Self-Fulfilling Runs and Endogenous Liquidity Creation*

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Abstract

This paper incorporates endogenous money creation into the liquidity mismatch problem of Diamond and Dybvig (1983). We characterize a nominal economy where demandable deposits are created through lending. Depositors use sight deposits to buy consumption goods and the banks manage reserves to clear payments and to offset liquidity risk. We show that deposit contracts are suboptimal in terms of liquidity risk-sharing. We also observe that the self-fulfilling run depends on the refinancing rate of the central bank. Our analysis emphasizes the importance of effective lender of last resort policies to prevent expectational banking panics.

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Figure 1: Timing of events

1 Introduction

The prevailing view in banking theory is, as Diamond and Dybvig [9] pointed out, that “illiquidity of assets provides the rationale both for the existence of banks and for their vulnerability to runs”.¹ According to this theory, banks channel pre-existing real assets from savers to finance illiquid entrepreneurial projects. In this intermediation process, banks create liquidity, that is, they offer liabilities (deposits) that are immediately available while the assets they hold (loans) are not. In other words, banks promise full recovery of deposits at anytime although disposing of their assets before maturity may only come at a cost. This liquidity creation is responsible for the ability of banks to reproduce the socially optimal allocation among depositors with random liquidity needs. However, the liquidity mismatch between redeemable deposits and illiquid assets also explains the inherent fragility of banks.

Recently, a body of literature has explored the implications of introducing fiat money over the withdrawal incentives of depositors and the optimality of deposit contracts in terms of liquidity risk-sharing. An important feature of these models has to do with the idea that, in modern monetary systems, payments and withdrawals usually take place as electronic transfers in inside money among depository institutions which means these funds are not drained out of the banking sector.² Within this literature, Skeie [28] includes these features and shows the existence of a unique and efficient equilibrium when nominal deposits are repayable in inside money. Provided banks choose the right amount of liquidity, price adjustments in the goods market rule out the existence of an equilibrium with a bank run. Along the same line, Allen et al. [3] incorporate fiat money issued by the central bank into Allen et al. [2] and find that the efficiency and uniqueness of the equilibrium also hold with aggregate return uncertainty, aggregate liquidity shocks, and bank specific liquidity shocks. Freixas et al. [14] also characterize a nominal banking system in which the central bank can implement the efficient allocation by choosing the policy rate.³

Both in Skeie [28] and Allen et al. [3], the central bank prints cash and lends it to commercial banks which in turn use it to provide loans to the private sector. Once in the hands of private agents, this cash is then employed to buy goods and deposited back in commercial banks. To support the efficient allocation, the central bank commits to supply whatever money is needed to cover liquidity needs. It is this commitment by the central bank to inject outside money that is behind the price adjustments responsible for efficiency and uniqueness of the

¹See also Bryant [5], Holmstrom and Tirole [18], and Diamond and Rajan [10], [11].

²The term inside money refers to the money-like liabilities privately created within the banking industry, whereas outside money deals with liquidity created by central banks. See Lagos [19] for a general description of the distinction between inside and outside money.

³Other papers have developed nominal frameworks to explore other issues for financial stability. Allen and Gale [1] introduce fiat money in a model of banking and show that variations in the price level allow nominal debt to become effectively state contingent so that risk-sharing is improved. Diamond and Rajan [12] find that nominal contracts cannot prevent bank runs when there is idiosyncratic risk on the bank’s asset side caused by delays in asset returns.

equilibrium. Furthermore, in these models, the pre-existence of outside money is a necessary condition for financial intermediation to take place. Without the initial injection of outside money, banks cannot operate.

Despite the significant contribution of incorporating nominal contracts to the analysis of financial intermediation, the role these papers assign to outside money is at odds with the one it plays in our current monetary systems. In particular, holding outside money is not a necessary precondition for commercial banks to intermediate, namely, to extend credit and produce deposits in the process. From a technological point of view, depository institutions are completely autonomous in that respect. Neither reserves nor cash are needed *ex-ante*. They may be needed *ex-post* but this demand of outside money is a consequence of the intermediation process itself. Regarding reserves, banks demand them for two reasons. The first reason is to satisfy reserve requirements. One could think that reserves need to be accumulated before deposit creation to fulfill these obligations.⁴ However, this is not the case. Many countries effectively do not impose these requirements and for those that do, these requirements are lagged, that is, they have to be fulfilled after deposits are created, not before. The second reason to hold reserves is to clear payment transactions among clients of different banks. Again, this obligation comes after those deposits are created. Regarding cash, banks hold it to honor the convertibility of sight deposits. However, once more, deposits are created before they are converted into cash. Of course, this is not to mean that banks do not accumulate outside money before producing deposits. They do so mainly for precautionary reasons to prevent illiquidity problems when fulfilling the obligations just mentioned. But causality goes from deposit creation to outside money demand and not the other way around. Furthermore, central banks implement monetary policy by managing a short term interest rate. Given this cost of liquidity, commercial banks decide their level of intermediation which produces a demand for reserves. The central bank then uses this demand to determine the quantity of reserves compatible with its interest rate target. This is to say, central banks do not determine prices by managing the quantity of outside money.

In this paper we reformulate the model of Diamond and Dybvig [9] (DD for short) to incorporate a set of elements aimed at reproducing these basic features of a modern monetary system. First, as in Skeie [28] and Allen et al. [3] instead of sticking to the traditional description of banks as financial vehicles that take real assets from savers to lend them to ultimate borrowers, we only consider nominal financial contracts where the intermediation is done in fiat money. However, unlike these papers, this intermediation is not done with pre-existing outside money. Instead, our model reflects the fact that nominal inside money (i.e. deposits) is created when banks originate a new loan. Thus, the intermediation process performed by banks starts on the asset side of their balance sheets not on its liabilities. With this in mind, the maturity and liquidity mismatch between bank assets and liabilities arises automatically when a loan

⁴The idea that reserves influence the quantity of deposits and loans is based on the concept of the money multiplier. As noted by Disyatat [13], though, there is no causal relationship from reserves to bank lending.

is originated. This is because the counterpart to the provision of a long term loan is the creation of an overnight liability in the form of a disposable deposit which by nature is designed to exchange hands in the form of a payment. In this sense, the liquidity risk faced by the banking institution is due to the transfer of funds between banks, which is solved by managing a demand for outside money produced by the central bank.

A second feature of the model is that the demand for outside money (reserves) is a consequence of, and not a condition for, deposit production. As mentioned above, the role outside money plays in this model differs from that in the existing literature at least in two instances. *Ex ante*, central bank liquidity is not needed for the creation of loans and deposits by banks. *Ex post*, once payments are made, the liquidity risk banks face, and therefore, the need to borrow outside money from the central bank relates to the net transfer of funds between banks and into cash the use of these deposits imply.

Finally, a third feature that separates our model from the existing literature, is that the vast majority of bank loans cannot be recalled nor banks have any say about the liquidation decision of the investment projects pursued by borrowers. This is the case of basically all mortgages and, according to the Board of Governors of the Federal Reserve System, of 87.5 percent of all C&I loans.⁵ As will be made clear below, this will be important when analyzing the welfare effects of a bank run.

We show how our environment has significant implications on the equilibrium of the model and its predictions on financial fragility as compared with traditional banking theories based on the seminal work of DD or on the nominal contract theories of Skeie [28] and Allen et al. [3]. In this respect, we state that expectational runs of the Diamond and Dybvig type can be prevented by effective, lender of last resort, interest policies that offset the incentives of depositors to coordinate in a run. In particular, we show that the possibility of a run only appears at relatively high refinancing rates. This result is independent on whether the run takes place at a single bank or at the banking system as a whole, two cases we distinguish in the analysis below.

On the policy front, the connection between panics and the response of the central bank has empirical support. The view that purely self-fulfilling runs could be possible was endorsed by Friedman and Schwartz [15]’s explanation of the bank panics that occurred in the United States up to the 1930s but was disputed by Gorton [17] and Calomiris and Mason [6], [7]. More recently, the collapses of Bear Stearns and Lehman Brothers (Lucas and Stokey [20]), and the run on the UK bank Northern Rock in 2007 (Shin [27]) do not seem to be related to a coordination failure, so additional elements on the bank’s fundamentals are required to explain bank instability.⁶ In our case, this element is the refinancing

⁵See Board of Governors of the Federal Reserve System, Statistical Release E2 [4]. This figure is the average fraction of noncallable loans, weighted by volume, between the second quarter of 1997 and the first quarter of 2003. In 2003 the Board stopped including the amount of C&I loans that are callable because, representing a small fraction of total loans, their behavior did not significantly differed from loans which are not callable.

⁶Goldstein and Pauzner [16], and Rochet and Vives [24] provide the theoretical foundations

policy of the central bank.

Furthermore, unlike the existing literature on nominal deposit contracts, we cast doubt on the capacity of nominal deposit contracts to implement the optimal amount of real liquidity. We show that the equilibrium is always inefficient from a social point of view. This result holds independently of who decides on the real liquidity of the economy, namely the nonfinancial sector or the banking system. A key feature for this result is the lack of commitment of depository institutions to ensure future consumption needs of depositors. With nominal deposit contracts repayable in cash, banks cannot set in the present credible promises about the real value of future payoffs because of their inability to set prices.

The paper is structured as follows. Section 2 describes the real economy and connects with the DD model. Section 3 introduces money in the model and shows the main results. Finally, section 4 concludes.

2 The real setup

The real model reproduces the maturity transformation problem described in DD but introducing labor in the production technology. As will be made clear in this and next sections, this modification does not alter the production possibility frontier in any respect but allows for the existence of inside money.

2.1 The environment

Consider an economy characterized by a circle with measure 1 and three dates, indexed by $t = 0, 1, 2$. Locations are continuously distributed over the circle. On each location there is a continuum of identical risk averse households with measure 1. Households are composed of a worker and an entrepreneur. Each worker is endowed in period 0 with a unit of time which they supply inelastically. Entrepreneurs have access to risk-free productive technologies to transform labor into output.

At period $t = 0$, households face uncertainty about future liquidity needs. With probability $\lambda \in (0, 1)$ the household becomes impatient ($h = 1$) and prefers to consume in period 1, while with probability $(1 - \lambda)$ the household is patient ($h = 2$) and consumes at $t = 2$. Once households observe types at the beginning of period 1 they make choices and obtain a utility $u(c^h)$, where c^h denotes the consumption of a household of type $h \in \{1, 2\}$ either at period 1 (those of type $h = 1$) or at period 2 (those of type $h = 2$).⁷ The function $u(c)$ has the following properties

$$u'(c) > 0, u''(c) < 0, \lim_{c \rightarrow 0} u'(c) = \infty, \text{ and } \lim_{c \rightarrow \infty} u'(c) = 0.$$

to fill the gap between both literatures.

⁷Throughout the paper, subscripts will refer to periods ($t = 1, 2$) and superscripts to types ($h = 1, 2$).

We assume further that the coefficient of relative risk aversion satisfies

$$-c \frac{u''(c)}{u'(c)} > 1 \quad (1)$$

everywhere.

Households can use both a long as well as a short productive technologies. The long productive technology transforms each unit of labor employed at $t = 0$ into $\rho_2 > 1$ units of the good at $t = 2$. If a fraction $y \in [0, 1]$ of the production is interrupted at $t = 1$ it will produce a scrap value equal to $\rho_1 y$ with $0 \leq \rho_1 \leq 1$. The remaining fraction left until maturity of the production process will yield $\rho_2(1 - y)$ in period 2. In addition to the long productive technology, at any time $t = \{0, 1\}$ households could also invest in a short technology providing with a unit return at $t + 1$.

