On Cosmology, the Quantum Vacuum, and Zeta Functions

Emilio Elizalde

17th RUSSIAN GRAVITATIONAL CONFERENCE — INTERNATIONAL CONFERENCE ON GRAVITATION, COSMOLOGY AND ASTROPHYSICS (RUSGRAV-17)

JUNE 29 – JULY 3, 2020

SAINT PETERSBURG, RUSSIA

GOBIERNO DE ESPAÑA MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD

CSIC

IEEC

Saint Petersburg, 2.7.2020

Outline

Cosmological constant

Possible quantum vacuum contributions

How to use zeta functions

Apr 26, 1920: The Scale of the Universe

Vienna 2018: the Hubble-Lemaître Law

2019+: What is the value of the Hubble constant?

1917

- ✓ Universe: eternal
- ✓ Universe = Milky Way
- ✓ Universe: static *why?*

Einstein field equations with Λ

"Die Gröβte Eselei meines Lebens..." (Gamov)

➡ Beginning of (Theoretical) Modern Cosmology

Year 1912

The Beginning of Modern Cosmology

- ✓ Distances Henrietta S. Leavitt (Cepheids)
- ✓ Velocities Vesto M. Slipher (redshifts)
  Carl Wirtz (later)

7 April 1912: Victor Hess discovers cosmic rays

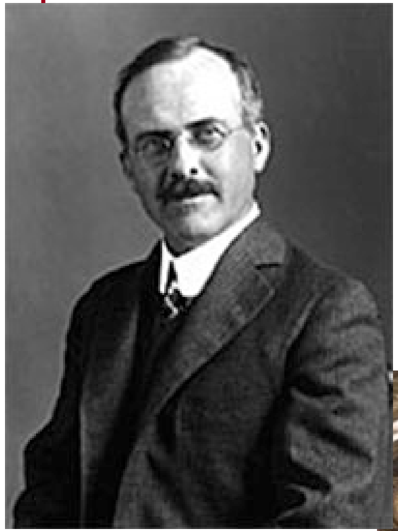
By 1917 Slipher ...

had 25 results, 4 of them blueshifts, and he gave an interpretation on the enormous receding mean velocity, of nearly 500 km/s, of these objects:

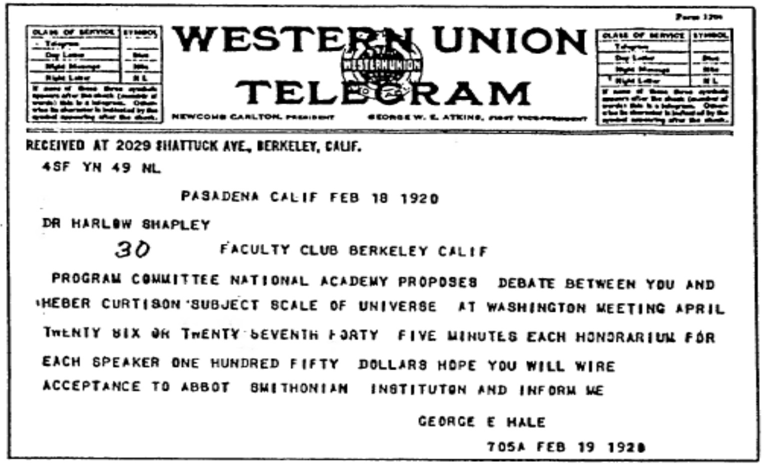
"This might suggest that the spiral nebulae are scattering but their distribution on the sky is not in accord with this since they are inclined to cluster."

And he added that:

"... our whole stellar system moves and carries us with it. It has for a long time been suggested that the spiral nebulae are stellar systems seen at great distances ... This theory, it seems to me, gains favor in the present observations."



Heber D. Curtis
(1872-1942).


WESTERN UNION
TELEGRAM

RECEIVED AT 2029 SHATTUCK AVE., BERKELEY, CALIF.
4SF YN 49 NL

PASADENA CALIF FEB 18 1920

DR HARLOW SHAPLEY
30 FACULTY CLUB BERKELEY CALIF

PROGRAM COMMITTEE NATIONAL ACADEMY PROPOSES DEBATE BETWEEN YOU AND HEBER CURTISON SUBJECT SCALE OF UNIVERSE AT WASHINGTON MEETING APRIL TWENTY SIX OR TWENTY SEVENTH FORTY FIVE MINUTES EACH HONORARIUM FOR EACH SPEAKER ONE HUNDRED FIFTY DOLLARS HOPE YOU WILL WIRE ACCEPTANCE TO ABBOT SMITHONIAN INSTITUTON AND INFORM ME

GEORGE E HALE

705A FEB 19 1920

Harlow Shapley
(1885-1972)

Apr 26, 1920

Baird
Auditorium

Museum of Natural History, Smithsonian
Institution Washington DC., USA

THE GREAT DEBATE

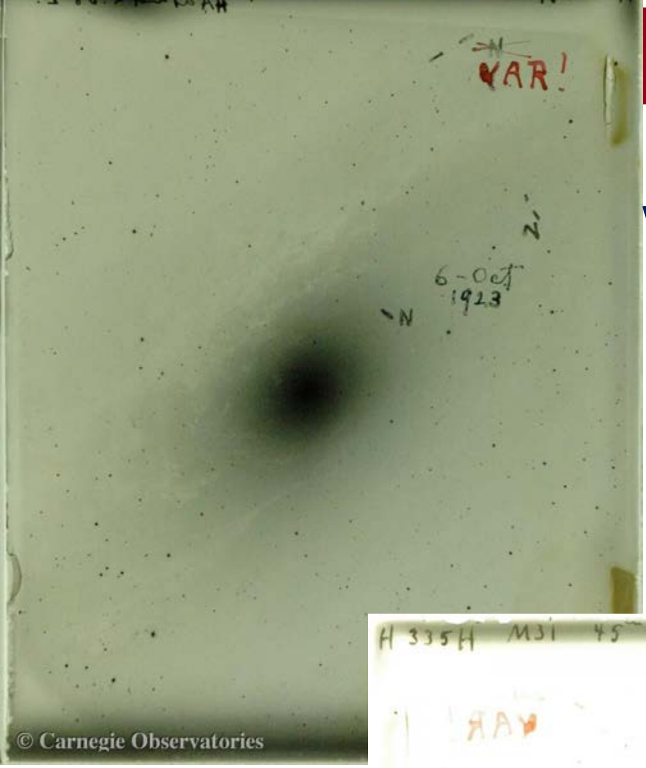
"The scale of the universe"

