Large-scale transport in the oceans

Emilio Hernández-García
IFISC (CSIC-UIB)
Palma de Mallorca, Spain
The Climate System

Simplification of Bretherton (1988) horrendogram
OUTLINE

- Ocean spatial and temporal scales
- Ocean dynamics: basic ingredients and equations
- Global thermohaline and wind-driven circulation
- Geostrophic currents and vorticity
- Eddies and other Lagrangian Coherent Structures
- Waves
- El Niño-Southern Oscillation (ENSO): a coupled atmosphere-ocean phenomenon
REFERENCES

- Ocean circulation
  Angela Colling and the Open University course team

- Introduction to geophysical fluid dynamics
  Benoit Cushman-Roisin & Jean-Marie Beckers

- Geophysical fluid dynamics
  Joseph Pedlosky
  Springer, 1982
OUTLINE

- Ocean spatial and temporal scales
- Ocean dynamics: basic ingredients and equations
- Global thermohaline and wind-driven circulation
- Geostrophic currents and vorticity
- Eddies and other Lagrangian Coherent Structures
- Waves
- El Niño-Southern Oscillation (ENSO): a coupled atmosphere-ocean phenomenon
NASA Views Our Perpetual Ocean

https://www.nasa.gov/topics/earth/features/perpetual-ocean.html
https://www.youtube.com/watch?v=Wxt7ziHh1-I

June 2005 to December 2007. Model + data
NASA project Estimating the Circulation and Climate of the Ocean (ECCO).
Massachusetts Institute of Technology and NASA's Jet Propulsion Laboratory in Pasadena, Calif.
Scales of motion

Seawifs climatology (1997-2010), vegetative index, chlorophyll

source: nasa.gov
Scales of motion

Seawifs climatology (1997-2010), vegetative index, chlorophyll

Source: nasa.gov
Scales of motion

scale (meters)

Suomi VIIRS false color

nasa.gov
Scales of motion

scale (meters)

Suomi VIIRS false color

nasa.gov
Scales of motion

Suomi VIIRS false color

source: nasa.gov
Scales of motion

scale (meters)

10^6 10^7 10^6 10^5 10^4 10^3 10^2 10^1 10^0

Suomi VIIRS false color

source: nasa.gov
Scales of motion

scale (meters)

Gulf of Mexico, 2010
Scales of motion

scale (meters)

$\approx 75 \text{ cm}$

Woods, JFM, 1968
Scales of (non-wave) motion

- **3D turbulence**
  - Unbalanced: $R_o \gg 1$

- **Sub-mesoscale**
  - $R_o = O(1)$

- **Meso-scale**
  - $R_o \ll 1$

- **General circulation**

```
R_o \equiv \frac{U}{fL}
```
Temporal scales

Energy spectra

10 months 1 month 1 day

Slope $= -2.32$

Inertial fraction 33%

$15^\circ N, z=500m.$

Slope $= -1.98$

$M_2$ fraction 34%

S2S

Slope $= -2.72$

Inertial fraction 66%

$51^\circ S, z=1000m.$

Slope $= -2.36$

$M_2$ fraction 14%


Annu. Rev. Fluid Mech. 41:253–82
OUTLINE

- Ocean spatial and temporal scales
- **Ocean dynamics: basic ingredients and equations**
- Global thermohaline and wind-driven circulation
- Geostrophic currents and vorticity
- Eddies and other Lagrangian Coherent Structures
- Waves
- El Niño-Southern Oscillation (ENSO): a coupled atmosphere-ocean phenomenon
Energy sources / reservoirs

Arrows: energy fluxes (Terrawatts)

Wunsch and Ferrari 2004
SIMPLIFICATION: MAIN FORCINGS

- **Atmosphere**
  - HEAT and MOISTURE FLUXES
  - HEAT, EVAPORATION (thermohaline circulation)

- **Oceans**
  - WINDS, PRECIPITATION

- **SUN**

- **TIDES**
Beyond wind forcing, the basic ingredients:

**Coriolis force:** \[-2\Omega \times u\] , \(u = (u, v, w)\)

Always acts to the **right** of the trajectory in the **Northern** hemisphere
And to the **left** in the **Southern** one.

\[
\Omega = \frac{2\pi}{1\text{day}} = 7.29 \times 10^{-5} \text{ s}^{-1}
\]

\[
f = 1.46 \times 10^{-4} \text{ s}^{-1} \text{ in the poles}
\]
\[
f = 0.88 \times 10^{-4} \text{ s}^{-1} \text{ in Sitges}
\]
\[
f = 0 \text{ s}^{-1} \text{ in the equator}
\]

