Dimension dependence of the thermomechanical noise of microcantilevers

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Thermomechanical noise determines the lowest detection limits of microcantilever-based devices for measuring forces and surface stress variations. In this work, arrays of 334-nm-thick single-crystalline silicon microcantilevers with dissimilar lengths and widths from 50 to 500 μm and 20 to 200 μm , respectively, have been fabricated to calculate the minimal detectable force and surface stress on the basis of the measurement of the spring constant, resonance frequency, and quality factor. The calculated minimal detectable force and surface stress are of the orders of $10^{-15} \text{ N Hz}^{-1/2}$ and $10^{-7} \text{ N m}^{-1} \text{ Hz}^{-1/2}$, respectively, and both follow a nonintuitive dependence on the dimensions. The minimal detectable force decreases as the cantilevers are shorter and narrower, whereas the minimal detectable surface stress decreases by making the cantilevers shorter and wider. Theoretical expressions of the minimal detectable force and surface stress are provided as a function of the material properties, cantilever dimensions, and quality factor, which allow us to interpret the results. Both force and surface stress noises follow the same dependence on the quality factor and material properties, however, exhibit striking differences in the dimension dependences. The force and surface stress noises enhance with the quality factor. If the quality factor is kept constant, the force noise enhances as the cantilever is longer and wider, whereas the surface stress noise enhances by making the cantilever shorter and wider. The observed increase of the force noise with the length is attributed to the strong decrease of the quality factor. The results imply that the design of cantilevers for surface stress measurements in general should be different than for atomic force microscopy probes. © 2006 American Institute of Physics. [DOI: [10.1063/1.2164537](https://doi.org/10.1063/1.2164537)]

I. INTRODUCTION

Silicon micromachined cantilevers are widely employed as transducers of external forces, surface stress, and mass and heat fluxes in a variety of applications such as atomic force microscopy (AFM), magnetic-resonance force microscopy (MRFM), chemical and biological sensors, and photo-thermal spectroscopy, to name a few.^{1–6} The principle is the measurement of the position and motion of the free cantilever end that approximately follows a Hookean behavior. Depending on the application and the magnitude measured, cantilevers can be operated in two modes; static and dynamic. In the static mode, the average deflection is directly related to the external interaction, usually a force or variation of the surface stress.^{2–8} In the dynamic mode, the cantilever is set into oscillation, and the changes of the amplitude and phase of the oscillation are usually related to the gradient of an external force or the added mass on the cantilever.^{1,2,9–12}

This work is focused on the sensitivity of microcantilevers in the static mode in air, to measure tiny forces and surface stress variations. The first application is particularly

relevant to measure intermolecular forces between an AFM tip and a surface at the single molecule level.¹³ The second application is relevant for chemical and biological sensings, in which selective binding of a substance on a previously sensitized cantilever side produces a cantilever bending due to the differential surface stress between opposite cantilever sides.^{14–18} In both applications, cantilevers with high length-to-thickness ratio are used due to the low spring constant. The ultimate sensitivity is determined by the thermomechanical noise that manifests as the Brownian motion of the cantilever.^{19,20} Thermomechanical noise depends on the spring constant, resonance frequency, and quality factor (Q). Although there are accurate analytical models to predict the size dependence of the spring constant and resonance frequency, the quality factor cannot be readily predicted. The quality factor is defined as $2\pi U_s/U_d$, where U_s and U_d are the stored vibrational energy and the energy lost per oscillation cycle, respectively. Since the energy loss is due to several dissipation mechanisms, Q can be expressed as $1/Q = \sum_i 1/Q_i$ where Q_i is the quality factor due to each dissipation mechanism. The dissipation mechanisms include the viscous damping, the internal friction of the material, the

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clamping losses, and the thermoelastic dissipation.^{21,22} However, for pressures higher than 1 mtorr, the dominant source of dissipation is the viscous damping.²¹

Here, microcantilever arrays composed of 334-nm-thick silicon microcantilevers with various lengths and widths from 50 to 500 μm and from 20 to 200 μm , respectively, have been fabricated by using micromachining silicon technology. The spring constant, resonance frequency and Q factor have been measured to determine the dimension dependence of the thermomechanical noise. From this data, the minimal detectable force and surface stress have been calculated.

