Predominance of Triaxial Shapes in Transitional Super-Heavy Nuclei: Ground-State Deformation and Shape Coexistence along the Flerovium ($Z = 114$) Chain of Isotopes

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We present the first triaxial beyond-mean-field study of even-even super-heavy nuclei. Calculations for the even Flerovium isotopes towards the supposed $N = 184$ neutron shell closure were performed using the effective finite-range density-dependent Gogny force. They include the restoration of the particle-number and angular-momentum symmetries and the mixing of different shapes using the generator coordinate method. The importance of the γ degree of freedom is highlighted by comparing the triaxial to axial-symmetric calculations performed within the same framework. For the three even Fl isotopes between the prolate ^{288}Fl and the oblate ^{296}Fl triaxial ground-state shapes are predicted, whereas axial-symmetric calculations suggest a sharp prolate-oblate shape transition between ^{290}Fl and ^{292}Fl . A novel type of shape coexistence, namely, that between two different triaxial shapes, is predicted to occur in ^{290}Fl . Finally, the existence of a neutron shell closure at $N = 184$ is confirmed, while no evidence is found for $Z = 114$ being a proton magic number.

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In the last two decades, the nuclear chart has been significantly extended at its upper end through the synthesis of new elements with proton numbers up to $Z = 118$ (e.g., see Ref. [1] and references therein). The experimental study of these heaviest nuclei is extremely difficult and a huge effort is currently spent on this showcase of nuclear physics research. Already now, sophisticated experimental techniques allow to obtain first nuclear structure information beyond the global properties such as α -decay half-lives and Q_α values [2,3] and also the chemical properties of some of the new elements have already been studied [4]. In the future, dedicated new facilities will not only enable the search for new elements with even higher Z , but also the production of the already synthesized isotopes at significantly higher rates, a prerequisite for more detailed studies of their nuclear structure. On the theoretical side, the region of super-heavy nuclei (SHN) has so far mainly been studied using either macroscopic-microscopic models or self-consistent mean-field approaches (MFA) employing a variety of nuclear forces, see Refs. [5,6] and references therein. In general, the description of the heaviest nuclei is a challenge to nuclear theory since it requires the extrapolation of model parameters, which are usually adjusted to stable nuclei, towards an unknown region. It is therefore no surprise that still no firm consensus has been reached concerning the location of the next proton and neutron shell closures above doubly magic ^{208}Pb .

While most theoretical studies concentrated on global properties, in some MFA studies [7–13] the coexistence of

prolate and oblate minima in the potential energy surfaces (PES) and the evolution from prolate to oblate ground-state shapes were observed for nuclei with $Z \geq 106$, $N \geq 170$, i.e., in the transition between the region of stable quadrupole deformation centered around ^{254}No and the supposed spherical shell gap at $N = 184$. This behavior resembles that observed in other regions of the nuclear chart in which shape transitions and shape coexistence phenomena have already been discussed in detail. Examples are the shape transitions in the W, Os, and Pt isotopes [14–16], the evolution of deformation and shape coexistence along the Kr isotopic chain [17–19], and the triple shape coexistence in Pb isotopes [20–22]. In general, these phenomena are observed in regions in which energy gaps on both the prolate and the oblate side of the single-particle energy diagram exist for nearby proton and neutron numbers. In Refs. [8,13], the occurrence of prolate and oblate minima in the PES of super-heavy nuclei was related to the single-particle level structure around the Fermi level calculated with Skyrme forces. We note, however, that in these heavy systems, the energy gaps are in general much smaller than those obtained in lighter nuclei. In addition, the PES of these weakly deformed nuclei are rather soft so that configuration mixing is important. This was demonstrated in the first *axial-symmetric* (in the following just *axial*) beyond-mean-field study of SHN [12,13]. Still, besides the configuration mixing, a second crucial ingredient has to be added to the calculations before meaningful theoretical predictions can be made. *Triaxial* MFA calculations have

clearly shown that the nuclei in the region $Z = 112\text{--}120$, $N = 170\text{--}176$ are characterized by potential energy surfaces which are very soft in the γ degree of freedom [8,10,23]. Therefore, correlation effects associated with the large-amplitude collective motion in the γ direction are expected to play a particularly important role. Hence, it seems necessary to consider both triaxial shapes and correlations beyond the mean field *at the same time* in order to make reliable theoretical predictions regarding the shape evolution and the possible occurrence of shape coexistence in this region.

