A NEW TECHNIQUE OF MEASURING LOW-POWER PICOSECOND OPTICAL PULSE TRAINS.

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ABSTRACT

We present a theoretic approach to the characterization of low-power bright ultrashort optical pulses with an internal frequency modulation simultaneously in both time and frequency domains. The analysis and computer simulations are applied to studying the capability of Wigner distribution to characterize solitary pulses in practically important case of the sech-pulses. Then, the simplest two-beam scanning Michelson interferometer is selected for shaping the field-strength auto-correlation function of low-power picosecond pulse trains. We are proposing the key features of a new interferometric experimental technique for accurate and reliable measurements of the train-average width as well as the value and sign of the frequency chirp of pulses in high-repetition-rate trains. This technique is founded on an ingenious algorithm for the advanced metrology, assumes using a specially designed supplementary semiconductor cell, and suggests carrying out a pair of additional measures with exploiting this semiconductor cell.

AUTO-CORRELATION FUNCTION FOR A SECH-PULSE WITH THE FREQUENCY CHIRP

We can consider a two-beam scanning Michelson interferometer, which is the simplest optical auto-correlator. Such a device makes it possible to register the field strength auto-correlation function, which can be exploited via the inverse Fourier transform for finding the spectral power density $|S_S(\omega)|^2$ and measuring the width of the spectral contour. In so doing, one has to use a square-law photodiode detecting an interference of two incident field strengths $U_S(t)$ and $U_S(t-\tau)$, where the delay time τ of the second field can be varied by the corresponding movable mirror of the scanning interferometer. The issuing electronic signal is proportional to the energy E under registration, if the integration time of that photodiode is sufficiently long. Generally, this energy includes a background $G_0(0)$ and is proportional to the value

$$E \sim \int_{-\infty}^{\infty} [U_S(t) + U_S(t-\tau)]^2 dt \sim G_0(0) + 2G_A(\tau), \quad (1)$$

$$G_A(\tau) = \int_{-\infty}^{\infty} [U_S(t) \times U_S(t-\tau)] dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |B_S(\omega)|^2 \exp(-i\omega\tau) d\omega. \quad (2)$$

Equation (2) is true only when the field strength $U_S(t)$ is real-valued as for a sech-pulse. So, using Eq.(2), the function $G_A(\tau)$ can be calculated due to the Fourier transform of the spectral intensity contour

$$G_A(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |B_S(\omega)|^2 \exp(-i\omega\tau) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} [X_2^2(\omega) + Y_2^2(\omega)] \exp(-i\omega\tau) d\omega \quad (3)$$

In fact, the function $G_A(\tau)$ includes two terms

$$a) \quad G_1(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_2^2(\omega) \exp(-i\omega\tau) d\omega, \quad b) \quad G_2(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y_2^2(\omega) \exp(-i\omega\tau) d\omega. \quad (4)$$

The integral in Eq.(4) can be formally rewritten as

$$G_1(\tau) = \frac{1}{32\pi^3} \int_{-\infty}^{\infty} dt_1 \operatorname{sech}\left(\frac{t_1}{\tau_0}\right) \cos\left(\frac{bt_1^2}{2\tau_0^2}\right) \times \left\{ \int_{-\infty}^{\infty} dt_2 \operatorname{sech}\left(\frac{t_2}{\tau_0}\right) \cos\left(\frac{bt_2^2}{2\tau_0^2}\right) \times \int_{-\infty}^{\infty} d\omega \exp(-i\omega\tau) \cos[(\omega-\Omega)t_1] \cos[(\omega-\Omega)t_2] \right\}. \quad (5)$$

Because of $\frac{1}{2\pi} \int_{-\infty}^{\infty} dx \exp(-ixy) \cos(ax) = \frac{1}{2} [\delta(y+a) + \delta(y-a)]$, one can find that

$$H = \frac{\pi}{2} \exp(-i\tau\Omega) [\delta(\tau+t_1+t_2) + \delta(\tau+t_1-t_2) + \delta(\tau-t_1-t_2) + \delta(\tau-t_1+t_2)]. \quad (6)$$

Then, Eq.(5) can be substituted into Eq.(4)

$$G_1(\tau) = \frac{1}{32\pi^2} \exp(-i\tau\Omega) \int_{-\infty}^{\infty} dt_1 \operatorname{sech}\left(\frac{t_1}{\tau_0}\right) \cos\left(\frac{bt_1^2}{2\tau_0^2}\right) \times \left\{ \operatorname{sech}\left(\frac{\tau+t_1}{\tau_0}\right) \cos\left[\frac{b(\tau+t_1)^2}{2\tau_0^2}\right] + \operatorname{sech}\left(\frac{\tau-t_1}{\tau_0}\right) \cos\left[\frac{b(\tau-t_1)^2}{2\tau_0^2}\right] \right\}. \quad (7)$$