II. MATERIALS AND METHODS

A. Cantilever fabrication

The cantilevers were fabricated from commercially available Unibond silicon-on-insulators (SOI) (100) wafers (SOITEC). The top silicon layer, used as structural material, was 0.334 μm thick with uniformity better than 5%. The film was free of stress gradient and allowed the fabrication of up to 800- μm -long straight cantilever beams. The 1- μm -thick buried silicon oxide layer was used as an etch-stop layer for deep reactive ion etching (DRIE). A 1- μm -thick aluminum layer was deposited and patterned at the backside of the wafer to be used as a mask material during the DRIE process. The exposed 30-nm-thick thermal silicon oxide was removed by short etching in HF (49%) using the same mask as for the aluminum patterning. The top silicon layer was patterned by dry etching to define the cantilevers. The DRIE process from the backside was performed to define the cavity until the buried silicon oxide layer. The membranes of silicon oxide with cantilevers on top were formed at this stage. A silicon oxide etching in HF was performed to release the cantilevers.

B. Resonant frequency and Q measurements

The cantilever vibration was measured by using the optical beam deflection technique in a homebuilt setup. Briefly, a laser beam from a 3 mW laser diode (Edmund Optics Ltd) is focused on the free end of the cantilever and the reflected beam is collected by a four-segmented photodetector. The photocurrents from the upper and lower segments are amplified and filtered by current-voltage amplifiers (HMS Elektronik). The voltage signals are connected to an analog-to-digital data-acquisition card (National Instruments) for processing and monitoring. The acquisition software was programmed in LABVIEW (National Instruments). The resonance frequency and the quality factor were determined by fitting the frequency spectra of the Brownian motion with the harmonic-oscillator theory.

III. THEORY

A. Damped vibrating beam

Let us consider a vibrating beam subjected to a drag force due the interaction between the beam and the surrounding fluid (i.e., air). The drag force per unit length can be written as $f_1 \partial z / \partial t + f_2 \partial^2 z / \partial t^2$, where the first term represents

the viscous damping that leads to energy dissipation; and the second term, proportional to the acceleration, is the inertial force.^{23,24} The transversal vibration of the beam $z(x, t)$ can be described using the following partial differential equation:

$$EI \frac{\partial^4 z(x, t)}{\partial x^4} + f_1 \frac{\partial z(x, t)}{\partial t} + (\rho S + f_2) \frac{\partial^2 z(x, t)}{\partial t^2} = 0, \quad (1)$$

where E represents Young's modulus, ρ the cantilever density, S the cross-section area of the beam, and I the moment of inertia. For a rectangular cantilever of length L , width W , and thickness T , the cross-section area and the moment of inertia are $S = WT$ and $I = WT^3/12$, respectively. The boundary conditions for a singly clamped beam are $z(0, t) = \partial z / \partial t(0, t) = 0$ and $\partial^2 z / \partial t^2(L, t) = \partial^3 z / \partial t^3(L, t) = 0$. Applying the boundary conditions to the solution of Eq. (1), the eigenfrequencies are obtained,

$$\omega_n = \sqrt{\frac{EI k_n^4}{(\rho A + f_2) L^4} - \frac{1}{4} \left(\frac{f_1}{\rho A + f_2} \right)^2}, \quad (2)$$

where the subscript $n=0, 1, 2, \dots$ represents the different eigenmodes and the coefficient k_n^2 is 3.51, 22.01, 61.7, ..., for the first three vibration modes. Since the damping parameter f_1 is small for the vibration in gases and $f_2 \ll \rho S$, the resonance frequency of the vibrating beam can be approximated by

$$\omega_n \cong k_n^2 \left(\frac{E}{12\rho} \right)^{1/2} \frac{T}{L^2}. \quad (3)$$

