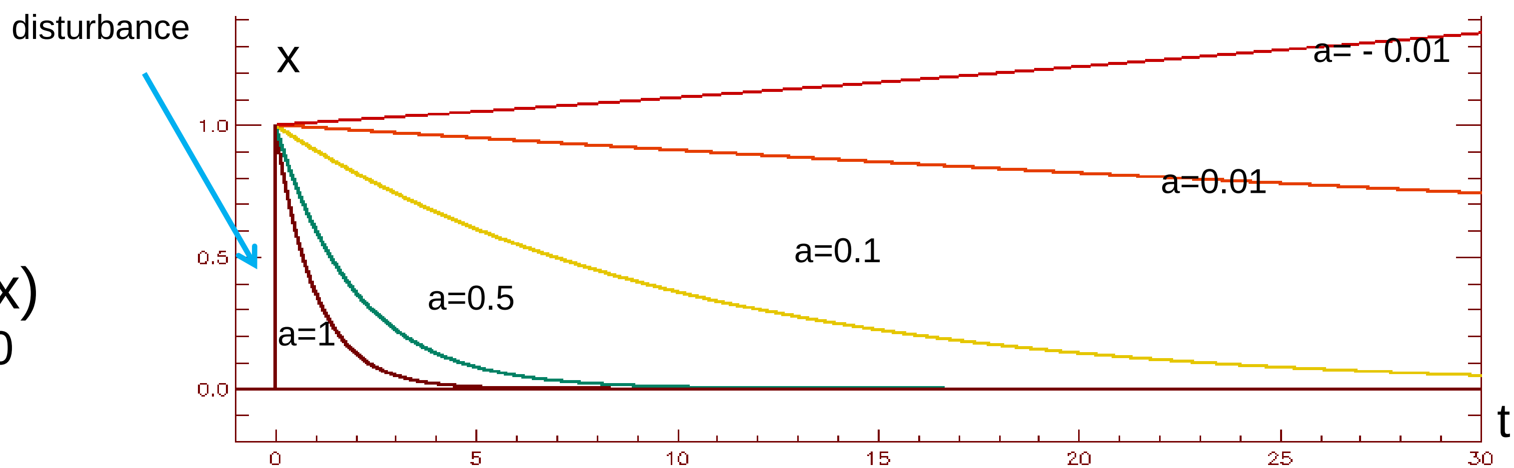
so that

$$x(t) \approx x(0)e^{-at}$$

Exponential relaxation to equilibrium when it is stable ($a > 0$). Exponential escape from equilibrium when it is unstable ($a < 0$).

The Linear Decay Rate LDR=*a* approaches zero when approaching the bifurcation. Relaxation time *1/LDR* diverges

CRITICAL SLOWING DOWN



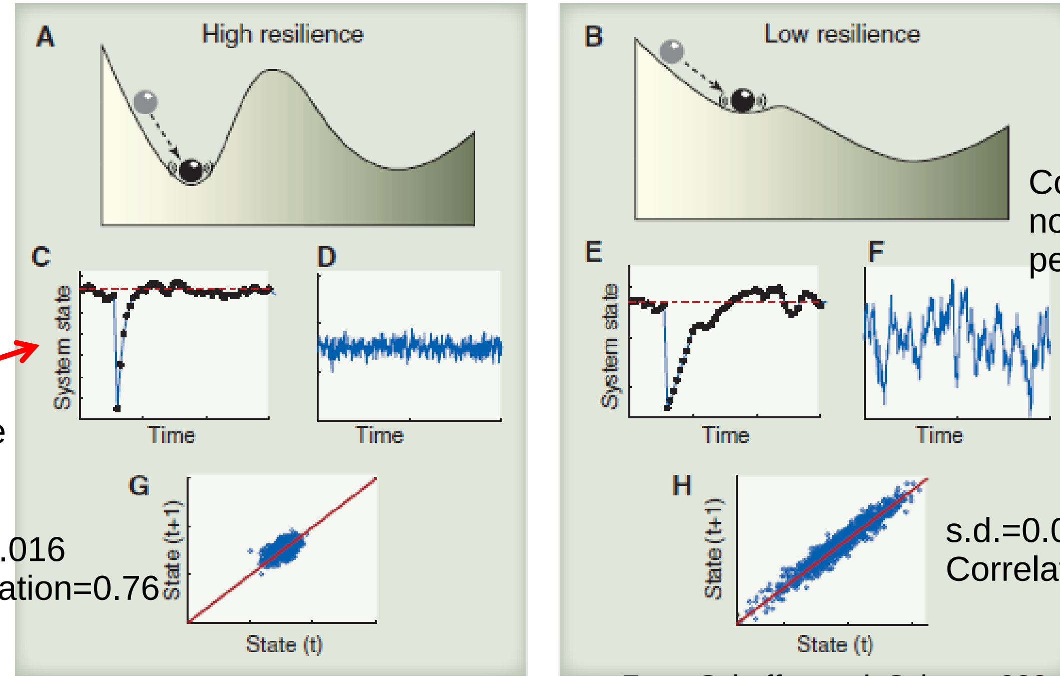
Critical slowing down

Critical slowing down :

- vanishing of Linear Decay Rate
- increased autocorrelation
- increased variance

Discretize and add noise

Single disturbance



From Scheffer et al. Science 338, 344 (2012)

Continuous noise perturbation

s.d.=0.016
Correlation=0.76

s.d.=0.091
Correlation=0.90

$$x(t) - x_{eq} \approx (x(0) - x_{eq})e^{-at}$$

discretizing in steps Δt:

$$y_{n+1} = e^{-a\Delta t} y_n \ , \quad y_n = x(t_n) - x_{eq}$$

adding white noise:

$$y_{n+1} = c y_n + \sigma \epsilon_n \quad \text{AR(1) process.} \quad c = e^{-a\Delta t}$$

is the propagator, or autocorrelation function or factor (ACF), or first-order autoregressive coefficient

$$< y_n > \ \to \ 0 \ , \ n \to \infty$$

$$< y_{n+1} y_n > \ = \ c < y_n^2 > + \sigma < \epsilon_n y_n >$$

$$c \ = \ \frac{< y_{n+1} y_n >}{< y_n^2 >}$$

$$\text{variance} = < y_n^2 > - < y_n >^2 \to \frac{\sigma^2}{1 - c^2} \ , \ \text{as } n \to \infty$$

Thus *c* is the lag-1 autocorrelation.
Critical slowing down → $c \to 1$

Then, variance diverges when approaching the bifurcation

But remember that the linear approximation is no longer valid when *a=0* or *c=1*

Q: Do all relevant bifurcations display critical slowing down?
A: Not all. But most of them:

Precursors of Codimension-1 Bifurcations

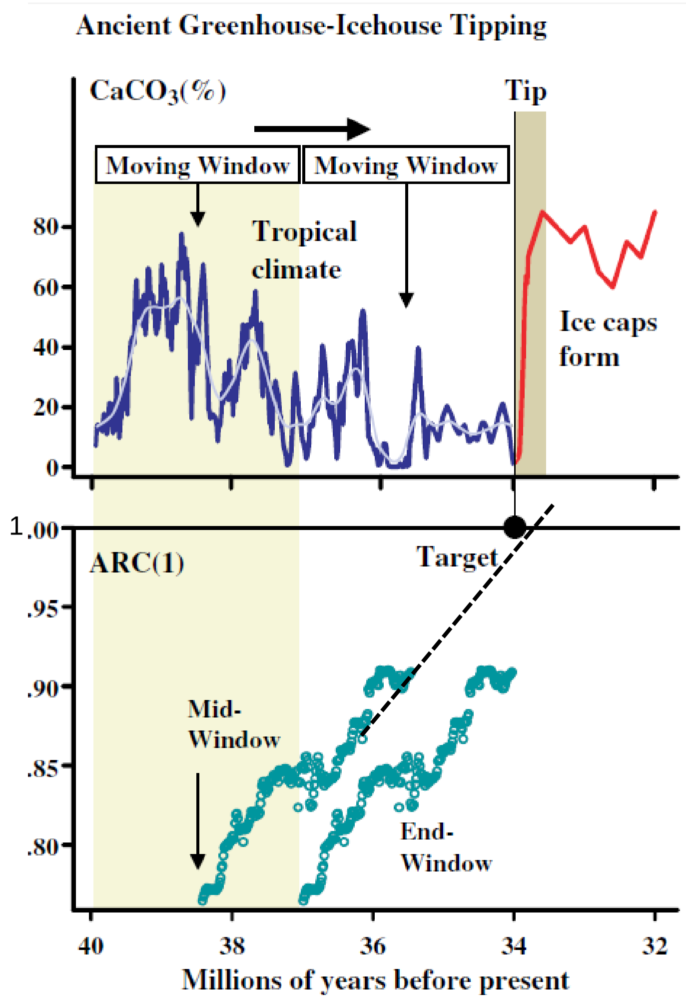
Supercritical Hopf	S: point to cycle	LDR $\to$ 0 linearly with control	★
Supercritical Neimark	S: cycle to torus	LDR $\to$ 0 linearly with control	★
Supercritical flip	S: cycle to cycle	LDR $\to$ 0 linearly with control	★
Band merging	S: chaos to chaos	separation decreases linearly	
Flow explosion	E: point to cycle	Path folds. LDR $\to$ 0 linearly along path	★
Map explosion	E: cycle to torus	Path folds. LDR $\to$ 0 linearly along path	★
Intermittency expl: flow	E: point to chaos	LDR $\to$ 0 linearly with control	★
Intermittency expl: map	E: cycle to chaos	LDR $\to$ 0 as trigger (fold, flip, Neimark)	★
Regular interior crisis	E: chaos to chaos	lingering near impinging saddle cycle	
Chaotic interior crisis	E: chaos to chaos	lingering near impinging chaotic saddle	
Static fold	D: from point	Path folds. LDR $\to$ 0 linearly along path	★
Cyclic fold	D: from cycle	Path folds. LDR $\to$ 0 linearly along path	★
Subcritical Hopf	D: from point	LDR $\to$ 0 linearly with control	★
Subcritical Neimark	D: from cycle	LDR $\to$ 0 linearly with control	★
Subcritical flip	D: from cycle	LDR $\to$ 0 linearly with control	★
Saddle connection	D: from cycle	period of cycle tends to infinity	
Regular exterior crisis	D: from chaos	lingering near impinging saddle cycle	
Chaotic exterior crisis	D: from chaos	lingering near impinging accessible saddle	

From Thompson and Sieber, IJBC 21, 399 (2011)

Practicalities

Q: How to extract the Linear Decay Rate or the propagator *c* from a time series?

A1: Held and Kleinen, GRL 31, L23207 (degenerate fingerprinting); Dakos et al., PNAS 105, 14308 (2008)



1. **Interpolation:** if the time series measurements are not equidistant, interpolate it to equidistant time steps Δt. One should have $1/LDR \gg \Delta t \gg 1/$decay rate of all other modes
2. **Detrending:** Remove slow drifts of the possibly moving equilibrium. For example, calculate $X(t_n)$ as an average of the series values with a Gaussian kernel of witdth *d* centered at t_n, or use a local polinomial fit, and replace x_n by $y_n = x_n - X(t_n)$.
3. **Fit data** to the AR(1) model $y_{n+1} = cy_n + \sigma\varepsilon_n$ in a moving window of width 2k centered at t_n. This can be done by least squares of data to $y_{n+1} = cy_n$. Assign the $c(t_n)$ value to the middle of the sliding window (in some papers it is assigned to the end point).

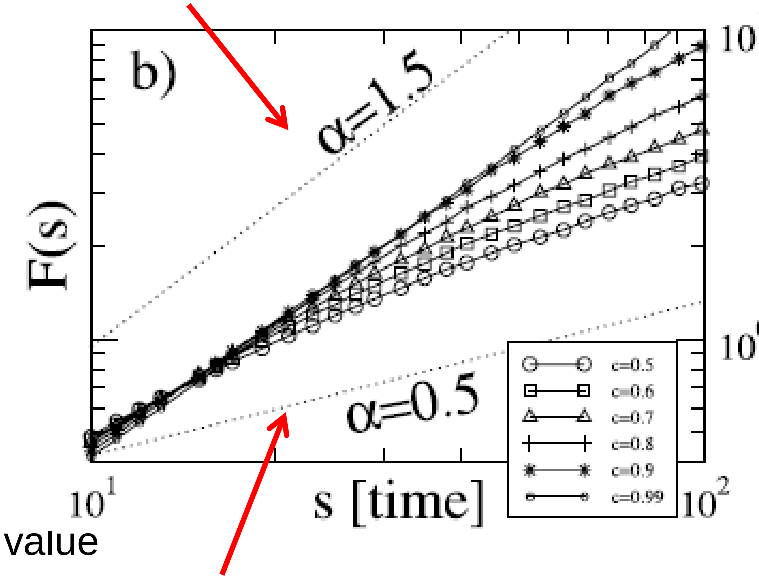
Increasing $c(t_n)$ indicates increasing slowing down. One can extrapolate to $c \to 1$ to make a prediction of the time for the transition.

A2: Fit an increase in variance as $\sigma^2/(1-c^2)$

Q: How to extract the Linear Decay Rate or the propagator *c* from a time series?

A3: Livina and Lenton, GRL 34, L03712 : Detrended fluctuation analysis (DFA) propagator

yn=correlated random walk



yn=uncorrelated white noise

1. **Interpolation**: as before
2. **Detrending:** $y_n = x_n - X(t_n)$ as before
3. **Calculate** the variance in a window of width 2k:

$$F^2(t_n, k) = \frac{1}{2k} \sum_{i=n-k}^{n+k} y_i^2$$

4. and fit a local exponent α_n by varying window size 2k:

$$F^2(t_n, k) \sim k^{2\alpha_n}$$

α_n =0.5 for white noise, and α_n >0.5 when there are power-law correlations. The idea (nonrigurous) is that, since correlations will increase as *c* approaches 1, one should see an increasing value of α_n when approaching a bifurcation.
Livina and Lenton proposed an empirical formula to obtain c_n from α_n
.

Apparently this method is good when the time series is short

Critical slowing down examples

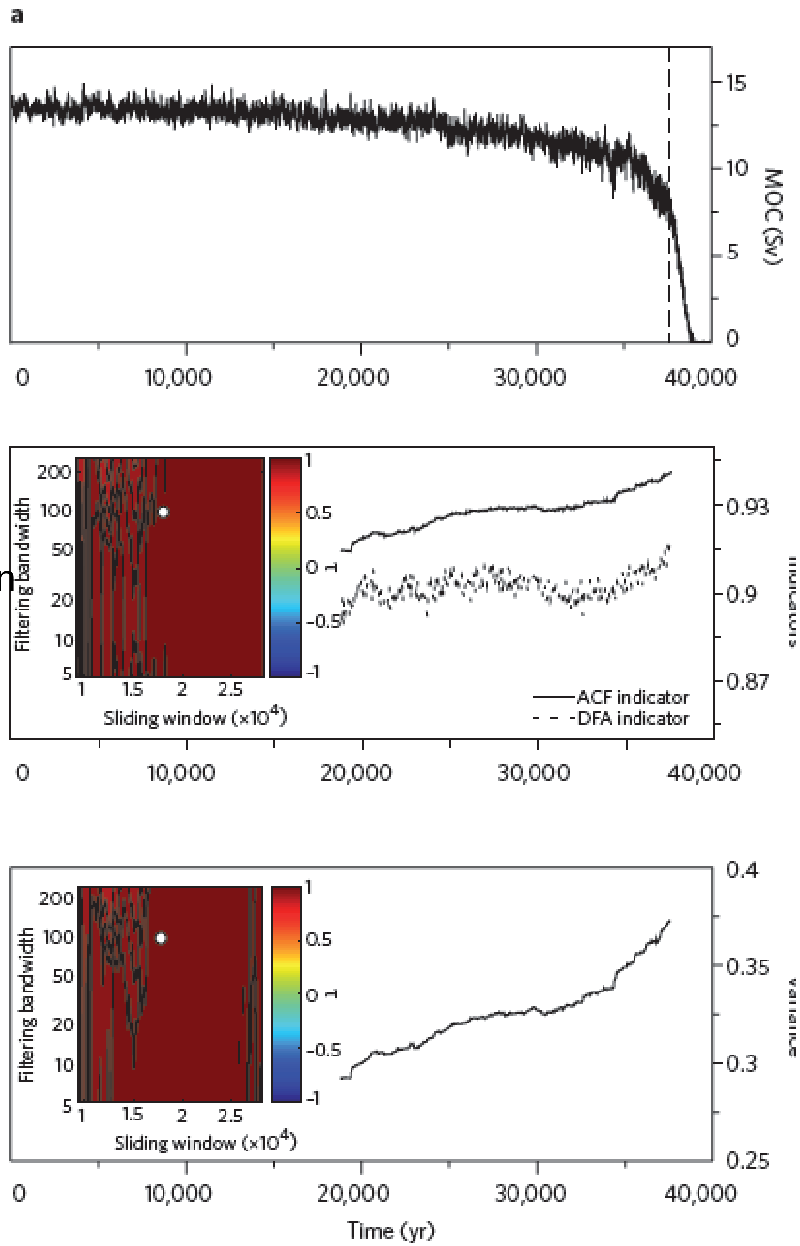
Examples (from Lenton, Nature Clim. Change 1, 201 (2008))

MOC ⇨ collapse (from GENIE-1 simulation)