**Rossby number** = \(\frac{U}{\Omega L}\)

Ro < 1 : rotation relevant for displacements \(L\) at speed \(U\)
**Stratification (gravity, bouyancy)**

Water density in the Adriatic (43°32’N, 14°03’E) on 27 May 2003. From Cushman-Roisin book

- Pycnoclines (and thermoclines)
- In the ocean, density differences rarely exceed 2%

![Graph showing density vs depth](image)

Comparision of the kinetic energy (per unit volume) available in a current and the potential energy (per unit volume) needed to change the depth of a fluid element:

$$\frac{1}{2}\rho_0 U^2}{\Delta \rho g H} \approx (\text{Froude number})^2$$

$$Fr = \frac{U}{NH}$$

$$N = \text{Brunt–Väisälä frequency}$$

$$N^2 = -\frac{g \frac{d \rho}{\rho_0} dz}{\rho_0 H} \sim \frac{g \Delta \rho}{\rho_0 H}$$

Reduced gravity, $g'$

Fr $< 1$ : stratification relevant for displacements $H$ at speed $U$
Rotation and stratification:

\[ L = \frac{Fr \cdot NH}{Ro \cdot \Omega} \]

When \( Fr \approx Ro \), Rossby radius of deformation (changing \( \Omega \) by \( f \))

\[ L_R = \frac{NH}{\Omega} \]

Above this horizontal scale, rotation dominates. Below it, stratification (bouyancy) dominates

(\text{or} \quad L_R = \frac{1}{\Omega} \sqrt{\frac{\Delta \rho}{\rho_0 gH}})

Example: for \( Ro \approx Fr \approx 1 \)

\begin{align*}
\text{atmosphere} & \quad (\rho_0 = 1.2 \text{ kg/m}^3, \Delta \rho = 0.03 \text{ kg/m}^3, H = 5000 \text{ m}) \\
L_{\text{atmosphere}} & \sim 500 \text{ km} \\
L_{\text{ocean}} & \sim 60 \text{ km}
\end{align*}

\begin{align*}
\text{ocean} & \quad (\rho_0 = 1028 \text{ kg/m}^3, \Delta \rho = 2 \text{ kg/m}^3, H = 1000 \text{ m}) \\
U_{\text{atmosphere}} & \sim 30 \text{ m/s} \\
U_{\text{ocean}} & \sim 4 \text{ m/s}
\end{align*}

Typical width of major currents, eddies, …
Equations of motion

\[ \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \]

- Conservation of momentum:
  \[ \frac{d\mathbf{u}}{dt} = -\frac{1}{\rho} \nabla p - 2\Omega \times \mathbf{u} + g + F \]
  \( \text{O}(1/\text{Ro}) \quad \text{O}(1/\text{Fr}^2) \)
  Forcing, friction, ...
  Eddy viscosity, ...
  ... anisotropic ...

- Conservation of mass:
  \[ \frac{1}{\rho} \frac{d\rho}{dt} + \nabla \cdot \mathbf{u} = 0 \]

- Equation of state:
  \[ \rho = \rho_0 [1 - \alpha (T - T_0) + \beta (S - S_0)] \]
  Typical seawater values: \( \rho_0 = 1028 \text{ kg/m}^3, \ T_0 = 10^\circ \text{C} = 283 \text{ K}, \ S_0 = 35^\circ/00 \)
  \( \alpha = 1.7 \times 10^{-4} \text{ K}^{-1}, \ \beta = 7.6 \times 10^{-4} \)
  (Nearly independent of pressure)

- Energy conservation:
  \[ \rho C_v \frac{dT}{dt} + \rho \nabla \cdot \mathbf{u} = k \nabla^2 T \]

- Salt conservation:
  \[ \frac{dS}{dt} = \kappa_S \nabla^2 S \]
Basic ingredients

\[ \frac{du}{dt} = -\frac{1}{\rho} \nabla p - 2\Omega \times u + g + F \]

- O(1/Ro)
- O(1/Re) or O(1/Ek)
- O(1/\text{Fr}^2)

Process:

Scale (meters):

- 3D Turbulence
- Stratified Turbulence
- Submesoscales
- Mesoscale
- Gyres
- MOC