The viscous damping part of the drag force provides a mechanism for energy dissipation, as it is proportional to the beam velocity. The Q factor for each vibration mode can be calculated by making use of the equation $Q = 2\pi U_s / U_d$, where U_s and U_d are the stored vibrational energy and the energy lost per oscillation cycle, respectively,²⁵

$$Q_n \cong \frac{\rho S \omega_n}{f_1}. \quad (4)$$

When the beam is driven by a sinusoidal force at a frequency ω , $F_0 e^{i\omega t}$, the steady oscillation of the free end of the cantilever can be written as $z(L, t) = \sum_n A_n e^{i(\omega t - \varphi_n)}$, where A_n and φ_n are the amplitude and phase shift of the n th mode, respectively. A_n is described by a frequency-dependent function analogous to the case of a damped harmonic oscillator,

$$A_n(\omega) = |\chi_n(\omega)| F_0 = \frac{\omega_n^2}{D_n \sqrt{(\omega^2 - \omega_n^2)^2 + (\omega \omega_n / Q_n)^2}} F_0, \quad (5)$$

where $|\chi_n(\omega)|$ is the modulus of the response function of the free end of the vibrating beam for the n th mode, and D_n is $m \omega_n^2$ ($m = \rho WTL$) and represents the mode-dependent spring constant. The spring constant of the first mode is usually referred to as k , and relates the force exerted on the free end of the beam to its displacement in equilibrium $k = D_1 = 3EI/L^3$. Approximately, $D_2 \cong 121.3 D_1$ and $D_3 \cong 951.6 D_1$.¹⁹

B. Thermal noise

Since the cantilever is in a thermal bath with the surrounding molecules at temperature θ , the cantilever is sub-

jected to a random and uncorrelated time-dependent force that results into the Brownian motion. The thermal force has a white spectral density and a Gaussian distribution with zero mean. The spectral density of the thermal force for the n th mode is given by^{20–25}

$$\psi_n(\omega) = \frac{2 k_B \theta D_n}{\pi Q_n \omega_n} \quad (6)$$

The frequency distribution of the thermal noise, i.e., the Brownian motion, is given by the modulus of the response function $|\chi_n(\omega)|$. The thermal noise for a bandwidth $\Delta\omega$ is described by

$$\langle z_{\text{th}}^2 \rangle = \sum_n \int_0^{\Delta\omega} |\chi_n(\omega)|^2 \psi_n d\omega. \quad (7)$$

For the case in which the bandwidth is much smaller than the resonance frequency of the first mode, the thermal noise is approximately given by

$$\langle z_{\text{th}}^2 \rangle = \sum_n \frac{2}{\pi} \frac{k_B \theta}{Q_n \omega_n D_n} \Delta\omega. \quad (8)$$

The thermal noise corresponding with the higher flexural modes can be neglected with respect to the first mode. For instance, the second mode thermal noise is 253 times smaller than the first mode noise. Therefore the thermal noise can be approximately described by

$$\langle z_{\text{th}}^2 \rangle \cong \frac{2}{\pi} \frac{k_B \theta}{Q \omega_r k} \Delta\omega, \quad (9)$$

where $Q=Q_1$, $\omega_r=\omega_1$, and $k=D_1$.

C. Minimal detectable force and surface stress

The minimal detectable force acting on the free end of the cantilever for a bandwidth $\Delta\omega$ is $F_{\text{min}} = k \langle z_{\text{th}}^2 \rangle^{1/2}$,

$$F_{\text{min}} = \left(\frac{2 k_B \theta k \Delta\omega}{\pi Q \omega_r} \right)^{1/2}. \quad (10)$$

In sensing applications, the cantilever bending is due to the differential surface stress between opposite sides of the cantilever. The differential surface stress arises from the uneven adsorption of molecules on opposite surfaces of the cantilever. The relationship between the differential surface stress ($\Delta\sigma$) and the cantilever deflection (z) is²⁶

$$\Delta\sigma \cong \frac{E}{3(1-\nu)} \left(\frac{T}{L} \right)^2 z. \quad (11)$$