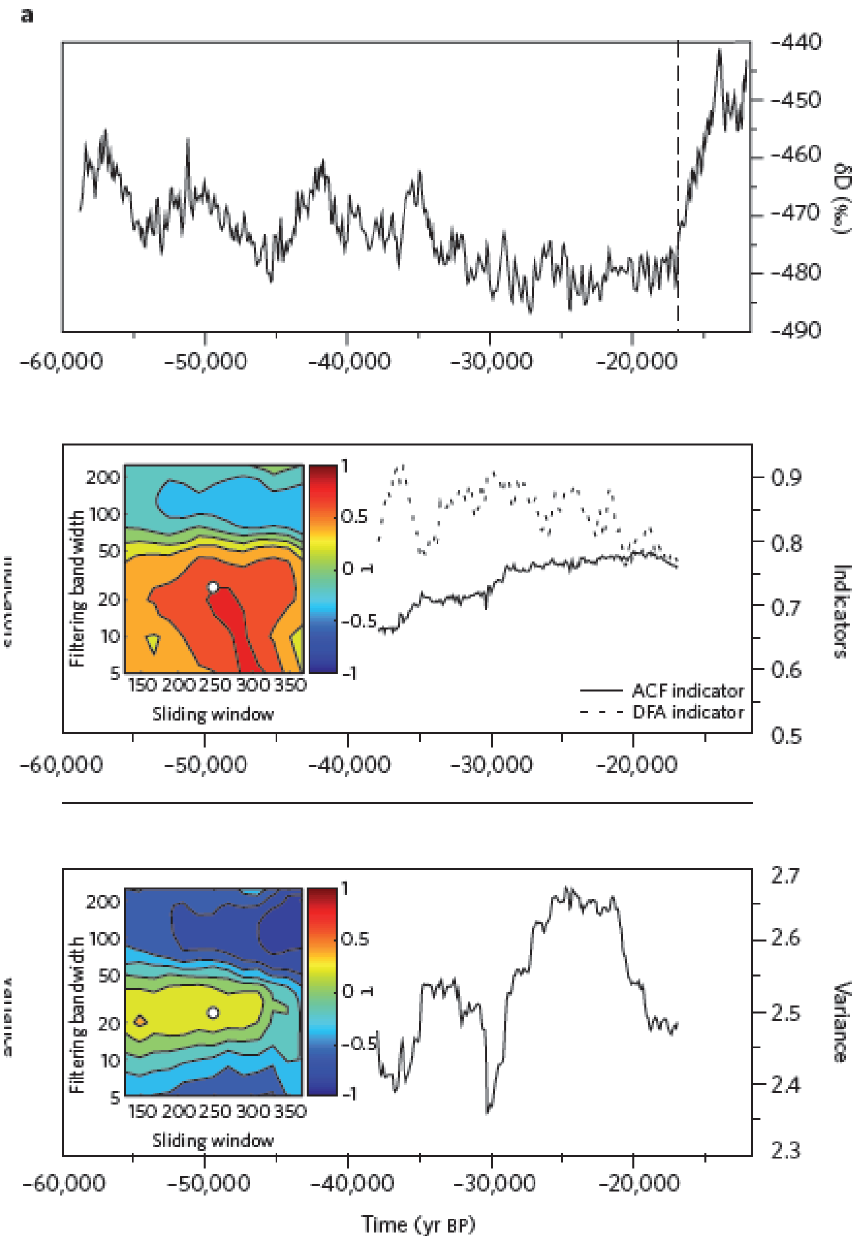
ACF: Autocorrelation

DFA: Detrended Fluct. analysis

Variance

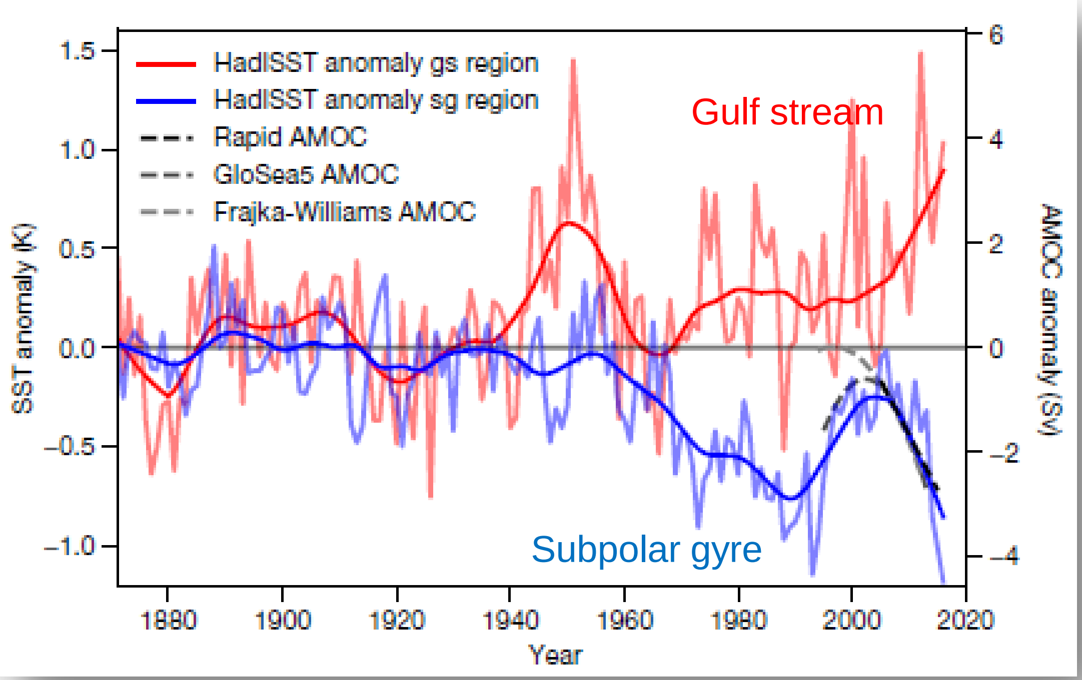


Vostok ice core data



Atlantic Meridional Overturning Circulation (AMOC or MOC)

Caesar et al. Nature 2018
Nature 556,191 (2018)



Testing the methods

Table 1 | Early warning indicators of approaching bifurcation points and tests thereof.

Phenomenon	Indicator	System	Data Source	Signal	Reference(s)
Critical slowing down	Increasing autocorrelation, AR(1) coefficient	Climate	Models	+	8, 10, 12, 53
			Palaeorecord	+	10, 12, 53
				0	12, 13
		Ecological	Models	+	44
	Increasing return time from perturbations	Ecological	Models	+	39, 40, 45, 51
			Lab experiments	+	6, 52
	Increasing DFA exponent	Climate	Models	+	9, 11, 12
			Palaeorecord	+	9, 12
				–	12
	Spectral reddening	Climate	Models	+	7
		Ecological	Model	0	79
	Increasing spatial correlation	Ecological	Models	+	47
			Lab experiments	+	52
Increased variability	Increasing variance	Climate	Models	+	12
				0	12
			Palaeorecord	+	12
				0	13
				–	12
		Ecological	Models	+	43–45, 79
			Lab experiments	+	52
	Increasing spatial variance	Ecological	Model	+	48
			Data	+	49
			Lab experiments	+	52
Skewed responses	Increasing skewness	Climate	Palaeodata	0	46
		Ecological	Model	+	44–46
			Lab experiments	+	52
	Increasing spatial skewness	Ecological	Model	+	48

'+' means indicator increased as expected; '–' means indicator decreased, contrary to expectation; '0' means there was no significant change in the indicator.

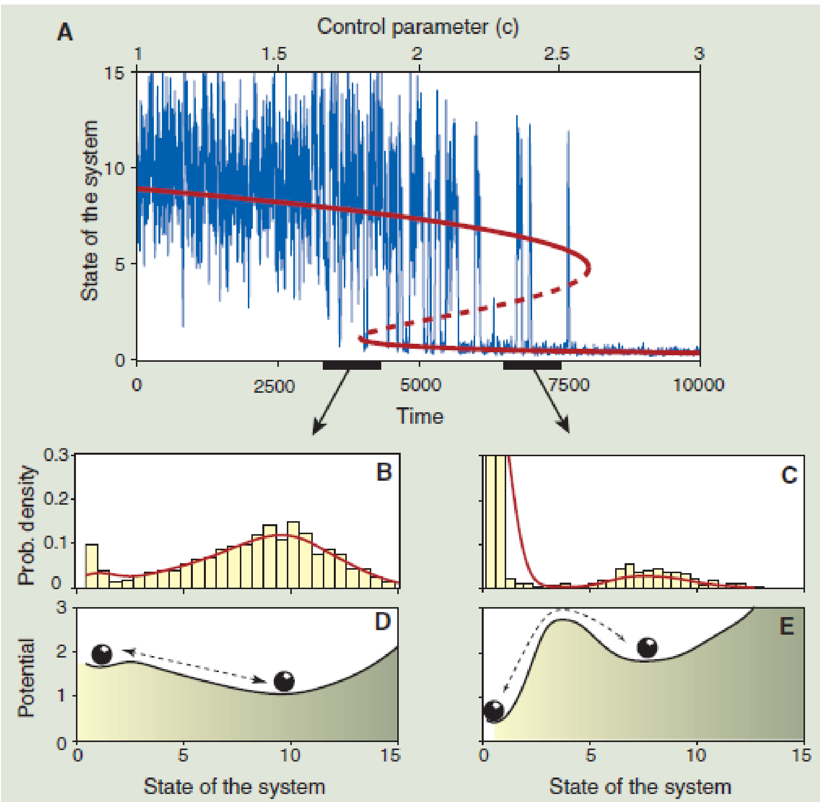
from Lenton, Nature Clim. Change 1, 201 (2008))

Q: What can go wrong with the bifurcation approach (B-tipping)?

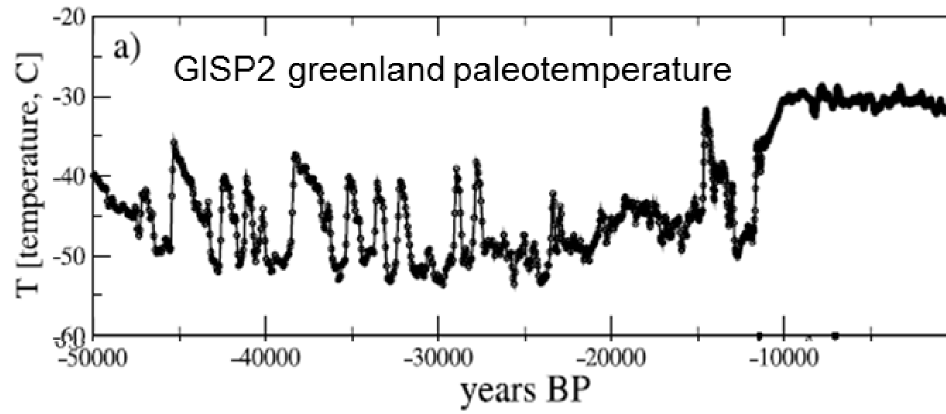
A: Many things:

- Critical slowing down is present in many bifurcation types, but not in all.
- Several parameters to tune: interpolation step, filter width, window width, …
- There is an assumption of quasistationarity: control parameter changing much slower than system's time scales. For anthropogenic changes this is certainly not true: bifurcation effects will be seen well after crossing the tipping point (R-tipping – Ashwin et al. Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system, Phil. Trans. R. Soc. A.370: 1166 (2012)).
- After all, perhaps abrupt changes do not come from a bifurcation:
  - External causes (asteroid, volcanism, geological changes, … outside the monitored climatic variables)
  - Noise induced transitions (N-tipping)

Noise induced jumps between metastable states: no advancement towards bifurcation, so **no** critical slowing down indicators.

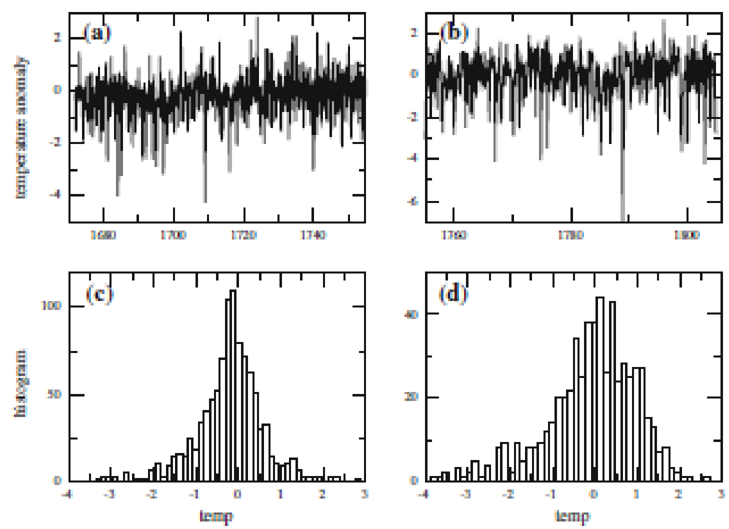


Scheffer et al, Science 338, 345 (2012)



Flikering: jumping between alternate states. Is not really an ‘early warning’. It is a signal that transitions are already occurring.

$$\text{skewness} = \left\langle \left(\frac{x - \langle x \rangle}{\sigma} \right)^3 \right\rangle$$



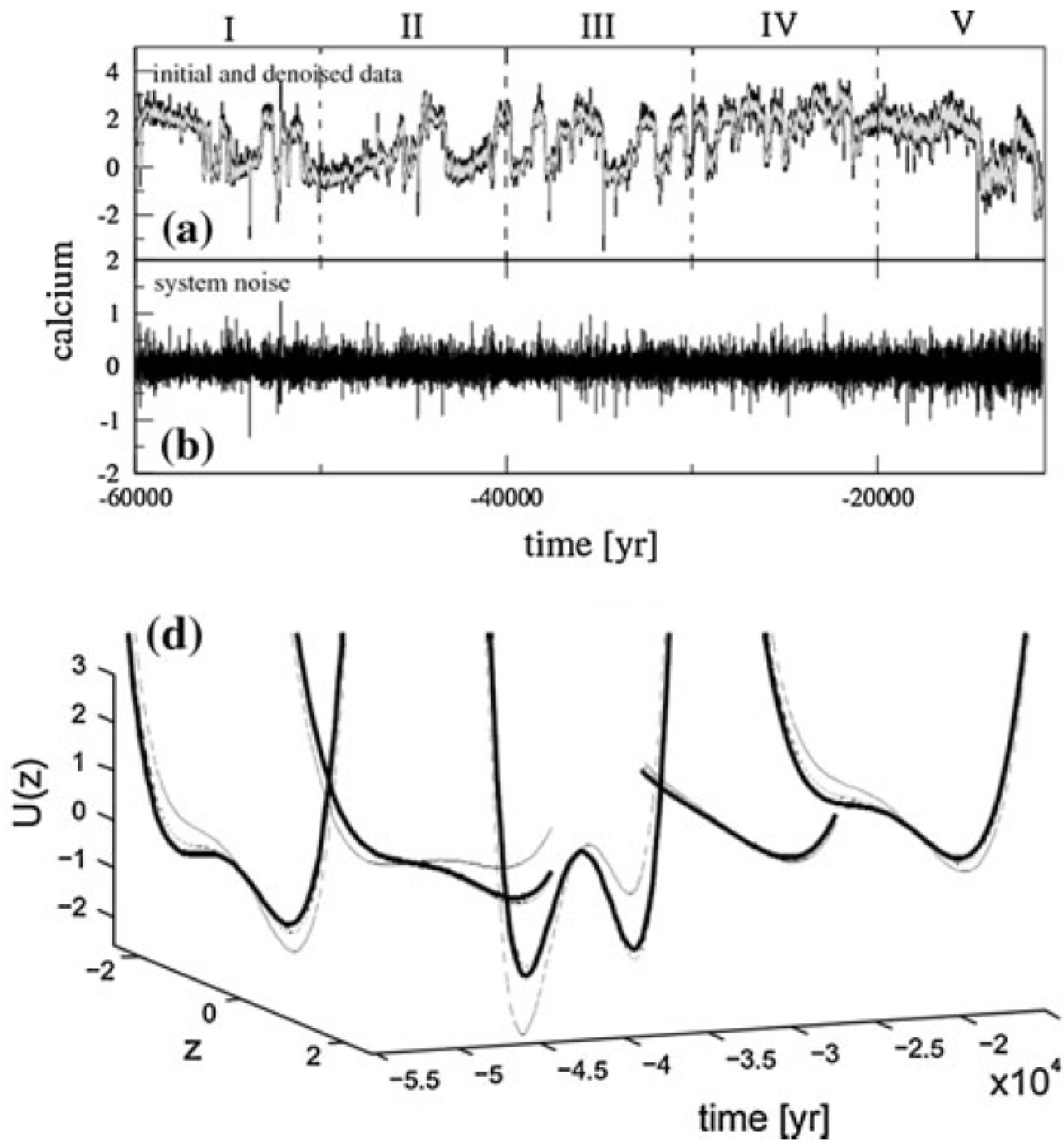
Skewness: Asymmetry in fluctuations indicate the proximity to an alternate state

Beyond increased variance, skewness, **… full reconstruction of the probability distribution and landscape potential**.

Other developments:

Livina, Kwasniok, Lohman, Kantelhardt, Lenton, Clim. Dyn 37, 2437 (2011)

Livina, Lohmann, Mudelsee, Lenton, Physica A 392, 3891 (2013).



Indicators

- Critical slowing-down
  - Slower recovery from perturbations
  - Increased autocorrelation
  - Increased variance
- Skewness, flickering, potential recovery
- **Spatial indicators**
  - Increased spatial variance
  - Increased correlation length
- Network indicators
  - Degree, clustering, …
  - Percolation-based methods

Dakos et al. *Slowing down as an early warning singal for abrupt climate change,* PNAS 105, 14308 (2008)

Thompson and Sieber, *Predicting Climate tipping as a noisy bifurcation: A review,* International Journal of Bifurcation and Chaos 21, 399 (2011).

Lenton, *Early warning of climate tipping points.* Nature Clim. Change 1, 201 (2008)

Dijkstra, Hernandez-Garcia, Masoller, Barreiro. *Networks in Climate.* Cambridge (2019).