Now, applying the same approach to Eq.(4b), one can obtain

$$G_2(\tau) = \frac{1}{32\pi^2} \exp(-i\tau\Omega) \int_{-\infty}^{\infty} dt_1 \operatorname{sech}\left(\frac{t_1}{\tau_0}\right) \sin\left(\frac{bt_1^2}{2\tau_0^2}\right) \times$$

$$\times \left\{ \operatorname{sech} \left(\frac{\tau + t_1}{\tau_0} \right) \sin \left[\frac{b(\tau + t_1)^2}{2\tau_0^2} \right] + \operatorname{sech} \left(\frac{\tau - t_1}{\tau_0} \right) \sin \left[\frac{b(\tau - t_1)^2}{2\tau_0^2} \right] \right\}. \quad (8)$$

To simplify a sum of Eq.(7) and (8) one can use the standard ratios and find

$$G_A(\tau) = G_1(\tau) + G_2(\tau) = \frac{1}{32\pi^2} \exp(-i\tau\Omega) \int_{-\infty}^{\infty} dt_1 \operatorname{sech} \left(\frac{t_1}{\tau_0} \right) \times \\ \times \left\{ \operatorname{sech} \left(\frac{\tau + t_1}{\tau_0} \right) \cos \left[\frac{b(\tau^2 + 2t_1\tau)}{2\tau_0^2} \right] + \operatorname{sech} \left(\frac{\tau - t_1}{\tau_0} \right) \cos \left[\frac{b(\tau^2 - 2t_1\tau)}{2\tau_0^2} \right] \right\}. \quad (9)$$

Then, one can apply the ratios $\cos \left[\frac{b(\tau^2 \pm 2t_1\tau)}{2\tau_0^2} \right] = \cos \left(\frac{b\tau^2}{2\tau_0^2} \right) \cos \left(\frac{bt_1\tau}{\tau_0^2} \right) \mp \sin \left(\frac{b\tau^2}{2\tau_0^2} \right) \sin \left(\frac{bt_1\tau}{\tau_0^2} \right)$ to Eq.(9). Two terms with $\sin \left(\frac{bt_1\tau}{\tau_0^2} \right)$ give the odd functions under the integral signs in symmetrical limits, so that the corresponding integrals equal to zero. That is why with $G_A(\tau) = G_1(\tau) + G_2(\tau)$ we arrive at

$$G_A(\tau) = \frac{\exp(-i\tau\Omega)}{32\pi^2} \cos \left(\frac{b\tau^2}{2\tau_0^2} \right) \int_{-\infty}^{\infty} dt_1 \operatorname{sech} \left(\frac{t_1}{\tau_0} \right) \cos \left(\frac{bt_1\tau}{\tau_0^2} \right) \times \left\{ \operatorname{sech} \left(\frac{\tau + t_1}{\tau_0} \right) + \operatorname{sech} \left(\frac{\tau - t_1}{\tau_0} \right) \right\}. \quad (10)$$

At this stage, one can take the ratios $\operatorname{sech} \left(\frac{t_1}{\tau_0} \right) \operatorname{sech} \left(\frac{\tau \pm t_1}{\tau_0} \right) = 2 \left[\cosh \left(\frac{t_1 + \tau \pm t_1}{\tau_0} \right) + \cosh \left(\frac{t_1 - \tau \mp t_1}{\tau_0} \right) \right]^{-1}$ and include them into Eq.(10) to write $G_A(\tau) = g_1(\tau) + g_2(\tau)$, where

$$g_1(\tau) = \frac{\exp(-i\tau\Omega)}{16\pi^2} \cos \left(\frac{b\tau^2}{2\tau_0^2} \right) \int_{-\infty}^{\infty} dt_1 \cos \left(\frac{bt_1\tau}{\tau_0^2} \right) \times \left\{ \cosh \left(\frac{\tau + 2t_1}{\tau_0} \right) + \cosh \left(\frac{\tau}{\tau_0} \right) \right\}^{-1}. \quad (11)$$

$$g_2(\tau) = \frac{\exp(-i\tau\Omega)}{16\pi^2} \cos \left(\frac{b\tau^2}{2\tau_0^2} \right) \int_{-\infty}^{\infty} dt_1 \cos \left(\frac{bt_1\tau}{\tau_0^2} \right) \times \left\{ \cosh \left(\frac{\tau}{\tau_0} \right) + \cosh \left(\frac{2t_1 - \tau}{\tau_0} \right) \right\}^{-1}. \quad (12)$$

