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ABSTRACT

We investigate electronic thermal rectification in ferromagnetic insulator-based superconducting tunnel junctions. Ferromagnetic insulators coupled to superconductors are known to induce sizable spin splitting in the superconducting density of states and also lead to efficient spin filtering if used as tunnel barriers. The combination of spin splitting and spin filtering is shown to yield a substantial amount of *self-amplification* of the electronic heat diode effect due to breaking of the electron-hole symmetry in the system, which is added to the thermal asymmetry of the junction. Large spin splitting and large spin polarization ($\geq 90\%$) can potentially lead to thermal rectification efficiencies exceeding $\sim 5 \times 10^4\%$ for realistic parameters in a suitable temperature range, thereby outperforming up to a factor of ~ 250 , the heat diode effect achievable with conventional superconducting tunnel junctions. These results are relevant for improved control of heat currents in innovative phase-coherent caloritronic nanodevices and for enhanced thermal management of quantum circuits at the nanoscale.

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A *thermal rectifier*, or a heat diode,^{1,2} is a device in which the heat current depends on the sign and the amplitude of the temperature gradient imposed across it. The implementation of efficient heat diodes would represent a breakthrough in the realization of improved thermal circuits for cooling,³ thermal isolation, energy harvesting, radiation sensing, and several other applications.⁴ Recently, the control of thermal transport at the nanoscale has been attracting great interest.^{3–6} From the theoretical side, strong effort has been put to conceive thermal rectification setups dealing with phonons,^{7–10} electrons,^{6,11–20} and photons.²¹ Experimentally, promising results were obtained so far in the context of electronic^{22–24} and phononic^{25–27} heat transport. Rectification of electronic heat currents has been studied in several types of tunneling junctions between different materials such as, for instance, normal metals,¹⁷ Josephson junctions,¹⁸ and superconductor-normal metal structures.^{6,19} In all cases, heat rectification stems from thermal asymmetry of the structure in the direction of the current flow. In conventional superconducting tunnel junctions, this asymmetry together with the highly non-

linear temperature dependence of the density of states leads to a heat rectification up to $\sim 800\%$ in Josephson junctions.¹⁸

In this Letter, we study thermal rectification effects in systems based on ferromagnetic insulator (FI) superconducting tunnel junctions. The combination of spin splitting and spin polarization induced by FIs breaks the electron-hole symmetry of the electronic transport. This electron-hole symmetry breaking (EHSB) together with the non-linear temperature dependence of the superconducting spectral properties leads to an unrivaled heat diode effect with a suitable choice of realistic parameters. Specifically, a rectification efficiency exceeding $\sim 5 \times 10^4\%$ can be obtained, outperforming up to a factor of ~ 250 that is achievable in conventional superconducting tunnel junctions. These results could be relevant for improved thermal isolation of cryogenic quantum circuitry at the nanoscale.

Figure 1(a) schematizes the generic system under investigation, which consists of two electronic reservoirs (either superconducting or normal metallic) residing at temperature $T_{L,R}$. The density of states $N_{L,R}$ depends on the temperature and applied exchange field. The two

FIG. 1. (a) Sketch of two electronic reservoirs (either normal or superconducting) with different temperature-dependent densities of states $N_{L,R}$ residing at temperature $T_{L,R}$. The reservoirs are coupled to an element that breaks electron-hole symmetry. This leads to a thermoelectric response and enhanced thermal rectification. J_+ and J_- represent the heat flow in the forward ($T_L > T_R$) and reverse ($T_L < T_R$) thermal-bias configuration, respectively, while V_+ and V_- denote the corresponding thermovoltages developed across the system. (b) Prototypical ferromagnetic insulator (FI)-based superconductor (S_L)-normal metal (N) and (c) superconductor (S_L)-superconductor (S_R) tunnel junctions. The FI is inserted in the structures as to both induce a spin splitting in the density of states of the superconductor and provide a spin filtering effect. $\Delta_L(\Delta_R)$ denotes the energy gap in $S_L(S_R)$.