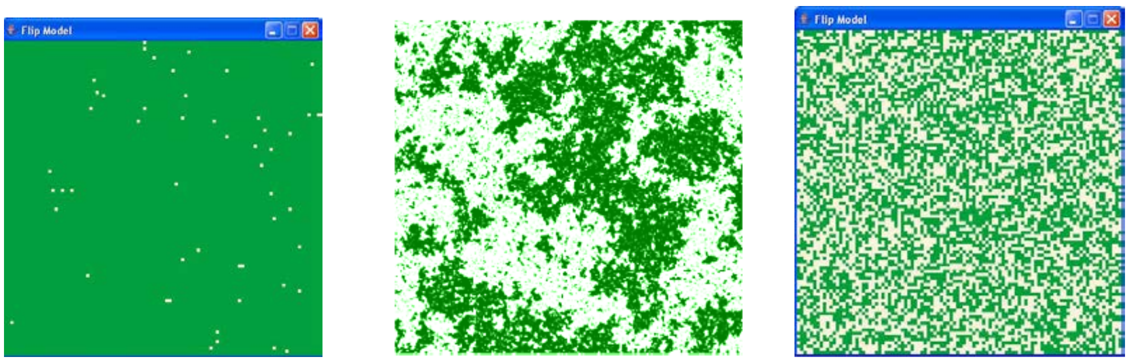
van der Mheen et al. *Interaction network based early warning indicators for the Atlantic Meridional Overturning Circulation collapse.* Geophys. Res. Lett., 40, 2714 (2013).

Rodriguez-Mendez et al. *Percolation-based precursors of transitions in extended systems*. Scientific Reports 6, 29552 (2016).

Spatial indicators

Historically, divergence of SPATIAL correlations and spatial variance was recognized as an indicator of phase transitions before TEMPORAL ones.

T<Tc T=Tc T>Tc



Ising model at H=0

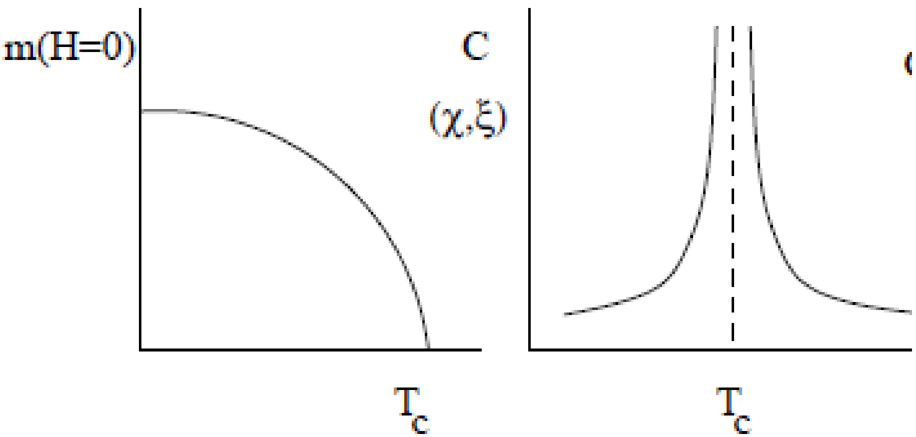
$$\chi_\pm(T, H\to 0^+) = \frac{\partial m}{\partial H}\Big|_{H=0^+} \propto |t|^{-\gamma_\pm},$$

t=T-Tc/Tc

Susceptibility diverges at the critical point

$$\frac{V\chi}{\beta} = \mathrm{var}(M) \equiv \langle M^2\rangle - \langle M\rangle^2. =$$

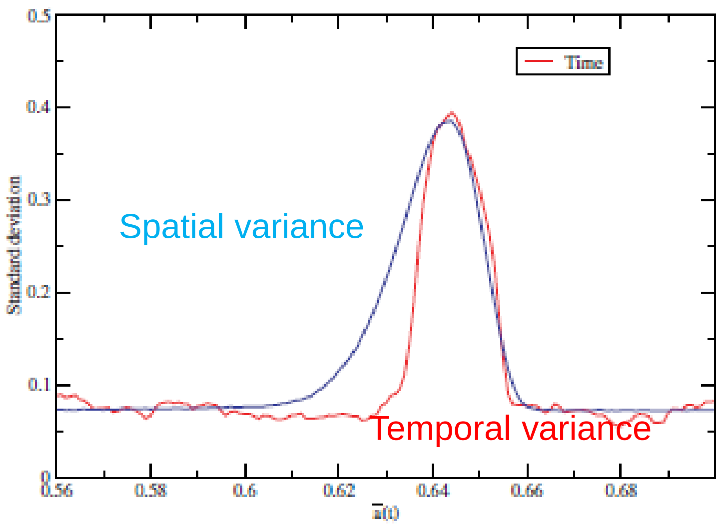
$$\int d\mathbf{x} \int d\mathbf{x}' \left[\langle m(\mathbf{x})m(\mathbf{x}')\rangle - \langle m(\mathbf{x})\rangle \langle m(\mathbf{x}')\rangle\right].$$



Implying divergence of spatial variance and of the correlation lenght of the magnetization

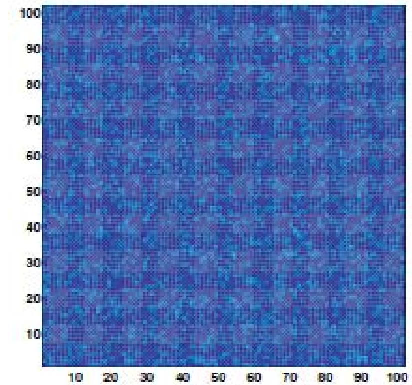
The reason is that this type of transition is towards long-range order (together with the definition of connected correlations)

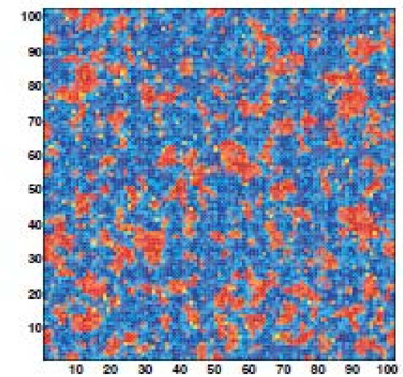
Divergence of spatial variance/correlation has been proposed as early warning indicator of critical transitions in some ecological systems

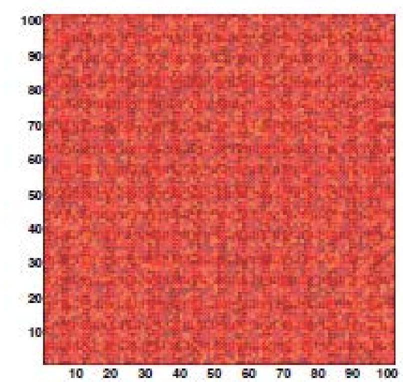


Donangelo et al. IJBC 20, 315 (2010)
Dakos et al. Theor. Ecol. *3*, 163 (2009)

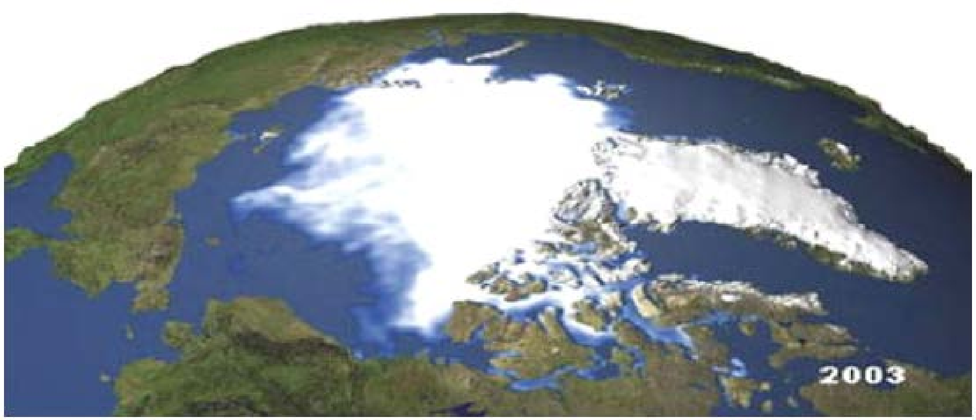
This works here because the type of **transition is between ordered homogeneous states**, so that maximum variance occurs during the transition. There are also **enhanced spatial correlations**







Because of inhomogeneities and boundary conditions**, it is unlikely that this will be pertinent to climatic transitions** (it could be useful for vegetation transitions)



Q: Why spatial correlations are enhanced near a critical transition?

$$\partial_t A(x,t) = F(A,\nabla)$$

Linearization with respect to a (x-dependent) steady state

$$\partial_t \delta A(x,t) = L(\nabla)\delta A(x,t)$$

$$\delta A(x,t) = \sum_i a_i e^{-\lambda_i t} v_i(x) \;,\; L v_i(x) = -\lambda_i v_i(x)$$

Fluctuating deviations with respect to the steady state in the eigenbasis of the linearization close to that state (discretizing x):

$$\varphi_x(t) = \sum_{i=1}^{N} f_i\,(t)\, {v^i}_x$$

Close to the bifurcation, one eigenmode dominates, the bifurcating one (critical slowing down). The others decay much faster:

$$\varphi_x(t) \approx f_1\,(t)\,{v^1}_x \; + \ldots \quad \text{and also} \quad \varphi_y(t) \approx f_1\,(t)\,{v^1}_y \; + \ldots$$

Then

$$\varphi_x(t) \approx \frac{{v^1}_y}{{v^1}_x}\,\varphi_y(t)$$

Perfect coherence:
Pearson correlation $c_{xy} = \pm 1$

This mode $v_1(x)$ will be also be easily excited by noise and detected, for example, by EOFs

Q: How to detect the increased spatial coherence when approaching a bifurcation?

A1: If the linear operator L is of diffusive type, and boundary conditions are not too disturbing, the steady state and $v_1(x)$ will be rather homogeneous, and standard spatial correlation function and correlation length will be adequate to detect when it becomes dominant.

But if advection is dominant (strange eigenmodes) and/or complex geometries …

- A2: **Network idea: use network links to represent the increased coherence.**

- Critical slowing-down
  - Slower recovery from perturbations
  - Increased autocorrelation
  - Increased variance
- Skewness, flickering, potential recovery
- Spatial indicators
  - Increased spatial variance
  - Increased correlation length
- **Network indicators**
  - Degree, clustering, …
  - Percolation-based methods

Dakos et al. *Slowing down as an early warning singal for abrupt climate change,* PNAS 105, 14308 (2008)

Thompson and Sieber, *Predicting Climate tipping as a noisy bifurcation: A review,* International Journal of Bifurcation and Chaos 21, 399 (2011).

Lenton, *Early warning of climate tipping points.* Nature Clim. Change 1, 201 (2008)

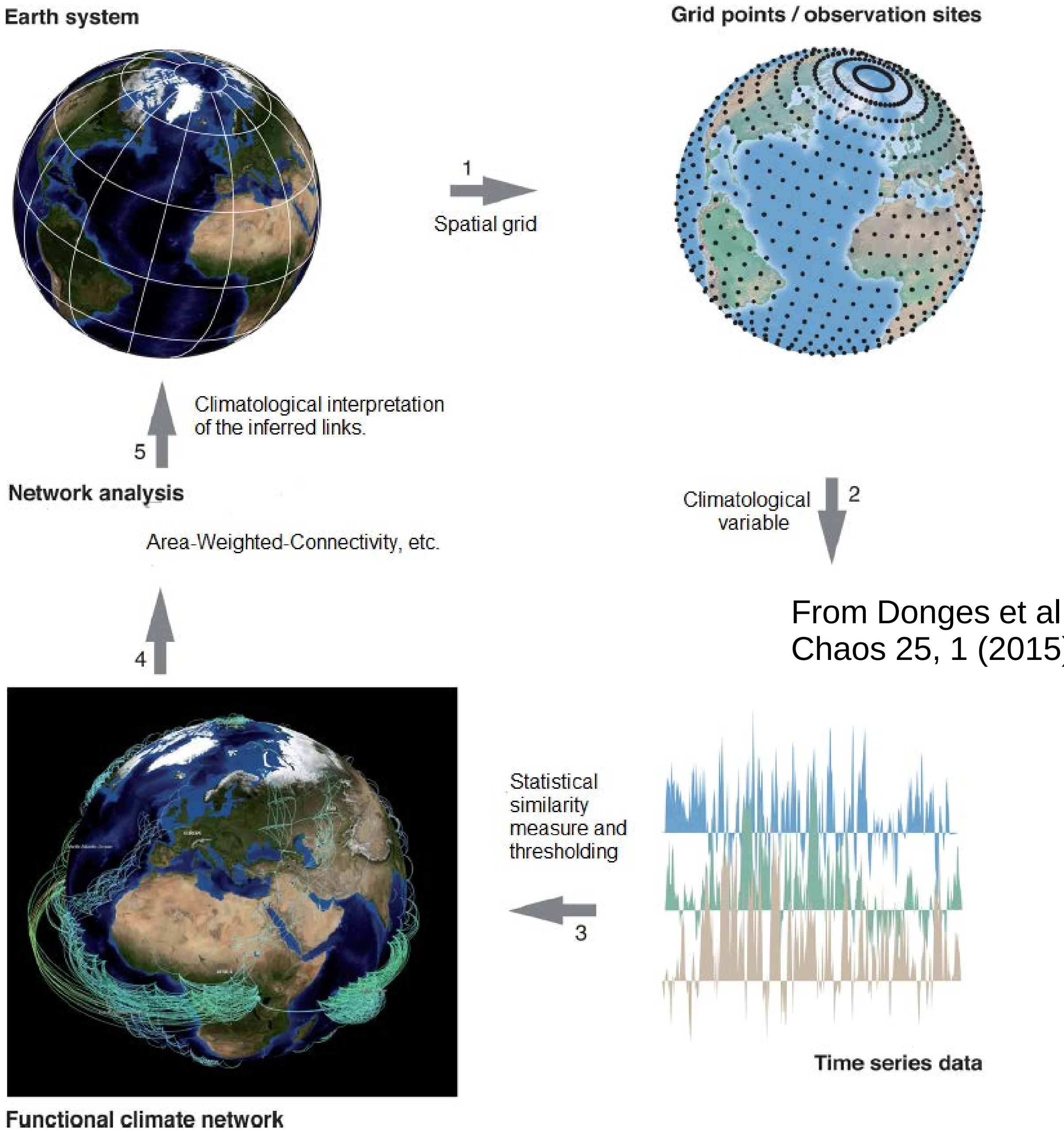
Dijkstra, Hernandez-Garcia, Masoller, Barreiro. *Networks in Climate.* Cambridge (2019).

van der Mheen et al. *Interaction network based early warning indicators for the Atlantic Meridional Overturning Circulation collapse.* Geophys. Res. Lett., 40, 2714 (2013).

Rodriguez-Mendez et al. *Percolation-based precursors of transitions in extended systems*. Scientific Reports 6, 29552 (2016).

CLIMATE NETWORK CONSTRUCTION

Dijkstra, Hernandez-Garcia, Masoller, Barreiro. *Networks in Climate.* Cambridge (2019).



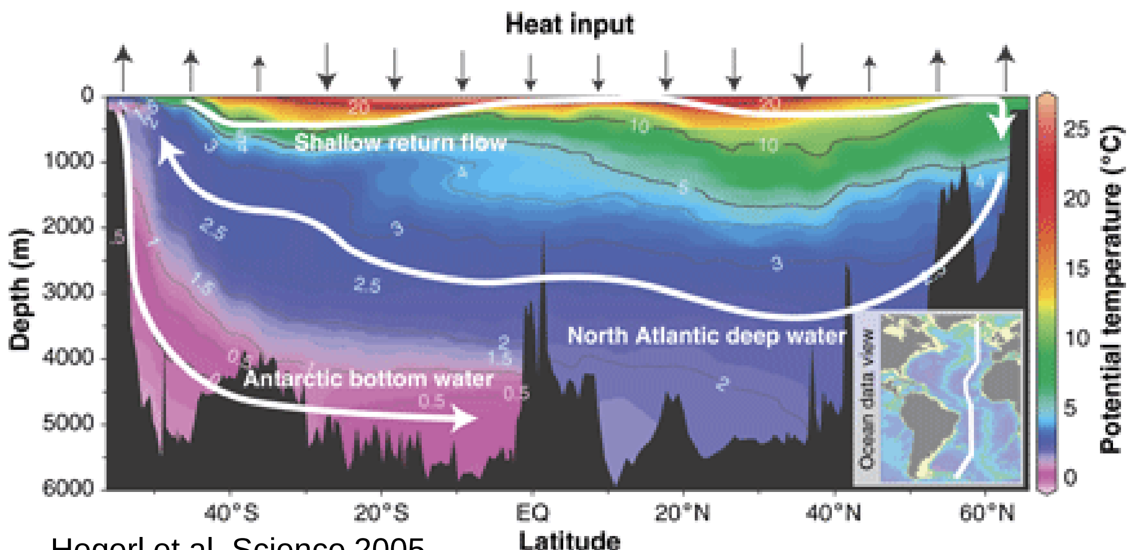
From Donges et al. Chaos 25, 1 (2015)

If links constructed from correlations, they measure statistical coherence

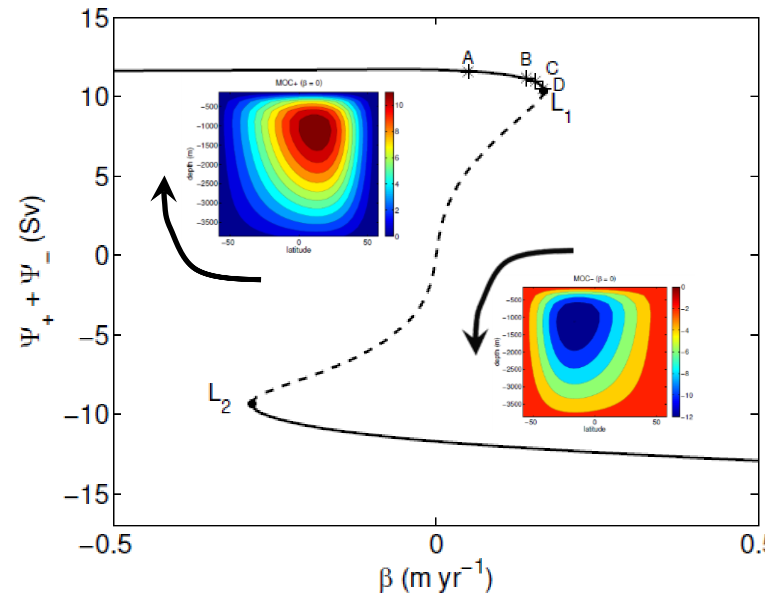
Degree: number of links of a node
Clustering: number of triangles adjacent to a node

Two-dimensional version (meridional-depth) of the Thermohaline Circulation Model (de Niet et al 2007). Fluctuating (noise) freshwater input + anomalous excess β in the North of the domain.