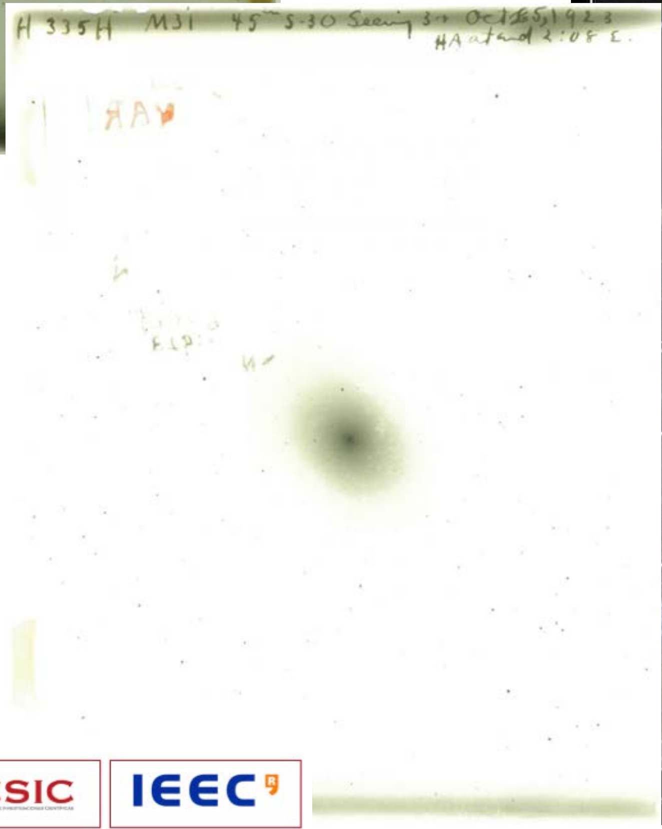
Apr 26, 1920: The Great Debate – Shapley vs Curtis

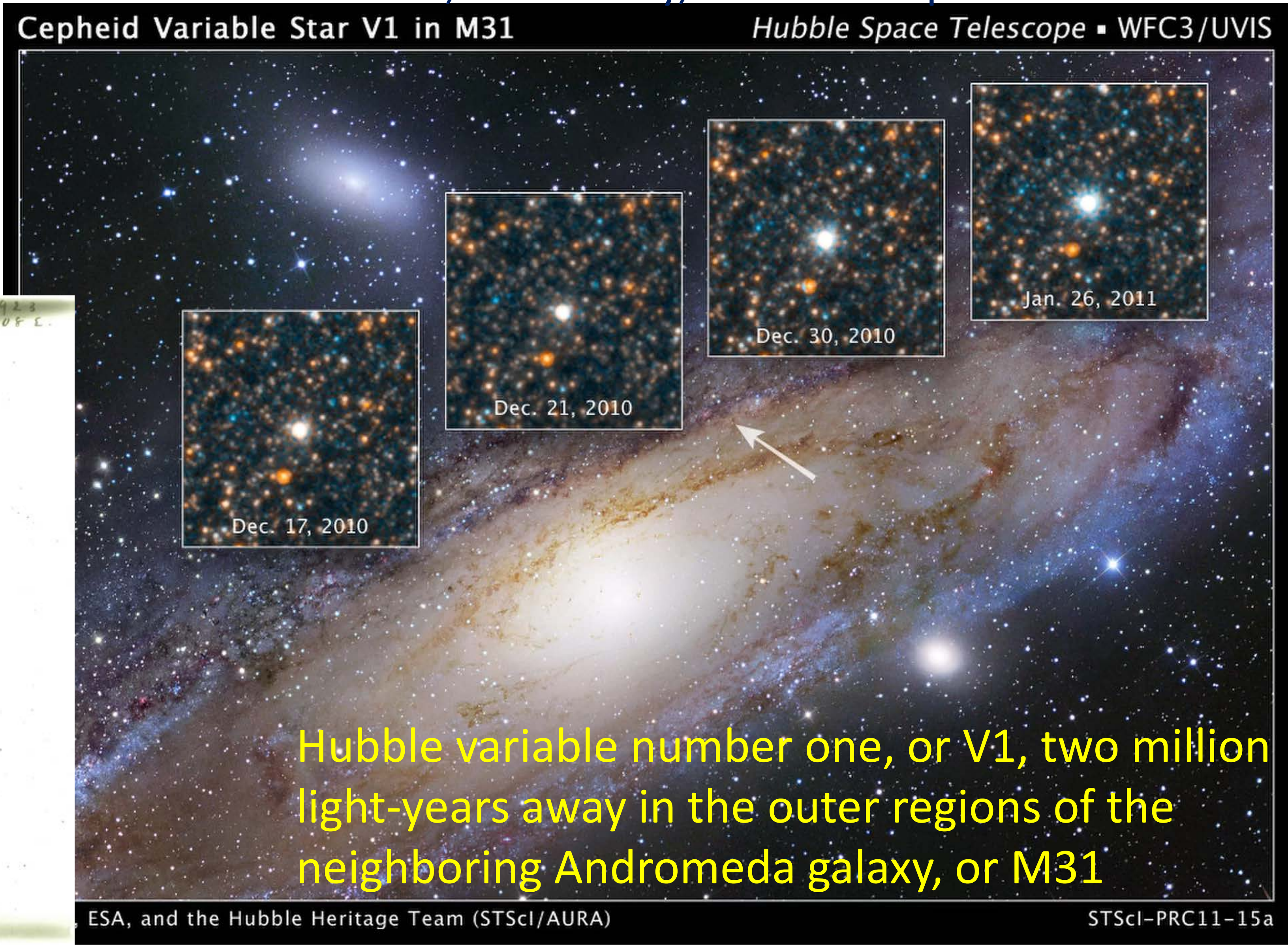
Harlow Shapley – the Milky Way was the entire Universe

Heber Curtis – many novae in Andromeda: "island Universe" (I Kant)

Image of H335H shows the glass side of the photographic plate, on which Hubble marked novae and, eventually, the first Cepheid in ink