To integrate Eqs.(11) and (12) one has to introduce a pair of the new independent variables $\vartheta_{1,2} = 2t_1 \pm \tau$, so that $t_1 = (\vartheta_{1,2} \mp \tau)/2$ and $dt_1 = d\vartheta_{1,2}/2$. Again, one can be exploit the standard ratios

$$\cos \left(\frac{bt_1\tau}{\tau_0^2} \right) = \cos \left[\frac{b\tau(\vartheta_{1,2} \mp \tau)}{2\tau_0^2} \right] = \cos \left(\frac{b\tau\vartheta_{1,2}}{2\tau_0^2} \right) \cos \left(\frac{b\tau^2}{2\tau_0^2} \right) \pm \sin \left(\frac{b\tau\vartheta_{1,2}}{2\tau_0^2} \right) \sin \left(\frac{b\tau^2}{2\tau_0^2} \right)$$

Function $g_1(\tau)$ and $g_2(\tau)$ take the same form in terms of the corresponding new variable, ϑ_1 or ϑ_2 , namely,

$$g_{1,2}(\tau) = \frac{\exp(-i\tau\Omega)}{32\pi^2} \cos^2 \left(\frac{b\tau^2}{2\tau_0^2} \right) \int_{-\infty}^{\infty} d\vartheta_{1,2} \cos \left(\frac{b\vartheta_{1,2}\tau}{2\tau_0^2} \right) \times \left\{ \cosh \left(\frac{\vartheta_{1,2}}{\tau_0} \right) + \cosh \left(\frac{\tau}{\tau_0} \right) \right\}^{-1}. \quad (13)$$

The odd terms with $\sin[b\tau\vartheta_{1,2}/(2\tau_0^2)]$ gave zero. Using Ref.[2], the number 2.5.48-2, one can integrate Eq.(13)

$$\int_0^\infty d\vartheta_{1,2} \cos\left(\frac{b\vartheta_{1,2}\tau}{2\tau_0^2}\right) \times \left\{ \cosh\left(\frac{\vartheta_{1,2}}{\tau_0}\right) + \cosh\left(\frac{\tau}{\tau_0}\right) \right\}^{-1} = \frac{\pi\tau_0 \sin\left[b\tau^2/(2\tau_0^2)\right]}{\sinh(\tau/\tau_0) \sinh[\pi b\tau/(2\tau_0)]}. \quad (14)$$

Using Eqs.(11) and (12), one can express the field strength auto-correlation function inherent in a sech-like pulse as

$$G_A(\tau) = \frac{\tau_0 \exp(-i\tau\Omega)}{8\pi} \cos^2\left(\frac{b\tau^2}{2\tau_0^2}\right) \frac{\sin[b\tau^2/(2\tau_0^2)]}{\sinh(\tau/\tau_0) \sinh[\pi b\tau/(2\tau_0)]}. \quad (15)$$

The normalized traces for the real parts of this field strength auto-correlation function are shown in Fig.1.

Figure 1 The normalized real parts of field-strength auto-correlation functions for the sech-pulses with:

(a) $b = 0$, $\tau_0 = 1$, $\Omega = 40$; (b) $b = 1$, $\tau_0 = 1$, $\Omega = 40$.

Now, using Eq.(7), one can estimate the square-average duration of the field strength auto-correlation function as

$$a) \tau_A = \sqrt{T_{A2} - (T_{A1})^2}, \quad b) T_{An} = E_A^{-1} \int_{-\infty}^{\infty} \tau^n |G_A(\tau)|^2 d\tau, \quad c) E_A = \int_{-\infty}^{\infty} |G_A(\tau)|^2 d\tau. \quad (16)$$

It follows from Eq.(16b) that $T_{A1} \equiv 0$. Both the integrals in Eqs.(16) cannot be for the present calculated analytically in a closed form, so that the duration τ_A can be presented as the graphic function of the parameters τ_0 and b . That is why the variable τ and another values in Eqs.(16) will be simply normalized by τ_0 , and one can write with $\theta = \tau/\tau_0$

$$a) \frac{\tau_A}{\tau_0} = \left[\frac{1}{E_A} \int_{-\infty}^{\infty} \theta^2 \frac{\cos^4(b\theta^2/2) \sin^2(b\theta^2/2) d\theta}{\sinh^2(\theta) \sinh^2(\pi b\theta/2)} \right]^{1/2}, \quad b) E_A = \int_{-\infty}^{\infty} \frac{\cos^4(b\theta^2/2) \sin^2(b\theta^2/2) d\theta}{\sinh^2(\theta) \sinh^2(\pi b\theta/2)}. \quad (17)$$

The corresponding plots, divided in two parts for the convenience of practical usage, are depicted in Fig.2.