Model of the Atlantic overturning circulation
van der Mheen, Dijkstra, Gozolchiani, den Toom, Feng, Kurths, Hernandez-Garcia,GRL **40**, 2714 (2013)



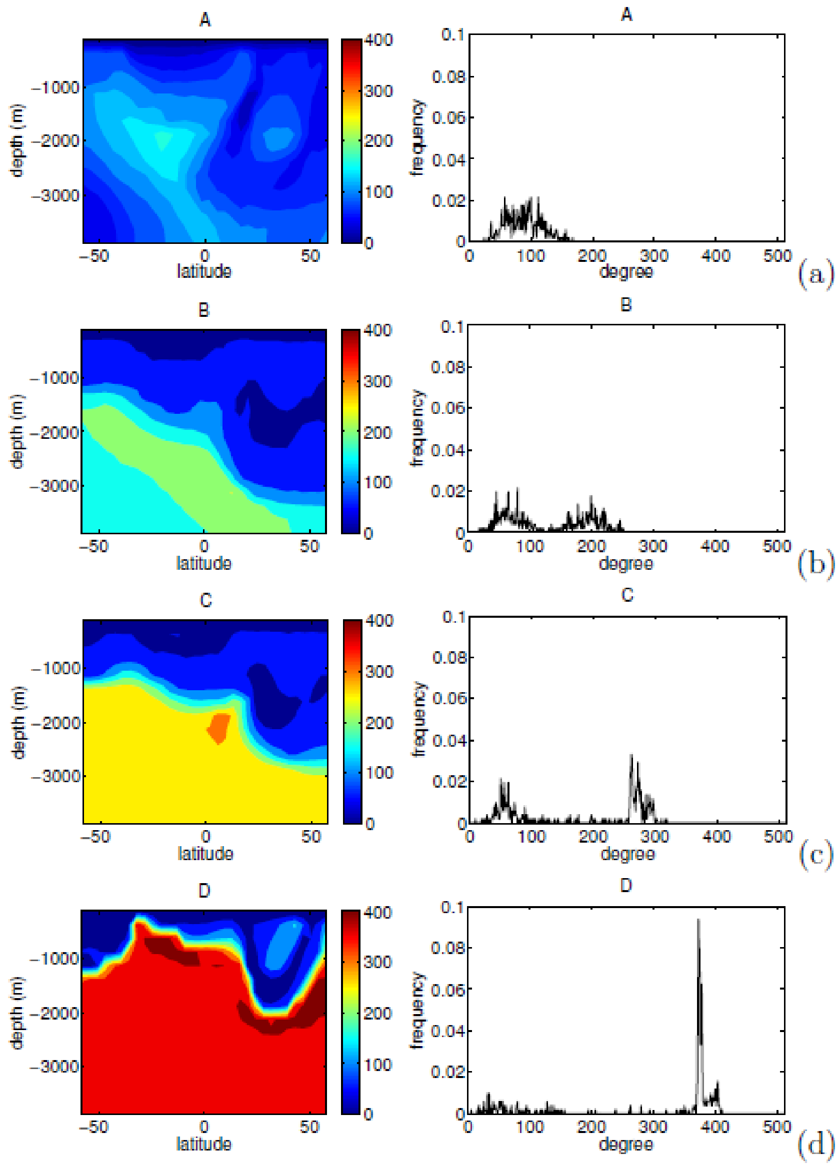
Hegerl et al. Science 2005



Network constructed by linking points where zero-lag (Pearson) correlations between temperature fluctuations exceed 0.7

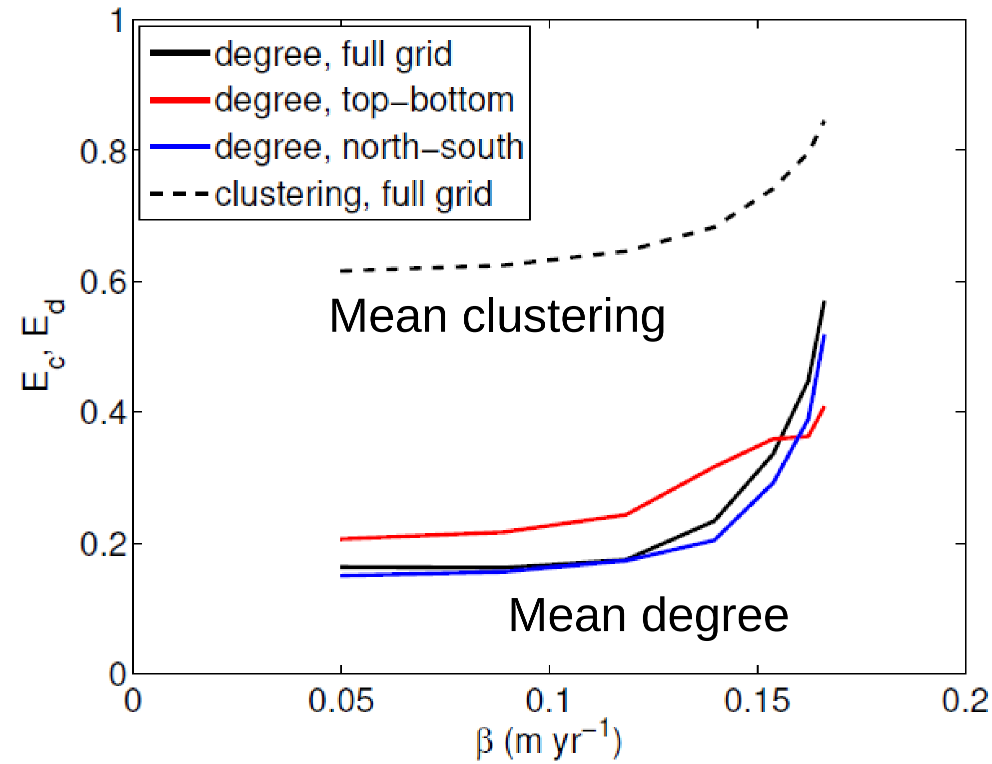
Further examples: Feng,Viebahn, Dijkstra, GRL (2014) MOC from the FAMOUS model
Tirabassi, Viebahn, Dakos, Dijkstra, Masoller, Rietkerk, Ecol. Complexity (2014): desertification transition

Degree fields and degree distributions



Approaching the bifurcation

Mean clustering and mean normalized degree



Steady-state simulations

Changing-forcing simulations

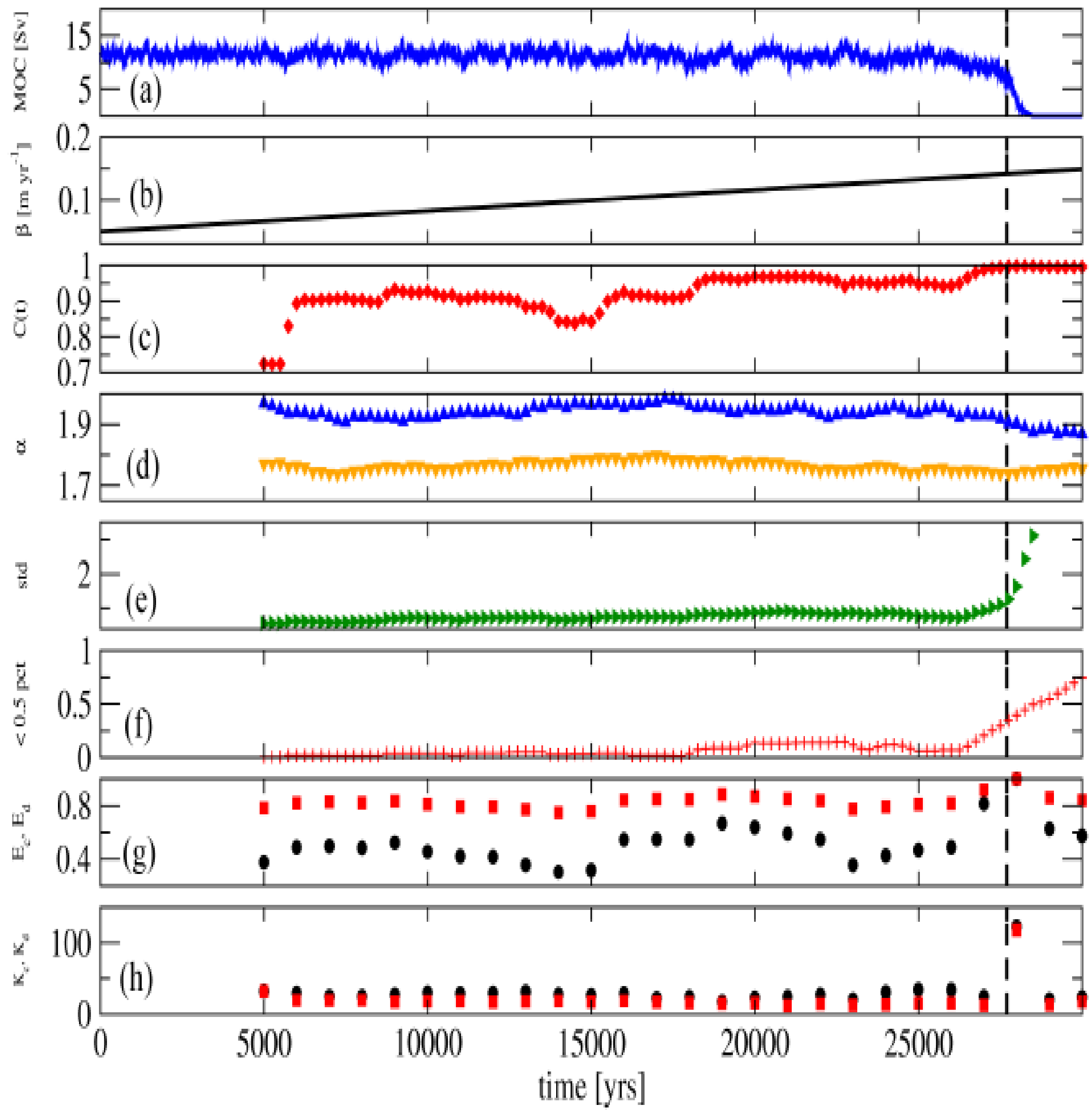
ACF

DFA

variance

Mean clustering and normalized **degree**

Kurtosis of degree and clustering distributions



- Critical slowing-down
  - Slower recovery from perturbations
  - Increased autocorrelation
  - Increased variance
- Skewness, flickering, potential recovery
- Spatial indicators
  - Increased spatial variance
  - Increased correlation length
- Network indicators
  - Degree, clustering, …
  - **Percolation-based methods**

Dakos et al. *Slowing down as an early warning singal for abrupt climate change,* PNAS 105, 14308 (2008)

Thompson and Sieber, *Predicting Climate tipping as a noisy bifurcation: A review,* International Journal of Bifurcation and Chaos 21, 399 (2011).

Lenton, *Early warning of climate tipping points.* Nature Clim. Change 1, 201 (2008)

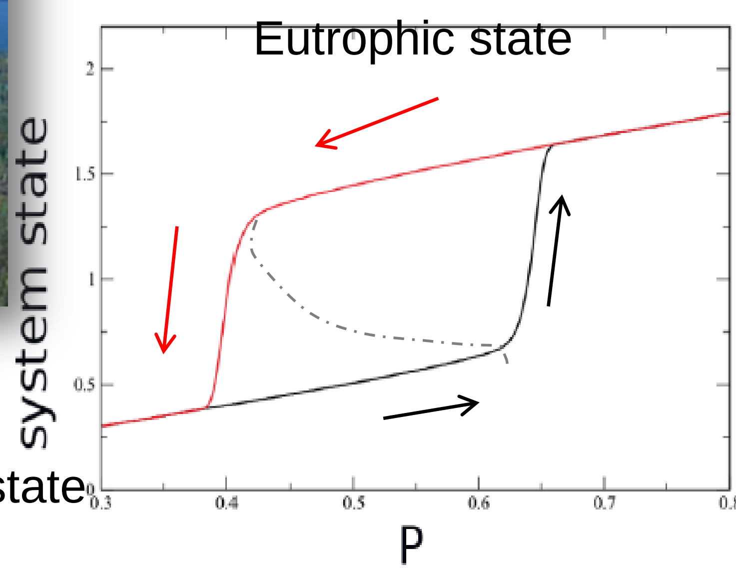
Dijkstra, Hernandez-Garcia, Masoller, Barreiro. *Networks in Climate.* Cambridge (2019).

van der Mheen et al. *Interaction network based early warning indicators for the Atlantic Meridional Overturning Circulation collapse.* Geophys. Res. Lett., 40, 2714 (2013).

Rodriguez-Mendez et al. *Percolation-based precursors of transitions in extended systems*. Scientific Reports 6, 29552 (2016).

Lake eutrophication model (2d)

$$\frac{\partial \phi(x,y;t)}{\partial t} = p - b\phi(x,y;t) + \frac{\phi^8}{\phi^8+1} + \epsilon \nabla^2 \phi(x,y;t) + \eta(x,y;t)$$

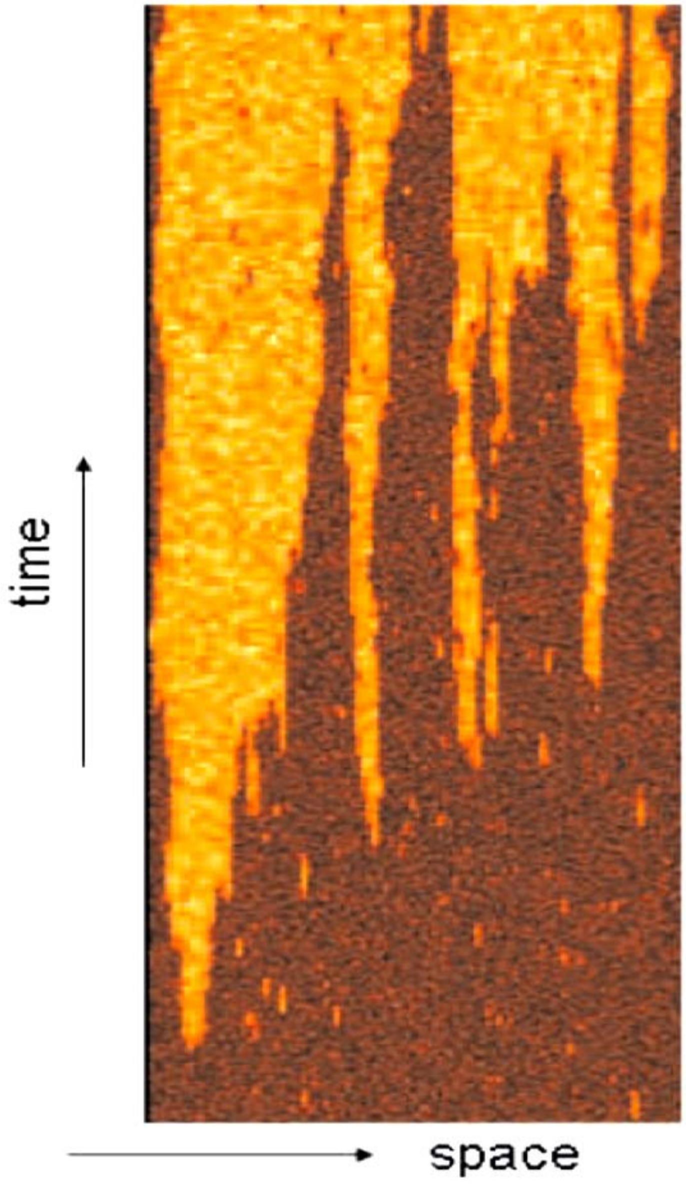


= phosphorus concentration

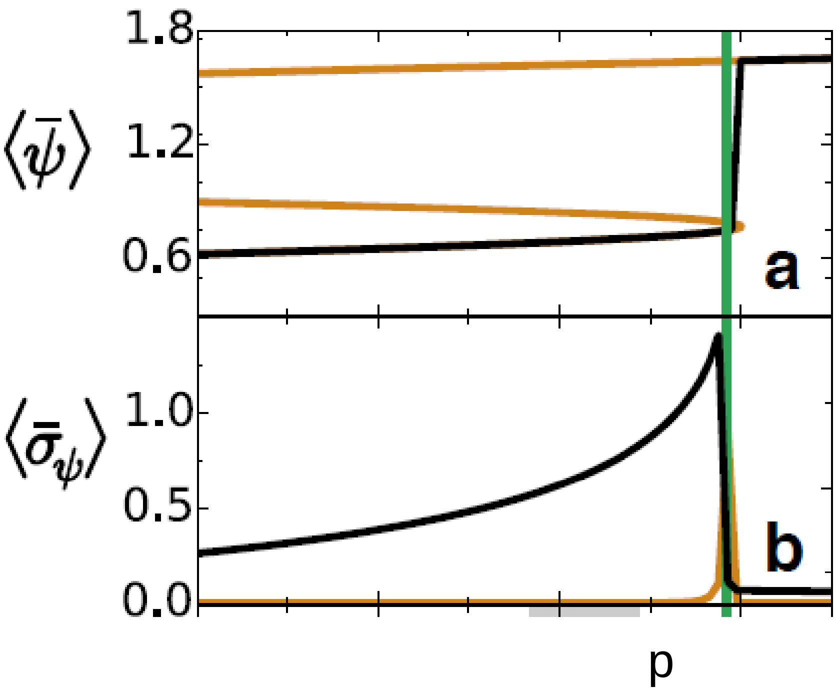
- $\phi(x,y;t) \rightarrow$ state of the system.
- $s(x,y;t)$ Phosphorous decay
- $\frac{\phi^8}{\phi^8+1}$ Recycling from the sediment.
- $\eta(x;t)$ is a white noise.
- p is the control parameter (phosphorous input).

Carpenter, Ludwig, Brock, Management of eutrophication for lakes subject to potentially irreversible change, Ecological Applications 9, 751–771 (1999).