2.2 Optimal risk sharing

Let $\alpha \in [0, 1]$ be the fraction of labor used at time $t = 0$ by the household in the short technology. It is easy to see that a planner would choose not to store, while determining c^1 and c^2 , together with the aggregate fraction of labor to be used in the short technology, α , as well as the part of the long productive technology to be liquidated prematurely, y , to maximize

$$\lambda u(c^1) + (1 - \lambda)u(c^2) \quad (2)$$

subject to the feasibility constraints

$$\lambda c^1 \leq \alpha + \rho_1 y \quad (3)$$

and

$$(1 - \lambda)c^2 = (1 - \alpha)\rho_2 + \alpha + \rho_1 y - \lambda c^1. \quad (4)$$

The first order conditions of this problem to determine the optimal choice $\{c^{1*}, c^{2*}, \alpha^*, y^*\}$ are

$$u'(c^{1*}) = \rho_2 u'(c^{2*}), \quad (5)$$

$$\lambda c^{1*} = \alpha^* \quad (6)$$

$$(1 - \lambda)c^{2*} = (1 - \alpha^*)\rho_2, \quad (7)$$

and

$$y^* = 0. \quad (8)$$

These expressions characterize the efficient risk-sharing for this economy and are equivalent to the ones in DD. Since $\rho_2 > 1$ and because of the degree of risk aversion considered in (1), it turns out that $1 < c^{1*} < c^{2*} < \rho_2$, which means households would prefer to share *ex ante* the risk associated with the timing of consumption. Notice this allocation satisfies the incentive compatibility constraint so the planner does not need to know types when implementing it.

2.3 A time bank

Insurance against consumption uncertainty could be provided by introducing a contingent time bank. Workers can deposit their time at $t = 0$ in a time depository institution. The time bank then designs a contingent deposit contract at $t = 0$ providing c^{1b} units of consumption at $t = 1$ to those withdrawing their deposits in that period, or c^{2b} units at $t = 2$ for those who wait to withdraw at that period. At time 0 the time bank then puts a fraction α^b of the deposited time to work in the short productive technology and the fraction $1 - \alpha^b$ in the long technology while choosing the aggregate liquidation of the long productive investment, y^b , to maximize the expected utility of depositors

$$\lambda u(c^{1b}) + (1 - \lambda)u(c^{2b}) \quad (9)$$

subject to the feasibility constraint

$$\lambda c^{1b} \leq \alpha + \rho_1 y^b,$$

and

$$(1 - \lambda)c^{2b} \leq (1 - \alpha^b)\rho_2 + \alpha^b - \rho_1 y^b - \lambda c^{1b}.$$

Obviously, this problem yields the optimal allocation found in the planner's problem above.

This time deposit contract $\{c^{1b}, c^{2b}\}$ provides efficient risk-sharing because it determines implicit contingent wages to be paid to households at $t = 1$ depending on their realized types. The $t = 1$ equivalent contingent wages depositors are receiving for providing time at $t = 0$ are

$$w_1^1 = \frac{\alpha^b}{\lambda} = c^{1b} \quad \text{and} \quad w_1^2 = \frac{1 - \alpha^b}{1 - \lambda} = \frac{c^{2b}}{\rho_2},$$

for workers belonging to impatient and patient households, respectively. Because of the assumed degree of risk aversion (1), it must be the case that

$$w_1^1 = \frac{\alpha^b}{\lambda} > 1 > \frac{1 - \alpha^b}{1 - \lambda} = w_1^2,$$

which allows to obtain the first-best allocation.

The contingent time deposit contract supports, however, a suboptimal equilibria. The bank designs such contract inferring that there will be λ withdrawals in period 1. As in the seminal work of DD, patient households have incentives to withdraw before time if they anticipate that the bank will be forced to liquidate a significant amount of its long term investment to service the increasing demand of early withdrawals. Let $\bar{\lambda}$ be the minimum amount of early withdrawals in period 1 to have a self-fulfilling run. This threshold satisfies

$$\bar{\lambda} = \frac{\alpha^b}{\rho_2(1 - \alpha^b) + \alpha^b}.$$

If late consumers expect the fraction of early withdrawals to be larger than $\bar{\lambda}$, it will be optimal for them to withdraw at date 1 and store the proceeds until $t = 2$. In this second equilibrium the bank suffers a run since anyone who waits until the last period will get nothing.

3 The nominal economy

3.1 The setup

In this section we add nominal deposit contracts and a flow of nominal funds into the real economy of Section 2. In particular, we include banks that (i) create endogenous money in the form of deposits when providing loans, and (ii) manage central bank reserves to honor the convertibility promise associated with deposits. We also separate the liquidation decision of financial positions from that of real investments. We then explore the extent to which these nominal contracts achieve optimal risk-sharing and study whether self-fulfilling panics do occur, distinguishing whether entrepreneurs or banks take productive decisions.

Production technologies and preferences remain equal as described in Section 2. That is, entrepreneurs use labor hired at $t = 0$ in the short and long technologies. In this economy, workers are in charge of buying consumption goods for the household either at $t = 1$ or $t = 2$ depending on the realization of the idiosyncratic liquidity shock. Entrepreneurs sell the goods produced at $t = 1$ and $t = 2$ and store goods for the household.

To introduce a role for banks, assume entrepreneurs cannot use the worker in their household and need to hire them from other households in a competitive labor market. Furthermore, when workers are hired, at $t = 0$, these entrepreneurs lack the credibility to convince those workers they will get paid in the future, when production takes place either at $t = 1$ or $t = 2$. Assume each location is served by a continuum of banks with measure 1. The function of banks under this setting is to intermediate between households in this payment process.

The timing of events is described in Figure 1. At $t = 0$, entrepreneurs borrow inside money, W , from one of the banks located in the same location they live in. This loan produces a double entry in the bank's balance sheet. On the asset side, the bank annotates the right associated with the loan taken by the entrepreneur. On the other hand, means of payments are created, and the liability side reflects the right of the entrepreneur to dispose of those funds to make payments. The interest rate of these loans is i^l to be paid at the end of period $t = 2$. Still at time $t = 0$, the loan is used by entrepreneurs to pay workers in advance for their labor services. Importantly, the introduction of banks solves the commitment problem on the part of both entrepreneurs and workers since wage payment is done at the time labor is delivered, $t = 0$. Entrepreneurs obtain labor for production and workers obtain an asset which will have value in equilibrium. Notice, deposits are homogeneous units of account that can be used by workers to buy goods from any entrepreneur at either $t = 1$ or $t = 2$. When workers pay for consumption goods, they transfer these deposits to an entrepreneur. A nominal price for consumption goods will be formed as deposits are exchanged for goods. Entrepreneurs then use these revenues from selling the goods they produce to pay back the loan they asked for at period 0. That is the reason these deposits are accepted back by entrepreneurs in exchange for consumption goods. Thus, with the introduction of depository institutions, loan and deposit

creation by banks are used to bridge the intertemporal gap between the wage and goods payments in this economy.

3.1.1 Households' problem

Households make choices to maximize utility. At period $t = 0$ each entrepreneur hires 1 (and only 1) worker from a different household and borrows the nominal wage rate W from the local bank to make the wage payment. The only choice to be made at this moment is the fraction of the hired labor to be used in the short technology, α . If this choice is made by the entrepreneur, they will make it to maximize expected utility⁸

$$\lambda u(c^1) + (1 - \lambda)u(c^2).$$

Workers receive the income W from the payment of wages as a sight deposit. Assume each household maintains a single account, shared by its worker and entrepreneur, in one of the banks in the location the pair lives on. At the beginning of the interim period, $t = 1$, households receive the remuneration from their sight deposits at the interest rate i_1^s . Then, the liquidity shock realizes and households learn whether they are of the patient or impatient type. At this point, workers have the opportunity to buy goods for consumption, c . This means these workers will transfer part of their liquid funds to entrepreneurs of different households in exchange for goods produced at $t = 1$. These liquid funds consist of the gross sight deposits, $(1 + i_1^s)W$. Let P_t be the nominal price of the consumption good in period t .

Because by $t = 1$ households already know their type, their remaining choices will depend on whether the household is impatient, $h = 1$, or patient, $h = 2$. They will make these choices to maximize their corresponding utility $u(c^h)$. First, only workers belonging to impatient households will buy goods at $t = 1$. These purchases are then subject to the following cash in advance constraint

$$P_1 c^1 \leq (1 + i_1^s)W \quad (10)$$

while patient households save those funds to be used in the following period.

Once goods purchases take place, the household makes a portfolio choice allocating $t = 1$ resources into either cash, M_2^h , or sight deposits, S_2^h . The available resources are whatever funds are left from the purchases of goods plus the revenues from selling goods obtained from the short technology α and from the liquidation of the long productive technology, y^h . Thus, in making these choices, households face the portfolio constraint

$$M_2^1 + S_2^1 \leq (1 + i_1^s)W - P_1 c^1 + P_1 q_1^1 \quad (11)$$

and

$$M_2^2 + S_2^2 \leq (1 + i_1^s)W + P_1 q_1^2. \quad (12)$$

⁸Below we will analyze both the cases where α is chosen by the entrepreneur and by the bank.

where q_1^h is the amount of goods sold by the entrepreneur of a household of type $h = \{1, 2\}$. This amount has to satisfy the resource constraint

$$q_1^h \leq \alpha + \rho_1 y^h \quad (13)$$

for $h = \{1, 2\}$.⁹

At the beginning of period $t = 2$, only workers belonging to patient households are buying goods. Therefore, they face the cash-in-advance constraint

$$P_2 c^2 \leq M_2^2 + (1 + i_2^s) S_2^2 + P_2 (\alpha + \rho_1 y^2 - q_1^2). \quad (14)$$

In this case, liquid funds are the cash hoarded from period $t = 1$, M_2^2 , plus the return on sight deposits, $(1 + i_2^s) S_2^2$. The household can also consume the goods stored in the previous period, $\alpha + \rho_1 y^2 - q_1^2$. Then, after these goods are consumed, the household has to pay back the original loan taken by the entrepreneur, so, it must be the case that

$$(1 + i^l) W \leq M_2^2 + (1 + i_2^s) S_2^2 + P_2 (\alpha + \rho_1 y^2 - q_1^2) - P_2 c^2 + P_2 (1 - \alpha - y^2) \rho_2. \quad (15)$$

In this expression $(1 - \alpha - y^2) \rho_2$ is the amount of goods produced by the entrepreneur of the household of type $h = 2$ on period 2 with the long technology. The entrepreneur will sell in the market whatever goods are produced at $t = 2$ plus those not sold in the first period and not consumed by the household. That is, the loan needs to be repaid with whatever resources are left from buying goods, plus revenues from selling goods. On the other hand, impatient agents do not buy goods at $t = 2$ and they only care about repaying back the loan, that is,

$$(1 + i^l) W \leq M_2^1 + (1 + i_2^s) S_2^1 + P_2 [(1 - \alpha - y^1) \rho_2 + (\alpha + \rho_1 y^1 - q_1^1)] - P_2 c^1. \quad (16)$$

Notice we allow the household to store between period 1 and 2 by keeping production not sold at $t = 1$. In this case, the constraint (13) will be slack for that household.¹⁰

3.1.2 Banks' problem

Banks make choices to maximize their net worth at $t = 2$ keeping net worth at $t = 1$ positive. To work out that net worth, notice that at $t = 0$ each bank has W in assets (the loans provided to entrepreneurs) and also W in liabilities (the deposits of workers). Thus their net worth at $t = 0$ is 0. At $t = 1$ the bank maintains assets equal to W (the loans to be paid at $t = 2$). On the other hand, banks demand reserves at $t = 1$ to solve liquidity shortages they can anticipate. Each bank has

$$\lambda (M_2^1 + S_2^1) + (1 - \lambda) (M_2^2 + S_2^2)$$

⁹Because α is chosen at $t = 0$, it is not dependent on the household's type.