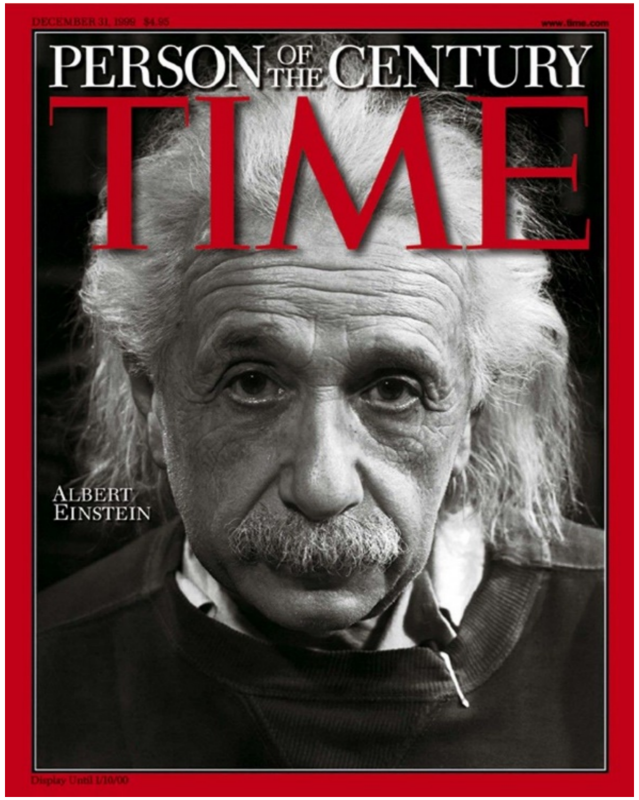
Hubble variable number one, or V1, two million light-years away in the outer regions of the neighboring Andromeda galaxy, or M31

Ernst J. Öpik

Estonian astronomer and astrophys. (1893-1985) worked at the Armagh Observatory in Northern Ireland. In 1922 published a paper estimating the distance to Andromeda using an original method based on observed rotational velocities of the galaxy: 450 kpc. Was the first to calculate the density of a white dwarf.

His result was closer to recent estimates (775 kpc) than Hubble's result (285 kpc) of Nov 23, 1924; E Öpik, ApJ 55, 406, 1922.

Institute of
Space Sciences



Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta\phi = 4\pi K\rho \qquad (1)$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unendlichen das Potential ϕ einem festen Grenzwerte zustrebt. Analog verhält es sich bei der Gravitationstheorie der allgemeinen Relativität; auch hier müssen zu den Differentialgleichungen Grenzbedingungen

der an sich nicht beansprucht, ernst genommen zu werden; er dient nur dazu, das Folgende besser hervortreten zu lassen. An die Stelle der Poissonschen Gleichung setzen wir

$$\Delta\phi - \lambda\phi = 4\pi K\rho . \qquad (2)$$

wobei λ eine universelle Konstante bedeutet. Ist ρ_0 die (gleichmäßige) Dichte einer Massenverteilung, so ist

$$\phi = -\frac{4\pi K}{\lambda}\rho_0 \qquad (3)$$

eine Lösung der Gleichung (2). Diese Lösung entspräche dem Falle, daß die Materie der Fixsterne gleichmäßig über den Raum verteilt wäre, wobei die Dichte ρ_0 gleich der tatsächlichen mittleren Dichte der Materie des Weltraumes sein möge. Die Lösung entspricht einer unendlichen Ausdehnung des im Mittel gleichmäßig mit Materie erfüllten Raumes. Denkt man sich, ohne an der mittleren Verteilungs-

müßten wir wohl schließen, daß die Relativitätstheorie die Hypothese von einer räumlichen Geschlossenheit der Welt nicht zulasse.

Das Gleichungssystem (14) erlaubt jedoch eine naheliegende, mit dem Relativitätspostulat vereinbare Erweiterung, welche der durch Gleichung (2) gegebenen Erweiterung der Poissonschen Gleichung vollkommen analog ist. Wir können nämlich auf der linken Seite der Feldgleichung (13) den mit einer vorläufig unbekannten universellen Konstante $-\lambda$ multiplizierten Fundamentaltensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right). \qquad (13a)$$

Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn man gelangt zu (13a) an Stelle von (13), wenn man statt des Skalars des Riemannschen Tensors diesen Skalar, vermehrt um eine universelle Konstante, in das Hamiltonsche Prinzip einführt, welches Prinzip ja die Gültigkeit von Erhaltungssätzen gewährleistet. Daß die Feldgleichung (13a) mit unseren Ansätzen über Feld und Materie vereinbar ist, wird im folgenden gezeigt.

E Gaztañaga
2003.11544

- Case of Newton's gravitation, same phenomenon, tries to find a solution introducing, next to the Laplacian of Poisson’s equation, a “universal constant” λ
- And proceeding analogously, p 151 he introduces the same type of universal constant, *“eine vorläufig unbekannte universelle Konstante”* (“a universal constant, unknown for the moment”), the cosmological constant

GOBIERNO DE ESPAÑA MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD
CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
IEEC

A. Einstein, “Zum kosmologischen Problem” (On the cosmological problem) prob. Jan, 1931

Note, Archives of the Hebrew University of Jerusalem

A failed attempt at constructing a steady state model

Hoyle, Bondi, Gold, 1948

"If you consider a physically limited volume, the particles of matter will continually leave it. In order for the density to remain constant, it is necessary for new particles of matter to be continuously produced in the volume, from the space itself”.