In this Letter, we present the first study of super-heavy nuclei within the *triaxial* symmetry-conserving configuration-mixing (SCCM) approach [24–27]. As an example, we have chosen the even $N = 174\text{--}184$ Flerovium ($Z = 114$) isotopes. This chain is representative for others in the range $Z = 108\text{--}118$ since the dependence of the shape evolution on the atomic number is weak (see, for example, Figs. 10–12 of Ref. [9]). To clearly isolate the effects of the consideration of nonaxial shapes in the description of the transitional SHN, we compare in this work axial and triaxial SCCM calculations performed within the same theoretical framework.

The SCCM approach considers linear combinations of product wave functions, previously projected to the appropriate quantum numbers, corresponding to different configurations obtained by taking the nuclear deformations as generator coordinates in the sense of the generator coordinate method (GCM):

$$\begin{aligned} |\Phi_M^\sigma\rangle &= \sum_{\{\xi,K\}} f_{\{\xi,K\}}^\sigma P^Z P^N P_{MK}^I |\phi(\xi)\rangle \\ &= \sum_{\{\xi,K\}} f_{\{\xi,K\}}^\sigma |IMK; NZ; \{\xi\}\rangle, \end{aligned} \quad (1)$$

where $\{\xi\}$ are the shape parameters, $\{\beta, \gamma\}$ in the triaxial approach and $\{\beta\}$ in the axial one, additionally $K = 0$ in this case. The operators P^Z , P^N , and P_{MK}^I are projector operators associated with the particle number and the angular momentum, respectively, see Ref. [26], and $\sigma = 1, 2, \dots$ labels the states for a given value of the angular momentum I . The Hartree-Fock-Bogoliubov wave functions $|\phi(\xi)\rangle$ of Eq. (1) are determined by particle number restoration using the variation after projection approach (PNVAP) [28,29], i.e., minimizing the functional

$$E[\xi] = \frac{\langle \phi\{\xi\} | H P^Z P^N | \phi\{\xi\} \rangle}{\langle \phi\{\xi\} | P^Z P^N | \phi\{\xi\} \rangle} - \sum_\mu \lambda_{q_\mu} \langle \phi\{\xi\} | \hat{Q}_{2\mu} | \phi\{\xi\} \rangle, \quad (2)$$

where $\hat{Q}_{2\mu}$ are the quadrupole moment operators and λ_{q_μ} the Lagrange multipliers determined by the constraints $\langle \phi | \hat{Q}_{2\mu} | \phi \rangle = q_{2\mu}$. In the axial calculations $\mu = 0$ and in the triaxial case $\mu = 0, 2$. The deformation parameters (β, γ)

are defined by $\beta = \sqrt{20\pi(q_{20}^2 + 2q_{22}^2)}/3r_0^2 A^{5/3}$ and $\gamma = \arctan(\sqrt{2}q_{22}/q_{20})$ with $r_0 = 1.2$ fm and the mass number A . Finally, the coefficients $f_{\{\xi,K\}}^\sigma$ of the linear combination in Eq. (1) are found by a minimization of the energy in the Hilbert space spanned by the *linearly dependent* wave functions $|IMK; \{\xi\}\rangle$. The variational principle provides the Hill-Wheeler (HW) equation [30]

$$\sum_{\{\xi',K'\}} (\mathcal{H}_{\{\xi,K\},\{\xi',K'\}}^I - E^{I\sigma} \mathcal{N}_{\{\xi,K\},\{\xi',K'\}}^I) f_{\{\xi',K'\}}^{I\sigma} = 0. \quad (3)$$

Here, we have introduced the norm overlaps $\mathcal{N}_{\{\xi,K\},\{\xi',K'\}}^I = \langle IMK; \{\xi\} | IMK'; \{\xi'\} \rangle$ and the Hamiltonian overlap defined by a similar expression, see Ref. [26]. Note that the N , Z labels have been suppressed. Equation (3) is solved by standard techniques [24,26,30]: First, the norm matrix is diagonalized and its eigenvalues n_k^I and eigenvectors $u_k^I(\xi, K)$ provide the basis of the so-called natural states. The diagonalization of the Hamiltonian in this basis gives the eigenvalues $E^{I\sigma}$ of Eq. (3) and the eigenvectors $g_k^{I\sigma}$. In addition, the collective wave functions

$$p^{I\sigma}(\xi) = \sum_K p_K^{I\sigma}(\xi) = \sum_{k,K} g_k^{I\sigma} u_k^I(\xi, K), \quad (4)$$

are orthogonal and $|p^{I\sigma}(\xi)|^2$ can be interpreted as a probability amplitude.