¹⁰As stated in expressions (10) and (14) households cannot use the current period production for consumption in the same period. This is assumed both for patient and impatient agents. The only possibility for a household to consume its own production is to store at $t = 1$ and consume those goods in the second period (patient households).

in liabilities. That is, the sight deposits to be maintained until period $t = 2$, S_2^h , plus the outside money which has to be borrowed from the central bank. Thus, the net worth of banks at $t = 1$ is

$$NW_1^B = W - [\lambda (M_2^1 + S_2^1) + (1 - \lambda) (M_2^2 + S_2^2)]. \quad (17)$$

Finally, at $t = 2$, each bank receives $(1 + i^l)W$ in loan repayment while facing liabilities equal to

$$(1 + i_2^s) [\lambda S_2^1 + (1 - \lambda) S_2^2] - (1 - \lambda) P_2 c^2 + P_2 [\lambda q_2^1 + (1 - \lambda) q_2^2] \\ + (1 + i^o) [\lambda M_2^1 + (1 - \lambda) M_2^2]$$

where q_2^h is equal to the amount of goods sold by household h at period $t = 2$. Thus, liabilities are equal to the value of sight deposits carried over from period $t = 1$, minus the use of those deposits for consumption by patient households, plus the deposits of entrepreneurs selling goods, plus the cost of providing cash to each type of household, $(1 + i^o)M_2^h$. This cash had to be borrowed at $t = 1$ from the central bank accruing the rate i^o to be paid at $t = 2$.¹¹ Thus, the net worth of banks at $t = 2$, the objective function they try to maximize, equals

$$NW_2^B = (1 + i^l)W - (1 + i_2^s) [\lambda S_2^1 + (1 - \lambda) S_2^2] + (1 - \lambda) P_2 c^2 \\ - P_2 [\lambda q_2^1 + (1 - \lambda) q_2^2] - (1 + i^o) [\lambda M_2^1 + (1 - \lambda) M_2^2]. \quad (18)$$

Clearly, both (17) and (18) are to be positive for each bank to be solvent. In particular, having NW_2^B positive is the condition households evaluate at $t = 1$ to predict whether the bank will be solvent at $t = 2$. Anticipation that the net worth of the bank could be negative at $t = 2$ may trigger a run at $t = 1$. Given that rates i^l and i_1^s are set on period $t = 0$, high policy rates i^o together with a strong enough desire to convert sight deposits into cash, could render the bank insolvent and, therefore, to derive in a run. Notice, loans cannot be recalled at $t = 1$. They are only paid at $t = 2$. With this we incorporate the long maturity of bank assets against the short maturity of their liabilities.

3.1.3 Discussion of the setup

As mentioned in the Introduction, this setup includes several features worth mentioning. First, the intermediation role played by banks starts when a borrower (entrepreneur) asks for a loan at time $t = 0$. The loan is produced because the borrower lacks the means of payment to make a purchase (wage payment). Then, once these means of payment, in the form of deposits, are created, the liquidity problem of the household is financial rather than technological. That is, to make a purchase, either at $t = 1$ or $t = 2$, households need to maintain financial claims which are generally acceptable as a mean of payment. These liquid funds then circulate in the economy as broad money as long as the loan

¹¹We could interpret i^o literally as the refinancing rate of the central bank or as the opportunity cost faced by the bank when funding reserves.

does not mature. Notice this sequence of events is opposite to the one in DD and the subsequent literature where the intermediation process starts when a saver deposit pre-existing assets in the bank to be loaned out to a borrower.

Second, the intermediation role of depository institutions when producing these deposits implies the obligation to service the payment orders of depositors. Obviously, if the owner of the deposits make payments to other clients of the same bank, this service obligation is very easy to fulfill. The bank just renames the owner of the deposits. However, if the destination of the payment is a client of another bank, then the transfer of deposits must be met by a transfer of liquid assets. This will also happen if a household wants to convert the deposit into outside money (cash). Unlike DD, these liquid assets are not deposits that are left idle or invested in an inferior short term technology (remember the intermediation process starts on the asset side of the bank's balance sheet). These liquid assets are borrowed from the central bank either in the form of cash in banks' vaults or in the form of reserves (current accounts in the central bank). The refinancing rate of the central bank, i^o , represents the cost of this obligation.

In the model here, under the assumption that all wage and goods payments are distributed evenly across all banks, depositors will be disposing of their deposits but there would not be any need for banks to hold liquid assets as all net flows between them will be zero.¹² In other words, there is no liquidity risk associated with wage and goods payments. Nevertheless, households may demand the conversion of deposits into cash or the transfer of those deposits to another depository institution. In such instances, the corresponding bank would need outside money. Notice this demand for outside money will only show up in a bank run. In either case, when confronted with a run, banks will search for this outside liquidity supporting the role of the central bank as a lender of last resort (LOLR). In this sense, we could have included also a demand for cash at $t = 0$ adding the possibility of a run to happen already at that initial period. We have not done so for two reasons. First, without a run, households would still weakly prefer sight deposits to cash as long as deposits pay an interest $i_1^s \geq 0$. Second, we wanted to be as close as possible to DD which only include the possibility of a run at $t = 1$ so as better compare our results to theirs.

The third point to stress is that, contrary to the literature based on DD and the intermediation of real assets, this model separates the liquidation of financial positions from the liquidation of investment projects. The loan the bank holds provides a right to a future flow of funds for the bank. This flow of funds originates from the production and selling activities of the borrowers (the entrepreneurs in the model). In the event of insolvency, forcing the bank to liquidate that asset to respond to a deposit outflow does not necessarily imply the production activity financed with that loan has to be liquidated too. In fact, banks cannot force borrowers to pay earlier than what is established in the loan contract. What the bank can do is to try to sell the asset in the market,

¹²It is precisely the fact that payments are unevenly distributed across banks one of the reasons why outside money is needed to settle accounts among them in the real world.

possibly at a discount. This means the bank is only obtaining a fraction of the present discounted value of the cash flows produced by the loan. Therefore, the liquidation of bank assets is just a redistribution of future flows between market participants and does not need to imply a real cost for society as a whole. The anticipation that banks will not be able to honor these promises, either because they will not raise enough liquid funds in the market or from the central bank or because they will sell assets at a significant discount, is the reason why holders of its short term debt run the bank. But selling the loan this way, *per se*, has nothing to do with the ability of the borrower to pay back the loan. The borrower will continue with his/her investment with the loan payments now accruing to a different creditor.

Of course, the fact that financial liquidation is separated from real investment liquidation does not mean these two decisions are not linked. But any connection between these liquidation decisions must go through general equilibrium effects as changes in prices and interest rates in response to a generalized failure of the banking system may induce households to take them simultaneously. We look at this possibility in the solution of the model below.

3.2 Solution

3.2.1 Individual problems

Looking at the households' problems recursively, at $t = 1$ and $t = 2$ each household $h \in \{1, 2\}$ chooses consumption, c^h , liquidation of the long productive technology, y^h , as well as the portfolio allocation at $t = 1$, between cash, M_2^h , and sight deposits, S_2^h , to maximize utility $u(c^h)$ subject to constraints (10) through (16) depending on whether the household is impatient, $h = 1$, or patient, $h = 2$. In making these choices, households take as given prices, namely, nominal good prices in each period, P_1 and P_2 , and interest rates on loans, i^l , and sight deposits, i_1^s and i_2^s as well as the fraction of hired labor committed to the short, α , and long, $1 - \alpha$, technologies chosen at $t = 0$.

To describe the problem of households, notice that all nominal variables can be normalized by the initial level of the loan, W .¹³ Denote normalized nominal variables by the corresponding lower case letter. Take now a household of type h , entering period $t = 1$ with a fraction α of the hired labor in the short technology. Let $v^h(\alpha)$ be the maximum level of utility this household is going to obtain as a function of the technology portfolio. The problem the household solves is then

$$v^1(\alpha) = \max u(c^1) \quad (19)$$

subject to

$$p_1 c^1 \leq 1 + i_1^s, \quad (20)$$

$$m_2^1 + s_2^1 \leq 1 + i_1^s - p_1 c^1 + p_1 q_1^1, \quad (21)$$

¹³This economy is neutral with respect to the choice of the nominal bank size as measured by W . For a model in which this form of neutrality is not present see Rodriguez Mendizabal [25] and Rodriguez Mendizabal [26].

$$0 \leq q_1^1 \leq \alpha + \rho_1 y^1 \quad (22)$$

and

$$1 + i^l \leq p_2 [(1 - \alpha - y^1)\rho_2 + \alpha + \rho_1 y^1 - q_1^1] + m_2^1 + (1 + i_2^s)s_2^1 \quad (23)$$

if the household is impatient ($h = 1$), or

$$v^2(\alpha) = \max u(c^2) \quad (24)$$

subject to

$$m_2^2 + s_2^2 \leq 1 + i_1^s + p_1 q_1^2, \quad (25)$$

$$0 \leq q_1^2 \leq \alpha + \rho_1 y^2 \quad (26)$$

$$p_2 c_2^2 \leq m_2^2 + (1 + i_2^s)s_2^2 + p_2(\alpha + \rho_1 y^2 - q_1^2), \quad (27)$$

and

$$1 + i^l \leq p_2 [(1 - \alpha - y^2)\rho_2 + \alpha + \rho_1 y^2 - q_1^2] + m_2^2 + (1 + i_2^s)s_2^2 - p_2 c_2^2 \quad (28)$$

if the household is patient ($h = 2$).

On period $t = 0$ the household, if allowed to do so, chooses the split of hired labor between the short and long technology, α , to solve the problem

$$v = \max \lambda v^1(\alpha) + (1 - \lambda)v^2(\alpha) \quad (29)$$

subject to

$$0 \leq \alpha \leq 1. \quad (30)$$

On the other hand, banks make choices to maximize their net worth at $t = 2$, NW_2^B , specified in (18). This expression can also be normalized by W , so banks decide on their supply of (normalized) sight deposits (and possibly α) to maximize

$$\begin{aligned} nw_2^B = & 1 + i^l + (1 - \lambda)p_2 c^2 - p_2 [\lambda q_2^1 + (1 - \lambda)q_2^2] \\ & - (1 + i_2^s) [\lambda s_2^1 + (1 - \lambda)s_2^2] - (1 + i^o) [\lambda m_2^1 + (1 - \lambda)m_2^2]. \end{aligned} \quad (31)$$

Furthermore, the normalized net worth at $t = 1$ reads, from (17)

$$nw_1^B = 1 - [\lambda (m_2^1 + s_2^1) + (1 - \lambda) (m_2^2 + s_2^2)]. \quad (32)$$

We assume perfect competition in the banking sector. When banks make decisions, they take choices of households and prices as given, and ensure that net worth at $t = 1$ is positive.

3.2.2 Equilibrium with valued deposits

In this economy, an equilibrium without a bank run is defined as follows.