The jump process (example in 1d)

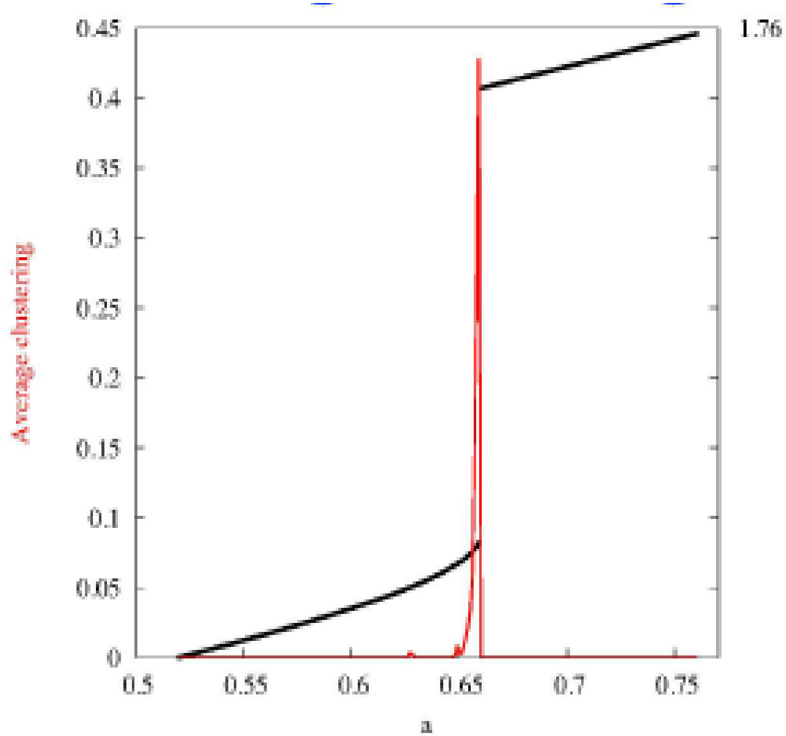


variance

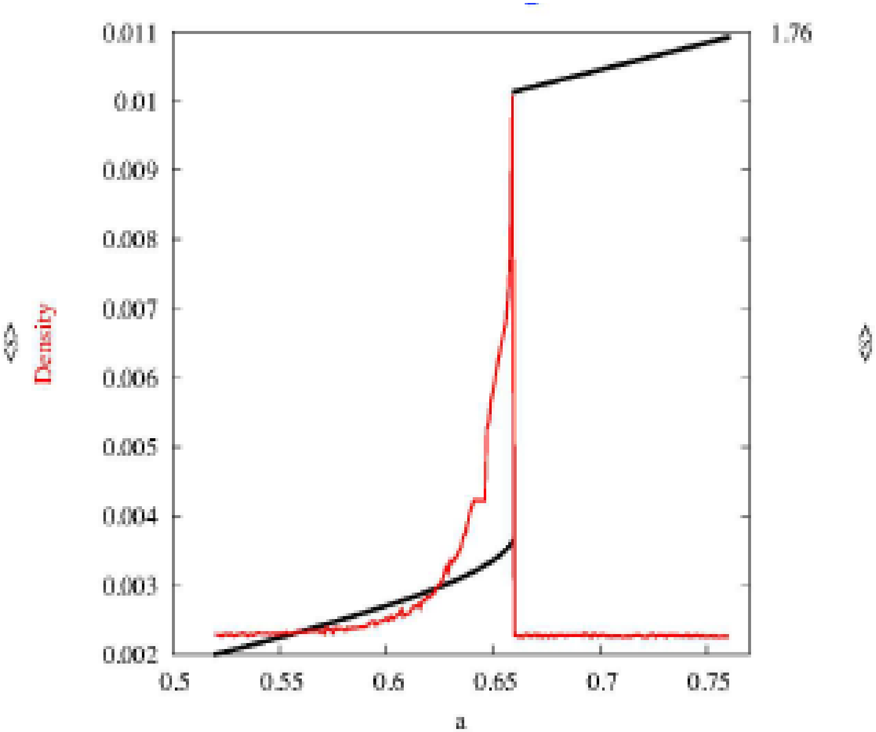


Identifies. **Not really anticipates.**

Average clustering

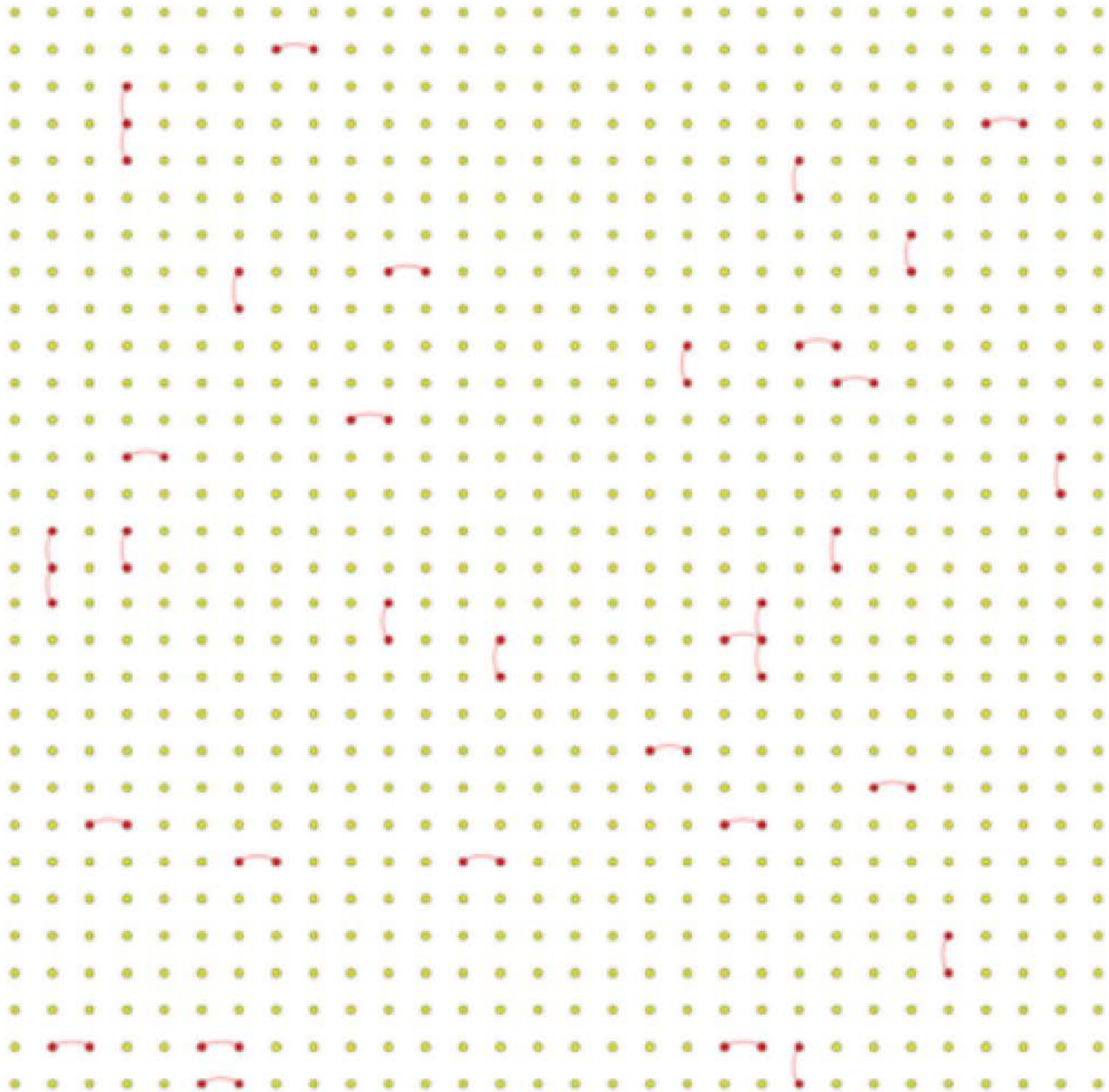


Link density

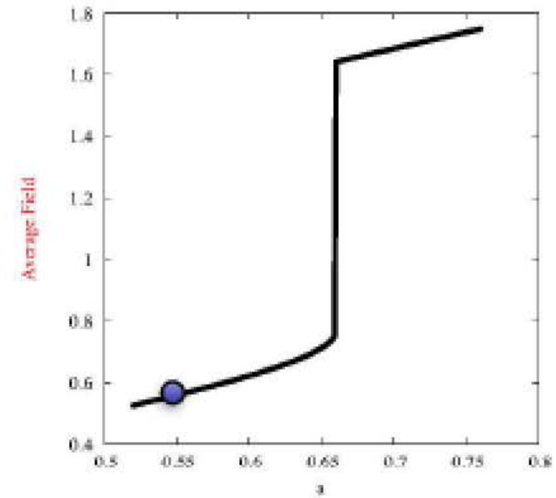


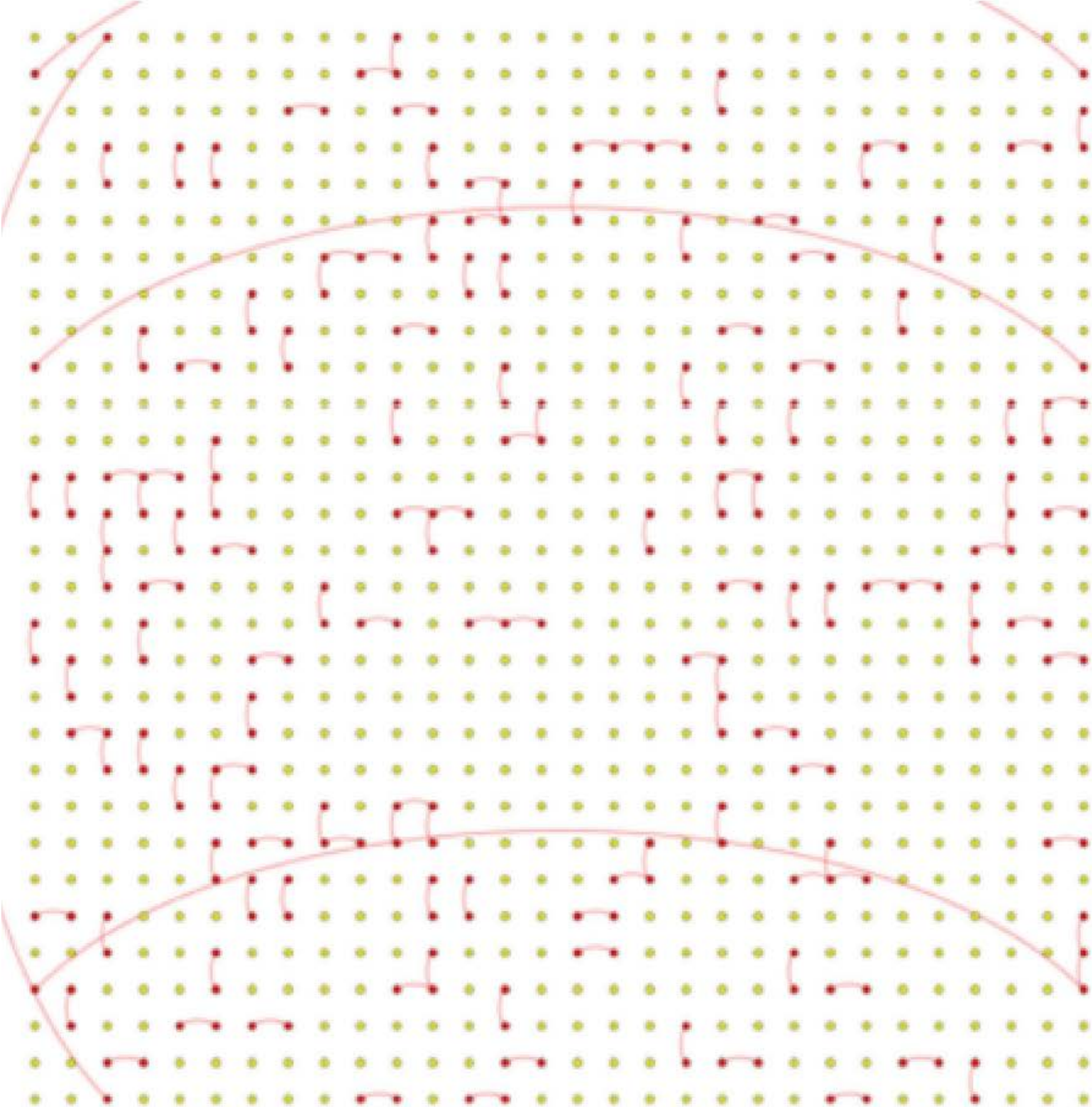
Parameter p

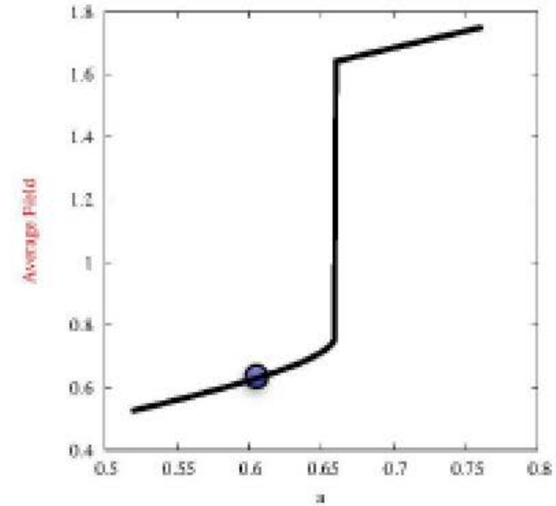
Still: Identify. Not really anticipate.

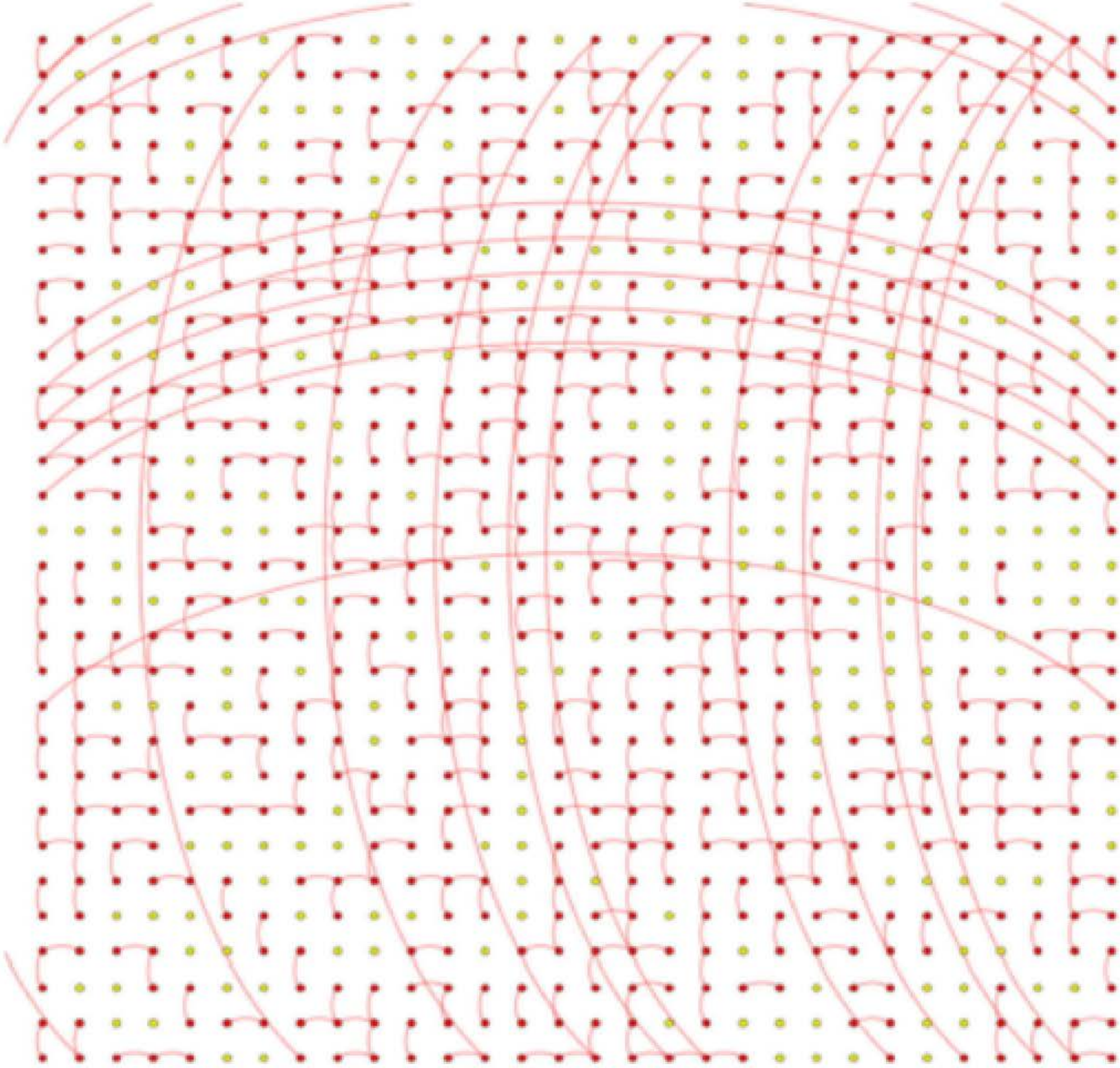


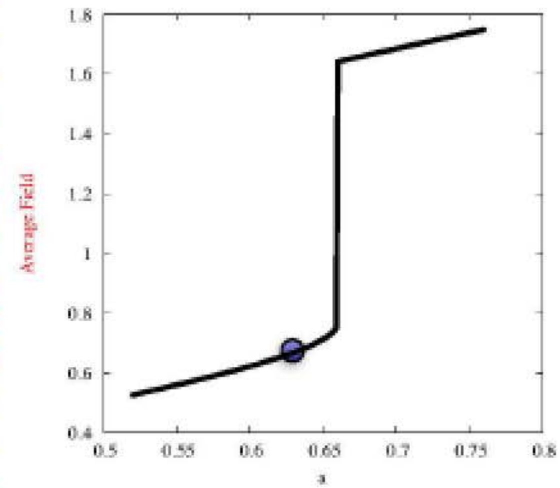
Correlation network for the lake eutrophication model