electrodes are electrically coupled via an EHSB mechanism. The latter gives rise to a thermoelectric response in the system and to a thermovoltage amplitude $V_+(V_-)$ developed for $T_L > T_R(T_L < T_R)$. The heat current flowing in the forward thermal-bias configuration is denoted with J_+ , whereas J_- denotes the one flowing in the reverse thermal-bias configuration. As we shall show, the presence of EHSB in the system yields a *self-amplification* of thermal rectification efficiency up to unparalleled values for suitable parameters of the structure and proper thermal bias conditions. The EHSB mechanism can be achieved by placing an S-FI building block in two possible configurations: S_L FIN [Fig. 1(b)] and S_L FIS $_R$ [Fig. 1(c)] tunnel junctions. The presence of the FI layer yields both spin splitting of the density of states in S_L and spin filtering at the barrier. The combination results in the EHSB mechanism.^{28–34}

The interaction between the spin of conducting electrons in the superconductor and the localized magnetic moments in the adjacent FI leads to an effective exchange interaction (h_{exc}) in the superconductor. This field decays away from the S/FI interface over the superconducting coherence length ξ_0 .³⁵ Yet, we assume that the superconducting layer thickness is smaller than ξ_0 , so that the induced h_{exc} in the superconductor by FI is spatially homogeneous. In this situation, the spin-dependent normalized density of states of the superconductor is simply given by $N^{\uparrow,\downarrow}(E) = \frac{1}{2} |\text{Re}[\frac{E + i\Gamma \pm h_{exc}}{\sqrt{(E + i\Gamma \pm h_{exc})^2 - \Delta^2}}]|$,

where Γ is the Dynes parameter and Δ is the superconducting gap, which depends on T and h_{exc} via the self-consistency equation $\ln(\frac{\Delta_0}{\Delta}) = \int_0^{\omega_D} dE \frac{f_+(E) + f_-(E)}{\sqrt{E^2 + \Delta^2}}$, where $f_{\pm}(E) = \{1 + \exp[\frac{1}{k_B T}(\sqrt{E^2 + \Delta^2} \mp h_{exc})]\}^{-1}$, ω_D is the Debye frequency of the superconductor, $\Delta_0 = 1.764 k_B T_c$ is the zero-temperature, zero-exchange field superconducting pairing potential, T_c is the critical temperature, and k_B is the Boltzmann constant. The parameter Γ accounts for the broadening of the coherent peaks in the density of states due to inelastic scattering, and for an ideal superconductor, $\Gamma \rightarrow 0^+$.³⁶ In all the calculations, we set $\Gamma = 10^{-4} \Delta_0$, unless differently stated. We are interested in both the DC charge and electronic heat currents through the junctions, which are given by³⁰ $I = \frac{1}{eR_T} \int_{-\infty}^{\infty} dE [N_+ + PN_-][f_L(V, T_L) - f_R(T_R)]$ and $J = \frac{1}{eR_T} \int_{-\infty}^{\infty} dE (E + eV) [N_+ + PN_-][f_L(V, T_L) - f_R(T_R)]$, respectively. Here, R_T is the normal-state tunneling resistance of the junction, $N_{\pm} = (N_L^{\uparrow} N_R^{\uparrow} \pm N_L^{\downarrow} N_R^{\downarrow})$, and $0 \leq P \leq 1$ is the barrier spin polarization provided by the FI layer.³⁷ Moreover, $f_L(V, T_L) = [1 + \exp[(E + eV)/k_B T_L]]^{-1}$ and $f_R(T_R) = [1 + \exp(E/k_B T_R)]^{-1}$ are the equilibrium quasiparticle distribution functions, and e is the electron charge. In principle, the expressions for both currents may contain additional phase-dependent contributions due to the Josephson coupling present in the junction.^{4,5,38} However, since our system exhibits a thermovoltage across the junction, the phase becomes time-dependent, thereby not contributing to DC transport. It is also worth emphasizing that the values of both spin polarization and spin splitting can be extracted from experiments, as demonstrated in several works.^{39–48}