Definition 1 *An equilibrium without a bank run is a collection of allocations $\{\alpha, y^h, q_1^h, c^h, m_2^h, \text{ and } s_2^h\}$ for $h \in \{1, 2\}$ and prices $\{p_1, p_2, i^l, i_1^s, \text{ and } i_2^s\}$ such that:*

1. *given prices, allocations solve individual problems both of households and banks, and*
2. *prices are such that goods markets clear, for $t = 1$*

$$\lambda c^1 = \lambda q_1^1 + (1 - \lambda)q_1^2,$$

and $t = 2$

$$(1 - \lambda)c^2 = \rho_2 - (\rho_2 - 1)\alpha - (\rho_2 - \rho_1) [\lambda y^1 + (1 - \lambda)y^2] - [\lambda q_1^1 + (1 - \lambda)q_1^2].$$

When there is not a bank run, at $t = 1$, only impatient households, representing a fraction λ of the population, demand goods to cover for consumption, c^1 , while supplying goods q_1^1 . On the other hand, patient households, representing a fraction $1 - \lambda$ of the population, do not demand goods and supply q_1^2 . At $t = 2$, only impatient agents demand goods, c^2 , while both impatient and patient households supply goods from the return of the long technology not liquidated at $t = 1$ together with the storing carried over from the previous period. Notice the definition imposes that labor markets clear as workers supply labor inelastically. Also, financial markets clear because the objective function of banks is linear in choice variables, so that the supply of financial services is perfectly elastic at market rates and effectively is demand determined.

We are interested in figuring out whether an equilibrium with valued deposits exists and whether it is unique or not. In such an equilibrium households are willing to hold bank liabilities at $t = 1$. Because at $t = 1$ competition is between cash and sight deposits, we define an equilibrium with valued deposits as follows:

Definition 2 *An equilibrium with valued deposits is an equilibrium in which $s_2^1 > 0$ and $s_2^2 > 0$.*

We can now show the following proposition.

Proposition 1 *When entrepreneurs choose the size of the short term technology, α , there exists a unique equilibrium with valued deposits in which consumption levels are*

$$c^1 = 1, c^2 = \rho_2, \tag{33}$$

production decisions satisfy

$$y^1 = y^2 = 0, \tag{34}$$

$$q_1^1 = q_1^2 = \alpha = \lambda, \tag{35}$$

$$q_2^1 = q_2^2 = (1 - \lambda)\rho_2, \tag{36}$$

while

$$m_1^1 = m_1^2 = 0. \tag{37}$$

Furthermore, interest rates obey

$$1 \leq 1 + i_2^s = \rho_2 \frac{p_2}{p_1} = 1 + i^l, \tag{38}$$

$$i_1^s = 0, \quad (39)$$

and the price level for the first period is

$$p_1 = 1. \quad (40)$$

Proof. See the Appendix.

This is as much of the equilibrium as it can be characterized. Nevertheless, some conclusions can be drawn from this proposition. First, as shown in (33), the equilibrium in the nominal economy does not provide households with any degree of insurance. The intuition of this result is as follows. Banks in this economy cannot make any promise about the real value of deposits, nor they can condition the rates on sight deposits on household types. Thus, effectively they cannot write deposit contracts with real contingent payouts.

Second, as expressed in (38), notice the price level for the second period, p_2 , together with the nominal interest rate for the sight deposit from $t = 1$ to $t = 2$, i_2^s , and the lending rate, i^l , are not determined. In an equilibrium with valued deposits, those nominal levels do not matter as long as the real rate of those deposits is equal to ρ_2 . This is because, for sight deposits to be demanded between $t = 1$ and $t = 2$, they should pay at least the real return of the long technology. However, the equilibrium prices cannot be such that the real rate of sight deposits is above ρ_2 . If that were the case, entrepreneurs would not invest in the long technology, expecting to sell all output at $t = 1$ and carry on income to $t = 2$ in sight deposits. Furthermore, nobody will store at $t = 1$ to supply goods at $t = 2$ because storing provides with a real return of 1. This means, those prices cannot be an equilibrium so that the real return on sight deposits have to be exactly equal to ρ_2 .

Third, because there is no price uncertainty, it is not optimal for entrepreneurs to liquidate the long term technology beforehand, thus, having result (34). However, having sight deposits with a real return equal to ρ_2 produces two effects on the goods market. On the one hand, at those prices individual entrepreneurs are indifferent between any $\alpha \in (0, 1)$. They could invest more in the short technology, sell goods at $t = 1$, and save the proceeds in the sight deposit until $t = 2$ or else, they could invest more in the long technology and obtain the same real revenue at $t = 2$. On the other hand, using the budget constraints (23) and (28) for households to be able to payback the loan at the equilibrium prices, relative demand for consumption should be such that $c^2 = \rho_2 c^1$. For market clearing to happen with elastic supplies, we should have $c^1 = 1$, $c^2 = \rho_2$ and $\alpha = \lambda$ as described in (33), (35), and (36).

Fourth, in an equilibrium with valued deposits, $i_2^s \geq 0$. For deposits to be valued they should dominate in rate of return to cash. Furthermore, $i_2^s < 0$ is not an equilibrium if the refinancing rate of the central bank is positive, $i^o \geq 0$. This is because, if $i_2^s < 0$, the resulting demand for cash will generate a cost for banks as they will have to borrow the outside money from the central bank. Competition in the banking industry will push the deposit rate above 0 to reduce the cost of payment intermediation, thus, effectively making cash to be dominated in rate of return, providing result (37).

Fifth, the levels of the lending rate, $i^l = i_2^s$, and the deposit rate between $t = 0$ and $t = 1$, $i_1^s = 0$, in (38) and (39), are the ones ensuring that the net worth of banks is zero both at $t = 1$ and $t = 2$. Thus, banks are solvent in both periods and maintaining funds in them is an equilibrium outcome. Given these rates, prices in period 1 are equal to $p_1 = 1$ to support equilibrium in the good market as reflected in (40).

Some authors, like Skeie [28], argued that efficiency could be restored if banks were to choose the level of real liquidity, α . In this sense, we have the following result.

Proposition 2 *The equilibrium is independent of who chooses the size of the short term technology, α , either the banks or the entrepreneurs.*

Proof. See the Appendix.

Unlike Skeie [28], having banks choosing the level of real liquidity, α , not only does not produce the efficient allocation but it cannot improve upon the allocation obtained when α is chosen by entrepreneurs. The reason is that banks face the same trade-off as entrepreneurs but on the other side of its balance sheet. Increasing α makes entrepreneurs sell more at $t = 1$ which implies a marginal cost of $(1 + i_2^s)p_1$ in terms of deposit remuneration at $t = 2$. On the other hand, at that period, entrepreneurs will sell $p_2\rho_2$ less goods which represent a marginal revenue. Again, for a solution to exist, these marginal revenue and marginal cost should equate which makes each bank indifferent among possible values of α . With a real deposit rate equal to ρ_2 , households will pay back the loan maintaining a consumption ratio of $c^2 = \rho_2 c^1$ which produces an equilibrium value of $\alpha = \lambda$ as before.

3.2.3 The possibility of a run

In a bank run, households decide in $t = 1$ they do not trust their bank to be solvent and withdraw their funds. Notice both impatient and patient households need funds at $t = 2$ to pay back the loan they asked for at $t = 0$. Thus, when households are concerned about solvency, all of them, independent of their type, may have incentives to transfer funds between $t = 1$ and $t = 2$ by means of a different asset than the deposits at their bank. This could be cash, deposits at a different banking institution or goods stored.

To make the run comparable with the one in DD, we assume it takes place at the beginning of $t = 1$, before households start purchasing goods but after they know their types. This means that the decision on the split between the short and long technologies has already been taken which means $\alpha = \lambda$. Furthermore, the loan rate, i^l , as well as the rate of the deposits between $t = 0$ and $t = 1$, $i_1^s = 0$, are also determined and cannot be changed. Given this deposit rate, all agents start period $t = 1$ holding a normalized value of 1 in bank liabilities in the form of deposits. In the event of a run, depositors will evaluate the extent to which the bank will be solvent the following period. When a household withdraws its deposits at $t = 1$, the bank is forced to borrow that amount in

reserves from the central bank at the rate i^o . Notice this should be independent of whether the withdrawal is in cash or as an electronic transfer to another financial institution. Thus, if all depositors withdraw their deposits from a bank, this institution will remain solvent as long as

$$1 + i^l - (1 + i^o) \geq 0. \quad (41)$$

This is the condition households evaluate to join the run or not. If condition (41) is not satisfied, any household will anticipate there will be no funds left in the bank at $t = 2$ to honor its deposit and will prefer to withdraw at $t = 1$. Because i^l has already been set at $t = 0$, the existence of a run should be a purely self-fulfilling phenomenon provided condition (41) does not hold. This condition also shows that, once lending rates i^l are known, low enough refinancing rates i^o could prevent a self-fulfilling run. We then have the following result.

Proposition 3 *Households do not have incentives to coordinate in a run in their bank, given that the remaining banks are solvent, as long as the refinancing rate is low enough, in particular, as long as*

$$1 + i^o \leq \rho_2 p_2. \quad (42)$$

If this condition is not satisfied, a self-fulfilling run on any of the banks is supported in equilibrium.

Proof: See the Appendix.

Notice this result is independent on whether the run occurs in a bank or in the banking system as a whole. It does not matter either whether the amount of real liquidity, α , is chosen by the entrepreneurs or the bank. The consequences for the equilibrium of the existence of a bank run may depend on its extent and the initial choice of α . This is what we analyze below distinguishing between the case in which the bank run occurs in a single institution from the case in which there is an economy-wide bank run.

Individual bank run When the run affects only a single bank, depositors of that depository institution decide to withdraw their deposits, totally liquidating them. Because there are no solvency concerns with respect to other banks, and because deposits still dominate cash in rate of return, these funds are then transferred to a different financial institution as sight deposits. As the bank that is run is an atomistic agent in the economy, this means prices remain at their equilibrium values

$$1 + i^l = 1 + i_2^s = \rho_2 \frac{p_2}{p_1} = \rho_2 p_2 > 1.$$

Also, assuming all customers of the running bank distribute themselves among the rest of banks in the economy, the balance sheets of the recipient banks are not altered.

The bank that is run faces a different situation, though. Now as withdrawals take place, the bank demands funds to the central bank in the form of outside money at the price i^o to be paid at $t = 2$. Because these withdrawals happen at the beginning of period $t = 1$, before consumption takes place, the normalized amount consumers demand to be withdrawn from the bank and transferred to a different financial institution will be $1 + i_1^s = 1$, independently of household's types.¹⁴ Notice with this, the bank is still solvent at $t = 1$. At $t = 2$, for the bank to be solvent it needs to be the case that

$$nw^r = 1 + i^l - (1 + i^o)m_1^r \geq 0,$$

where m_1^r is the amount of outside money obtained from the central bank to service withdrawals by customers. Here, the superscript r indicates that the run is an individual bank. Thus, the aggregate withdrawal customers of the bank can make should be equal to

$$m_1^r = \min \left\{ 1, \frac{1 + i^l}{1 + i^o} \right\} = \min \left\{ 1, \frac{\rho_2}{1 + i^o} p_2 \right\}.$$

That is, in the event of a run at a bank, m_1^r is the maximum amount of reserves the central bank will be willing to lend to that bank. Here, we assume the bank collects all these withdrawal orders and distribute m_1^r equally among its customers when the institution is liquidated.