To gain additional insight in the interpretation of the HW solution, the PES obtained by considering the simplified ansatz $|\Psi_M^{I\sigma}\{\xi\}\rangle = \sum_K F_K^{I\sigma} P^Z P^N P_{MK}^I |\phi(\xi)\rangle$, which amounts just to recover the symmetries of each wave function $|\phi(\xi)\rangle$, can be inspected. The particle number and angular momentum projected (PNAMP) PES as a function of $\{\xi\}$ is given by

$$E^{I\sigma}(\xi) = \frac{\langle \Psi_M^{I\sigma}\{\xi\} | H | \Psi_M^{I\sigma}\{\xi\} \rangle}{\langle \Psi_M^{I\sigma}\{\xi\} | \Psi_M^{I\sigma}\{\xi\} \rangle}. \quad (5)$$

As in the case of the full HW equation, this expression applies for both axial and triaxial calculations.

In the numerical applications the finite-range density-dependent Gogny interaction [31] with the D1S parametrization [32] is used together with a configuration space of thirteen major harmonic oscillator shells. Though the ground states of the Flerovium isotopes under study do have small deformations, $\beta < 0.2$, we have performed triaxial PNVAP calculations with seventeen shells to check the convergence. We found good agreement, in particular, with respect to the (β, γ) values at the minima of the PES. The only difference is that the PES obtained with 17 shells are slightly softer for larger β deformations. Although this could possibly lead to small changes of the excited-state energies, it will not affect the general features discussed in this work. For the numerical solution of the

FIG. 1. PNVAP (dashed thin) and PNAMP (solid thick) PES as a function of β for the even isotopes $^{288-298}\text{Fl}$. Positive (negative) β values indicate prolate (oblate) shapes. The squared collective wave functions of the ground and first-excited 0^+ states are shown in red and blue, respectively. For better visibility, they were arbitrarily scaled and shifted vertically in each panel and zero components were suppressed.

HW equation, we used a grid of 30 points which are uniformly distributed over the sector $0 \leq \beta \leq 0.3$, $0^\circ \leq \gamma \leq 60^\circ$. For the particle number projection 11 integration points were used while the projection on angular momentum was done employing a grid of 4096 points (16 points for each of the three Euler angles). The limitation of the generator coordinates to the quadrupole deformation is justified because higher multipoles, such as octupole, hexadecupole, etc., do not play an important role in these nuclei, see Fig. 3 of Ref. [9].

In the following, we will discuss the results of the calculations, first for the axial and then for the triaxial case, in three steps. First, the PNVAP energy surfaces, determined minimizing Eq. (2), are presented. Then, the PNAMP PES obtained after symmetry restoration, i.e., calculated using Eq. (5) for $I = 0$, are reported. Finally, the collective wave functions, Eq. (4), and energies of the first two 0^+ states, obtained solving the Hill-Wheeler equation, Eq. (3), are presented. Starting with the axial calculations, the PNVAP curves for the nuclei under study are shown in Fig. 1. A transition is observed from prolate minima for ^{288}Fl and ^{290}Fl to an oblate minimum for ^{292}Fl . For $^{294,296}\text{Fl}$ shallow minima are found at small values of β while for the presumably semi-magic isotope ^{298}Fl , a spherical minimum is obtained. After symmetry restoration, the PNAMP curves display significantly more pronounced minima (see Fig. 1), but the positions of the minima remain

FIG. 2. Excitation energies of the 0^+ states in $^{288-298}\text{Fl}$ from the axial (connected by a dashed line) and triaxial (connected by a solid line) calculations, displaced to the left and right of the mass number, respectively. Different shapes of the 0^+ states are tagged by symbols and colors according to the legend.

practically unchanged for all nuclei except ^{298}Fl . In this case, two nearly degenerate minima on the prolate and oblate sides show up, a behavior which is characteristic of spherical nuclei [33]. Finally, the collective wave functions shown in Fig. 1, or their average β values, provide direct information with respect to the shapes of the 0^+ states. Prolate ground states and oblate first excited 0^+ states are predicted for $^{288,290}\text{Fl}$, while for ^{292}Fl , the calculations yield an oblate ground and prolate excited state. For ^{294}Fl and ^{296}Fl , ground states of nearly spherical and oblate shape, respectively, are expected before perfect sphericity is reached at $N = 184$. The excitation energies of the 0^+ states are shown in Fig. 2 exhibiting a minimum for $^{290,292}\text{Fl}$. In summary, the axial calculations predict a prolate-oblate shape transition between ^{290}Fl and ^{292}Fl . In these two isotopes, prolate and oblate 0^+ states are expected within roughly 1 MeV.