By substituting z by $\langle z_{\text{th}}^2 \rangle^{1/2}$ [Eq. (8)] and writing T/L as a function of the spring constant and beam width in Eq. (11), the minimal detectable surface stress can be written as

$$\Delta\sigma_{\text{min}} \cong \frac{2^{4/3}}{3} \frac{E^{1/3}}{1-\nu} \frac{k^{1/6}}{W^{2/3}} \left(\frac{2 k_B \theta \Delta\omega}{\pi Q \omega_r} \right)^{1/2}. \quad (12)$$

The minimal detection limits for both surface stress and force can be enhanced by decreasing the spring constant and increasing the quality factor and resonance frequency. However, small spring constants are more important for force detection ($F_{\text{min}} \sim k^{1/2}$) than for surface stress ($\Delta\sigma_{\text{min}} \sim k^{1/6}$).

FIG. 1. Optical micrograph of the two kinds of cantilever arrays used in this work. (a) An array composed of seven cantilevers with varying length from 50 to 500 μm . The width and thickness are 40 and 0.334 μm , respectively. (b) An array of five cantilevers with varying width from 20 to 200 μm . The length and the thickness are 200 and 0.334 μm , respectively.

IV. RESULTS

Two kinds of arrays with dissimilar cantilevers were fabricated; one composed of six cantilevers with different lengths (50, 100, 200, 300, 400, and 500 μm) and a width of 40 μm and the other composed of five cantilevers with different widths (20, 40, 60, 100, and 200 μm) and a length of 200 μm (Fig. 1). The thickness of the cantilevers is 334 nm. The error in the cantilever dimensions was about 5% for the thickness and about $\pm 0.3 \mu\text{m}$ for the width and length dimensions. The measured values of the spring constant, resonance frequency, and quality factor are summarized in Table I. The spring constant was measured by exerting a controlled force on the cantilever with an atomic force microscopy cantilever.^{27,28} The error in this measurement mainly arises from the uncertainty in the position where the force is exerted. The spring constant was calculated by assuming that the force is exerted at $10 \pm 5 \mu\text{m}$ from the free end. The resonance frequency and the quality factor were determined by fitting the frequency spectrum of the thermal noise to the driven damped beam model described in the previous section [Eq. (5)]. The errors in both magnitudes are given by the fitting.

The measured spring constant ranges from 3.3×10^{-4} to 0.08 N/m. A discrepancy between the experimental values and the theory described by the formula $k = EWT^3/4L^3$ (dotted line) is found (Fig. 2). This is more noticeable for the shorter and wider microcantilevers. The deviation is attributed to the compliance of the chip region

TABLE I. Data of the cantilever dimensions, experimental, and theoretical values of the resonance frequency, spring constant, and experimental values of the quality factor.

$L \times W$ (μm)	k Expt. (N/m)	k Theor. (N/m)	Q	f Expt. (kHz)	f Theor. (kHz)
50 $\times$ 40	0.078 5	0.5038	18.92	81.82	184.33
100 $\times$ 40	0.034 0	0.0630	7.19	29.78	46.08
200 $\times$ 40	0.006 7	0.0079	3.29	8.79	11.52
300 $\times$ 40	0.002 2	0.0023	2.1	4.08	5.12
400 $\times$ 40	0.001 1	0.0010	1.53	2.34	2.88
500 $\times$ 40	0.000 33	0.0005	1.40	1.90	1.84
200 $\times$ 20	0.024 6	0.0039	10.19	8.15	11.52
200 $\times$ 40	0.014	0.0079	9.77	8.20	11.52
200 $\times$ 60	0.012 0	0.0118	9.42	8.52	11.52
200 $\times$ 100	0.008 0	0.0197	8.03	8.65	11.52
200 $\times$ 200	0.003 9	0.0394	8.00	8.99	11.52

FIG. 2. Experimental values of the spring constant as a function of the length (a) and width (b) of the cantilever. The dotted and solid lines represent the theoretical expression $k=EWT^3/4L^3$ and to the fitting curve to the “two spring model,” respectively.