With this, the form of problem for each of the households involved in the run does not change because transferring funds between banks is costless for them and prices are the same. The only difference is that all households will have liquid funds at $t = 1$ equal to $m_1^r \leq 1$. Then households choose c^{hr} , y^{hr} , q_1^{hr} , s_2^{hr} , and m_2^{hr} to maximize

$$u(c^{1r}) \tag{43}$$

subject to

$$p_1 c^{1r} \leq m_1^r \tag{44}$$

$$m_2^{1r} + s_2^{1r} \leq m_1^r - p_1 c^{1r} + p_1 q_1^{1r} \tag{45}$$

$$q_1^{1r} \leq \alpha + \rho_1 y^{1r} \tag{46}$$

and

$$1 + i^l \leq p_2 [(1 - \alpha - y^{1r})\rho_2 + \alpha + \rho_1 y^{1r} - q_1^{1r}] + m_2^{1r} + (1 + i_2^s)s_2^{1r} \tag{47}$$

if the household turns out to be impatient, or

$$u(c^{2r}) \tag{48}$$

subject to

$$m_2^{2r} + s_2^{2r} \leq m_1^r + p_1 q_1^{2r}, \tag{49}$$

$$q_1^{2r} \leq \alpha + \rho_1 y^{2r} \tag{50}$$

¹⁴ As seen above, in equilibrium $i_1^s = 0$.

$$p_2 c^{2r} \leq m_2^{2r} + (1 + i_2^s) s_2^{2r} + p_2 (\alpha + \rho_1 y^{2r} - q_1^{2r}), \quad (51)$$

and

$$1 + i^l \leq p_2 [(1 - \alpha - y^{2r}) \rho_2 + \alpha + \rho_1 y^{2r} - q_1^{2r}] + m_2^{2r} + (1 + i_2^s) s_2^{2r} - p_2 c^{2r}. \quad (52)$$

Now, the deposits s_2^{hr} are at a different financial institution than the one the household started with at $t = 0$. Remember, because this is a run in an individual bank, prices $\{i^l, i_1^s, i_2^s, p_1, \text{ and } p_2\}$ remain at their equilibrium levels with valued deposits.

Proposition 4 *When condition (42) is not satisfied and a run happens in an individual bank, its depositors will consume*

$$c^{1r} = \frac{\rho_2 p_2}{1 + i^o} < 1 \text{ and } c^{2r} = \rho_2 \frac{\rho_2 p_2}{1 + i^o} < \rho_2, \quad (53)$$

while

$$y^{1r} = y^{2r} = 0, \quad (54)$$

and

$$q_1^{1r} = q_1^{2r} = \lambda. \quad (55)$$

No household has an incentive not to join the run.

In summary, when households decide on the possibility of joining a run in their bank, they evaluate the extent to which the bank will be solvent at $t = 2$. Solvency will now depend on the relative costs of the reserves needed to satisfy the transfers demanded by the bank's customers. According to Proposition 4, multiplicity of equilibria, and the possibility of a self-fulfilling run on a particular bank, depends on the level of the official rate of the central bank. For relatively low rates, there is only one equilibrium with solvent banks, while for relatively high rates, there is also an inferior equilibrium in which depositors coordinate in a run. Condition (42) specifies the threshold for the refinancing rate, above which the bank becomes insolvent at $t = 2$ and the run may take place. In such a run, both types of households are worse off. This is because, being atomistic within the whole economy, the supply of goods and prices in both periods are the same while the value of their deposits at $t = 1$ falls for all households. Notice even in this case, there is neither liquidation of the long technology nor storage. This is needed for households to pay back the loan. In this sense, the recovery of the loan is not enough to drive the bank solvent because the problem is the cost of liquidity.

Aggregate bank run Unlike the run on a single bank, in a system-wide run there is no other bank to turn into and the withdrawals are done in cash and households transfer resources from $t = 1$ either in cash or in goods. Notice that, as the run is assumed to happen at $t = 1$, all households have already chosen the investment in the short technology, $\alpha = \lambda$. These choices, together with the interest for loans, i^l , and $t = 0$ sight deposits, $i_1^s = 0$, stay at their equilibrium

values as they were determined at $t = 0$. Furthermore, because all banks are affected by a run, aggregate prices, p_1 and p_2 could be affected.

As before, let m_1^R the normalized amount withdrawn from any bank at the beginning of $t = 1$ and converted into cash. This is the aggregate amount each bank needs to borrow from the central bank. Here, the superscript R denotes the fact that there is a run in the whole banking system of the economy. As before, at $t = 2$, for the bank to be solvent it needs to be the case that

$$nw_2^R = 1 + i^l - (1 + i^o)m_1^R \geq 0.$$

Thus, the withdrawal customers can make should be

$$m_1^R = \min \left\{ 1, \frac{1 + i^l}{1 + i^o} \right\}.$$

Again, assume this amount is distributed equally among depositors as cash when the bank is liquidated.

With this, the individual problems become choosing c^{hR} , y^{hR} , and m_2^{hR} to maximize

$$u(c^{1R}) \tag{56}$$

subject to

$$p_1^R c^{1R} \leq m_1^R \tag{57}$$

$$m_2^{1R} = m_1^R - p_1^R c^{1R} + p_1^R q_1^{1R} \tag{58}$$

$$q_1^{1R} \leq \alpha + \rho_1 y^{1R} \tag{59}$$

and

$$1 + i^l \leq p_2^R [(1 - \alpha - y^{1R})\rho_2 + \alpha + \rho_1 y^{1R} - q_1^{1R}] + m_2^{1R} \tag{60}$$

if the household turns out to be impatient, or

$$u(c^{2R}) \tag{61}$$

subject to

$$m_2^{2R} = m_1^R + p_1^R q_1^{2R}, \tag{62}$$

$$q_1^{2R} \leq \alpha + \rho_1 y^{2R} \tag{63}$$

$$p_2^R c^{2R} \leq m_2^{2R} + p_2^R (\alpha + \rho_1 y^{2R} - q_1^{2R}), \tag{64}$$

and

$$1 + i^l \leq p_2^R [(1 - \alpha - y^{2R})\rho_2 + \alpha + \rho_1 y^{2R} - q_1^{2R}] + m_2^{1R} - p_2^R c^{2R}, \tag{65}$$

if the household turns out to be patient. Here, potentially all endogenous variables are affected with the exception of α , i^l , and i_1^s which are determined at $t = 0$.

Proposition 5 *When condition (42) is not satisfied and an aggregate run happens in the whole banking system, the equilibrium is*

$$c^{1R} = 1, c^{2R} = \rho, \quad (66)$$

production decisions satisfy

$$y^{1R} = y^{2R} = 0, \quad (67)$$

$$q_1^{1R} = q_1^{2R} = \alpha = \lambda, \quad (68)$$

$$q_2^{1R} = q_2^{2R} = (1 - \lambda)\rho_2, \quad (69)$$

while

$$m_1^{1R} = p_1^R \lambda, \quad (70)$$

and

$$m_1^{2R} = \frac{\rho_2 p_2^R}{1 + i^o} + p_1^R \lambda. \quad (71)$$

Furthermore, prices obey

$$p_1^R = \rho_2 p_2^R. \quad (72)$$

Notice the run does not change the real allocation of the economy because prices adjust accordingly. It does not impose storing or liquidation of the long technology. However, households will always join the run as not doing so will mean losing nominal wealth, m_1^R , which has a real value either at $t = 1$ or $t = 2$.

4 Conclusions and Implications

In this paper we have provided a model for analyzing the maturity transformation and liquidity insurance dispensed by banks in a nominal economy with endogenous money creation. In the model banks commit to implement a payment system. In this setup, we make an explicit separation between the liquidity created through lending and the reserves held by depository institutions at the central bank. Sight deposits are used by depositors to acquire consumption goods while reserves are employed by the banks to service payment orders and offset liquidity risk. Moreover, we also differentiate between the liquidation of financial (nominal) assets and the liquidation of real investments. Withdrawals generally imply the convertibility to sight deposits produced by other banks. The liquidation of bank assets to honor this commitment does not imply, however, the extinction of the real investment loans finance. The reason is that most of the financial assets so originated are noncallable, and the liquidation of loans results in selling the right to the future cash flows they generate.

Our paper can be viewed as corroborating the hypothesis, already included in Diamond and Dybvig [9], about the equivalence between deposit insurance and the lender of last resort function of central banks when technology is riskless. There is one caveat, though. We observe that, in equilibrium, nominal

deposit contracts do not reproduce the efficient allocation traditionally found in the previous literature. The interpretation of this result is that, in a nominal setup, depository institutions cannot commit to support a particular consumption bundle according to the future liquidity needs of their depositors. Since inside money creation in our model is linked to loan origination, the counterpart to loan provision is the creation of debt contracts redeemable on demand. The debt holder chooses the demandability of these liabilities, namely sight and time deposits, but it does not achieve the optimality in terms of liquidity risk-sharing.

On the policy front, we show that the existence of a self-fulfilling equilibria depends on the refinancing rate of the central bank. In the model, depositors only have incentives to withdraw early if they anticipate the insolvency of the depository institution. We study the possibility of an individual bank run as in Diamond and Dybvig [9], as well as the existence of a system-wide run in which depositors of the entire banking system coordinate in a run. In both cases, we state that whenever the central bank provides outside money at proper rates, depositors will anticipate the solvency of the banking system and the eventual illiquidity of the bank will not imply its bankruptcy.¹⁵

The failing response of the Fed to offset the banking panics during the Depression can be considered as a case study to support the view that an effective discount-window lending policy can prevent bank runs. As noted by Meltzer [21], [22], the Fed was at the center of the forces creating the banking panics of 1930-1933. The misreading of monetary conditions was explained by the implementation of an ineffective discount window mechanism, manifested by the massive decline in the money supply, a collapse of the public's deposit currency ratio, and the exclusive access to this facility to member banks.

Further evidence of the role of central banks to prevent a wave of panics can be found in the response of the Federal Reserve of Atlanta in the period 1929-1933. The extraordinary and aggressive measures adopted by the Atlanta Fed to inject liquidity in the region helped to prevent the banking panic in Florida in 1929 (Carlson et al. [8]), and to contain the failures of Mississippi's banks during the initial banking panic of the 1930s (Richardson and Troost [23]). This evidence highlights the importance of the Bagehot's doctrine in the event of expectational panics of the Diamond and Dybvig type: a lender of last resort policy that commits to provide liquidity to banks can be effective to prevent a system-wide run and restore the confidence of depositors in the banking system.

Finally, another interesting feature of the model described here is that it provides an explicit bridge between banking and monetary theories. At least from a quantitative point of view, bridging this gap seems important since liabilities of depository institutions make the bulk of broad monetary aggregates. For the US, travelers checks and checkable deposits have represented about 67 percent of M1 over the period 1959-2017, while these items together with small time deposits and savings deposits have represented an average 85 percent of M2 over

¹⁵This eventuality was also anticipated by Diamond and Dybvig [9] when they stated that the central bank would buy bank assets "... for prices greater than their liquidating value."

the same period. For the euro area, the liabilities of depository institutions have represented an average of 84, 91 and 83 percent of M1, M2 and M3, respectively between 1997 and 2017. In the model, both the size of banks' balance sheet and the split between monetary and nonmonetary liabilities are endogenous. Furthermore, the model introduces reserves as a liability of the central bank which is different from the monetary units depository institutions create and connects it with the payment flows between banks. These connections make the model a promising tool to open the box as of how central banks are able to manage economic activity by controlling the cost of expanding bank's balance sheet through changes in the level of the refinancing rate.

References

- [1] Allen, F., and D. Gale (1998): "Optimal financial crises" *Journal of Finance* 53, pp. 1245-1283.
- [2] Allen, F., E. Carletti, and D. Gale (2009): "Interbank market liquidity and central bank intervention" *Journal of Monetary Economics* 56(5), pp. 639-652.
- [3] Allen, F., E. Carletti, and D. Gale (2014): "Money, financial stability and efficiency" *Journal of Economic Theory* 149, pp. 100-127.
- [4] Board of Governors of the Federal Reserve System (2003): "Changes to the E.2 Statistical Release", Federal Reserve Statistical release, June 25, available at <https://www.federalreserve.gov/releases/e2/200306/e2.pdf>.
- [5] Bryant, J. (1980): "A model of reserves, bank runs, and deposit insurance" *Journal of Banking and Finance* 4, pp. 335-344.
- [6] Calomiris, C. W. and J. R. Mason (1997): "Contagion and bank failures during the Great Depression: The June 1932 Chicago Bank panic" *American Economic Review* 87, pp. 863-883.
- [7] Calomiris, C. W. and J. R.. Mason (2003): "Fundamentals, panics, and bank distress during the Depression" *American Economic Review* 93, pp. 1615-1647.
- [8] Carlson, M. A., K. J. Mitchener and G. Richardson (2011): "Arresting banking panics: Federal Reserve liquidity provision and the forgotten panic of 1929" *Journal of Political Economy* 119(5), pp. 889-924.
- [9] Diamond, D. W., and P. H. Dybvig (1983): "Bank runs, deposit insurance and liquidity" *Journal of Political Economy* 91(3), pp. 401-19.
- [10] Diamond, D. W., and R. G. Rajan (2001): "Liquidity risk, liquidity creation, and financial fragility: A theory of banking " *Journal of Political Economy* 109(2), pp. 287-327.