O’Raifeartaigh, et al. *Einstein’s steady-state model of the universe*, ArXiv 1402.0132;

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering (harm oscill):

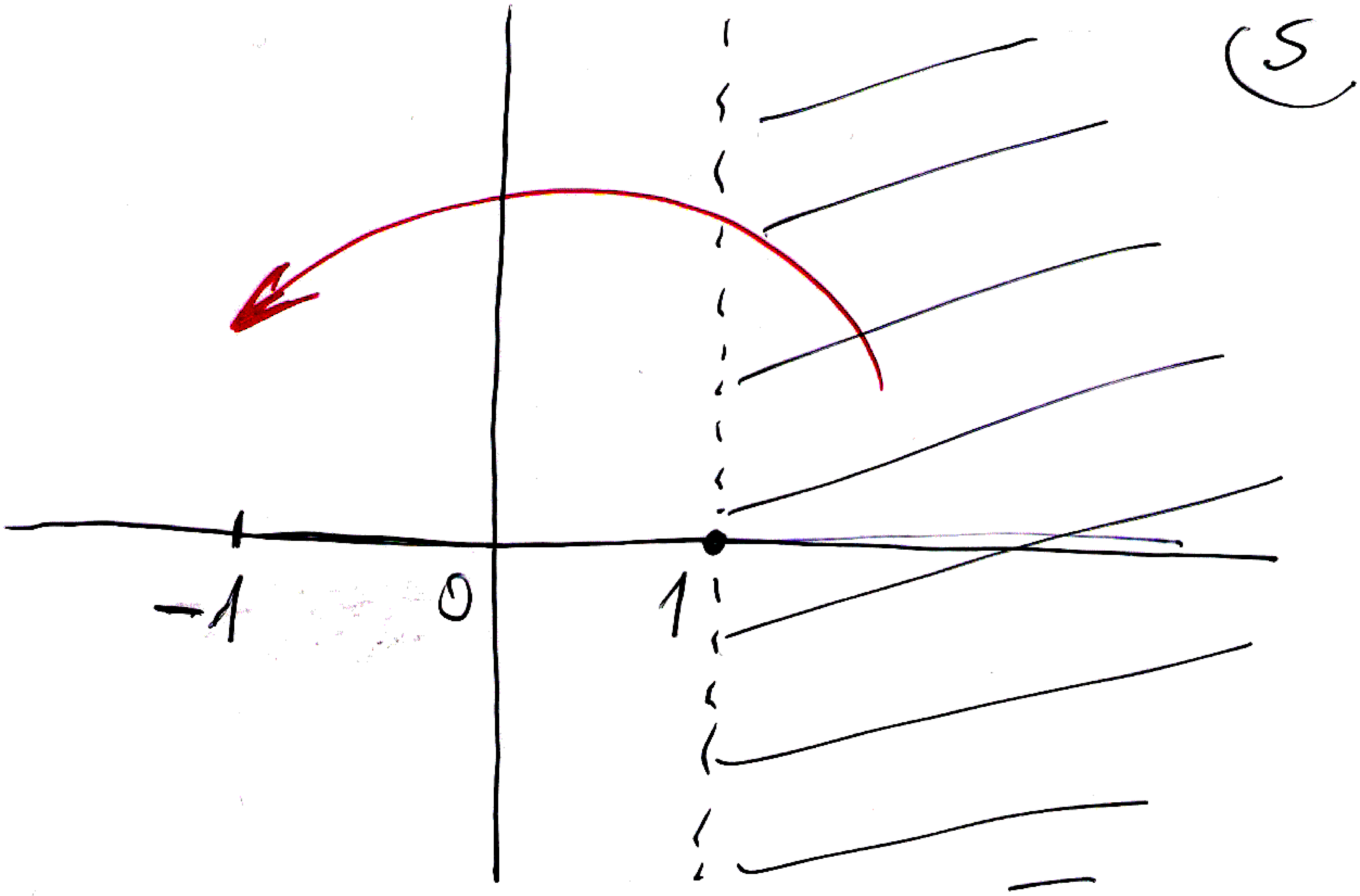
$$H = \left(n + \frac{1}{2}\right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle = \frac{\hbar\, c}{2} \sum_n \lambda_n = \frac{1}{2} \text{ tr } H$$

gives ∞ physical meaning?

Regularization $+$ Renormalization (cut-off, dim, ζ)

Even then: Has the final value real sense ?



$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots = \infty$$

$$\zeta(0) = -\frac{1}{2} \quad \text{or} \quad 1 + 1 + 1 + \cdots \underset{\zeta}{=} -\frac{1}{2}$$

$$\zeta(-1) = -\frac{1}{12} \quad \text{or} \quad 1 + 2 + 3 + \cdots \underset{\zeta}{=} -\frac{1}{12}$$

$$\vdots$$

F Yndurain, A Slavnov
"As everybody knows ..."

Commun. math. Phys. 55, 133—148 (1977)

Communications in
Mathematical
Physics

Zeta Function Regularization of Path Integrals in Curved Spacetime

S. W. Hawking

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, England

Abstract. This paper describes a technique for regularizing quadratic path integrals on a curved background spacetime. One forms a generalized zeta function from the eigenvalues of the differential operator that appears in the action integral. The zeta function is a meromorphic function and its gradient at the origin is defined to be the determinant of the operator. This technique agrees with dimensional regularization where one generalises to n dimensions by adding extra flat dimensions. The generalized zeta function can be expressed as a Mellin transform of the kernel of the heat equation which describes diffusion over the four dimensional spacetime manifold in a fith dimension of parameter time. Using the asymptotic expansion for the heat kernel, one can deduce the behaviour of the path integral under scale transformations of the background metric. This suggests that there may be a natural cut off in the integral over all black hole background metrics. By functionally differentiating the path integral one obtains an energy momentum tensor which is finite even on the horizon of a black hole. This energy momentum tensor has an anomalous trace.

1. Introduction

The purpose of this paper is to describe a technique for obtaining finite values to path integrals for fields (including the gravitational field) on a curved spacetime background or, equivalently, for evaluating the determinants of differential operators such as the four-dimensional Laplacian or D'Alembertian. One forms a gemeralised zeta function from the eigenvalues λ_n of the operator

$$\zeta(s)=\sum_n \lambda_n^{-s} . \tag{1.1}$$

In four dimensions this converges for $\mathrm{Re}(s)>2$ and can be analytically extended to a meromorphic function with poles only at $s=2$ and $s=1$. It is regular at $s=0$. The derivative at $s=0$ is formally equal to $-\sum_n \log\lambda_n$. Thus one can define the determinant of the operator to be $\exp(-d\zeta/ds)|_{s=0}$.