Re = O(1) \quad \Rightarrow \quad Re \gg 1

Fr >> 1 \quad \Rightarrow \quad Fr = O(1) \quad \Rightarrow \quad Fr \ll 1

Ro >> 1 \quad \Rightarrow \quad Ro = O(1) \quad \Rightarrow \quad Ro \ll 1

From J. Taylor
Basic ingredients

From Cushman-Roisin book
Ocean slower than atmosphere, and with smaller scales
Bussinesq approximation: \( \rho = \rho_0 + \rho'(x, y, z, t) \) with \( |\rho'| \ll \rho_0 \)

\( p = p_0(z) + p'(x, y, z, t) \) with \( p_0(z) = P_0 - \rho_0 gz \)

\( H \ll L \) and equal eddy viscosities of heat and salt

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - fu &= -\frac{1}{\rho_c} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2}, \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + fu &= -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu \frac{\partial^2 v}{\partial z^2}, \\
0 &= -\frac{\partial p}{\partial z} - \rho g, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0, \\
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} &= \kappa \frac{\partial^2 p}{\partial z^2},
\end{align*}
\]

+ Earth curvature terms + forcing, boundaries …
OUTLINE

- Ocean spatial and temporal scales
- Ocean dynamics: basic ingredients and equations
- **Global thermohaline and wind-driven circulation**
- Geostrophic currents and vorticity
- Eddies and other Lagrangian Coherent Structures
- Waves
- El Niño-Southern Oscillation (ENSO): a coupled atmosphere-ocean phenomenon
Large-scale and slow circulation driven by planetary driven by **global density gradients** created by **surface heat** and **freshwater fluxes**

Times for return in the order of millenia
Average winds drives the major mean ocean currents. But in a non-trivial way elucidated by Sverdrup.

Western-boundary current intensification
OUTLINE

- Ocean spatial and temporal scales
- Ocean dynamics: basic ingredients and equations
- Global thermohaline and wind-driven circulation
- Geostrophic currents and vorticity
- Eddies and other Lagrangian Coherent Structures
- Waves
- El Niño-Southern Oscillation (ENSO): a coupled atmosphere-ocean phenomenon
THE GEOSTROPHIC APPROXIMATION: 

(geostrophic balance, geostrophic adjustment … )

Valid for L’s and U’s such that $Ro=U/fL<<1$:

If we have an ocean with a single density $\rho_0$ its steady state:

From which:

$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0.$

Taylor-Proudman theorem: horizontal velocities do not change with depth in a rapidly rotating homogeneous fluid

Flow perpendicular to horizontal $p$-gradients: follows isobars

If $f$ is constant: non-divergent horizontal flow:

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Over flat bottom or surface, $w=0$.

In general, flow along isobaths

$u = \frac{-1}{\rho_0 f} \frac{\partial p}{\partial y}, \quad v = \frac{+1}{\rho_0 f} \frac{\partial p}{\partial x}$

$P_T(x,y,z) = -\rho g z + P(x,y)$

$P_0 = P_T(x,y,2=h(x,y)) = -\rho g h(x,y) + P(x,y)$

$\Rightarrow P(x,y) = P_0 + \rho g h$
Examples of geostrophic currents
The larger the slope, the stronger the current...

SEOS project  
(Northern hemisphere, slopes not to scale)

\[ u = -\frac{g \partial h'}{f \partial y}, \quad v = \frac{g \partial h'}{f \partial x} \]

Northern hemisphere

Really observed when \( Ro \ll 1 \)
GENERALIZATION TO STRATIFIED OCEAN

Barotropic

Baroclinic

Adjustment process described by quasigeostrophic theory

Thermal wind along fronts. Frontal jets

pressure
depth
density
Dynamic Topography (DT) = Sea Surface Heigh (SSH) – Geoid (G)

\[ \text{SSH} \approx \text{cm} \quad \text{G} \approx \text{meters} \ldots \]

Sea Level Anomalies (SLA) = \( \text{SSH} - <\text{SSH}>_t = \text{DT} - <\text{DT}>_t \)

Dynamic topography determines, via the Coriolis force, the velocity field (at large scales, geostrophic approximation)