where the cantilevers are clamped. The DRIE process employed for the cantilever fabrication produces a thinning of the chip region where the array is supported. Thus the cantilevers are held by a 22–26- μm -long and 334-nm-thick region of the chip. The effect of the clamping on the spring constant can be understood by modeling the system as two springs in series: one for the clamping of the beam to the support k_{clamping} and the other for the cantilever $k_{\text{cant}}=EWT^3/4L^3$. The measured spring constant is $k_{\text{meas}}=[(1/k_{\text{cant}})+(1/k_{\text{clamping}})]^{-1}$. As can be seen in Fig. 2, a very good fitting of the experimental values was obtained by using the “two spring model” (solid line), in which the clamping effective spring constant is 0.08 ± 0.01 N/m. The thickness and Young’s modulus values used in the fitting were 0.334 μm and 169 GPa (nominal value of silicon in the directions $\langle 110 \rangle$), respectively.

Figure 3 shows the resonance frequency of the funda-

FIG. 3. Experimental values of the resonance frequency as a function of the cantilever length (triangles) and width (circles). The resonance frequency is calculated from the fitting of the frequency spectrum of the thermal noise to the harmonic-oscillator model. The inset shows the thermal noise frequency spectrum for a 300- μm -long and 40- μm -wide cantilever. The dotted and solid lines represent the theoretical resonance frequency for the singly clamped beam model, $f_0=1.020(E/\rho)^{1/2}T/L^2$, and the fitting curve to the “two spring model.”

FIG. 4. Experimental values of the quality factor as a function of the cantilever length (a) and width (b). The dashed line represents the theoretical values.

mental vibration mode as a function of the length and the width. The figure inset shows the thermal noise spectrum for a 300- μm -long and 40- μm -wide cantilever in air. Two resonance peaks can be clearly distinguished at 4.14 and 26.3 kHz that correspond with the first and second vibration modes. The ratio between the resonance frequencies of the first two modes is $\omega_0/\omega_1=0.157$ that is very close to the theoretical value of 0.159. On the other hand, the values of the fundamental resonance frequency are smaller than those predicted by the theory [Eq. (3)]. This deviation increases as the cantilevers are stiffer. Thus the experimental resonance frequencies for the 100- μm and 50- μm -long cantilevers are of about 20% and 50% smaller than the theoretical values, respectively (see Table I). To explain this discrepancy, we recall the “two spring” model in which the measured resonance frequency is given by $\omega_{\text{meas}}^2=(1/m_{\text{clamping}}^*+m_{\text{cant}}^*)\times[(1/m_{\text{cant}}^*\omega_{\text{cant}}^2)+(1/m_{\text{clamping}}^*\omega_{\text{clamping}}^2)]^{-1}$, where m_{clamping}^* and m_{cant}^* are the effective masses of the clamping region and the cantilever, respectively. For an infinitely rigid clamping region $k_{\text{meas}}\cong k_{\text{cant}}$ and $\omega_{\text{meas}}\cong\omega_{\text{cant}}$. A good fitting of the experimental values is obtained by using this model, which gives a resonance frequency of the clamping region of 334.6 kHz and a clamping-to-cantilever mass ratio of 0.19, approximately. The effect of the compliant clamping region is the decrease of the spring constant and resonance frequency. This effect is particularly important on the stiffest cantilevers, i.e., the shorter and wider.