Multipl or N-Comm Anomaly, or Defect

- Given A, B, and AB ψDOs, even if ζ_A, ζ_B, and ζ_{AB} exist, it turns out that, in general,

$$\det{}_\zeta(AB) \neq \det{}_\zeta A \, \det{}_\zeta B$$

- The multiplicative (or noncommutative) anomaly (defect) is def ned as

$$\delta(A,B) = \ln \left[\frac{\det_\zeta(AB)}{\det_\zeta A \ \det_\zeta B} \right] = -\zeta'_{AB}(0) + \zeta'_A(0) + \zeta'_B(0)$$

- Wodzicki formula

$$\delta(A,B) = \frac{\text{res}\left\{ [\ln \sigma(A,B)]^2 \right\}}{2 \ \text{ord}\, A \ \text{ord}\, B \ (\text{ord}\, A + \text{ord}\, B)}$$

where $\quad \sigma(A,B) = A^{\text{ord}\, B} B^{-\text{ord}\, A}$

Electronic vote on the Resolution B4
"on a suggested renaming of the Hubble Law"

The Chair of the Resolution Committee presenting the Resolution B4

Voting Results for the Vote on Resolution B4 "on a suggested renaming of the Hubble Law"

Option	No. of Votes	
I approve the Resolution B4	3169	78%
I reject the Resolution B4	798	20%
Abstain	93	2%
Total Number of Votes	4,060	

Background

Five Resolutions were proposed for approval at the XXXth IAU General Assembly (Vienna, August 20th – 31st, 2018). They were announced and posted on the IAU web site on June 20th (see https://www.iau.org/news/announcements/detail/ann18029/) and initially they did not generate any comments by the members.

Institute of
Space Sciences

Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics Beyond LambdaCDM

Adam G. Riess, Stefano Casertano, Wenlong Yuan, Lucas M. Macri, Dan Scolnic *(Submitted on 18 Mar 2019 (v1), last revised 27 Mar 2019 (this version, v2))*

We present an improved determination of the Hubble constant (H0) from Hubble Space Telescope (HST) observations of 70 long-period Cepheids in the Large Magellanic Cloud. These were obtained with the same WFC3 photometric system used to measure Cepheids in the hosts of Type Ia supernovae. Gyroscopic control of HST was employed to reduce overheads while collecting a large sample of widely-separated Cepheids. The Cepheid Period-Luminosity relation provides a zeropoint-free link with 0.4% precision between the new 1.2% geometric distance to the LMC from Detached Eclipsing Binaries (DEBs) measured by Pietrzynski et al (2019) and the luminosity of SNe Ia. Measurements and analysis of the LMC Cepheids were completed prior to knowledge of the new LMC distance. Combined with a refined calibration of the count-rate linearity of WFC3-IR with 0.1% precision (Riess et al 2019), these three improved elements together reduce the full uncertainty in the LMC geometric calibration of the Cepheid distance ladder from 2.5% to 1.3%. Using only the LMC DEBs to calibrate the ladder we find H0=74.22 +/- 1.82 km/s/Mpc including systematic uncertainties, 3% higher than before for this particular anchor. Combining the LMC DEBs, masers in NGC 4258 and Milky Way parallaxes yields our best estimate: H0 = 74.03 +/- 1.42 km/s/Mpc, including systematics, an uncertainty of 1.91%---15% lower than our best previous result. Removing any one of these anchors changes H0 by < 0.7%. The difference between H0 measured locally and the value inferred from Planck CMB+LCDM is 6.6+/-1.5 km/s/Mpc or 4.4 sigma (P=99.999% for Gaussian errors) in significance, raising the discrepancy beyond a plausible level of chance.

The Astrophysical Journal, 876:85, 2019 May 1
https://doi.org/10.3847/1538-4357/ab1422

GOBIERNO DE ESPAÑA · MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD · CSIC · IEEC

Planck 2018 results. VI. Cosmological parameters

Planck Collaboration *(Submitted on 17 Jul 2018)*

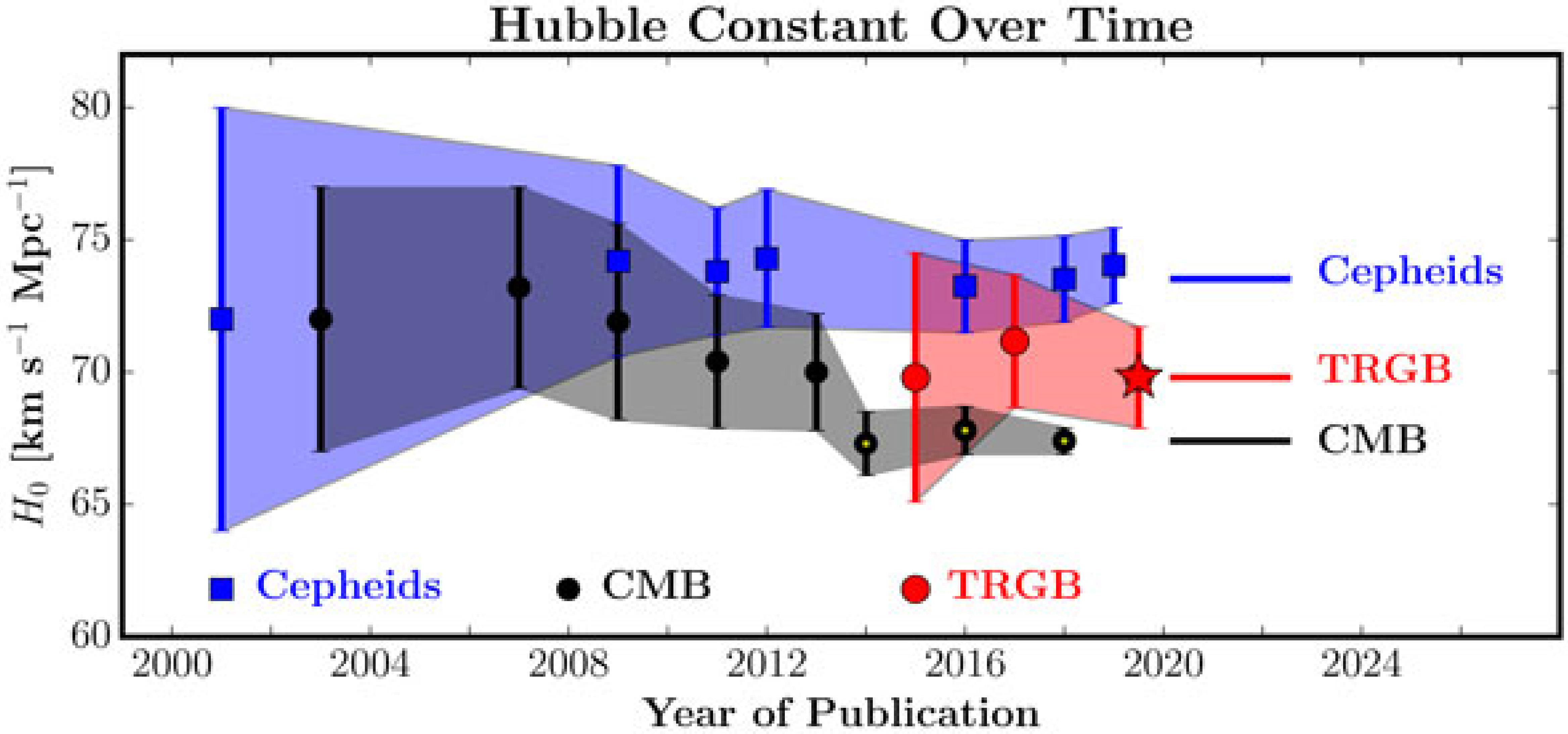
We present cosmological parameter results from the final full-mission Planck measurements of the CMB anisotropies. We find good consistency with the standard spatially-flat 6-parameter ΛCDM cosmology having a power-law spectrum of adiabatic scalar perturbations (denoted "base ΛCDM" in this paper), from polarization, temperature, and lensing, separately and in combination. A combined analysis gives dark matter density $\Omega ch2=0.120\pm0.001$, baryon density $\Omega bh2=0.0224\pm0.0001$, scalar spectral index $ns=0.965\pm0.004$, and optical depth $\tau=0.054\pm0.007$ (in this abstract we quote 68% confidence regions on measured parameters and 95% on upper limits). The angular acoustic scale is measured to 0.03% precision, with $100\theta*=1.0411\pm0.0003$. These results are only weakly dependent on the cosmological model and remain stable, with somewhat increased errors, in many commonly considered extensions. Assuming the base-ΛCDM cosmology, the inferred late-Universe parameters are: Hubble constant $H0=(67.4\pm0.5)$km/s/Mpc; matter density parameter $\Omega m=0.315\pm0.007$; and matter fluctuation amplitude $\sigma 8=0.811\pm0.006$. We find no compelling evidence for extensions to the base-ΛCDM model. (Abridged)