We assume that both electronic reservoirs are thermalized with well-defined temperatures $T_{L,R}$. In the *forward* thermal bias configuration [see Fig. 1(a)], a thermal gradient is intentionally created at the junction by setting $T_L = T_{hot} > T_R = T_{cold}$ which leads to a total heat flux J_+ through the system. In the *reverse* thermal bias configuration, the heat gradient is inverted so that $T_L = T_{cold} < T_R = T_{hot}$, yielding a total heat current J_- flowing from the right to the left electrode. The electronic thermal rectification efficiency is defined as $R(\%) = 100 \times (|J_+| - |J_-|)/|J_-|$, such that the absence of heat rectification corresponds to $R = 0$ and $R > 0$ indicates a preferential heat flow from the left to the right side of the junction. In order to determine the thermal rectification, for example, in the forward thermal bias configuration, for arbitrary $T_L > T_R$, we need first to determine the thermovoltage V_+ across the junction by solving the equation $I(V_+, T_L, T_R) = 0$. Then, the obtained thermovoltage is used to compute the corresponding heat current $J_+(V_+, T_L, T_R)$ flowing through the junction. In the reverse thermal bias condition, $T_R > T_L$, the same procedure is performed to determine V_- and J_- . Following this procedure, we determine R in both junction setups depicted in Figs. 1(b) and 1(c).

It is instructive to start our discussion by analyzing the setup of Fig. 1(b), the S_L FIN superconducting tunnel junction. A thermal bias across the structure leads to a thermovoltage $V_{\pm}$, which depends on the sign of the thermal gradient itself, and stems from EHSB in the junction⁴⁹ [see Fig. 1(a)]. In particular, the thermovoltage V_+ vs T_L (at $T_R = 0.01 T_c$) is shown in the left panel of Fig. 2(a), whereas V_- vs T_R (at $T_L = 0.01 T_c$) is shown in the right panel of the same figure, both evaluated at $P = 1$ for selected values of the exchange field h_{exc} . Besides the substantial difference between the thermovoltage amplitudes V_+ and V_- for the same exchange field, V_+ is a non-monotonic function of T_L , vanishing when the superconducting pairing potential goes to zero, while V_- monotonically increases with T_R , saturating at the

FIG. 2. (a) Thermovoltage V_+ vs T_L (left panel) and V_- vs T_R (right panel) calculated for an S_2 FIN junction at $P=1$ and $T_R = 0.01T_c$ (left panel) and $T_L = 0.01T_c$ (right panel) for different values of the exchange field h_{exc} . (b) Absolute value of the heat current $|J|$ vs T_{hot} flowing through an S_2 FIN junction calculated at $P=1$ and $T_{cold} = 0.01T_c$ for two selected values of h_{exc} . (c) V_+ vs T_L (left panel) and V_- vs T_R (right panel) calculated for an S_2 FIN junction at $h_{exc} = 0.4\Delta_0$ and $T_R = 0.01T_c$ (left panel) and $T_L = 0.01T_c$ (right panel) for different values of barrier polarization P . (d) Absolute value of the heat current $|J|$ vs T_{hot} flowing through an S_2 FIN junction calculated at $h_{exc} = 0.4\Delta_0$ and $T_{cold} = 0.01T_c$ for two selected values of P .

asymptotic value $eV_- = h_{exc}$ at large temperature.⁴⁹ This sizable difference between the thermovoltages has a direct impact on the corresponding heat currents $J_{\pm}$ flowing through the junction. These are shown in Fig. 2(b) for two given values of the exchange field. For the larger value of h_{exc} , the difference between the forward and reverse heat currents is increased, thereby leading to enhanced thermal rectification in the S_2 FIN junction. This difference appears to be particularly pronounced for $0.1T_c \leq T_{hot} \leq 0.2T_c$, which is indeed the temperature range where the maximum thermovoltage amplitudes develop across the junction [cf. Fig. 2(a)].

The left panel of Fig. 2(c) displays V_+ vs T_L (at $T_R = 0.01T_c$), while the right one shows V_- vs T_R (at $T_L = 0.01T_c$), both calculated at $h_{exc} = 0.4\Delta_0$ for different values of P . The thermovoltage V_+ is negligibly affected by the polarization. By contrast, V_- is strongly affected by P and becomes larger by increasing the barrier polarization. These different behaviors can be understood by noticing that P changes only the effective barrier transmissivity and not the spectrum of the superconductor as h_{exc} does [cf. Fig. 2(a)]. When S_L is heated (V_+ situation), charge transport is weakly affected by interface transmissivity (i.e., by P) because of the large impedance provided by the gapped density of states. By contrast, P mostly determines the charge current in the V_- situation, because the N electrode has a large density of states at the Fermi energy, and, hence, very low impedance. Also, in the present case of finite barrier polarization, the large difference in thermovoltage amplitudes deeply affects the corresponding heat currents flowing through the structure, as shown in Fig. 2(d) for two selected values of P .