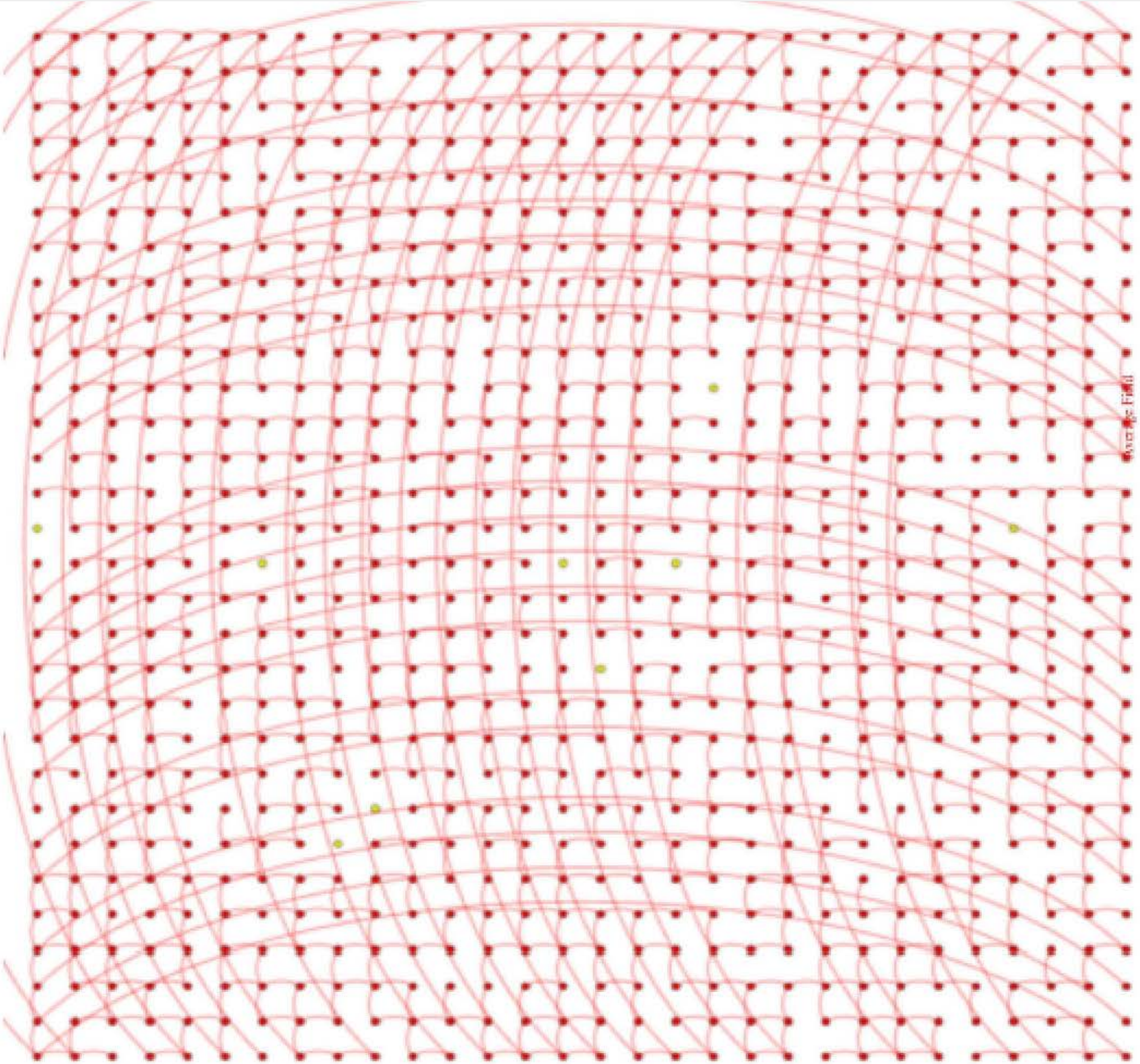


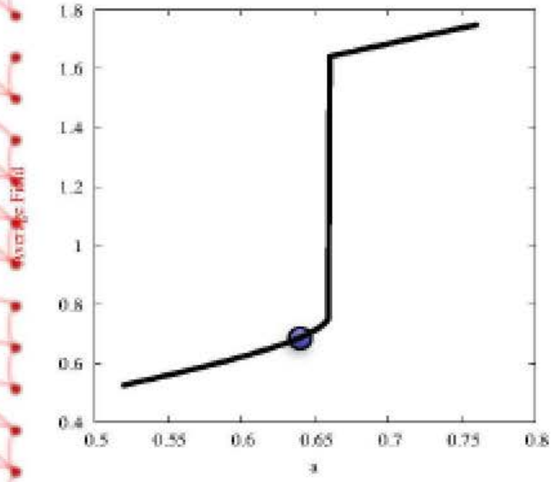


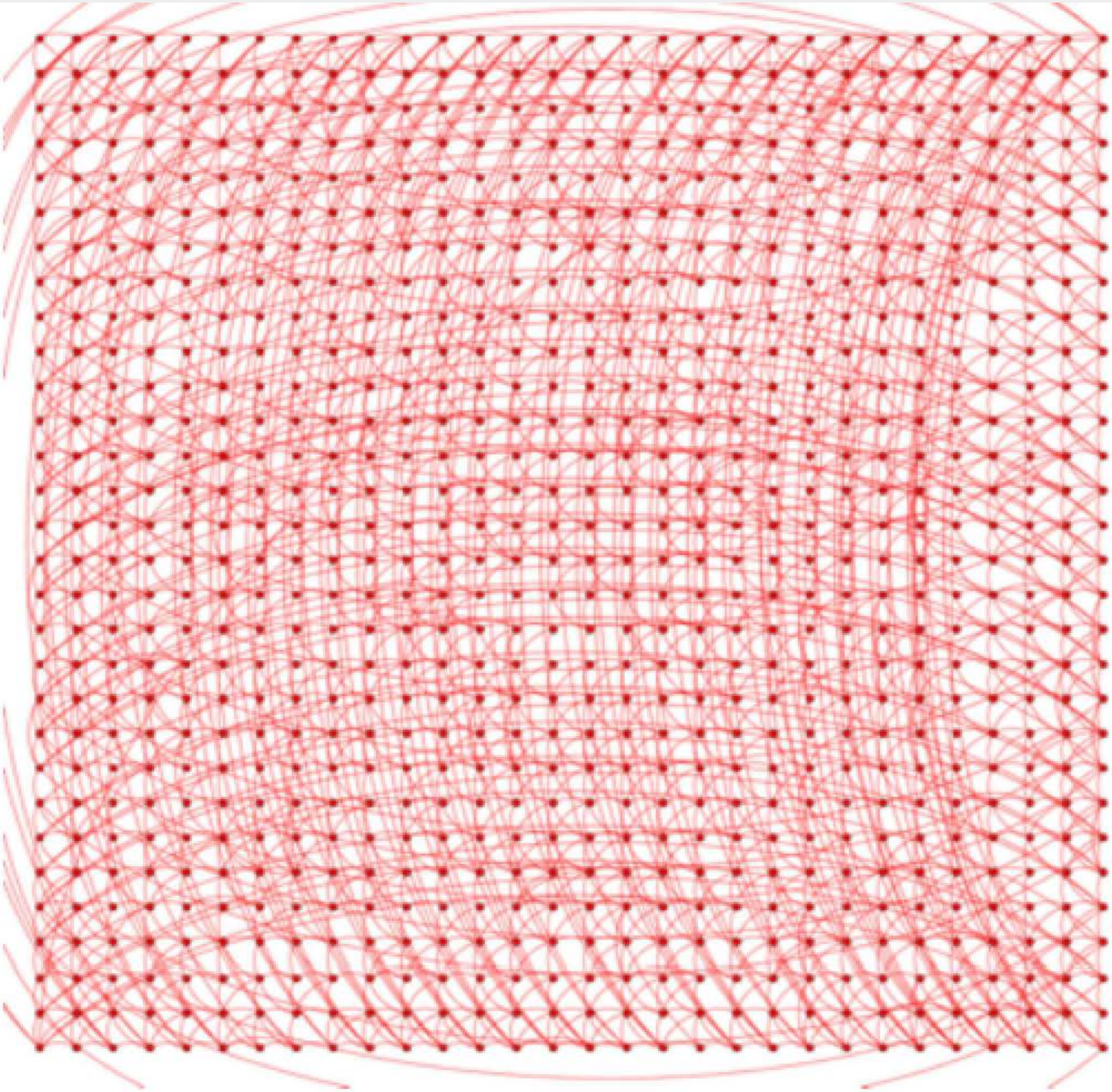

Average Field
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.5
0.55
0.6
0.65
0.7
0.75
0.8
a

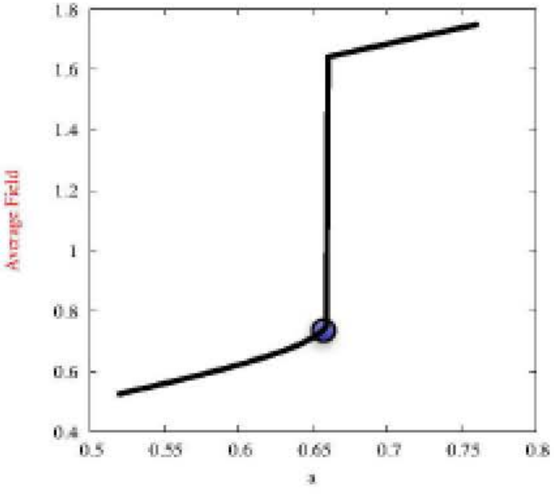


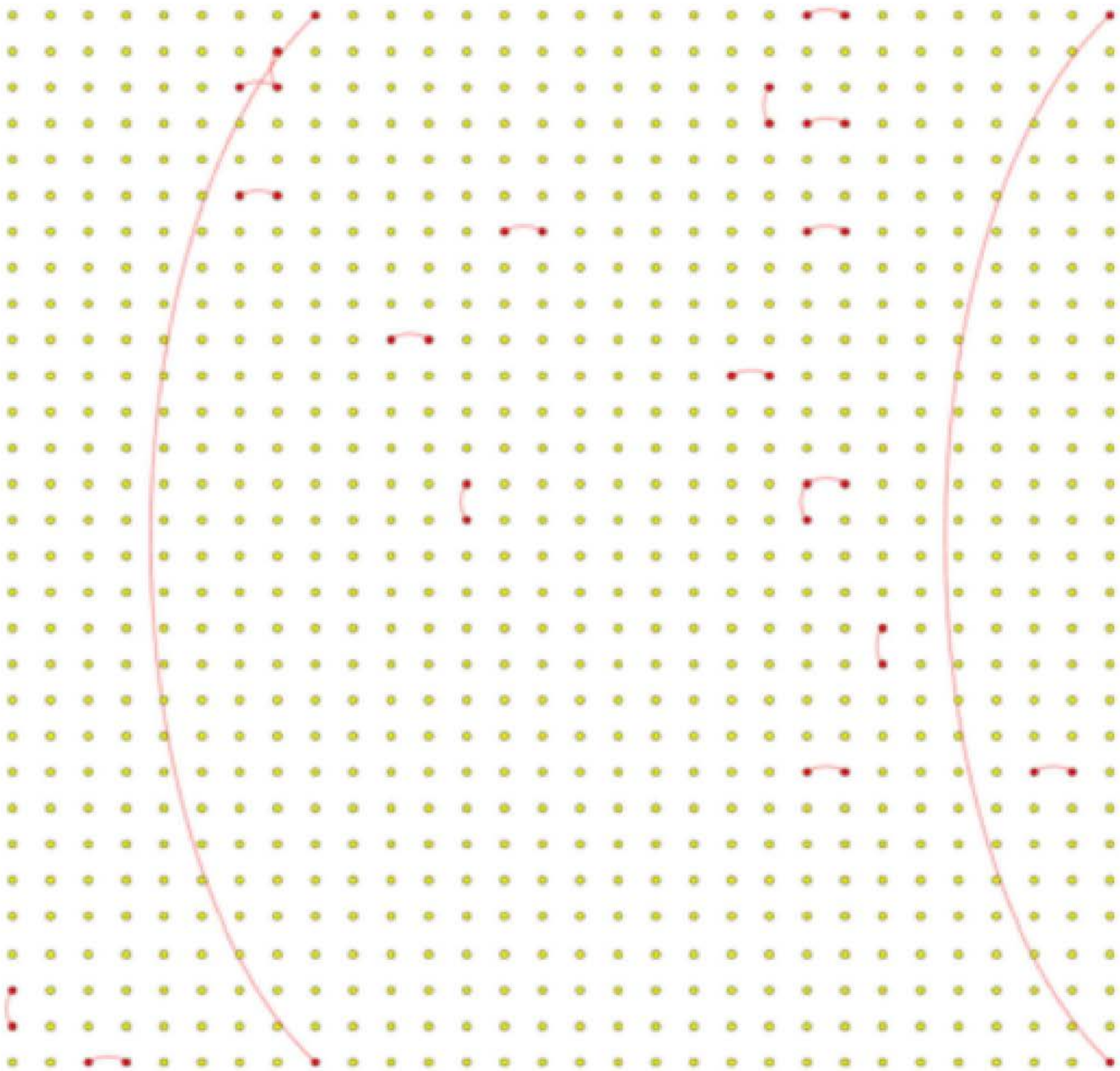

Average Field
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.5
0.55
0.6
0.65
0.7
0.75
0.8
a

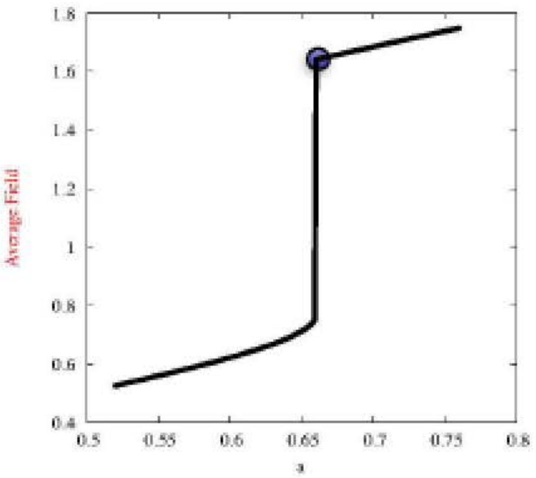



1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.5
0.55
0.6
0.65
0.7
0.75
0.8
a

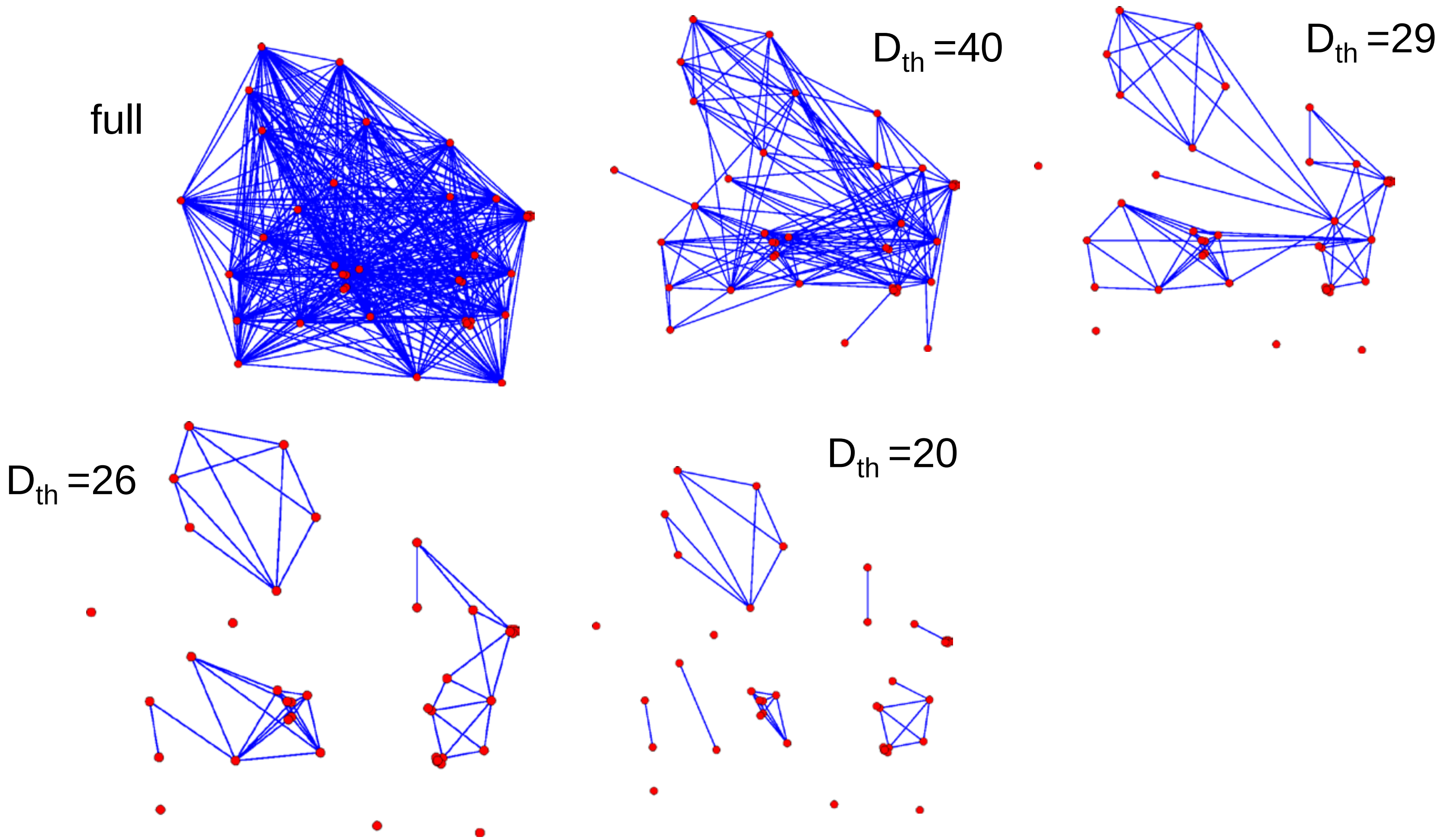



Average Field
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.5
0.55
0.6
0.65
0.7
0.75
0.8
a