Ageostrophic components Can be estimated from scatterometer data

(Surface roughness \( \rightarrow \) wind \( \rightarrow \) Eckman component)
Global Sea Surface Height from altimetry

https://www.youtube.com/watch?v=F8zYKb2GoR4
Indian ocean eddies

https://www.youtube.com/watch?v=J1NRN7gavgU
VORTICITY DYNAMICS

From the inviscid (shallow water) equations (any value of Ro) with homogeneous density:

\[ \frac{d}{dt} \left( \frac{f + \zeta}{h} \right) = 0, \]

Potential vorticity is conserved

If Ro<<1,

\[ q = \frac{f}{h} \]

If \( f \) is constant (small range of latitudes) fluid parcels must follow isobaths

(Rotation with positive relative vorticity (cyclonic) in the Northern homisphere)
Friction wind-sea gives correction to geostrophy: Ekman pumping, Ekman transport
OUTLINE

- Ocean spatial and temporal scales
- Ocean dynamics: basic ingredients and equations
- Global thermohaline and wind-driven circulation
- Geostrophic currents and vorticity
- Eddies and other Lagrangian Coherent Structures
- Waves
- El Niño-Southern Oscillation (ENSO): a coupled atmosphere-ocean phenomenon
Ocean mesoscale eddies:
Radius: 50 - 200 km
Rotation currents: 0.1 – 1 m/s
Period: days – month
Translation: few km/day

Atmospheric synoptic cyclons/anticyclons:
Radius: 100 - 5000 km
Rotation currents: 1 – 50 m/s
Period: 1 week
Translation: 1000 km/day

Discovered in the 70’s!
Eddies transport water (and thus heat, salt, substances, …) coherently over large distances

https://ifisc.uib-csic.es/users/emilio/presentations/CAFE_Sitges2019/M11.mp4

A El Aouni, H Yahia, K Daoudi, K Minaoui
Chaos: An Interdisciplinary Journal of Nonlinear Science 29 (9), 093106
Identifying relevant coherent structures (eddies, filaments, …) which control transport (Lagrangian Coherent Structures)

- Leaking, escape, or residence time methods
- Attracting or repelling material lines
- Distinguished hyperbolic trajectories and their manifolds
- Stretching-field methods: Finite-time Lyapunov exponents, Finite-size Lyapunov exponents, M function, …
- Variational approach to hyperbolic LCSs, black holes (Haller)
- Topological braiding (Thiffeault)
- Dimensions and ergodicity measures (Rypina)
- Lagrangian vorticity,
- Lagrangian-Averaged Vorticity Deviation (Haller)
- Transfer operators, network methods (Kantz, Froyland …)

... (biased) review: **Lagrangian Coherent Structures**
It is not sufficient to look at the velocity field?

https://ifisc.uib-csic.es/users/emilio/presentations/CAFE_Sitges2019/VelsAndParcelsAnimated.gif

Lagrangian vs Eulerian information
Finite-time Lyapunov exponent

Lyapunov exponent

Finite-size Lyapunov exponent

FSLE

All the quantities are also functions of the initial position and time:

$$\lambda(x, t, \delta_0, \delta_f)$$

STRETCHING FIELDS
The idea is that initial conditions close to the **stable manifold** of a hyperbolic trajectory or set will show strong divergence: **high FSLE**

The **unstable manifold** of hyperbolic sets (where material typically will accumulate) would be marked by **high FSLE in the time backwards direction**

*REMARK:* these are heuristic considerations. Theorems needed (some available for FTLE)

DieCAST model for the full Mediterranean Primitive equations, 48 vertical levels, 1/8° horizontal resolution, climatological forcings \( \rightarrow \) 5 years of daily velocity fields

\[
\delta_0 = 0.02^\circ \rightarrow \delta_f = 1^\circ \quad \text{(mesoscale transport)} \\
\delta_0 \approx 2 \text{ km} \rightarrow \delta_f \approx 110 \text{ km} \quad \text{twodimensional}
\]

Lyapunov values organized in strong lines

FSLE from forward and backwards integrations.

Are they really unstable manifolds of hyperbolic trajectories?