Turning now to the results of the triaxial calculations, the PNVAP energy surfaces for $^{288-298}\text{Fl}$ are shown in the first row of Fig. 3. Since the only difference between the axial and the triaxial calculations is the consideration of the γ degree of freedom in the latter, the axis with $\gamma = 0^\circ$ ($\gamma = 60^\circ$) of the “pie-like” PES in Fig. 3 corresponds to the positive (negative) β range of the curves shown in Fig. 1. For the three lightest isotopes, the lowest minima in the axial calculations, prolate in the cases of $^{288,290}\text{Fl}$ and oblate in ^{292}Fl , remain axial-symmetric minima also in the triaxial calculations. The second axial-symmetric minima, however, turn into saddle points in the (β, γ) surfaces. In addition, a new triaxial minimum around $\gamma \sim 40^\circ$ shows up in ^{290}Fl . For ^{294}Fl , both minima obtained in the axial calculations are now saddle points and instead a minimum at $\gamma \sim 30^\circ$ appears. For $^{296,298}\text{Fl}$ the PES show shallow

FIG. 3. PNVAP (first row) and PNAMP (second row) PES in the (β, γ) plane for $^{288-298}\text{Fm}$. In these rows, the palette to the left shows the color code common for all panels. The PES are normalized to the minimum of each surface and the dashed white and solid black contour lines are separated by 0.5 and 1.0 MeV, respectively. In the last two rows, the squared collective wave functions of the $0^+_{1,2}$ states are shown. For each panel, an individual color code is shown. The dashed white contour lines display probability densities of 0.02, 0.04, 0.06, and 0.08 while the solid black lines uniformly cover the range between 0.1 and the maximum value.

oblate, respectively, deep spherical minima as in the case of the axial calculations. Next, we examine the PNAMP PES shown in the second row of Fig. 3. Already at first sight it is conspicuous that, in contrast to the axial case, the symmetry restoration has a very strong effect on the topology of the PES. All axial-symmetric minima of the PNVAP surfaces disappeared and instead for each isotope exactly one triaxial minimum is observed [34]. The γ values at the minima vary between $\gamma \sim 20^\circ$ in ^{288}Fm and $\gamma \sim 40^\circ$ in ^{296}Fm , while the values of β at the minima barely change with respect to those of the PNVAP PES. They smoothly decrease from $\beta \sim 0.16$ in ^{288}Fm to $\beta \sim 0.04$ in ^{298}Fm . The observation of triaxial minima in the PNAMP energy surfaces does, however, not necessarily imply that all considered Fm isotopes have triaxial shapes in their ground states. In some cases, for example ^{288}Fm and ^{296}Fm , the barrier between the triaxial minimum and the nearest axial-symmetric saddle point is low so that configuration-mixing may drive the nuclei back towards axial-symmetric shapes. Inspection of the ground-state wave functions shown in the third row of Fig. 3 illustrates that this indeed happens. Axial symmetry is recovered in ^{288}Fm and ^{296}Fm with maxima of the wave functions around $(0.15, 0^\circ)$ and $(0.05, 60^\circ)$,

respectively. For $^{290-294}\text{Fm}$, in contrast, the deep triaxial minima in the PNAMP surfaces translate into triaxial maxima of the ground-state wave functions. Only for the $N = 184$ isotope ^{298}Fm a spherical ground state is obtained. Thus, in contrast to the sharp prolate-oblate shape transition between ^{290}Fm and ^{292}Fm featured by the axial calculations, a much smoother transition from prolate ^{288}Fm to oblate ^{296}Fm , with an extended region of isotopes with triaxial ground-state deformations in between, is predicted once the γ degree of freedom is taken into account. The excitation energies of the first excited 0^+ states obtained in the triaxial calculations are compared to those reported from the axial approach in Fig. 2. In general, a similar trend is observed. However, for $^{288, 290, 292}\text{Fm}$, i.e., the nuclei with the softest PES, significantly lower 0^+_2 energies are obtained when the γ degree of freedom is taken into account. Though, the more important difference between the results of the two calculations is that the axial approach predicts the coexistence of *prolate* and *oblate* shapes in $^{290, 292}\text{Fm}$, while based on the triaxial calculations the coexistence between two different triaxial shapes is expected. In the case of ^{292}Fm , the wave functions shown in Fig. 3 indicate a strong mixing between the two 0^+ states leading to a smearing of the wave