Figure 4 shows the experimental values of the quality factor (symbols) as a function of the length and the width in air. Q decreases from 19 to 1.9 when the length increases from 50 to 500 μm for a width of 40 μm , while it increases from 6.0 to 9.3 when the width increases from 20 to 200 μm for a length of 200 μm . The Q factor is inversely proportional to the energy loss per oscillation cycle. The dissipation mechanisms include the viscous damping, the internal friction, and the clamping losses, to name a few, however, in air, the dominant source of noise is the viscous damping.^{21,22} The Q factor due to the viscous damping has been theoretically calculated by Sader, and it can be written as²⁴

FIG. 5. Minimal detectable force per bandwidth unit as a function of the length (a) and the width (b) and minimal detectable surface stress per bandwidth unit as a function of the length (c) and width (d). The minimal detection values are obtained by calculating the thermomechanical noise from the experimental values of the spring constant, resonance frequency, and Q factor.

$$Q = \frac{(4\rho_c T / \pi \rho_{\text{air}} W) + \Gamma_r(\omega_r)}{\Gamma_i(\omega_r)}, \quad (13)$$

where ρ_c and ρ_{air} are the cantilever and air densities, respectively, and $\Gamma_r(\omega_r)$ and $\Gamma_i(\omega_r)$ are the real and imaginary parts of the hydrodynamic function given by

$$\Gamma(\omega_r) = 1 + \frac{4iK_1[-i\sqrt{i\text{Re}(\omega_r)}]}{\sqrt{i\text{Re}(\omega_r)}K_0[-i\sqrt{i\text{Re}(\omega_r)}]}, \quad (14)$$

where K_1 and K_0 are the third-order modified Bessel functions and Re is the Reynolds number, which determines the importance of the viscous effect, and is given by

$$\text{Re}(\omega_r) = \frac{\rho_{\text{air}} \omega_r W^2}{4\eta_{\text{air}}}, \quad (15)$$

where η_{air} is the air viscosity. The lines in Fig. 4 represent the values of the Q factor calculated by using Sader's model. Although, Sader's model qualitatively follows the experimental trend, i.e., the Q factor decreases with the length and it increases with the width, there are significant quantitative differences between the theoretical and experimental values. The discrepancy between the theory and experiments is attributed to that the hydrodynamic function used in the theory assumes that the cantilever behaves as an infinitely long cylinder. In addition, the theory does not take into account the effect of the semiflexible clamping region.

Figure 5 shows the minimal detectable force and surface stress as a function of the length and width of the cantilever beam. The values of the minimal force and surface stress are

calculated from the thermomechanical noise by substituting the experimentally determined values of k , Q , and ω_0 in Eqs. (10) and (12). The minimal detectable force is of the order of $10^{-15} \text{ N Hz}^{-1/2}$. The force resolution enhances as the width decreases, and surprisingly, as the length decreases (stiffer), except for the 500- μm -long cantilever whose sensitivity is similar to that of the 200- μm -long cantilever. The surface stress also exhibits a striking behavior. The minimal detectable surface stress is of the order of $10^{-7} \text{ N m}^{-1} \text{Hz}^{-1/2}$, at least four orders of magnitude smaller than the surface stress induced by the selective binding of organic and biological molecules. Counterintuitively, the surface stress resolution enhances as the length decreases and the width increases, i.e., the cantilever is stiffer.

V. DISCUSSION

By writing the spring constant and the resonance frequency as a function of the dimensions and the material properties [$k = EWT^3/4L^3$ and Eq. (3) in Eqs. (10) and (12)], the minimal detectable force and surface stress imposed by the thermomechanical noise can be explicitly written as a function of Q and the dimensions,

$$F_{\min} \cong 0.3963(E\rho)^{1/4} \left(\frac{k_B \theta \Delta\omega}{Q} \right)^{1/2} \left(\frac{WT^2}{L} \right)^{1/2}, \quad (16)$$

$$\Delta\sigma_{\min} \cong 0.5284 \frac{(E\rho)^{1/4}}{1-\nu} \left(\frac{k_B \theta \Delta\omega}{Q} \right)^{1/2} \left(\frac{L}{W} \right)^{1/2}. \quad (17)$$