The Carnegie-Chicago Hubble Program. VIII. An Independent Determination of the Hubble Constant Based on the Tip of the Red Giant Branch

Wendy L. Freedman, Barry F. Madore, Dylan Hatt, Taylor J. Hoyt, In-Sung Jang, Rachael L. Beaton, Christopher R. Burns, Myung Gyoon Lee, Andrew J. Monson, Jillian R. Neeley, Mark M. Phillips, Jeffrey A. Rich, Mark Seibert *(Submitted on 12 Jul 2019)*

We present a new and independent determination of the local value of the Hubble constant based on a calibration of the Tip of the Red Giant Branch (TRGB) applied to Type Ia supernovae (SNeIa). We find a value of Ho = 69.8 +/- 0.8 (+/-1.1\% stat) +/- 1.7 (+/-2.4\% sys) km/sec/Mpc. The TRGB method is both precise and accurate, and is parallel to, but independent of the Cepheid distance scale. Our value sits midway in the range defined by the current Hubble tension. It agrees at the 1.2-sigma level with that of the Planck 2018 estimate, and at the 1.7-sigma level with the SHoES measurement of Ho based on the Cepheid distance scale. The TRGB distances have been measured using deep Hubble Space Telescope (HST) Advanced Camera for Surveys (ACS) imaging of galaxy halos. The zero point of the TRGB calibration is set with a distance modulus to the Large Magellanic Cloud of 18.477 +/-0.004 (stat) +/-0.020 (sys) mag, based on measurement of 20 late-type detached eclipsing binary (DEB) stars, combined with an HST parallax calibration of a 3.6 micron Cepheid Leavitt law based on Spitzer observations. We anchor the TRGB distances to galaxies that extend our measurement into the Hubble flow using the recently completed Carnegie Supernova Project I sample containing about 100 well-observed SNeIa. There are several advantages of halo TRGB distance measurements relative to Cepheid variables: these include low halo reddening, minimal effects of crowding or blending of the photometry, only a shallow (calibrated) sensitivity to metallicity in the I-band, and no need for multiple epochs of observations or concerns of different slopes with period. In addition, the host masses of our TRGB host-galaxy sample are higher on average than the Cepheid sample, better matching the range of host-galaxy masses in the CSP distant sample, and reducing potential systematic effects in the SNeIa measurements.

The Astrophysical Journal, 882:34, 2019 Sep 1
https://doi.org/10.3847/1538-4357/ab2f73



The Astrophysical Journal, 882:34, 2019
https://doi.org/10.3847/1538-4357/ab2f73

arXiv:2006.01879

An analysis of the H_0 tension problem in a universe with a viscous dark fluid

Emilio Elizalde[1,2*], Martiros Khurshudyan[1,2†], Sergei D. Odintsov[1,3‡] and Ratbay Myrzakulov[4§]

[1] *Consejo Superior de Investigaciones Científicas,*
ICE/CSIC-IEEC, Campus UAB,
Carrer de Can Magrans s/n,
08193 Bellaterra (Barcelona) Spain

[2] *International Laboratory for Theoretical Cosmology,*
Tomsk State University of Control Systems and Radioelectronics (TUSUR),
634050 Tomsk, Russia

[3] *ICREA, Passeig Luis Companys,*
23, 08010 Barcelona, Spain

[4] *Eurasian International Center for Theoretical Physics*
and Department of General and Theoretical Physics,
Eurasian National University,
Nur-Sultan, 010008, Kazakhstan

In this paper, two inhomogeneous single fluid models for the Universe, which are able to naturally solve the H_0 tension problem, are discussed. The analysis is based on a Bayesian Machine Learning approach that uses a generative process. The adopted method allows to constrain the free parameters of each model by using the model itself, only. The observable is taken to be the Hubble parameter, obtained from the generative process. Using the full advantages of our method, the models are constrained for two redshift ranges. Namely, first this is done with mock $H(z)$ data over $z \in [0, 2.5]$, thus covering known $H(z)$ observational data, which are most helpful to validate the fit results. Then, aiming to extend to redshift ranges to be covered by the most recent ongoing and future planned missions, the models are constrained for the range $z \in [0, 5]$, too. Full validation of the results for this extended redshift range will have to wait for the near future, when higher redshift $H(z)$ data become available. This makes our models fully falsifiable. Finally, our second model here is able to explain the BOSS reported value for $H(z)$ at $z = 2.34$.

