Standard model radiative corrections in the pion form factor measurements do not explain the $a_\mu$ anomaly

Francisco Campanario,¹ Henryk Czyż,²,⁎ Janusz Głuza,²,³ Tomasz Jeliński,² Germán Rodrigo,¹ Szymon Tracz,¹ and Dmitry Zhuridov²

¹Instituto de Física Corpuscular, CSIC-Universitat de València, E-46980 Paterna, Valencia, Spain
²Institute of Physics, University of Silesia, PL-40-007 Katowice, Poland
³Faculty of Science, University of Hradec Králové, CZ-500 03 Hradec Králové, Czech Republic

(Received 26 March 2019; published 10 October 2019)

In this paper, we address the question of whether the almost four standard deviations difference between theory and experiment for the muon anomalous magnetic moment $a_\mu$ can be explained as a higher-order Standard Model perturbation effect in the pion form factor measurements. This question has, until now, remained open, obscuring the source of discrepancies between the measurements. We calculate the last radiative corrections for the extraction of the pion form factor, which were believed to be potentially substantial enough to explain the data within the Standard Model. We find that the corrections are too small to diminish existing discrepancies in the determination of the pion form factor for different kinematical configurations of low-energy BABAR, BES-III and KLOE experiments. Consequently, they cannot noticeably change the previous predictions for $a_\mu$ and decrease the deviations between theory and direct measurements. To solve the above issues, new data and better understanding of low-energy experimental setups are needed, especially as new direct $a_\mu$ measurements at Fermilab and J-PARC will provide new insights and substantially shrink the experimental error.

DOI: 10.1103/PhysRevD.100.076004

I. INTRODUCTION

The anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$ is predicted in the Standard Model (SM) with an accuracy at the level of 0.3 ppm [1–4] while the precision of the direct experimental measurement is of the order of 0.54 ppm [5]. Remarkably, the tension between the experimental measurement and the SM prediction, $a_\mu^{\exp} - a_\mu^{\text{SM}} = 268(63)/(43)\times 10^{-11}$ [4], corresponds to about 3.5 standard deviations. This is one of the largest and long-standing discrepancies between the SM and experiment. The central question is whether the discrepancy is due to unknown new physics effects beyond the SM (new particles and new interactions) or to theoretical and/or experimental errors not completely under control. Concerning the beyond the SM option, there are a few models, which are able to shift the theoretical prediction for $a_\mu$ in the direction of the experimental value in selected regions of the parameter space [6,7]. However, many commonly used nonstandard models have problems to accommodate this discrepancy and will have to be modified or rejected, when that harbinger of new physics is confirmed [8]. There are remarkable recent QCD + QED lattice calculations of $a_\mu$ [9–12]. However, their accuracy is still far from being competitive with the dispersive/experimental approach [1–4]. Nonetheless, a hybrid lattice/dispersive approach provides currently the most accurate prediction [12], and additional improvements are expected in the pure lattice calculations.

In this paper, we scrutinize possible flaws in the estimation of the theoretical and experimental errors by re-investigating the SM input into data analysis related to pion-photon interactions, including so far missing and potentially relevant radiative corrections.

This study is particularly timely due to the fact that new measurements at Fermilab [13] and J-PARC [14] aim to reduce the experimental error of the direct measurement by a factor of four. Therefore, the theoretical and experimental groups that contribute to the accurate determination of $a_\mu$ must point to a similar precision in the near future [15], and also to understand definitely the source of the present discrepancies.