Both, $F_{\min}$ and $\Delta\sigma_{\min}$ are inversely proportional to the square root of Q and have the same dependence on the material properties, in which the detection limits can be moderately enhanced by using materials with low values for Young's modulus and the density. However, the dimension dependences of $F_{\min}$ and $\Delta\sigma_{\min}$ are strikingly different. This should imply profound consequences in the design rules for cantilever sensors and AFM probes, which in general should be different. Equation (16) implies that the minimal detectable force decreases with the square root of the length (for Q constant), i.e., the softer the cantilever, the higher the force resolution. However, for the cantilevers fabricated here, the force resolution slightly degrades as the length increases from 50 to 400 μm , as shown in Fig. 5. This is due to the decrease of Q as the length increases (Fig. 4). The Q reduction gets smoother for lengths higher than 400 μm , in which the Q factors are very low, close to unity. This produces the force resolution enhancement for the 500- μm -long cantilever. Equation (16) also implies that the force resolution enhances as the cantilever is narrower (softer) if Q is kept constant. This behavior is experimentally observed (Fig. 5) as the quality factor shows small dependence on the width, as shown in Fig. 4. On the other hand, the minimal detectable surface stress follows a different dependence with the dimensions [Eq. (17)]. If Q remains constant, the surface stress resolution is independent of the thickness and enhances as the square root of the width-to-length ratio increases. In addition, as Q decreases with the length (Fig. 4), the minimal detectable surface stress significantly enhances as the length decreases (Fig. 5). Moreover, the surface stress resolution enhances by making the cantilevers wider, as Q shows small dependence on the width.

In this work, we have found that the minimal detection limits for measuring force and surface stress follow a nonintuitive behavior. Thus, stiffer cantilevers exhibit higher sensitivity for measuring surface stress, whereas the force sensitivity follows a more complicated behavior. These results are based on the intrinsic thermomechanical noise. The instrumental and environmental sources of noise such as the displacement sensor and variations in the laboratory temperature have not been considered. For sufficiently soft cantilevers, high measurement bandwidth, and sensitive displacement sensor, the thermomechanical noise is the dominant source of noise, and the cantilever must be designed with a geometry in order to minimize the force and surface stress noise due to the thermal kicking forces. The optimum design, as shown here, is not intuitive, and is highly dependent on the kind of measurement, surface stress or external forces (see Eqs. (16) and (17)). For cases, in which the extrinsic sources of noise (instrumental and environmental) are dominant, the cantilever dimensions should be chosen in order to minimize the cantilever stiffness. This can be easily achieved for both applications, surface stress and force measurements, by maximizing the length-to-thickness ratio. The extrinsic source of noises becomes dominant for stiff cantilevers, small bandwidths, and relatively noisy displacement sensors.

VI. CONCLUSIONS

Arrays of 334-nm-thick silicon microcantilevers have been fabricated with lengths from 50 to 500 μm and widths from 20 to 200 μm . The spring constant, resonance frequency, and quality factor have been measured in order to calculate the minimal detectable force and surface stress imposed by the thermomechanical limit. The experimental values of the spring constant and resonance frequency show small deviation with respect to the well-established continuum elastic theory of the beam. However, the behavior of the quality factor can be hardly predicted. The theoretically affordable model proposed by Sader significantly deviates from the experimental values. This discrepancy is attributed to that the hydrodynamic function used in the theory assumes that the cantilever behaves as an infinitely long cylinder. In addition, the theory does not take into account the effect of the semiflexible clamping region. The calculated values of the minimal detectable force and surface stress imposed by the thermomechanical limit are of the orders of 10^{-15} N and 10^{-7} N/m, respectively. Contrary to the expected behavior, the shorter (stiffer) cantilevers exhibit the lowest detection limits in force, and the wider and shorter cantilevers are the most sensitive for surface stress detection. The different dimension dependence of the minimal detectable force and surface stress highlights that the geometrical design of the microcantilevers should depend on the type of interaction to be measured, i.e., external forces or surface stress variations.

