Right to choose in oral auctions

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Abstract

Oral, right-to-choose auctions raise higher revenue when buyers are risk averse. When standard sequential and right-to-choose auctions allocate the objects in the same fashion, this means that sellers prefer the latter, which is consistent with normal practice in sales of real estate, for instance. Prices follow a declining pattern, as has also been observed.

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1 Introduction

In this paper I study the rational for using what is called "right-to-choose" or "pooled" auctions for selling a number of "similar" units of a good. This is, for instance, the usual way condominium units are auctioned (see Ashenfelter and Genesove (1992)). These are oral, ascending, sequential auctions where the winner of a round gets the unit that she chooses among the so far unsold units. The question arises as to whether there is any advantage in offering the right to choose instead of auctioning each individual unit at a time, in a more standard sequential auction.

My answer is that the right to choose allows the seller to cash on the buyers' risk aversion. The explanation is straightforward. In oral auctions, the bidding behavior of rival bidders may transmit information about their valuation for the item(s) being sold. In particular, if one observes a rival bidder bidding aggressively for one particular object, one can infer that this bidder has a high relative valuation for this particular object. Of course, this inference is possible if the unit being sold is individually identified, that is, if we are in an oral, standard auction. If, on the other hand, what is at stake is the right to choose between two objects, observing a bidder bidding aggressively is a signal of her having a high willingness to pay for "some" of the units, without further information as to which of the units this is. This makes the value of the subsequent auctions more uncertain, in as much as the two units are not perfect substitutes for the buyer. Thus, risk averse bidders are willing to pay a premium in order to secure their winning in the first auction.

As perhaps the simplest illustration of this effect, assume there are two bidders and two apartments, A and B. Each bidder is willing to pay 1 for one of the two apartments (and 0 for the other), but neither bidder knows which apartment the other prefers. Ex-ante, either buyer prefers apartment A (and is then willing to pay 1 for this and only this apartment) with a (independent) probability of

$\frac{1}{2}$. Now, if apartment A is auctioned first, a buyer who prefers apartment B will immediately drop out of the auction. If the buyer prefers apartment A, on the other hand, she will be willing to match all rival bids up to 1. Something similar will happen in the second auction, the one for apartment B. Then, with a probability of $\frac{1}{2}$ (the probability that both bidders like the same apartment), bidders will compete by raising their offers for the apartment they prefer until the price reaches 1, which will be the revenue that the seller will get (the other apartment will sell for 0). With a probability of $\frac{1}{2}$, the seller will get zero revenues, since each bidder will be interested in a different apartment and will then face no competition. Therefore, the expected revenue for the seller is $\frac{1}{2}$.

On the other hand, if the seller chooses to auction the right to choose, the bidders know that loosing means that the rival will choose the apartment they prefer and leave the other to the original bidder (for a price of zero). Assume preferences for a buyer are given by the von Neumann-Morgensten utility function $u(\theta - P)$ where $\theta = 0$ if the buyer gets the apartment she prefers less or gets no apartment at all, $\theta = 1$ if the buyer gets the apartment she prefers, and P is the amount of money paid. Then her best bid today is a price b such that

$$u(1-b) = \frac{1}{2}u(1) + \frac{1}{2}u(0).$$

Then, if the buyers are risk neutral, $b = 1/2$ (which coincides with the seller's revenue) and both auction formats, the right-to-choose and the standard sequential, give the seller an expected revenue of $1/2$. However, if the buyers are risk averse, $b > 1/2$, and then the right-to-choose format gives the seller a higher revenue in expected terms. Again, the key is that buyers are willing to pay a premium in the less informative right-to-choose auction.

What I do in this paper is to extend this simple intuition to a model that includes a continuum of "valuations" for each of the objects, and an arbitrary number of buyers for "stochastically equivalent" objects.