The QED and pure electroweak SM contributions to $a_\mu$ are known presently with a satisfactory precision and the biggest errors in the estimation of $a_\mu$ arise from the hadronic vacuum polarization [1–3]. One of the main obstacles to reduce the error of the hadronic contribution

⁎Corresponding author.
henryk.czyz@us.edu.pl

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI. Funded by SCOAP3.
to $a_{\mu}$ is the discrepancy between the experimental extrac-
tions of the pion form factor from the cross section of the 
reaction $e^+ e^- \to \pi^+\pi^- \gamma$ by using the initial state radiation 
(ISR) method [16–22]. The most relevant hadronic con-
tribution (about 70%) to the determination of $a_{\mu}$ comes 
from the region of the $\pi^+\pi^-$ invariant mass around the 
$\rho - \omega$ resonances. The biggest difference [18], between 
KLOE and BABAR measurements, amounts there to about 
2%. It goes even up to 10% around the $\omega$ resonance region, 
though that region is very narrow and its contribution to $a_{\mu}$ 
is smaller. For higher $\pi^+\pi^-$ invariant masses (at 0.9 GeV) 
the difference raises to 5%.

A possible source of the discrepancy might be attributed 
to missing radiative corrections in the event generator(s) 
used in the experimental analyses since radiative correc-
tions could be different at different energies of the exper-
iments. The Monte Carlo event generator PHOKHARA [23] 
was used by all the experiments, both for the mode 
$e^+ e^- \to \mu^+\mu^- \gamma$, which serves as a luminosity monitoring 
process, and the mode $e^+ e^- \to \pi^+\pi^- \gamma$, which was used to 
extract the cross section of the reaction $e^+ e^- \to \pi^+\pi^-$, and 
the corresponding pion form factor. In Ref. [24], the 
complete QED next-to-leading order (NLO) radiative 
corrections to the cross section of the reaction $e^+ e^- \to 
\mu^+\mu^- \gamma$ were calculated and implemented in PHOKHARA. It 
was shown there that the radiative corrections for that 
process, which were missing in the PHOKHARA event 
generator at the time of the experimental analyses, were 
at least one order of magnitude smaller than the discrep-
ancies between experiments. The still missing next-to-next-
to-leading order initial state radiative corrections were 
estimated in Ref. [23] to be at most 0.3%. That estimate was 
later confirmed in Ref. [25]. It is taken as a part of the 
intrinsic accuracy of the PHOKHARA event generator, which 
is 0.5%, and is added in the experimental analysis as a part 
of the systematic error. To improve and control errors in a 
better way, in what follows, we implement neglected so far 
corrections in the PHOKHARA event generator and discuss 
their impact on the determination of the pion form factor for 
the realistic experimental cuts used by BABAR, BES-III and KLOE.

II. SETTING

In Fig. 1, the two new classes of radiative corrections to 
the $e^+ e^- \to \pi^+\pi^- \gamma$ cross section discussed in this work 
are shown, namely, the final-state radiation corrections 
(FSRNLO) in diagrams (a) and (b), and the two-virtual-
photon (TVP) contributions in (c). These two types of 
contributions are separately gauge independent, allowing to 
show separate results for them.

The TVP contributions appear for the first time at NLO 
and consist of Feynman diagrams with two virtual photons 
exchanged between the initial-state electron-positron line 
and the final-state pions, and one extra real photon emitted

![FIG. 1. Representative final-state radiation (FSRNLO) and two-virtual-photon (TVP) diagrams discussed in the text, which describe the radiative corrections to the $e^+ e^- \to \pi^+\pi^- \gamma$ process.](image-url)

either from the initial or the final state. They include up to 
pentagon topologies. At the NLO level, only the interfer-
ence of these corrections with the Born diagrams contribute 
to the cross section. These corrections are ultraviolet finite 
and infrared divergent. To control the numerical accuracy, 
which is critical in some kinematical regions, the tensor 
integrals were calculated using the method described in 
Refs. [26,27] for the 5-point functions with the conventions 
defined in Ref. [28]. The scalar one-loop integrals were 
calculated with the QCDLoop library [29] and cross 
checked against the LoopTools library [30] with quadruple 
precision. The infrared divergences were regularized in 
dimensional regularization [31], and were canceled by the 
appropriate soft photon contributions.