An approach to the H_0 tension problem from Bayesian Learning and cosmic opacity

arXiv:2006.12913

Emilio Elizalde[1,2*] and Martiros Khurshudyan[1,2†]
[1] *Consejo Superior de Investigaciones Científicas, ICE/CSIC-IEEC, Campus UAB, Carrer de Can Magrans s/n, 08193 Bellaterra (Barcelona) Spain*
[2] *International Laboratory for Theoretical Cosmology, Tomsk State University of Control Systems and Radioelectronics (TUSUR), 634050 Tomsk, Russia*

Bayesian Learning and elements of probabilistic programming are used to probe the opacity of the Universe, as a way of addressing the H_0 tension issue. Bayesian Learning relies on what is called a generative process what allows to test the models well beyond the limits imposed by actual astronomical observations. In this approach, the model is the key object to generate the data, and the generative process involves the unknown parameters of the model, what permits to constrain them, too. In this way, prior beliefs of the unknown parameters can be incorporated, and be used to get the posterior results, and constrain them appropriately, in this way. Using Bayesian learning algorithms, beliefs on the parameters are thus updated, and a new distribution over them results. In our study, constraints on the cosmic opacity are determined for two flat models, ΛCDM and XCDM (this having $\omega_{de} \neq -1$), for three redshift ranges, $z \in [0, 2.5]$, $z \in [0, 5]$, and $z \in [0, 10]$, in each case.

GOBIERNO DE ESPAÑA MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD

CSIC

IEEC

galaxies

MDPI

Article

“All That Matter ... in One Big Bang ...,” & Other Cosmological Singularities

E Elizalde *

Galaxies 2018, 6, 25; doi:10.3390/galaxies6010025

arXiv:1801.09550v3 [physics.hist-ph] 25 Jan 2018
https://arxiv.org/pdf/1801.09550.pdf

symmetry

MDPI

Reasons in Favor of a Hubble-Lemaître-Slipher's (HLS) Law

Emilio Elizalde

Institute of Space Sciences (ICE/CSIC and IEEC), Campus UAB, Carrer de Can Magrans, s/n, 08193 Bellaterra (Barcelona), Spain

Symmetry **2019**, *11*(1), 35; https://doi.org/10.3390/sym11010035
arXiv:1810.12416 [physics.hist-ph]

Lecture Notes in Physics 855

Emilio Elizalde

Ten Physical Applications of Spectral Zeta Functions

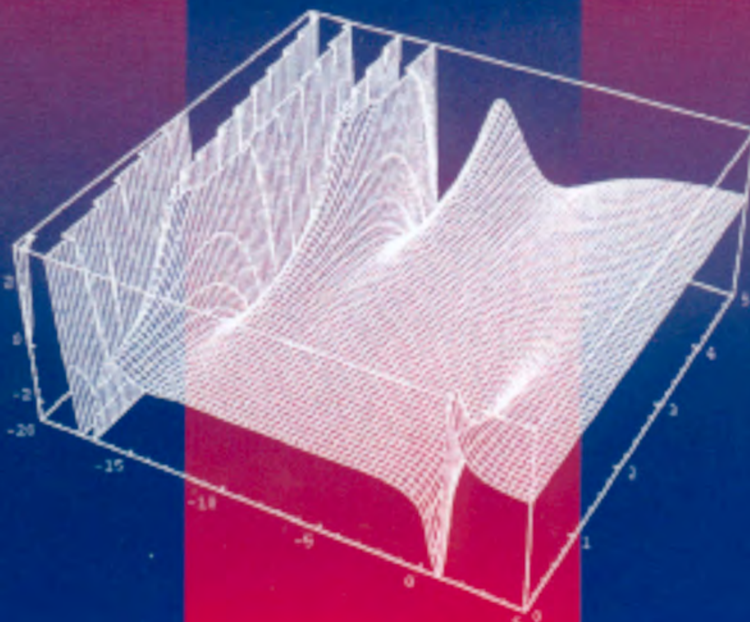
Second Edition

Springer

Elizalde
Odintsov
Romeo
Bytsenko
Zerbini

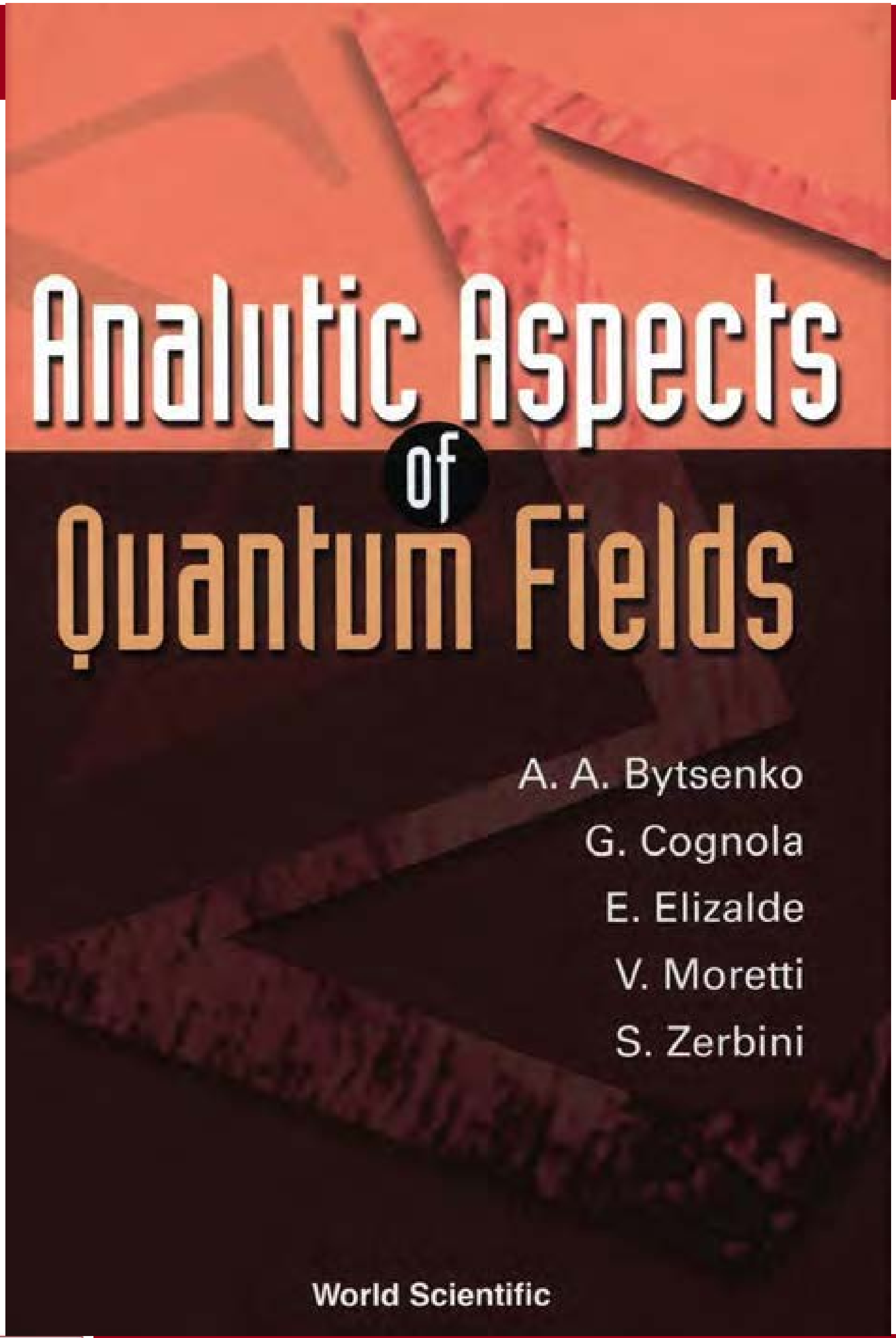
Zeta Regularization Techniques with Applications