- [11] Diamond, D. W., and R. G. Rajan (2005): “Liquidity shortages and banking crises” *Journal of Finance* 60(2), pp. 615-647.
- [12] Diamond, D. W., and R. G. Rajan (2006): “Money in a theory of banking” *American Economic Review* 96, pp. 30-53.
- [13] Disyatat, P. (2011): “The bank lending channel revisited”, *Journal of Money Credit and Banking* 43(4), pp. 711-734.
- [14] Freixas, X., A. Martin, and D. Skeie (2009): “Bank liquidity, interbank markets, and monetary policy”, Federal Reserve Bank of New York, Staff Reports, 371.
- [15] Friedman, M. and A. J. Schwartz (1963): *A monetary history of the United States, 1867-1960*. Princeton University Press.
- [16] Goldstein, I. and A. Pauzner (2005): “Demand-deposit contracts and the probability of bank runs” *Journal of Finance* 60(3), pp. 1293-1327.
- [17] Gorton, G. (1988): “Banking panics and business cycles” *Oxford Economic Papers* 40(4), pp.751-781.
- [18] Holmstrom, B., and J. Tirole (1998): “Private and public supply of liquidity” *Journal of Political Economy*, 106(1), pp.1-40.
- [19] Lagos, R. (2006): “Inside and outside money” Staff Report 374, Federal Reserve Bank of Minneapolis.
- [20] Lucas, R. E. Jr., and N. Stokey (2011): “Liquidity crises” Federal Reserve Bank of Minneapolis, Economic Policy Paper 11-3.
- [21] Meltzer, A. H. (1976): “Monetary and other explanations of the start of the Great Depression” *Journal of Monetary Economics* 2(4), pp. 455-471.
- [22] Meltzer, A. (2003): *A history of the Federal Reserve*, volume 1. University of Chicago Press, Chicago.
- [23] Richardson, G. and W. Troost (2009): “Monetary intervention mitigated banking panics during the Great Depression: Quasiexperimental evidence from a Federal Reserve district border, 1929-1933” *Journal Political Economy* 117(6), pp. 1031-1073.
- [24] Rochet, J. C. and X. Vives (2004): “Coordination failures and the lender of last resort: Was Bagehot right after all? *Journal of the European Economic Association* 2(6), pp. 1116-1147.
- [25] Rodriguez Mendizabal, H. (2016): “Risk diversification and the large-small Bank dichotomy in money markets”, ADEMU Working Paper 40
- [26] Rodriguez Mendizabal, H. (2017): “Narrow banking with modern monetary institutions: Is there a reason to panic?”, ADEMU Working Paper 52.

- [27] Shin, H. S. (2009): “Reflections on Northern Rock: The bank run that heralded the global financial crisis” *Journal of Economic Perspectives* 23(1), pp. 101-119.
- [28] Skeie, D. (2008): “Banking with nominal deposits and inside money” *Journal of Financial Intermediation* 17(4), pp.562-584.

A Proofs

A.1 Proof of Proposition 1

The FOCs associated with problems (19) and (24) are, with respect to consumption, c^h ,

$$\frac{u'(c^1)}{p_1} = \theta_1^1 + \eta_1^1, \quad (73)$$

and

$$\frac{u'(c^2)}{p_2} = \theta_2^2 + \eta_2^2, \quad (74)$$

where θ_1^1 is the Lagrange multiplier associated with (20), η_1^1 is the Lagrange multiplier associated with (21), θ_2^2 is the Lagrange multiplier associated with (27) and η_2^2 is the Lagrange multiplier associated with (28).

Moreover, differentiating with respect to the amount of goods sold at $t = 1$, q_1^h , we obtain

$$p_1 \eta_1^1 + \delta_1^1 = \gamma_1^1 + p_2 \eta_2^1, \quad (75)$$

and

$$p_1 \eta_1^2 + \delta_1^2 = \gamma_1^2 + p_2 (\theta_2^2 + \eta_2^2), \quad (76)$$

together with the slackness conditions

$$\delta_1^h q_1^h = 0 \quad (77)$$

and

$$\gamma_1^h (\alpha + \rho_1 y^h - q_1^h) = 0 \quad (78)$$

where η_2^1 is the Lagrange multiplier associated with (23), η_1^2 is the Lagrange multiplier associated with (25), and δ_1^h is the Lagrange multiplier associated with the left hand side of either (22) or (26). Finally, γ_1^h is the corresponding Lagrange multiplier associated with the right hand side of either (22) or (26).

At the same time, we can differentiate against the liquidation of the productive technology, y^h ,

$$[\gamma_1^1 \rho_1 - p_2 \eta_2^1 (\rho_2 - \rho_1)] y^1 = 0, \quad (79)$$

and

$$[\gamma_1^2 \rho_1 - p_2 \eta_2^2 \rho_2 + p_2 (\theta_2^2 + \eta_2^2) \rho_1] y^2 = 0; \quad (80)$$

with respect to cash holdings, m_2^h ,

$$(\eta_2^1 - \eta_1^1) m_2^1 = 0, \quad (81)$$

and

$$(\theta_2^2 + \eta_2^2 - \eta_1^2) m_2^2 = 0, \quad (82)$$

and finally, with respect to sight deposits, s_2^h ,

$$[(1 + i_2^s)\eta_2^1 - \eta_1^1] s_2^1 = 0, \quad (83)$$

and

$$[(1 + i_2^s)(\theta_2^2 + \eta_2^2) - \eta_1^2] s_2^2 = 0. \quad (84)$$

There is also the envelope conditions with respect to α ,

$$\frac{dv^1(\alpha)}{d\alpha} = \gamma_1^1 - p_2 \eta_2^1 (\rho_2 - 1), \quad (85)$$

and

$$\frac{dv^2(\alpha)}{d\alpha} = \gamma_1^2 - p_2 \eta_2^2 \rho_2 + p_2 (\theta_2^2 + \eta_2^2). \quad (86)$$

Finally, the first order condition with respect to α in problem (29) is

$$\lambda \frac{dv^1(\alpha)}{d\alpha} + (1 - \lambda) \frac{dv^2(\alpha)}{d\alpha} + \beta_0 - \beta_1 = 0, \quad (87)$$

where β_0 is the Lagrange multiplier associated with the left hand side of (30) and β_1 is the Lagrange multiplier associated with the right hand side of (30).

An equilibrium with valued deposits in which $s_2^h > 0$, implies from (83) and (84), respectively

$$(1 + i_2^s)\eta_2^1 = \eta_1^1, \quad (88)$$

and

$$(1 + i_2^s)(\theta_2^2 + \eta_2^2) = \eta_1^2. \quad (89)$$

Then, from (81) and (82) it should be that $m_2^h = 0$ as cash is dominated in rate of return by deposits. Also, from (75) and (76), and using (88) and (89)

$$\frac{p_1 \eta_1^1}{p_2 \eta_2^1} = 1 + \frac{\gamma_1^1 - \delta_1^1}{p_2 \eta_2^1} = (1 + i_2^s) \frac{p_1}{p_2}, \quad (90)$$

and

$$\frac{p_1 \eta_1^2}{p_2 (\theta_2^2 + \eta_2^2)} = 1 + \frac{\gamma_1^2 - \delta_1^2}{p_2 (\theta_2^2 + \eta_2^2)} = (1 + i_2^s) \frac{p_1}{p_2}. \quad (91)$$

Then, only three possibilities may happen:

- (i) $(1 + i_2^s)p_1 < p_2$ so that $\gamma_1^h = 0$, $\delta_1^h > 0$ and $q_1^h = 0$ for both $h = 1, 2$.
- (ii) $(1 + i_2^s)p_1 = p_2$ so that $\gamma_1^h = 0$, $\delta_1^h = 0$ and $0 < q_1^h < \alpha$ and $y^h = 0$ for both $h = 1, 2$, and
- (iii) $(1 + i_2^s)p_1 > p_2$ so that $\gamma_1^h > 0$, $\delta_1^h = 0$ and $q_1^h = \alpha + \rho_1 y^h$ for both $h = 1, 2$.

Possibility (i) will never be an equilibrium because impatient households will not consume. Thus, we have to consider only cases (ii) and (iii).

Assume first the solution for α is interior so that $0 < \alpha < 1$ and $\beta_0 = \beta_1 = 0$. Given (85) and (86), for (87) to hold one possibility could be that

$$\gamma_1^1 - p_2 \eta_2^1 (\rho_2 - 1) = 0 = \gamma_1^2 - p_2 \eta_2^2 (\rho_2 - 1) + p_2 \theta_2^2.$$

In this case

$$\gamma_1^1 = p_2 \eta_2^1 (\rho_2 - 1) > 0$$

so that, by point (iii) above, it must be the case that $q_1^h = \alpha + \rho_1 y^h$ for both $h = 1, 2$. But then, from (75) and (88)

$$(1 + i_2^s) \frac{p_1}{p_2} = \rho_2 > 1 \quad (92)$$

while from (76), and (89)

$$(1 + i_2^s) \frac{p_1}{p_2} \left(\frac{\eta_1^2}{\theta_2^2 + \eta_2^2} \right) = \rho_2 > 1$$

which means that $\theta_2^2 = 0$ and the CIA constraint of patient households at $t = 2$ is slack. Given this case, it is easy to show that

$$\gamma_1^1 \rho_1 < p_2 \eta_2^1 (\rho_2 - \rho_1),$$

and

$$\gamma_1^2 \rho_1 < p_2 \eta_2^2 \rho_2 - p_2 (\theta_2^2 + \eta_2^2) \rho_1$$

so that from (79) and (80), $y^h = 0$ for both $h = 1, 2$. This means that $q_1^h = \alpha$. Using the budget constraints for both agents at $t = 2$ and (92) we have that

$$c^2 = \rho_2 c^1.$$

Then, from the market clearing

$$\lambda c^1 = \lambda q_1^1 + (1 - \lambda) q_2^2 = \lambda$$

it implies that $c^1 = 1$ and $c^2 = \rho_2$.

However, (87) could also hold if

$$\gamma_1^1 - p_2 \eta_2^1 (\rho_2 - 1) > 0 > \gamma_1^2 - p_2 \eta_2^2 (\rho_2 - 1) + p_2 \theta_2^2.$$

In this case

$$\gamma_1^1 > p_2 \eta_2^1 (\rho_2 - 1) > 0$$

so that, again, by possibility (iii) above, it must be the case that $q_1^h = \alpha + \rho_1 y^h$ for both $h = 1, 2$. But then, from (75) and (88)

$$(1 + i_2^s) \frac{p_1}{p_2} > \rho_2 \quad (93)$$

while, on the other hand,

$$\gamma_1^2 < p_2 \eta_2^2 (\rho_2 - 1) + p_2 \theta_2^2$$

and from (76), and (89)

$$\rho_2 > (1 + i_2^s) \frac{p_1}{p_2} \left(\frac{\eta_2^2}{\theta_2^2 + \eta_2^2} \right) \geq (1 + i_2^s) \frac{p_1}{p_2}$$

which is a contradiction with (93). Thus, this cannot be an equilibrium.