FIG. 4. $E_x(2_1^+)$ in $^{288-298}\text{Fl}$ from the triaxial SCCM calculations (solid circles and lines). The same energies for $^{288-298}\text{Fl}$ calculated from the PNAMP PES are connected by dashed lines. The corresponding values for the $N = 184$ isotones ^{296}Cn ($Z = 112$) and ^{300}Lv ($Z = 116$) are included as red square and diamond, respectively.

functions over a wide range of γ . In contrast, the two 0^+ states in ^{290}Fl exhibit clearly distinct wave functions. Having very similar quadrupole deformations, $\beta \sim 0.13$, the wave function of the 0_1^+ state peaks around $\gamma \sim 20^\circ$ while that of the 0_2^+ state has a maximum around $\gamma \sim 40^\circ$. This type of shape coexistence, namely, that between two different triaxial shapes, has to our knowledge never been observed or predicted before in atomic nuclei. It is a prime example of *coexistence* because here two very similar sets of bands, i.e., two “ground-state” bands, two $K = 2$ bands, two $K = 4$ bands, etc., only shifted by a few hundred keV against each other, are expected to coexist side by side in one single excitation spectrum. Considering that a very similar shape evolution towards $N = 184$ is expected to occur in the neighboring isotopic chains over a wide range of Z (e.g., see Fig. 3 in Ref. [9] and Fig. 2 in Ref. [12]), the calculations presented in this work suggest the existence of an extended region of triaxial ground-state shapes in SHN. It is therefore very likely that ^{290}Fl is not the only candidate to show *triaxial-triaxial* shape coexistence.

Before closing this Letter, we briefly comment on the open question regarding the location of shell closures in the region of SHN. Often, the excitation energies of the first excited 2^+ states, $E_x(2_1^+)$, are used as indicators for shell or subshell closures. This quantity, calculated in the triaxial SCCM approach, is shown in Fig. 4. A smooth increase is observed for $A = 288-296$ before the 2_1^+ energy jumps to a value around 3.3 MeV for the $N = 184$ isotope ^{298}Fl . For comparison, we also include the corresponding energies calculated from the minima of the $I = 0$ and $I = 2$ PNAMP surfaces defined by Eq. (5). In view of the nice agreement between the two sets of values, we used the latter approach instead of very CPU-time consuming triaxial SCCM to calculate the 2_1^+ energies for $^{300}\text{Fl}_{186}$, $^{296}\text{Cn}_{184}$ and $^{300}\text{Lv}_{184}$, the even-even neighbors of $^{298}\text{Fl}_{184}$. As shown in Fig. 4, the 2_1^+ energy clearly drops in ^{300}Fl , i.e., beyond $N = 184$,

and thus confirms the $N = 184$ shell closure expected on the basis of the single-particle diagram (see Fig. 2 in Ref. [9]). In contrast, $E_x(2_1^+)$ remains nearly constant when two protons are added to or removed from ^{298}Fl . Hence, using the Gogny force no proton shell closure is predicted at $Z = 114$.

To summarize, we presented the first triaxial state-of-the-art calculations for super-heavy nuclei to study the shape transition in the region towards the $N = 184$ shell closure. These calculations predict six different ground-state deformations for the six isotopes under study, including an extended region of different triaxial ground-state deformations ($\gamma \sim 20^\circ$, $\gamma \sim 40^\circ$, and $\gamma \sim 30^\circ$, respectively) in the transition from the prolate nucleus ^{288}Fl to the oblate shape of ^{296}Fl . This finding is in contrast to the abrupt prolate-oblate shape transition between ^{290}Fl and ^{292}Fl predicted by the axial calculations. The predominance of triaxial shapes culminates in the prediction of a new type of shape coexistence in ^{290}Fl . In this nucleus, two 0^+ states with different triaxial shapes are expected within less than 500 keV. Finally, the calculated 2_1^+ energies along the Fl chain suggest a neutron shell closure at $N = 184$ while no evidence is found for a proton shell closure at $Z = 114$.