The strongest lines are seen to organize tracer flow.

https://ifisc.uib-csc.es/users/emilio/presentations/CAFE_Sitges2019/ForwBackAnimated.gif
Particular eddy enclosed by hyperbolic manifolds

FSLE methodology is giving the hyperbolic filamentation region, not the coherent core

Route of particle escape

J.H. Bettencourt, C. Lopez, E. Hernandez-Garcia
Ocean Modelling 51 (2012) 73–83
Seasonal and regional characterization of horizontal mixing in the global ocean
Hernández-Carrasco, I.; López, C.; Hernández-García, E.; Turiel, A.

https://ifisc.uib-csic.es/users/emilio/presentations/CAFE_Sitges2019/FSLE_Atlan_Sur_cada15dias_dia300.avi
OUTLINE

- Ocean spatial and temporal scales
- Ocean dynamics: basic ingredients and equations
- Global thermohaline and wind-driven circulation
- Geostrophic currents and vorticity
- Eddies and other Lagrangian Coherent Structures
- Waves
- El Niño-Southern Oscillation (ENSO): a coupled atmosphere-ocean phenomenon
WAVES (gravity, inertia-gravity (Poincaré), Kelvin, Rossby, topographic, …)

- Propagation of perturbations on the state of the ocean \((p, u, h, \ldots)\)
- Can travel much faster than fluid particles. Thus explain rapid long-distance changes in the ocean in response to remote winds.

Barotropic
(isobars // isopycnals)

Baroclinic
thermocline

Usually larger displacements
Kelvin waves

A coast to the right (Northern hemisphere) acts as a waveguide

Nondispersive Speed
\[ c = \sqrt{gH} \]

R = Rossby radius =

\[ R = \frac{\sqrt{gH}}{f} = \frac{c}{f} \]

midlatitudes:
Barotropic c=200m/s
Thermocline c=0.5-3 m/s
R=25km
Equatorial Kelvin wave

The Equator acts as a two-sided boundary / waveguide

Beta plane approximation

\[ f = 2\Omega \sin \phi \approx 2\Omega \sin \phi_0 + 2\Omega (\phi - \phi_0) \cos \phi_0 + ... = f_0 + \beta y \]

(Mid latitudes: \( f_0 \approx 8 \times 10^{-5} \text{ s}^{-1} \), \( \beta \approx 2 \times 10^{-11} \text{ m}^{-1}\text{s}^{-1} \))

Equatorial: \( f_0 \approx 0 \text{ s}^{-1} \), \( \beta \approx 2.28 \times 10^{-11} \text{ m}^{-1}\text{s}^{-1} \)

\( \Delta \rho / \rho_0 = 0.002 \) and thermocline depth \( H = 100 \text{ m.} \)

- Non-dispersive wave
- Travels to the east
- Trapped in the equator
- Equatorial deformation radius \( R_{eq} = \sqrt{c / \beta} \)

(100-200km)
Rossby waves (or planetary waves) Arise from the conservation of potential vorticity

\[
\frac{f + \xi}{H} = \text{constant}
\]

North

Lines of constant \( f \)

\[
f = 2\Omega \sin \phi \approx f_0 + \beta y
\]

State of rest

Toward higher values of potential vorticity

South

Greater \( f \)

Greater \( f \)

Smaller \( f \)

Smaller \( f \)

Squeezing

Squeezing

Rossby waves travel the Pacific from East to West in 3 months at the equator, and 10 years at 30° S or N

- Dispersive waves

\[
\omega = -\beta_0 R^2 \frac{k_x}{1 + R^2 \left( k_x^2 + k_y^2 \right)}
\]

- \( c = \frac{w}{k} \) towards West, NW, SW (with respect to the ambient flow)

- Group velocity \( c_g = \nabla_k \omega(k) \) different directions
Wave bouncing in an equatorial ocean model

10 day wind atmospheric anomaly

Kelvin

Rossby

UP

DOWN

equator

10 days

25 days

100 days

150 days

180 days

240 days

301 days

401 days

10 days

25 days

60 days

80 days

100 days
OUTLINE

- Ocean spatial and temporal scales
- Ocean dynamics: basic ingredients and equations
- Global thermohaline and wind-driven circulation
- Geostrophic currents and vorticity
- Eddies and other Lagrangian Coherent Structures
- Waves
- El Niño-Southern Oscillation (ENSO): a coupled atmosphere-ocean phenomenon
El Niño-Southern Oscillation (ENSO). Period: 2-8 years. System close to a noisy Hopf bifurcation. The most prominent climatic oscillation after day/night and yearly seasons.
The Oceanic Niño Index (ONI)

El Niño criterium:

An El Niño or La Niña event is identified if the 5-month running-average of the NINO3.4 index exceeds +0.4°C for El Niño or -0.4°C for La Niña for at least 6 consecutive months.

The application of Machine Learning Techniques to improve El Niño prediction skill
Frontiers in Physics 7, 153 (2019)
THANK YOU

for your attention