In order to speed up the Monte Carlo event generation, 
the distributed code works mostly in double precision, and 
quadruple precision is used only in tensor integrals of TVP 
contributions to assure the numerical stability. The numerical 
accuracy of the distributed event generator was also 
checked against an independent code that was generated 
with FEYNARTS [32] and FEYNCALC [33]. Both the scalar and 
the tensor integrals were calculated there in quadruple 
precision by using the LOOPTOOLS library [30]. A perfect 
agreement between the two codes was found in phase space 
points far from the collinear regions. In the collinear 
regions, where a real photon is emitted along the direction 
of an initial or final state particle, the numerical accuracy 
of the distributed code assures 5 significant digits of the result. 
Gauge independence tests were also performed, as well as 
tests checking the independence of the result on the slicing 
parameter separating the phase space of the photon emis-

sion into the soft part, where the integral is calculated 
analytically, and the hard part, where the integral is 
performed numerically. The gauge independence of the 
matrix elements holds at the level of $10^{-12}$ relative to the 
result, while the dependence on the separation parameter 
yields a numerical precision of 0.02%.

Similar tests were performed for the FSRNLO contribu-
tions, which consist of one-loop corrections to the final 
$\pi^+\pi^- \gamma$ state and the appropriate two photon real emission.
These corrections are both ultraviolet and infrared divergent. To cancel the ultraviolet singularities, the renormalization on-shell mass scheme was used. This part of the code is more stable numerically and was kept completely in double precision.

The new open source version 10.0 of the PHOKHARA event generator with complete NLO radiative corrections to the cross section of the reaction $e^+e^- \rightarrow \pi^+\pi^-\gamma$ is distributed from the web page [34].

A. Results and discussion

Having the complete radiative corrections implemented into the event generator PHOKHARA, we can examine how big are these corrections for the event selections used by the BABAR [16,17], KLOE [18–21] and BES-III [22] collaborations. Their contribution is compared to the predictions of PHOKHARA v9.0 [24], which was used in the experimental analysis of Ref. [22]. In all other analysis [16–21], earlier versions of PHOKHARA were used, which provide identical results as v9.0 for the channel $e^+e^- \rightarrow \pi^+\pi^-\gamma$.

The most relevant region for the evaluation of the muon anomalous magnetic moment is the pion pair invariant mass $Q^2$ in the range 0.6–0.9 GeV. This range was also used in the comparison between experiments in the latest KLOE compilation [18]. We will separately discuss the results within and outside this relevant range (RR).

For KLOE, the relative impact of the TVP and FSRNLO radiative corrections, as implemented in PHOKHARA v10.0, is shown in Fig. 2. For the KLOE 2008 event selection [21], they are both below 0.10% in the RR, and their sum amounts up to 0.18% at low pion pair invariant masses. At the $\rho$ peak, they are well below 0.05%. Similar results are expected for the event selections used by KLOE in Ref. [19] as the event selection is almost identical to KLOE 2008 [21]. For the KLOE 2010 measurement with a photon tagged in the detector [20], the radiative corrections can be larger, up to 0.5% in the RR, for both the TVP and the FSRNLO contributions. The sum is also at most 0.5%. Again around the $\rho$ peak they are smaller and amount up to 0.2%. Above 0.9 GeV, which is outside the RR, the corrections can be much bigger reaching up to 2.4%. It shows that if one aims to improve the accuracy in this region a dedicated study of the FSR by both experimental and theory groups is necessary.

For the BABAR [16,17] and BES-III [22] event selections, the size of the TVP corrections are shown in Fig. 3. They are below 0.10% in both cases. The FSRNLO corrections at the energies of these experiments are at least two orders of magnitude smaller than for KLOE, and are thus negligible. The reason for this is that they are proportional to the modulus square of the pion form factor evaluated at the energy of the given experiment and the form factor falls rapidly with the energy.

The radiative corrections involving pions are intrinsically model dependent. Yet, even if we conservatively assume

FIG. 2. The size of the TVP and FSRNLO radiative corrections for KLOE [21] (KLOE 2008; photon was not tagged) and [20] (KLOE 2010; tagged photon) event selections as a function of the $\pi^+\pi^-$ invariant mass $Q^2$.