Average Field
a
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0.5
0.55
0.6
0.65
0.7
0.75
0.8

Percolation in networks:



- Link density increases until …

  … total connectivity at the bifurcation point

- But at some intermediate connectivity …

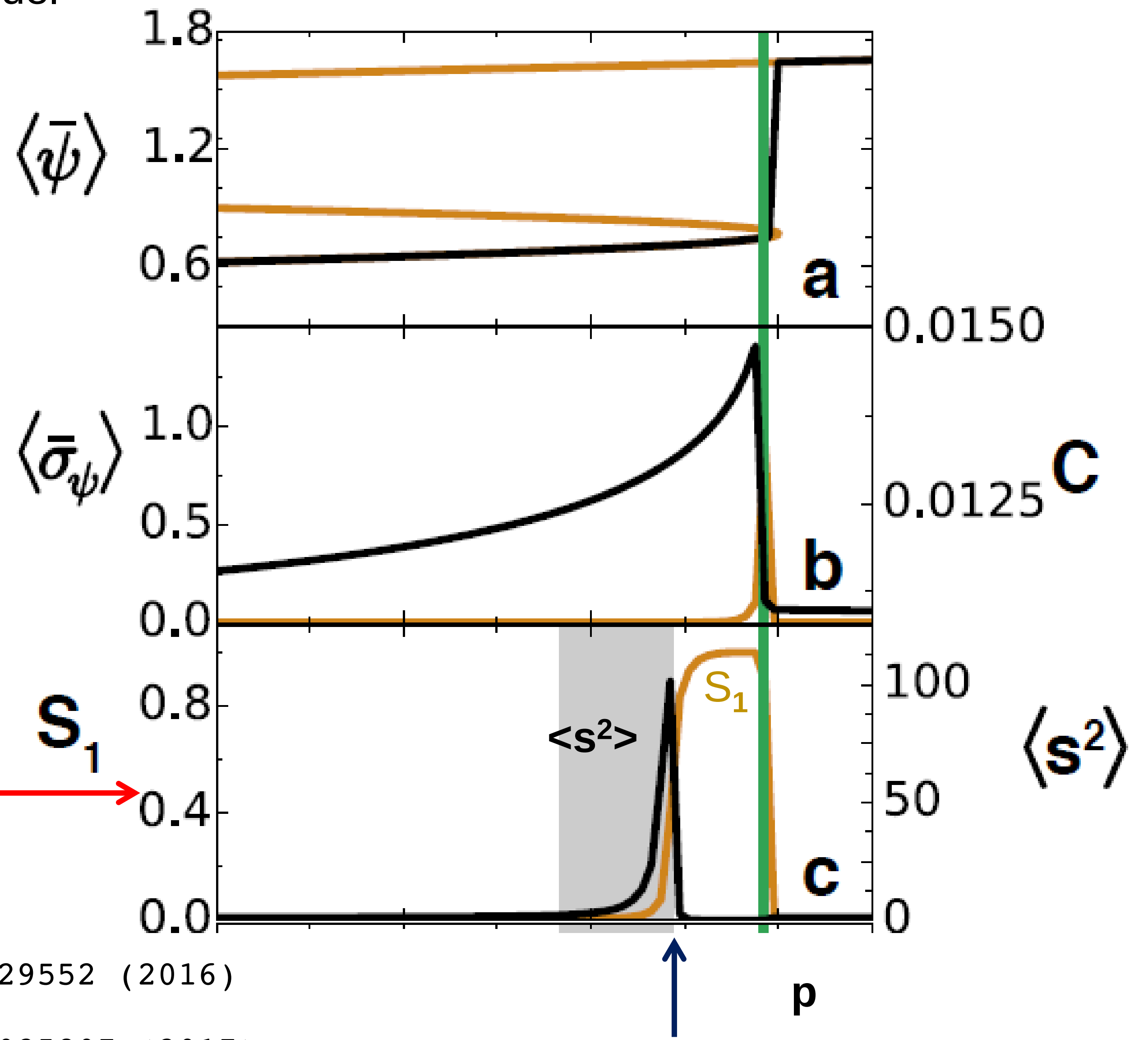
  … a **PERCOLATION TRANSITION** should occur

Percolation transition occurs BEFORE the dynamical transition
Thus, indicators of percolation **ANTICIPATE** the bifurcation: **early warning**

Standard percolation indicators: **S_1:** size (proportion of nodes) of the largest cluster
$\langle s^2 \rangle$: average size of the leftover clusters (related to the size of second largest one, or to th evariance of the cluster distribution

Percolation-based precursors of transitions in extended systems
Rodriguez-Mendez, Eguiluz, Hernandez-Garcia, Ramasco
Scientific Reports **6**, 29552 (2016)

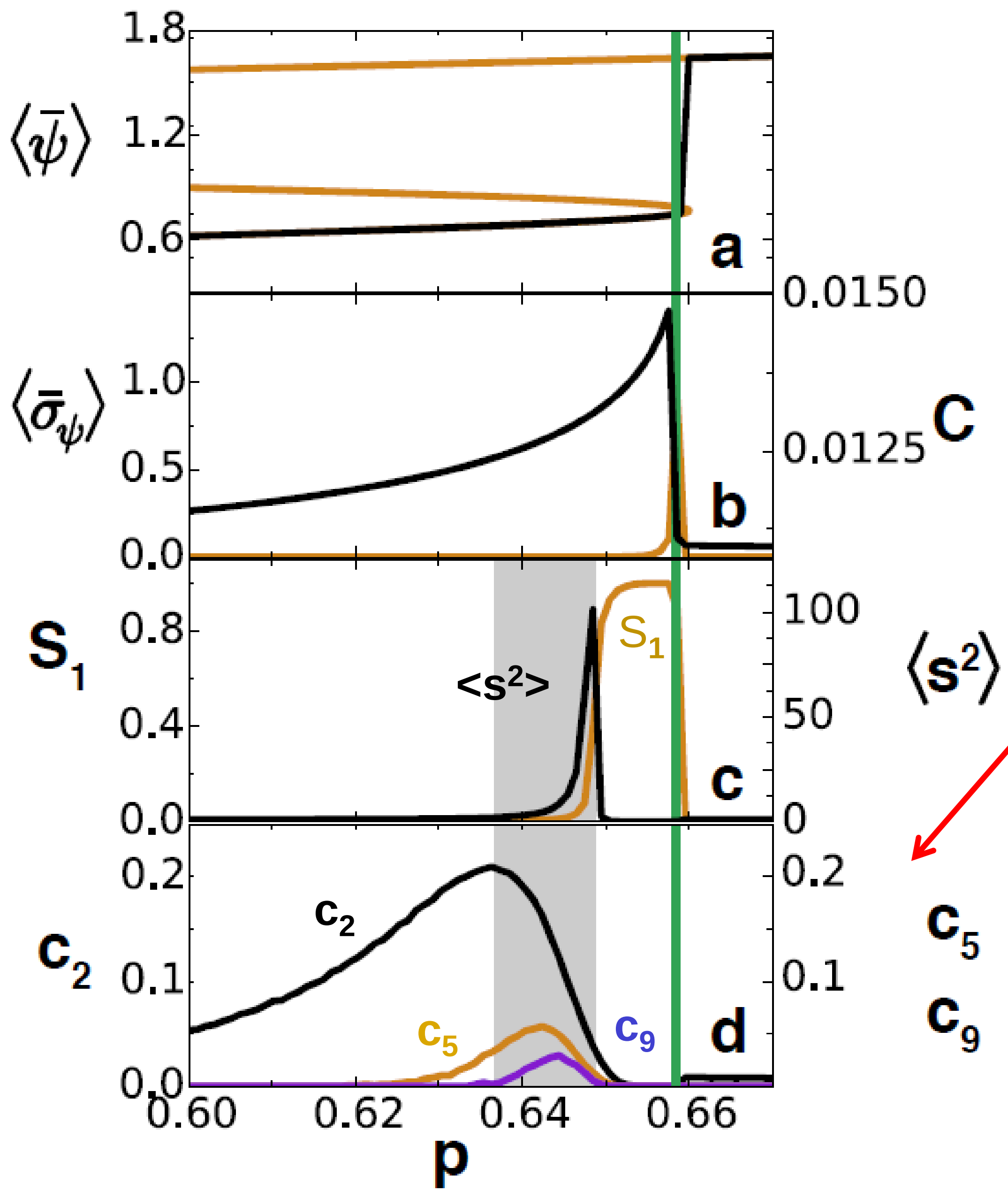
Lake eutrophication model



Percolation indicators:

Rodriguez-Mendez et al
Scientific Reports **6**, 29552 (2016)
also proposed in
Meng et al, Chaos 27, 035807 (2017)

Lake eutrophication model

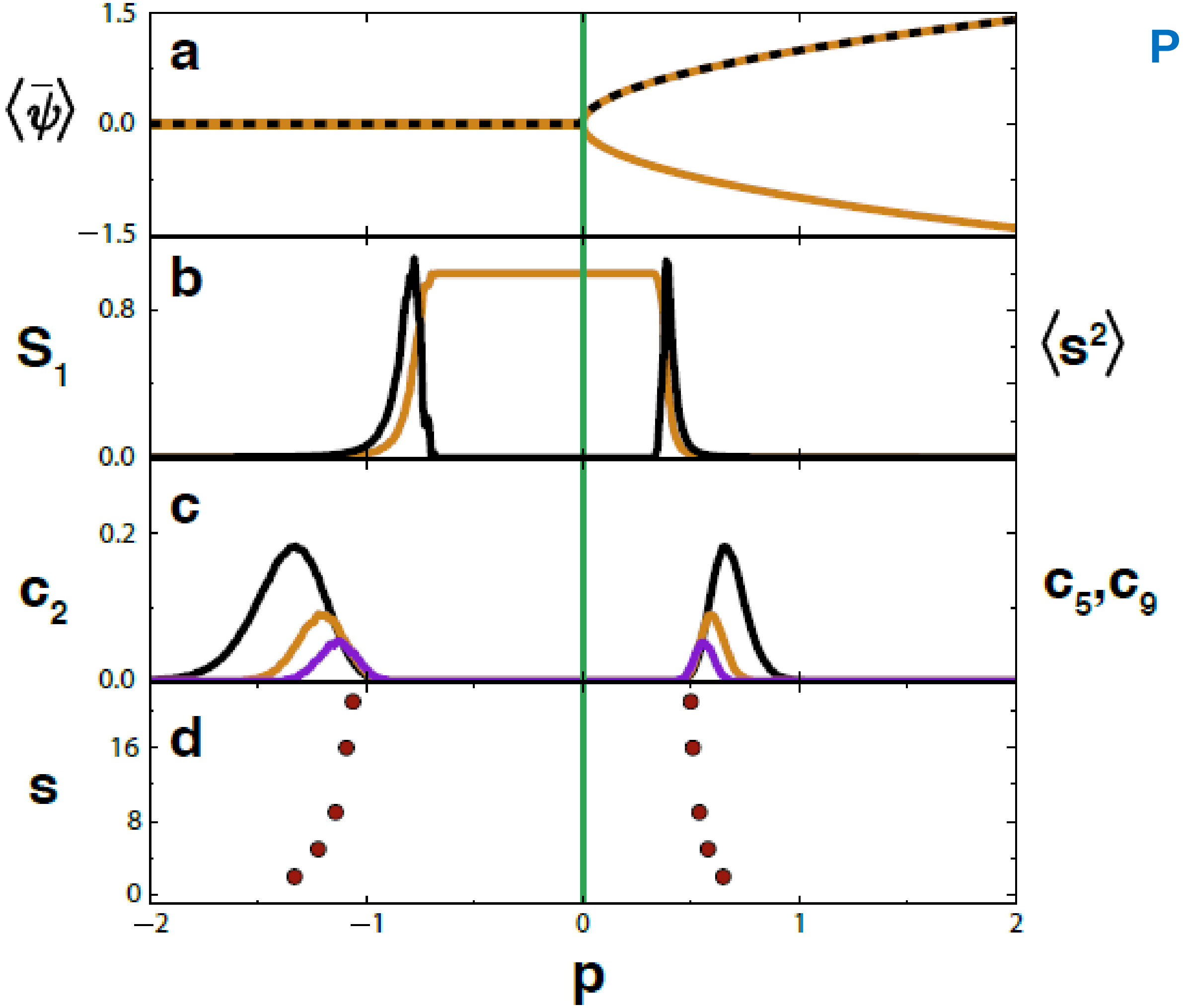


c_s= fraction of nodes in clusters of size s

Give early warning of the percolation transition which itself anticipates the bifurcation

Ginzburg-Landau model (1d)

$$\frac{\partial \phi(x,t)}{\partial t} = p\phi(x,t) - \phi(x,t)^3 + \epsilon\nabla^2\phi(x,t) + \eta(x;t).$$

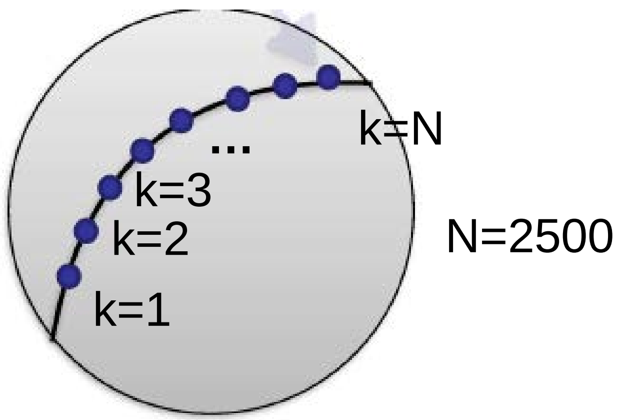


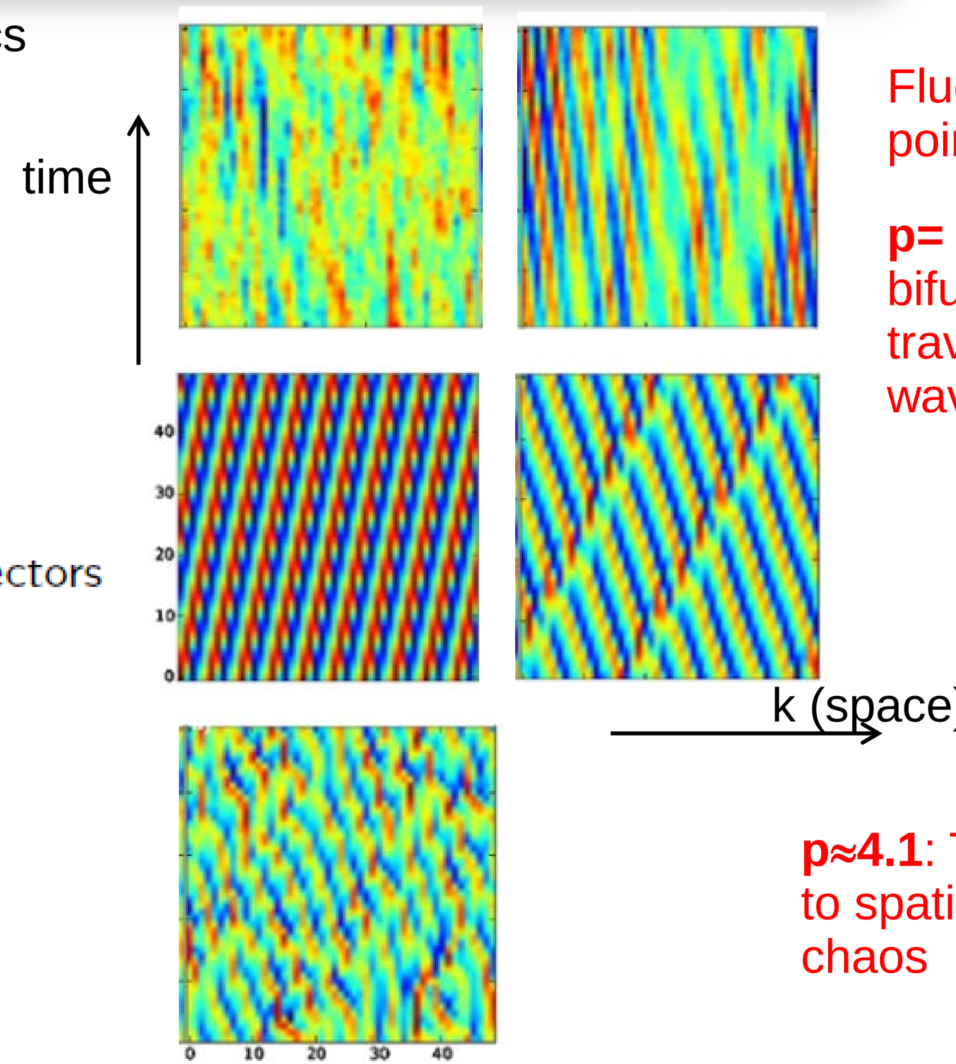
Pitchfork bifurcation

Lorenz96 model

$$\frac{d\phi_k(t)}{dt} = (\phi_{k+1}(t) - \phi_{k-2}(t))\phi_{k-1}(t) - \phi_k(t) + \eta_k(t) + p$$