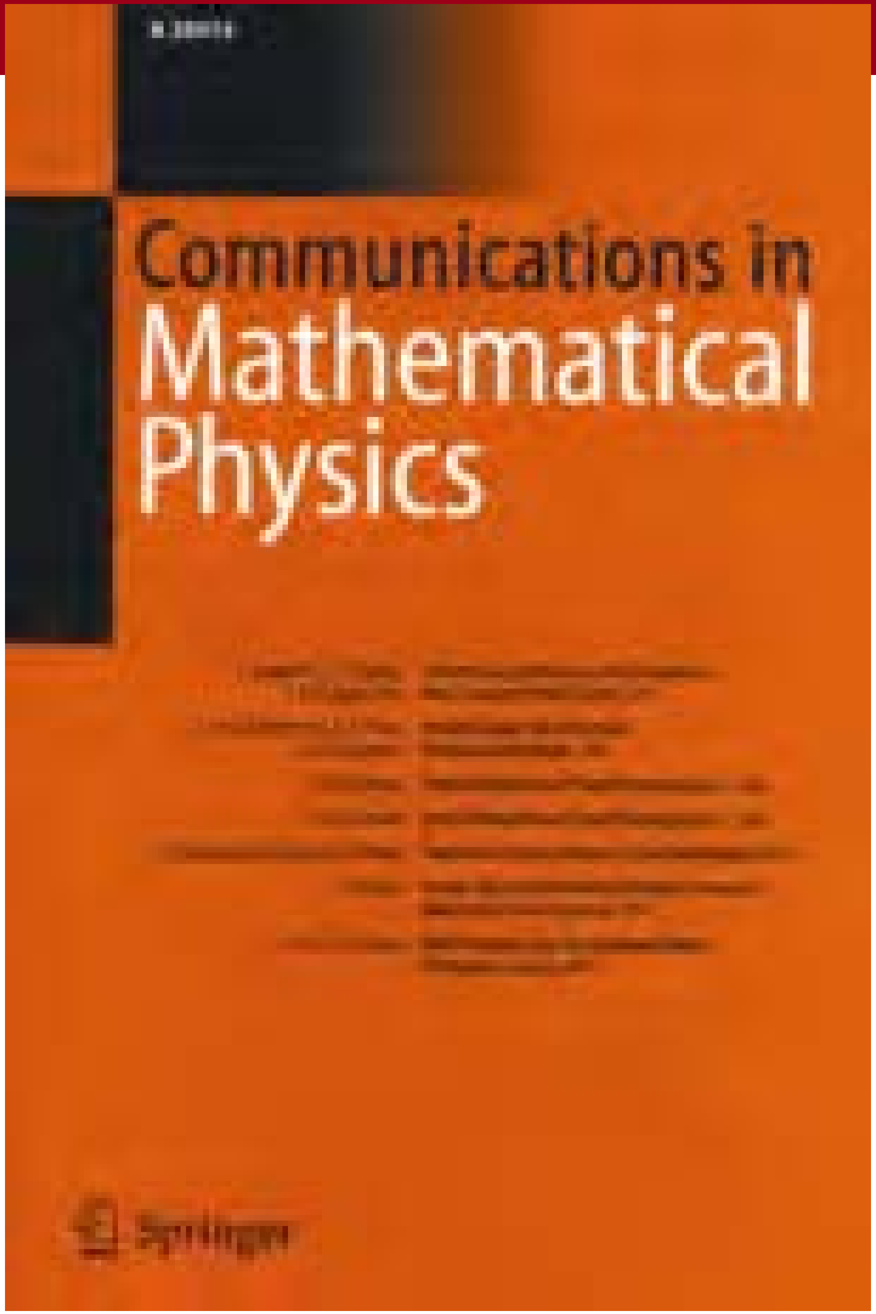
Zeta Regularization Techniques with Applications



E. Elizalde, S. D. Odintsov, A. Romeo,
A. A. Bytsenko and S. Zerbini

World Scientific


Analytic Aspects
of
Quantum Fields
A. A. Bytsenko
G. Cognola
E. Elizalde
V. Moretti
S. Zerbini
World Scientific


Communications in
Mathematical
Physics

Institute of
Space Sciences

MODERN COSMOLOGY

FROM THE VERY ORIGINS

Emilio Elizalde

(on the way…)

GOBIERNO DE ESPAÑA
MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD
CSIC
IEEC

Thank You !