A large barrier polarization ($P \sim 1$) strongly enhances the difference between forward and reverse heat currents, therefore leading to a sizable heat diode effect in the junction.

In Fig. 3, we show the rectification efficiency for the S_{LFIN} junction. Specifically, Fig. 3(a) shows the heat rectification efficiency R vs T_{hot} calculated for zero barrier polarization at $T_{cold} = 0.01T_c$ for different values of h_{exc} . The absence of spin polarization at the barrier leads to a zero thermoelectric voltage. The increase in h_{exc} yields a slight reduction of R compared to that of a conventional SIN tunnel junction (i.e., for $h_{exc} = 0$),^{6,18,19} allowing us to obtain a maximum rectification efficiency of $\sim 22\%$ for $h_{exc} = 0.5\Delta_0$ at $T_{hot} \sim 0.5T_c$. The above heat rectification reduction stems from a larger contribution to the heat transport for electrons with energy close to the Fermi level at large h_{exc} . This contribution restores the thermal symmetry of the junction and, hence, reduces the heat rectification. In addition, there are additional features appearing in the rectification characteristics at higher temperatures, which occur when superconductivity is quenched due to the presence of a finite exchange field. In short, Fig. 3(a) demonstrates that spin splitting alone cannot improve thermal rectification with respect to conventional superconducting junctions.

However, if the barrier spin polarization is finite, the situation changes drastically. The role of finite h_{exc} at $P=1$ is shown in Fig. 3(b). In particular, the thermoelectric response of the junction is increased in the presence of the high exchange field, allowing us to obtain both large heat rectification in the forward thermal-bias configuration for $h_{exc} = 0.5\Delta_0$ (i.e., up to $\sim 290\%$) and a sizable contribution in the reverse thermal-bias configuration (around $\sim -55\%$ for $h_{exc} = 0.3\Delta_0$). All the above results prove that the presence of EHSB mechanisms in the S_2 FIN junction leads to a substantial amount of self-amplification of the heat diode efficiency, which now yields values

FIG. 3. Thermal rectification coefficient R for the S_2 FIN junction. (a) Thermal rectification coefficient R vs T_{hot} calculated at $T_{cold} = 0.01T_c$ and $P=0$ for different values of the exchange field h_{exc} . (b) The same as in panel (a) but for barrier polarization $P=1$. (c) R vs T_{hot} calculated at $T_{cold} = 0.01T_c$ and $h_{exc} = 0.4\Delta_0$ for a few values of P . (d) R vs T_{hot} calculated at $h_{exc} = 0.4\Delta_0$ and $P=1$ for several values of T_{cold} . Dashed lines indicate $R=0$.

that are larger by more than a factor of ~ 10 than those typically achievable in conventional superconducting tunnel junctions.^{6,18,19}

The impact of a finite barrier polarization at $h_{exc} = 0.4\Delta_0$ is displayed in Fig. 3(c) for different values of P . The effect is similar to that caused by an increasing exchange field [see Fig. 3(c)] and shows the relevance of a large spin polarization in order to achieve a sizable heat rectification. For instance, R turns out to be suppressed by more than a factor of ~ 20 if P is reduced down to 50%. Finally, the effect of the smaller temperature T_{cold} on R is displayed in Fig. 3(d) as a function of T_{hot} . We notice, in particular, the strong suppression of R occurring by increasing T_{cold} : for instance, R is suppressed by roughly one order of magnitude at $T_{cold} = 0.1T_c$. This emphasizes the requirement of a sufficiently low T_{cold} in order to achieve large rectification effects.

We now discuss the heat diode effect in the junction setup sketched in Fig. 1(c). For simplicity, we assume that the two superconductors are identical such that they have the same zero-temperature and zero-exchange field energy gap Δ_0 . To maximize the thermal asymmetry, we also assume that only the density of states of the left electrode (N_L) is affected by h_{exc} so as to break the thermal symmetry of the system. Such an asymmetry can be achieved by inserting a very thin non-magnetic oxide layer at the FI/S_R interface.^{40,42} This implies that $N_R^{\uparrow} = N_R^{\downarrow}$. We now define the rectification efficiency as $R = 100 \times (|J_-| - |J_+|)/|J_+|$ so as to easily compare the two different junction setups since, as we shall show, J_- is typically much larger than J_+ in the $S_L FIS_R$ junction. Figures 4(a) and 4(b) show the impact of a finite h_{exc} on R at $P = 1$. In particular, we note the very large thermal rectification, which can be obtained, up to $\sim 5 \times 10^4\%$ for $h_{exc} = 0.5\Delta_0$ at $T_{hot} \simeq 0.15T_c$. This value is ~ 250 times larger than the one achievable in conventional all-superconducting tunnel junctions made with superconductors with different energy gaps.^{6,18} Yet, even for moderate h_{exc} values (i.e., $h_{exc} = 0.2\Delta_0$), R can reach values as large as $10^4\%$ in the suitable T_{hot} range. The above results demonstrate the effectiveness of $S_L FIS_R$ tunnel junctions to achieve very high thermal rectification efficiency.