Figure 2. The normalized square-average time duration τ_A/τ_0 of the field strength auto-correlation function versus the frequency chirp b for a sech-pulse: (a) $b \leq 1$ and $b \geq 1$.

A NEW TECHNIQUE OF MEASURING THE TRAIN-AVERAGE PULSE WIDTH AS WELL AS THE VALUE AND SIGN OF THE FREQUENCY CHIRP INHERENT IN PICOSECOND OPTICAL PULSES WITH A SECH-LIKE SHAPE IN HIGH-REPETITION-RATE TRAINS

Here, we demonstrate an opportunity of providing experimental conditions, under which the train-average auto-correlation function of the field strength can serve as a source of exact and reliable information on the average values of both duration and frequency chirp of a low-power optical pulses traveling in high-repetition-rate trains. We proceed from the assumption that all pulses in a train are identical pulses with a sech-envelope described by Eq.(1). For a sech-pulse, the relation between the pulse parameters, namely, the frequency chirp b and the square-average pulse duration $\tau_{SA} = \pi\tau_0/(2\sqrt{3})$, and the square-average duration τ_A of the corresponding auto-correlation function follows from Eq.(16a) and is expressed through the function $F(b)$ presented in Fig.2 as

$$\tau_A = \frac{2\sqrt{3}\tau_{SA}}{\pi} F(b) = \tau_0 F(b). \quad (18)$$

The auto-correlation function durations τ_{Am} ($m=1,2$) obtained from the repeated measurements are coupled with the new values of the pulse duration τ_m and the frequency chirp b_m through formula (17). We assume that $\tau_m = \alpha_m \tau_0$ and $b_m = b_0 + \beta_m$, where τ_0 and b_0 are unknown values of the duration and frequency chirp, while the quantities α_m and β_m are determined by supplementary optical components, and find

$$a) \tau_{A0} = \tau_0 F(b_0), \quad b) \alpha_1^{-1} \tau_{A1} = \tau_0 F(b_0 + \beta_1), \quad c) \alpha_2^{-1} \tau_{A2} = \tau_0 F(b_0 + \beta_2). \quad (19)$$

Using Eqs.(19b), (17c) and Fig.2, one can find four values of b_0 , of which two coincide with each other and correspond to just the true value of the train-average frequency chirp of the pulses.

Figure 3. Design of the supplementary semiconductor cell: I is the domain of linear amplification controlled by the pump current J ; II is the domain with a fast-saturable absorption.

Once the pulse frequency chirp b_0 is determined, one can use formula (18a) to calculate the pulse duration τ_0 . For the supplementary electronically controlled optical component, one can propose exploiting a specific device based on an InGaAsP single-mode traveling-wave semiconductor laser heterostructure, which is quite similar to a saturable-absorber laser with clarified facets. This device comprises two domains, see Fig.3. Domain I of the linear amplification controlled by pumping current J_m has the length L_1 and is characterized by the low-signal gain factor $\kappa_1(J_m)$. Domain II of a fast-absorption saturation, created by a deep implantation of oxygen ions into the output facet of the heterostructure, has the length L_2 and is characterized by the low-signal absorption factor κ_2 and the saturation power P_s . Domain I is able to modify the peak power P_m of pulses entering domain II, so that: $P_m = P \exp [\kappa_1(J_m)L_1]$. The peak power P_m determines, in its turn, the values of the parameters α_m and β_m , reflecting the action of domain II on the pulses. In the low-signal case, one can use the relations

$$\text{a) } \alpha_m = (\rho P_m \sqrt{2+1})^{-1/2}, \quad \text{b) } \beta_m = -\zeta \rho P_m \sqrt{2}, \quad (19)$$

where ζ is the line-width enhancement factor, which equals usually $\zeta = 3-8$ and $\rho = (2P_s)^{-1} [\kappa_1(J_m)L_1]$ is the absorption parameter, which may be of the order of $\rho \leq 1 \text{ W}^{-1}$. Such a device makes possible performing the repeated measurements without re-adjusting the optical circuit and ensures additions $\beta_m \leq 5$ to the frequency chirp. Our proposal consists in measuring variations in the duration τ_A of the corresponding auto-correlation function inherent in a sech-pulse with a frequency chirp after inserting the supplementary electronically controlled semiconductor optical cell into the measurement circuit.