FIG. 3. The size of the TVP radiative corrections for BABAR [17] and BES-III [22] event selections as a function of the $\pi^+\pi^-$ invariant mass $Q^2$. The FSRNLO contribution is well below $10^{-4}$ and thus TVP + FSRNLO contribution is almost identical with the TVP contribution.
that the discrepancy is about 50% of the obtained result, they cannot by any means explain the above mentioned differences between the experimental measurements. The actual accuracy of the presented results is much better than the 50% mentioned above, as the model used here was well tested experimentally leaving no space for substantial deviations (see Ref. [35] for discussion and further references). A good agreement with the data was found, while additional dedicated tests would be required if a more accurate estimate of the model dependence is needed. This is especially important for KLOE 2010 with the pion pair invariant mass range above 0.9 GeV.

III. SUMMARY AND OUTLOOK

We conclude that the last set of NLO radiative corrections not considered earlier in the event generator PHOKHARA, which was used by the BABAR, KLOE and BES-III collaborations, cannot be the source of the discrepancies between the different extractions of the pion form factor performed by these experiments. As a consequence, these corrections cannot be the origin of the discrepancy between the experimental measurement and the SM prediction of the muon anomalous magnetic moment $a_\mu$ because they are too small. More effort is needed on the experimental side and further, more accurate measurements of the pion form factor are needed to resolve that long standing puzzle, and also to match the expected precision attainable at the next generation of $a_\mu$ experiments. With this work, a new version of the event generator PHOKHARA with complete NLO radiative corrections is available for more refined future measurements of the pion form factor.

ACKNOWLEDGMENTS

The work supported in part by the Polish National Science Centre, Grant No. DEC-2012/07/B/ST2/03867, the German Research Foundation DFG under Contract No. Collaborative Research Center CRC-1044, the Generalitat Valenciana, Spanish Government and ERDF funds from the European Commission (Grants No. PROMETEO/2017/053, FPA2017-84445-P, FPA2017-84543-P and SEV-2014-0398), and the COST Action PARTICLEFACE (CA16201). F.C. acknowledges financial support by the Spanish Government and Generalitat Valenciana (Grants No. RYC-2014-16061 and SEII-2017/2017/019). J.G. is supported in part by the Polish National Science Centre, Grant No. DEC-2013/11/B/ST2/04023 and by international mobilities for research activities of the University of Hradec Králové, CZ.02.2.69/0.0/0.0/16_027/0008487.


[16] J. P. Lees et al., Precise measurement of the $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ cross section with the initial-state radiation method at BABAR, Phys. Rev. D 86, 032013 (2012).


[18] A. Anastasi et al., Combination of KLOE $\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma(\gamma))$ measurements and determination of $\sigma^{\pi^+\pi^-}$ in the energy range $0.10 < s < 0.95$ GeV$^2$, J. High Energy Phys. 03 (2018) 173.

[19] D. Babusci et al., Precision measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)/\sigma(e^+e^- \rightarrow \mu^+\mu^-\gamma)$ and determination of the $\pi^+\pi^-$ contribution to the muon anomaly with the KLOE detector, Phys. Lett. B 720, 336 (2013).

[20] F. Ambrosino et al., Measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ from threshold to 0.85 GeV$^2$ using Initial State Radiation with the KLOE detector, Phys. Lett. B 700, 102 (2011).

[21] F. Ambrosino et al., Measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma(\gamma))$ and the dipion contribution to the muon anomaly with the KLOE detector, Phys. Lett. B 670, 285 (2009).

[22] M. Ablikim et al., Measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma(\gamma))$ and the dipion contribution to the muon anomaly with the KLOE detector, Phys. Lett. B 753, 629 (2016).


[34] Monte Carlo event generator PHOKHARA, http://ific.uv.es/~rodrigo/phokhara/.