The role of P is displayed in Figs. 4(c) and 4(d), which reveal the increased robustness of the $S_L FIS_R$ setup with respect to the $S_L FIN$ one in terms of limited barrier polarization. For example, for a moderate spin polarization, $P = 75\%$, we still get a sizable $R \simeq 1370\%$, which is about two times larger than the maximum value achievable in conventional SIS tunnel junctions.^{6,18} Nowadays, for instance, state-of-the-art ferromagnetic europium (Eu) chalcogenide tunnel barriers can provide spin polarization close to 100%,⁵⁰ which would make thermal rectification efficiencies larger than $\sim 7.5 \times 10^3\%$ readily available in superconducting tunnel junction setups operating at cryogenic temperatures.

The impact of non-idealities of the junction is shown in Fig. 4(e), where R is plotted against T_{hot} for a few selected values of Γ . From a quantitative point of view, thermal rectification efficiency turns out to be less effective for the larger value of Γ . In particular, for a sizable $\Gamma = 10^{-2}\Delta_0$, the maximum of R is reduced down to $\sim 1140\%$ at $T_{hot} \simeq 0.25T_c$. This fact emphasizes the requirement of high-quality tunnel junctions in order to preserve a substantial heat diode effect. Finally, Fig. 4(f) shows how T_{cold} affects the rectification efficiency. Analogous to the $S_L FIN$ setup, increasing T_{cold} deeply suppresses the R coefficient, although in a reduced way. In particular, for $T_{cold} = 0.1T_c$, the rectification efficiency turns out to be decreased by almost a factor

FIG. 4. Thermal rectification coefficient R for the $S_L FIS_R$ junction. (a) R vs T_{hot} calculated at $T_{cold} = 0.01T_c$ and $P = 1$ for different values of the exchange field h_{exc} . (b) The same as in panel (a) but shown on a logarithmic scale. (c) R vs T_{hot} calculated at $T_{cold} = 0.01T_c$ and $h_{exc} = 0.4\Delta_0$ for a few values of P . (d) R vs T_{hot} calculated at $T_{cold} = 0.01T_c$ and $h_{exc} = 0.4\Delta_0$ for several high values of barrier polarization in the range of $0.92 \leq P \leq 1$. (e) R vs T_{hot} calculated at $T_{cold} = 0.01T_c$, $h_{exc} = 0.4\Delta_0$, and $P = 1$ for a few values of Γ . (f) R vs T_{hot} calculated at $h_{exc} = 0.4\Delta_0$ and $P = 1$ for different values of T_{cold} .

of 5 with respect to the lowest temperature, suggesting that the $S_L FIS_R$ junction is more efficient as a heat diode at higher temperatures than the $S_L FIN$ setup.

In summary, we have demonstrated theoretically the occurrence of an unparalleled thermal diode effect in superconducting tunnel junctions with ferromagnetic insulators. In particular, thermal rectification efficiencies above $\sim 5 \times 10^4\%$ could be achieved for realistic material parameters in a suitable temperature range. Such a heat rectifier efficiency exceeds by a factor of ~ 250 , the one obtained with conventional superconducting tunnel junctions. Ideal materials for the heat rectifier are europium chalcogenide layers (EuO and EuS), for which values of P ranging from 80% up to $\sim 100\%$ have been reported,^{37,40,42,51–55} in combination with Al superconducting thin films.^{46–48} Yet, very large spin-filtering has been reported in GdN barriers as well,^{56–58} with polarizations as large as 97% at low temperature. Our finding might be relevant for enhanced control of heat current in phase-coherent caloritronic devices^{4,5} as well as for general thermal management of nanoscale quantum circuits.³

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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