Finally, (87) could also hold if

$$\gamma_1^1 - p_2 \eta_2^1 (\rho_2 - 1) < 0 < \gamma_1^2 - p_2 \eta_2^2 (\rho_2 - 1) + p_2 \theta_2^2.$$

In this case assume first that $\gamma_1^1 > 0$, so that, again, by point (iii) above, it must be the case that $q_1^h = \alpha + \rho_1 y^h$ for both $h = 1, 2$. But then, from (75) and (88)

$$(1 + i_2^s) \frac{p_1}{p_2} < \rho_2 \quad (94)$$

while, on the other hand, again from condition in this case, (76), and (89)

$$\rho_2 < (1 + i_2^s) \frac{p_1}{p_2} \left(\frac{\eta_2^2}{\theta_2^2 + \eta_2^2} \right).$$

Clearly, if the CIA constraint for the patient household at $t = 2$ is not binding ($\theta_2^2 = 0$) this is a contradiction. But if that constraint is binding, so that $\theta_2^2 > 0$, both conditions may hold. However, from (79) it can be shown that $y^1 = 0$ and using the budget constraint for the impatient household at $t = 2$ yields

$$1 + i^l = p_2(1 - \alpha)\rho_2 + (1 + i_2^s)s_2^2 > p_2(1 - \alpha)\rho_2$$

while from the corresponding budget constraint for the patient household (remember, its CIA constraint is assumed to be binding)

$$1 + i^l = p_2(1 - \alpha - y^2)\rho_2 \leq p_2(1 - \alpha)\rho_2$$

which is a contradiction.

Then, the only interior solution involves $\alpha = \lambda$, $c^1 = 1$, $c^2 = \rho_2$, $y^1 = y^2 = m_2^1 = m_2^2 = 0$, and

$$(1 + i_2^s) \frac{p_1}{p_2} = \rho_2.$$

We can now rule out corner solutions in which either $\alpha = 0$ or $\alpha = 1$. For that, imagine first that $\alpha = 1$ so that $\beta_0 = 0$ and $\beta_1 > 0$ in (87). In this case, it is obvious that $y^1 = y^2 = 0$. Furthermore, it must be true that $0 < q_1^h < \alpha = 1$ for both $h = 1$. This is because, by (90) and (91), either both types of households store, so that $0 < q_1^h < 1$ for both $h = 1, 2$, or none of them store and $q_1^h = 1$ for both $h = 1, 2$. However in this last case, patient households waiting to consume

to period 2 will not have goods available and that cannot be an equilibrium. Then we must have that $0 < q_1^h < 1$ which means that $\delta_1^h = \gamma_1^h = 0$ for $h = 1, 2$. With this, the first term of (87) is negative

$$-p_2\eta_2^1(\rho_2 - 1) < 0.$$

For (87) to be equal to 0 it must be the case that

$$p_2(\theta_2^2 + \eta_2^2) > p_2\eta_2^2\rho_2.$$

Using (76) and (89) together with this condition implies

$$p_2(\theta_2^2 + \eta_2^2) = p_1\eta_1^2 = p_1(1 + i_2^s)(\theta_2^2 + \eta_2^2) > p_2\eta_2^2\rho_2$$

or

$$(1 + i_2^s)\frac{p_1}{p_2} = \frac{\eta_2^2}{\theta_2^2 + \eta_2^2}\rho_2. \quad (95)$$

On the other hand, from (75) and (88) we have in this case that

$$\frac{p_1\eta_1^1}{p_2\eta_2^1} = 1 = (1 + i_2^s)\frac{p_1}{p_2}. \quad (96)$$

Clearly, if the CIA constraint at $t = 2$ is not binding ($\theta_2^2 = 0$) there is a contradiction between (95), and (96). If, instead, the CIA constraint at $t = 2$ is binding ($\theta_2^2 > 0$) then, comparing the budget constraints at $t = 2$ for impatient households ($h = 1$)

$$1 + i^l = p_2(1 - q_1^1) + (1 + i_2^s)s_2^1$$

with those of patient households ($h = 2$)

$$1 + i^l = 0$$

there is an obvious contradiction. Thus, $\alpha = 1$ cannot be an equilibrium choice.

The last possibility is the other corner solution $\alpha = 0$ so that $\beta_0 > 0$ and $\beta_1 = 0$ in (87). Notice for expression (87) to be satisfied, the term $dv^h(\alpha)/d\alpha$ should be either negative for both types of households, or at least one of them. Above we have seen that it is not possible to have one positive and the other negative. Furthermore, if both are negative then we have that, at the same time,

$$\gamma_1^1 < p_2\eta_2^1(\rho_2 - 1)$$

and

$$\gamma_1^2 < p_2\eta_2^2(\rho_2 - 1) - p_2\theta_2^2.$$

However, it is easy to show then that

$$\gamma_1^1\rho_1 < p_2\eta_2^1(\rho_2 - \rho_1)$$

and

$$\gamma_1^2\rho_1 < p_2\eta_2^2(\rho_2 - \rho_1) - p_2\theta_2^2$$

which means, by (79) and (80), that $y^1 = y^2 = 0$. Again, this cannot be an equilibrium because there will not be goods available to consume at $t = 1$.

Thus, the only solution with valued deposits involves $\alpha = \lambda$, $c^1 = 1$, $c^2 = \rho_2$, $y^1 = y^2 = m_2^1 = m_2^2 = 0$, and

$$(1 + i_2^s) \frac{p_1}{p_2} = \rho_2.$$

With these values, the net worth of the banks at $t = 1$ and $t = 2$, respectively, are

$$nw_1^B = 1 - [\lambda(1 + i_1^s - p_1 + p_1\lambda) + (1 - \lambda)(1 + i_1^s + p_1\lambda)] = -i_1^s,$$

and

$$nw_2^B = 1 + i^l - (1 + i_2^s)(1 + i_1^s).$$

Equating these two expressions to 0 provides $i_1^s = 0$ and $i_2^s = i^l$. Furthermore, because the CIA constraint of impatient agents is binding, we get $p_1 = 1$. $\square$

A.2 Proof of Proposition 2

When banks choose α , they do so taking as given prices as well as choices by households of c^h , y^h , q_1^h , m_2^h , and s_2^h . This means conditions (73) to (84) are still valid. Given these choices, banks decide α at $t = 0$ to maximize net worth at $t = 2$, with the condition that net worth at $t = 1$ cannot be negative. Again, by (90), and (91) either both types of households store, so that $0 < q_1^h < \alpha$ with $y^h = 0$ for both $h = 1, 2$, or none of them store and $q_1^h = \alpha + \rho_1 y^h$ for both $h = 1, 2$. When choosing α banks take into account what consumers will do regarding the timing of buying and selling goods because these decisions affect the flow of funds each period and, therefore, the net worth of banks.

Let the net worth of a bank at $t = 1$ and $t = 2$ be, respectively,

$$nw_1^B = 1 - [\lambda(m_2^1 + s_2^1) + (1 - \lambda)(m_2^2 + s_2^2)]$$

$$\begin{aligned} nw_2^B &= 1 + i^l + (1 - \lambda)p_2c^2 - p_2[\lambda q_2^1 + (1 - \lambda)q_2^2] \\ &\quad - (1 + i_2^s)[\lambda s_2^1 + (1 - \lambda)s_2^2] - (1 + i^o)[\lambda m_2^1 + (1 - \lambda)m_2^2]. \end{aligned}$$

Then, the problem is to choose α to maximize nw_2^B subject to the constraint that net worth at $t = 1$ be positive

$$nw_1^B \geq 0,$$

the selling constraint of entrepreneurs at $t = 1$ and $t = 2$ which restricts their revenues and therefore affects savings at the bank,

$$q_1^h \leq \alpha + \rho_1 y^h,$$

$$q_2^h = (1 - \alpha - y^h)\rho_2 + \alpha + \rho_1 y^h - q_1^h$$

and

$$\alpha \in [0, 1]. \quad (97)$$

First assume households do not store so that $q_1^h = \alpha + \rho_1 y^h$ for both $h = 1, 2$. In this case,

$$q_2^h = (1 - \alpha - y^h)\rho_2.$$

Substituting for s_2^h from (21)

$$s_2^1 = 1 + i_1^s - p_1 c^1 + p_1 q_1^1 - m_2^1,$$

and (25)

$$s_2^2 = 1 + i_1^s + p_1 q_1^2 - m_2^2,$$

and using the values of $q_1^h = \alpha + \rho_1 y^h$ and $q_2^h = (1 - \alpha - y^h)\rho_2$, the net worth of the bank in the second period equals

$$\begin{aligned} nw_2^B &= 1 + i^l - (1 + i_2^s)(1 + i_1^s) + \lambda(1 + i_2^s)p_1 c^1 + (1 - \lambda)p_2 c^2 \\ &\quad - (1 + i_2^s)p_1 \rho_1 [\lambda y^1 + (1 - \lambda)y^2] + p_2 \rho_2 [\lambda y^1 + (1 - \lambda)y^2] \\ &\quad - (i^o - i_2^s) [\lambda m_2^1 + (1 - \lambda)m_2^2] - (1 + i_2^s)p_1 \alpha - p_2(1 - \alpha)\rho_2. \end{aligned}$$

Thus, the Lagrangian corresponding with maximizing this net worth subject to constraint (97) becomes

$$\begin{aligned} \mathcal{L}_2^B &= 1 + i^l - (1 + i_2^s)(1 + i_1^s) + \lambda(1 + i_2^s)p_1 c^1 + (1 - \lambda)p_2 c^2 \\ &\quad - (1 + i_2^s)p_1 \rho_1 [\lambda y^1 + (1 - \lambda)y^2] + p_2 \rho_2 [\lambda y^1 + (1 - \lambda)y^2] \\ &\quad - (i^o - i_2^s) [\lambda m_2^1 + (1 - \lambda)m_2^2] - (1 + i_2^s)p_1 \alpha - p_2(1 - \alpha)\rho_2 \\ &\quad + \beta_0 \alpha - \beta_1(\alpha - 1). \end{aligned}$$

The corresponding first order condition with respect to α is

$$p_2 \rho_2 - (1 + i_2^s)p_1 = \beta_1 - \beta_0.$$

If prices are such that $p_2 \rho_2 > (1 + i_2^s)p_1$, then $\beta_0 = 0$ and $\beta_1 > 0$. This means that $\alpha = 1$. Because we are assuming there is no storage, this cannot be an equilibrium since patient households will not get to consume. On the other hand, if prices are such that $p_2 \rho_2 < (1 + i_2^s)p_1$, then $\beta_0 > 0$ and $\beta_1 = 0$. This means that $\alpha = 0$. For this to be an equilibrium it must be the case that there is liquidation of the long technology so that impatient households have goods available to consume, that is, $y^h > 0$ for at least one $h = 1, 2$. Using (79) it must be the case that

$$\gamma_1^1 = p_2 \eta_2^1 (\rho_2 - \rho_1).$$

However, it is immediate to prove that

$$\gamma_1^1 > \gamma_1^1 \rho_1 > p_2 \eta_2^1 (\rho_2 - \rho_1) > p_1 \eta_1^1 - p_2 \eta_2^1$$

Thus, from (75) it follows that $\delta_1^1 > 0$ which means that $q_1^1 = 0$. This is a contradiction with the no storage assumption since the household is not selling

goods and liquidating the long technology. Similarly, using (80) it must be the case that

$$\gamma_1^2 > \gamma_1^2 \rho_1 \geq p_2 \eta_2^2 \rho_2 - p_2 (\theta_2^2 + \eta_2^2) \rho_1 > p_1 \eta_1^2 - p_2 (\theta_2^2 + \eta_2^2)$$

Thus, from (76) it follows that $\delta_1^2 > 0$ which means that $q_1^2 = 0$. Again, this is a contradiction with the no storage assumption since the household is not selling goods and liquidating the long technology.