Toy model for atmospheric dynamics



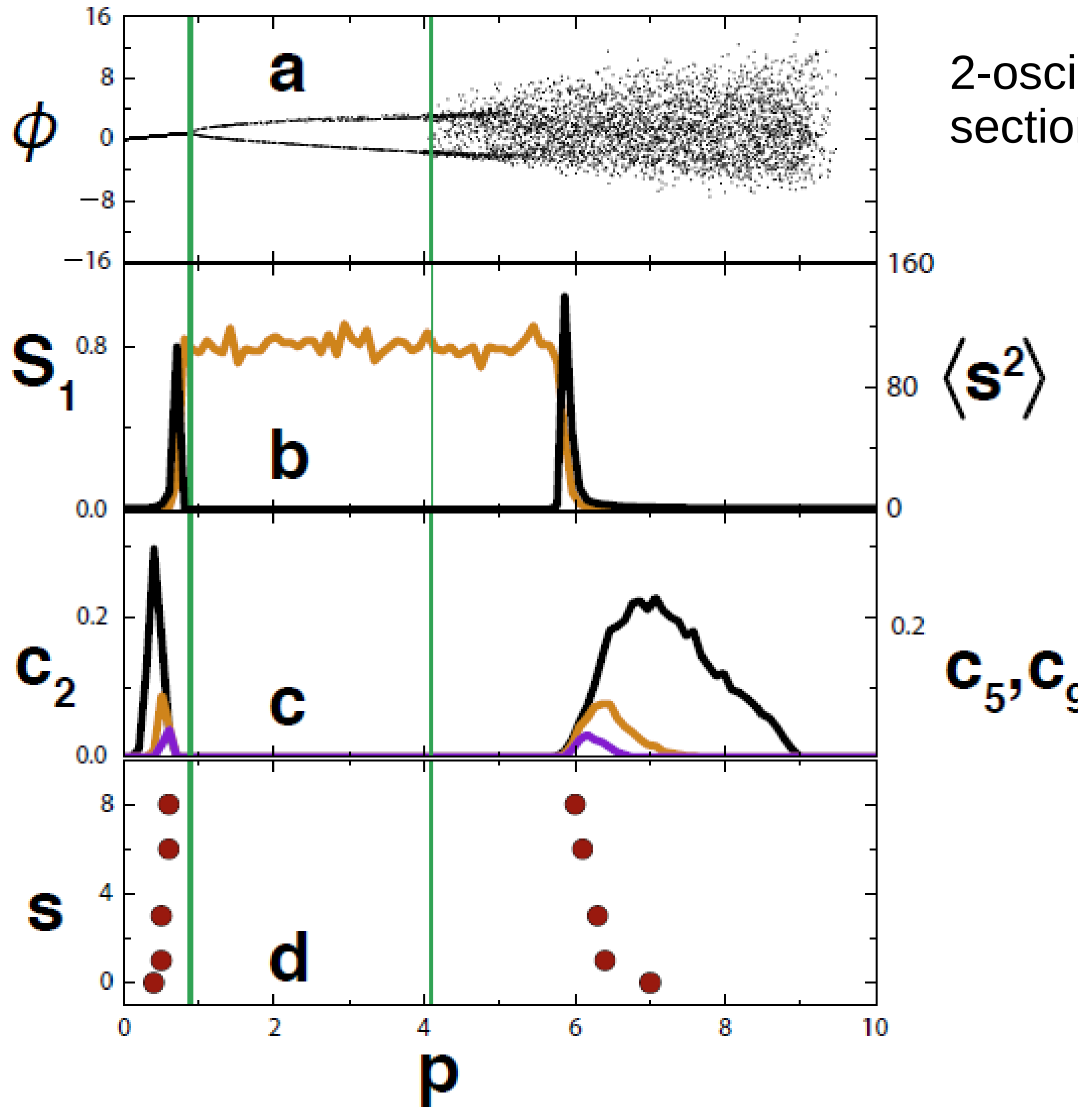


Fluctuating fix point

p= 3/8: Hopf bifurcation to travelling waves

p≈4.1: Transition to spatiotemporal chaos

- ϕ_k some atmospheric quantity in K sectors of a latitude circle.
- Linear terms $\rightarrow$ internal dissipation.
- Quadratic terms $\rightarrow$ advection.
- p is the control parameter (external forcing).
- $\eta_k(t)$ is a white noise.



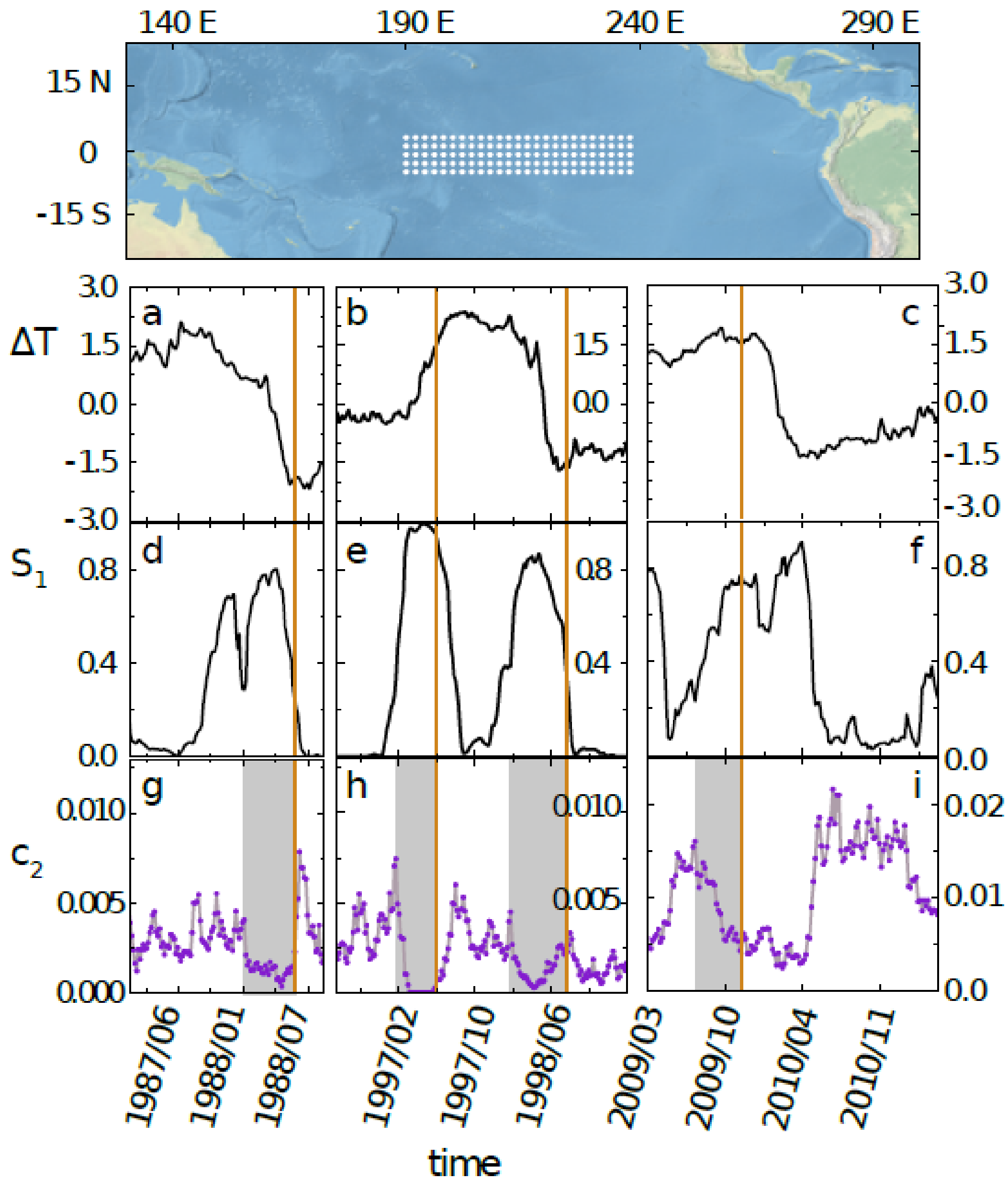
2-oscillator Poincaré section

Correlation network from El Niño3.4 index

ENSO dynamics:
System close to a Hopf bifurcation (**not clear if above or below**) forced by noise

Correlation network largest component

Fraction of nodes in clusters of size 2



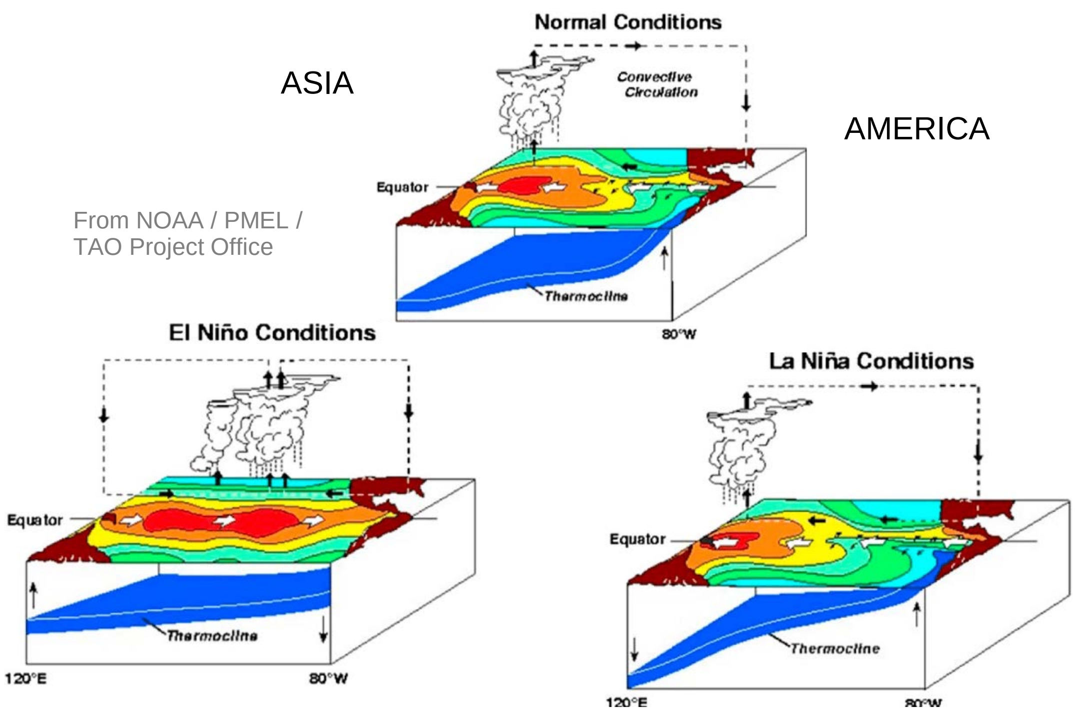
SST from ERA-interim, Δt=1 day, Δx=0.125 º , 1979-2014

Daily networks. Running window for correlations: 200 days

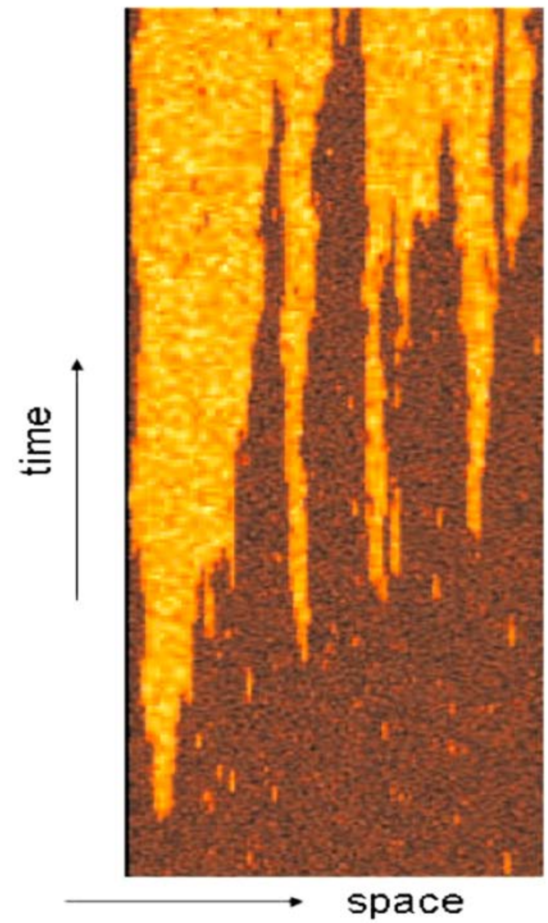
Anticipation: 240 days, 125 175 115 days

Why the approach seems to work for El Niño, if it is though to be an oscillation?

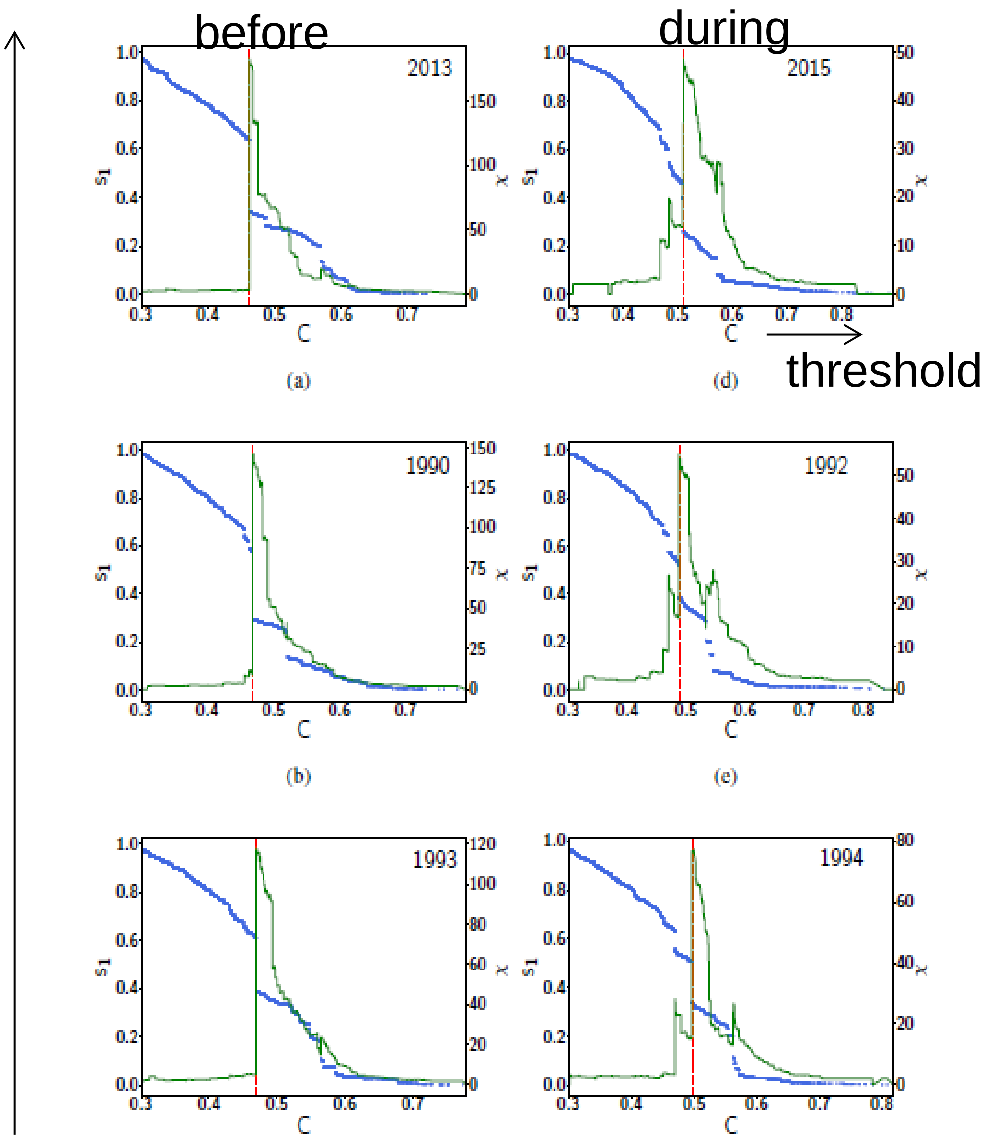
Especulative: The oscillation is between (simplifying) a cold state and a hot state (with associated thermocline depth). The transition between these states does not occur spatially at random, but with some coherence (in fact it involves wave propagation) that is detected by the correlation network



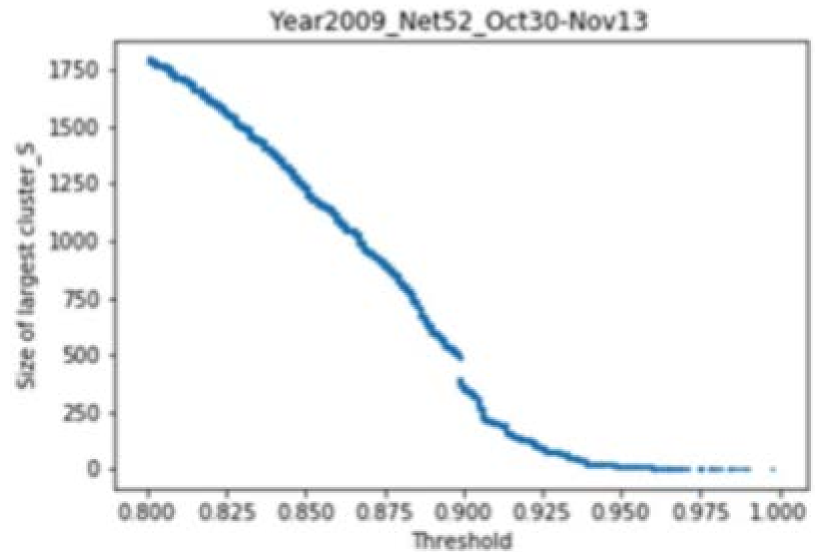
From NOAA / PMEL / TAO Project Office



Besides standard percolation, one can monitor ‘microtransitions’



Increasing intensity of El Niño event



Presence of a cyclone in the North-Indian ocean (Gupta, Kurths, Pappenberger, EGU 2020)

Sonone, Gupte, arXiv 2002.04530 (2020)
Gupte, Roy, Sonone, Indian Acad. Sci. Conf. Ser. 2:1 (2019)

Take-home message

- Climate transitions have occurred in the past and are likely to occur again. Within CAFE, extending the concept towards the subseasonal time scale would be a useful goal
- It is highly desirable to have early warning indicators which will announce us on the proximity of a close transition
- Many indicators have been proposed (most having in mind the hypothesis of an underlying bifurcation) and many work well with model transitions, but considerable improvement is needed to deal with real data
- Network approaches are a new line of research that is providing more efficient indicators

THANK YOU

for your attention

CAFE
Climate Advanced Forecasting
of sub-seasonal Extremes