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- ¹Y. Martin, C. C. Williams, and H. K. Wickramasinghe, *J. Appl. Phys.* **61**, 4723 (1987).
- ²H. P. Lang *et al.*, *Anal. Chim. Acta* **393**, 59 (1999).
- ³R. Raiteri, G. Nelles, H.-J. Butt, W. Knoll, and P. Skladal, *Sens. Actuators B* **61**, 213 (1999).
- ⁴J. K. Gimzewski, Ch. Gerber, E. Meyer, and R. R. Schlittler, *Chem. Phys. Lett.* **217**, 589 (1994).
- ⁵T. Thundat, E. A. Wachter, S. L. Sharp, and R. J. Warmack, *Appl. Phys. Lett.* **66**, 1695 (1995).
- ⁶D. Rugar, R. Budakian, H. J. Mamin, and B. W. Chui, *Nature (London)* **430**, 329 (2004).
- ⁷E. T. Arakawa, N. V. Lavrik, S. Rajic, and P. G. Datskos, *Ultramicroscopy* **97**, 459 (2003).
- ⁸H. J. Butt, *J. Colloid Interface Sci.* **180**, 251 (1996).
- ⁹B. Ilic, D. Czaplowsky, H. G. Craighead, P. Neuzil, C. Campagnolo, and C. Batt, *Appl. Phys. Lett.* **77**, 450 (2000).
- ¹⁰B. Ilic, H. G. Craighead, S. Krylov, W. Seneratne, C. Ober, and P. Neuzil, *J. Appl. Phys.* **95**, 3694 (2004).
- ¹¹N. V. Lavrik and P. G. Datskos, *Appl. Phys. Lett.* **82**, 2697 (2003).
- ¹²A. Gupta, D. Akin, and R. Bashir, *Appl. Phys. Lett.* **84**, 1976 (2004).
- ¹³P. Hinterdorfer, W. Baumgartner, H. J. Gruber, K. Schilder, and H. Schindler, *Proc. Natl. Acad. Sci. U.S.A.* **93**, 3477 (1996).
- ¹⁴G. Wu, R. H. Datar, K. M. Hansen, T. Thundat, R. J. Cole, and A. Majumdar, *Nat. Biotechnol.* **19**, 856 (2001).
- ¹⁵G. Wu *et al.*, *Proc. Natl. Acad. Sci. U.S.A.* **980**, 1560 (2001).
- ¹⁶R. McKendry *et al.*, *Proc. Natl. Acad. Sci. U.S.A.* **99**, 9783 (2002).
- ¹⁷M. Álvarez, A. Calle, J. Tamayo, L. Lechuga, A. Abad, and A. Montoya, *Biosens. Bioelectron.* **18**, 649 (2003).
- ¹⁸M. Álvarez, L. G. Carrascosa, M. Moreno, A. Calle, A. Zaballos, L. M.

- Lechuga, C. A. Martínez, and J. Tamayo, *Langmuir* **20**, 9663 (2004).
- ¹⁹S. Rast, C. Wattinger, U. Gysin, and E. Meyer, *Nanotechnology* **11**, 169 (2000).
- ²⁰F. Gittes and C. F. Schmidt, *Eur. Biophys. J.* **27**, 75 (1998).
- ²¹K. Y. Yasumura, T. D. Stowe, E. M. Chow, T. Pfafman, T. W. Kenny, B. C. Stipe, and D. Rugar, *J. Microelectromech. Syst.* **9**, 117 (2000).
- ²²F. R. Blom, S. Bouwstra, M. Elwenspoek, and J. H. J. Fluitman, *J. Vac. Sci. Technol. B* **10**, 19 (1992).
- ²³S. Kirstein, M. Mertesdorf, and M. Schönhoff, *J. Appl. Phys.* **84**, 1782 (1998).
- ²⁴J. E. Sader, *J. Appl. Phys.* **84**, 64 (1998).
- ²⁵R. B. Bhiladvala and Z. J. Wang, *Phys. Rev. E* **69**, 036307 (2004).
- ²⁶G. G. Stoney, *Proc. R. Soc. London, Ser. A* **82**, 172 (1909).
- ²⁷Ch. T. Gibson, G. S. Watson, and S. Myhra, *Nanotechnology* **7**, 259 (1996).
- ²⁸K. E. Peterson and C. R. Guarnieri, *J. Appl. Phys.* **50**, 6761 (1979).