Thus, the only option left is that prices are such that $p_2 \rho_2 = (1 + i_2^s) p_1$, then $\beta_0 = 0$ and $\beta_1 = 0$. This means that α has an interior solution and banks are indifferent among all values of α in $0 < \alpha < 1$. Because both $q_1^h = \alpha + \rho_1 y^h > 0$, we have that $\delta_1^h = 0$ and $\gamma_1^h > 0$ for both $h = 1, 2$. Then, from (79) and (80) it is easy to show that $y^h = 0$ for $h = 1, 2$. Then, comparing the budget constraints of the two households at $t = 2$, (23) and (28), at the equilibrium prices, we obtain that $c^2 = \rho_2 c^1$ and from the market clearing conditions we have, as before, that $\alpha = \lambda$, $c^1 = 1$, and $c^2 = \rho_2$.

The other option is for households to store so that $0 < q_1^h < \alpha$ for both $h = 1, 2$. Because there is storage it is not optimal to liquidate the long technology so $y^h = 0$ for $h = 1, 2$. Also, in this case, $\delta_1^h = \gamma_1^h = 0$ and the entrepreneurs sell at $t = 2$

$$q_2^h = (1 - \alpha) \rho_2 + \alpha - q_1^h,$$

for both $h = 1, 2$. The Lagrangian associated with the maximization problem of the bank becomes

$$\begin{aligned} \mathcal{L}_2^B = & 1 + i^l - (1 + i_2^s)(1 + i_1^s) + \lambda(1 + i_2^s)p_1 c^1 + (1 - \lambda)p_2 c^2 \\ & - (1 + i_2^s)p_1 [\lambda q_1^h + (1 - \lambda)q_1^h] + p_2 [\lambda q_1^h + (1 - \lambda)q_1^h] \\ & - (i^o - i_2^s) [\lambda m_2^1 + (1 - \lambda)m_2^2] - p_2 [(1 - \alpha)\rho_2 + \alpha] \\ & + \beta_0 \alpha - \beta_1 (\alpha - 1). \end{aligned}$$

The corresponding first order condition with respect to α is

$$-p_2(1 - \rho_2) = \beta_1 - \beta_0.$$

Because $\rho_2 > 1$ it must be the case that $\beta_0 = 0$ and $\beta_1 > 0$ which implies $\alpha = 1$. Because $\delta_1^h = \gamma_1^h = 0$ from (75), (76), (88), and (89)

$$\frac{p_2}{p_1} = \frac{\eta_1^1}{\eta_2^1} = \frac{\eta_1^2}{\theta_2^2 + \eta_2^2} = 1 + i_2^s$$

or

$$(1 + i_2^s) \frac{p_1}{p_2} = 1 < \rho_2.$$

Then, in equilibrium it must be the case that $c^1 = c^2 = 1$, and for net worth to be equal to zero both in $t = 1$ and $t = 2$ interest rates should be such that $i_1^s = 0$ and $i^l = i_2^s$. This situation, however, cannot be an equilibrium. This is because the real return on deposits (and real lending rate) is below the return on

the long technology. With these prices, any bank can decide to reduce α below 1. This reduces storage on the part of entrepreneurs, keeping their deposits constant. However, these entrepreneurs will sell $\rho_2 > 1$ more goods in period $t = 2$, increasing their revenues at that period. The bank can raise the lending rate splitting this surplus between households and entrepreneurs which should produce a positive profit. As all banks will do the same the economy converges to the previous equilibrium with $\alpha = \lambda$ which is the unique equilibrium here. $\square$

A.3 Proof of Proposition 3

To prove proposition 3 we start from the equilibrium with valued deposits characterized in Proposition 1. At equilibrium prices, if everyone joins the run at $t = 1$, banks affected will need to ask the central bank for an amount of liquidity equal to their liabilities at the time which are 1. Thus, with the revenues obtained at $t = 2$, $1 + i^l$, the bank should be able to pay back the refinancing provided by the central bank equal to $1 + i^l$. In the event that $1 + i^l < 1 + i^o$, any single client of a bank affected by a run will join in as the value of his/her deposits at $t = 2$ will be 0. $\square$

A.4 Proof of Proposition 4

When the run happens in a single bank (or a number of banks representing a measure 0 of the mass of banks) prices will stay at their equilibrium values included in Proposition 1, that is

$$1 \leq 1 + i_2^s = \rho_2 \frac{p_2}{p_1} = 1 + i^l,$$

$$i_1^s = 0,$$

and

$$p_1 = 1.$$

Also, notice what clients of the bank which is run do, is to transfer their deposits to another financial institution. Thus, problems (43) and (48) are identical to problems (19) and (24) except for the fact that now the wealth of all households affected by the run is $m_1^r < 1$ instead of 1. This means conditions (73) to (84) are still valid including the equilibrium choice of $\alpha = \lambda$ which was made at $t = 0$. At equilibrium prices, these conditions still imply that $q_1^h = \alpha = \lambda$ and $y^h = 0$, for both $h = 1, 2$. Furthermore, using the equilibrium prices in the budget constraints of the two households at $t = 2$ (23) and (28), together with these choices for q_1^h and y^h , we obtain that

$$c^{1r} = m_1^r = \frac{\rho_2 p_2}{1 + i^o} < 1 \text{ and } c^{2r} = \rho_2 m_1^r = \rho_2 \frac{\rho_2 p_2}{1 + i^o} < \rho_2. \quad \square$$

A.5 Proof of Proposition 5

With an aggregate run, the only predetermined choice for households is $\alpha = \lambda$ which is taken at $t = 0$. Also, prices i^l and i_1^s were set at $t = 0$ and stay fixed (whatever the value of i^l may have been). At $t = 1$, in the presence of an aggregate run, households reoptimize according to the programs (56) and (61), and new prices for goods p_1^R and p_2^R will be formed. Substituting for m_1^R from expression (58) in (57) and from (60) into (64) and (65), the first order conditions associated with these problems are those for consumption, c^{hR}

$$\frac{u'(c^{1R})}{p_1^R} = \theta_1^{1R} + \eta_2^{1R}, \quad (98)$$

and

$$\frac{u'(c^2)}{p_2} = \theta_2^2 + \eta_2^2, \quad (99)$$

where θ_1^{1R} is the Lagrange multiplier associated with (57), η_2^{1R} is the Lagrange multiplier associated with (60), θ_2^{2R} is the Lagrange multiplier associated with (64) and η_2^{2R} is the Lagrange multiplier associated with (65).

Moreover, differentiating with respect to the amount of goods sold at $t = 1$, q_1^{hR} , we obtain

$$\delta_1^{1R} = \gamma_1^{1R} + \eta_2^{1R} (p_2^R - p_1^R), \quad (100)$$

and

$$\delta_1^{2R} = \gamma_1^{2R} + (\theta_2^{2R} + \eta_2^{2R}) (p_2^R - p_1^R), \quad (101)$$

together with the slackness conditions

$$\delta_1^{hR} q_1^{hR} = 0 \quad (102)$$

and

$$\gamma_1^{hR} (\alpha + \rho_1 y^{hR}) = 0 \quad (103)$$

where δ_1^{hR} is the Lagrange multiplier associated with the left hand side of either (59) or (63) and γ_1^{hR} is the corresponding Lagrange multiplier associated with the right hand side of either (59) or (63).

Finally, we can differentiate against the liquidation of the productive technology, y^{hR} ,

$$[\gamma_1^{1R} \rho_1 - p_2^R \eta_2^{1R} (\rho_2 - \rho_1)] y^{1R} = 0, \quad (104)$$

and

$$[\gamma_1^{2R} \rho_1 - p_2^R \eta_2^{2R} \rho_2 + p_2^R (\theta_2^{2R} + \eta_2^{2R}) \rho_1] y^{2R} = 0. \quad (105)$$

From (100) and (101),

$$\frac{p_1^R}{p_2^R} = 1 + \frac{\gamma_1^{1R} - \delta_1^{1R}}{p_2^R \eta_2^{1R}}, \quad (106)$$

and

$$\frac{p_1^R}{p_2^R} = 1 + \frac{\gamma_1^{2R} - \delta_1^{2R}}{p_2^R (\theta_2^{2R} + \eta_2^{2R})}. \quad (107)$$

Then, only three possibilities may happen:

- (i) $p_1^R < p_2^R$ so that $\gamma_1^{hR} = 0$, $\delta_1^{hR} > 0$ and $q_1^{hR} = 0$ for both $h = 1, 2$.
- (ii) $p_1^R = p_2^R$ so that $\gamma_1^{hR} = \delta_1^h = 0$, and $0 < q_1^{hR} < \alpha$ and $y^{hR} = 0$ for both $h = 1, 2$, and
- (iii) $p_1^R > p_2^R$ so that $\gamma_1^{hR} > 0$, $\delta_1^{hR} = 0$ and $q_1^{hR} = \alpha + \rho_1 y^{hR}$ for both $h = 1, 2$.

As before, possibility (i) will never be an equilibrium because impatient households will not consume. Thus, we have to consider only cases (ii) and (iii). In case (ii) where $p_1^R = p_2^R$, using the budget constraints of the two households at $t = 2$ (23) and (28), together with these choices for q_1^{hR} and y^{hR} , we obtain that $c^{1R} = c^{2R}$. From market clearing the level of consumption should be

$$c^{1R} = c^{2R} = \lambda + (1 - \lambda)\rho_2.$$

Because this level of consumption should be smaller than 1 (notice in this case by feasibility there will be less than 1 unit of goods available at $t = 1$) it would imply that $\rho_2 < 1$ which is a contradiction.

Then we are left with the case (iii) in which $p_1^R > p_2^R$ and there is a deflation. On the one hand, from (100) we have that the term in brackets on the left hand side of (104) equals

$$\gamma_1^{1R} \rho_1 - p_2^R \eta_2^{1R} (\rho_2 - \rho_1) = \eta_2^{1R} (p_1^R \rho_1 - p_2^R \rho_2).$$

Furthermore, as before, the CIA constraint for patient households is not binding, so that $\theta_2^{2R} = 0$. Using this result, from (101) we have that the term in brackets on the left hand side of (105) equals

$$\gamma_1^{2R} \rho_1 - p_2^R \eta_2^{2R} \rho_2 + p_2^R \eta_2^{2R} \rho_1 = \eta_2^{2R} (p_1^R \rho_1 - p_2^R \rho_2).$$

Finally, using the budget constraints of the two households at $t = 2$ (23) and (28), together with these choices for q_1^{hR} and y^{hR} , we obtain that

$$(p_2^R \rho_2 - p_1^R \rho_1)(y^{2R} - y^{1R}) = p_1^R c^1 - p_2^R c^2.$$

Taking these three last expressions together we have that either $p_1^R \rho_1 < p_2^R \rho_2$ so that $y^{hR} = 0$ or else that $p_1^R \rho_1 = p_2^R \rho_2$ and $0 \leq y^{hR} < 1 - \alpha$ for both $h = 1, 2$. Notice that in either case $p_1^R c^1 = p_2^R c^2$. It is easy to see that the only possible equilibrium is one in which $c^{1R} = 1$, $c^{2R} = \rho_2$, and $p_1^R = p_2^R \rho_2$. $\square$