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- [1] Yu. Ts. Oganessian and V. K. Utyonkov, *Rep. Prog. Phys.* **78**, 036301 (2015).
- [2] D. Rudolph *et al.*, *Phys. Rev. Lett.* **111**, 112502 (2013).
- [3] U. Forsberg *et al.*, *Nucl. Phys.* **A953**, 117 (2016).
- [4] A. Yakushev *et al.*, *Inorg. Chem.* **53**, 1624 (2014).
- [5] A. Sobczewski and K. Pomorski, *Prog. Part. Nucl. Phys.* **58**, 292 (2007).
- [6] M. Bender, K. Rutz, P.-G. Reinhard, J. A. Maruhn, and W. Greiner, *Phys. Rev. C* **60**, 034304 (1999).
- [7] J.-F. Berger, L. Bitaud, J. Dechargé, M. Girod, and K. Dietrich, *Nucl. Phys.* **A685**, 1c (2001).
- [8] S. Cwiok, P.-H. Heenen, and W. Nazarewicz, *Nature (London)* **433**, 705 (2005).
- [9] M. Warda and J. L. Egidio, *Phys. Rev. C* **86**, 014322 (2012).
- [10] V. Prassa, T. Niksic, G. A. Lalazissis, and D. Vretenar, *Phys. Rev. C* **86**, 024317 (2012).
- [11] S. E. Agbemava, A. V. Afanasjev, T. Nakatsukasa, and P. Ring, *Phys. Rev. C* **92**, 054310 (2015).
- [12] P.-H. Heenen, J. Skalski, A. Staszczak, and D. Vretenar, *Nucl. Phys.* **A944**, 415 (2015).
- [13] P.-H. Heenen, B. Bally, M. Bender, and W. Ryssens, *EPJ Web Conf.* **131**, 02001 (2016).
- [14] P. Sarriuren, R. Rodríguez-Guzmán, and L. M. Robledo, *Phys. Rev. C* **77**, 064322 (2008).

- [15] R. Rodríguez-Guzmán, P. Sarriguren, L. M. Robledo, and J. E. García-Ramos, *Phys. Rev. C* **81**, 024310 (2010).
- [16] T. Nikšić, P. Ring, D. Vretenar, Y. Tian, and Z. Y. Ma, *Phys. Rev. C* **81**, 054318 (2010).
- [17] M. Bender, P. Bonche, and P.-H. Heenen, *Phys. Rev. C* **74**, 024312 (2006).
- [18] Y. Fu, H. Mei, J. Xiang, Z. P. Li, J. M. Yao, and J. Meng, *Phys. Rev. C* **87**, 054305 (2013).
- [19] T. R. Rodríguez, *Phys. Rev. C* **90**, 034306 (2014).
- [20] J. L. Egido, L. M. Robledo, and R. R. Rodríguez-Guzmán, *Phys. Rev. Lett.* **93**, 082502 (2004).
- [21] R. R. Rodríguez-Guzmán, J. L. Egido, and L. M. Robledo, *Phys. Rev. C* **69**, 054319 (2004).
- [22] J. M. Yao, M. Bender, and P.-H. Heenen, *Phys. Rev. C* **87**, 034322 (2013).
- [23] Z. Shi, A. V. Afanasjev, Z. P. Li, and J. Meng, *Phys. Rev. C* **99**, 064316 (2019).
- [24] M. Bender and P.-H. Heenen, *Phys. Rev. C* **78**, 024309 (2008).
- [25] J. M. Yao, J. Meng, P. Ring, and D. Vretenar, *Phys. Rev. C* **81**, 044311 (2010).
- [26] T. R. Rodríguez and J. L. Egido, *Phys. Rev. C* **81**, 064323 (2010).
- [27] J. L. Egido, *Phys. Scr.* **91**, 073003 (2016).
- [28] J. L. Egido and P. Ring, *Nucl. Phys.* **A383**, 189 (1982).
- [29] M. Anguiano, J. L. Egido, and L. M. Robledo, *Nucl. Phys.* **A696**, 467 (2001).
- [30] P. Ring and P. Schuck, *The Nuclear Many Body Problem* (Springer-Verlag, Berlin, 1980).
- [31] J. Dechargé and D. Gogny, *Phys. Rev. C* **21**, 1568 (1980).
- [32] J. F. Berger, M. Girod, and D. Gogny, *Nucl. Phys.* **A428**, 23c (1984).
- [33] J. L. Egido and L. M. Robledo, in *Extended Density Functionals in Nuclear Physics*, edited by G. A. Lalazissis, P. Ring, and D. Vretenar, Lecture Notes in Physics Vol. 641 (Springer-Verlag, Berlin, 2004), p. 269.
- [34] A. Hayashi, K. Hara, and P. Ring, *Phys. Rev. Lett.* **53**, 337 (1984).