To my knowledge, Gale and Hausch (1994) have been the only ones to analyze the right to choose in auctions. Their alternative explanation for its use is based on what they call "bottom-fishing". Consider the auctioning of two objects. In a standard sequential auction, a buyer who prefers the second object may still bid (the minimum bid) in the first auction. This bottom-fishing is justified ex-ante: if by bidding the lowest admissible bid the buyer gets the first good, this must be because other bidders also prefer the second object, in which case the expected price for that second object in the future auction is high. This bottom-fishing may reduce competition for the second unit even when buyers prefer this second unit, since they are assumed not to be able to participate in the second auction when they win the first. Then, if this effect is sufficiently severe, the seller's revenue is reduced as compared to that expected in a right-to-choose auction.

Gale and Hausch's insightful model can be objected to in several respects. First, they analyze sealed bid, second price auctions, whereas what one usually observes in auctions with the right to choose is oral, ascending auctions. Second, the buyers' preferences are peculiar: it is assumed that a buyer who gets the first unit will not try to get the second unit for even a very low price. Without assuming not only unit demands, but also a high cost of disposing second units (negative willingness to pay for second units), this does not seem to be a good feature of the model. Moreover, the revenue comparisons seem to hinge heavily on this assumption. Third, they study the two-buyer case. Whereas the insights they provide do not seem to depend on this assumption, the importance of the bottom-fishing on the seller's revenue is likely to decrease with the number of bidders, and therefore the likelihood that the right-to-choose auction gives a higher expected revenue to the seller.

I, on the contrary, consider oral auctions and need not assume (although for simplicity I will start by assuming) that a buyer can buy at most one of the objects. Buyers in general will have positive willingness to pay for each of the objects. In fact, buyers could even have multiple unit demands (see Section 6). This does not mean that I am considering a more general set-up than that of Gale and Hausch. In particular, for tractability, I will assume that buyers' preferences are one dimensional in the sense that a single parameter will describe the preferences over different combinations of objects and payments. I isolate the effect of risk aversion by considering a model in which the allocation of objects in each of the two auction formats is (if not the same) equivalent. Thus, and as in the example above, both auction formats will be equivalent under risk neutrality, but the right to choose will be a way for the seller to cash the buyers' aversion to risk

For the single unit case, Maskin and Riley (1984) have shown that it is never in the seller's interest to introduce uncertainty in the payments for given allocation of the good, but making the utility of buyers different when they win and lose is an appropriate screening device. The right to choose in auctions of substitutes, as we consider here, makes buyers' utility less random from an ex-ante point of view. In a way, it pools together the randomness which comes from the relative preferences of the objects: what a buyer pays for her most preferred good, if she wins the first auction, depends on other buyers' willingness to pay for the objects they most prefer, but does not depend on whether these buyers prefer one object or the other. In that sense, the results of this paper are consistent with the general intuition given by Maskin and Riley.

As an incidental result, in this paper I offer yet another explanation for declining prices in sequential auctions. In fact, I obtain this as a non ambiguous prediction for the case Ashenfelter and Genesove (1992) analyze. McAfee and Vincent (1992) have already shown that risk aversion can explain declining patterns of prices. Their explanation, however, is for standard sequential auctions. What I show here is that, even if it is hardly surprising, this tendency of prices to decline extends to auctions with the right to choose.

2 The model

There are $N \geq 3$ potential buyers for two distinct objects, A and B. Buyer i's preferences, $i = 1, 2, ..., N$ are characterized by two parameters (t_i, θ_i), where $t_i \in \{A, B\}$, the "type", and $\theta_i \in [0, 1]$, the "valuation". For each i, t_i takes the value A with a probability of $\frac{1}{2}$ and the value B with a probability of $\frac{1}{2}$. Also, θ_i is the realization of a random variable with c.d.f. F and density f which takes positive values on $[0, 1]$. All random variables are independent and common knowledge. In addition, buyer i knows the realization of (t_i, θ_i). The pay-off for buyer i with parameters (t_i, θ_i) who obtains object I at a price P is

$$U\left[I, P; (t_i, \theta_i)\right] = \begin{cases} u(\phi(\theta_i) - P) & \text{if } t_i = I \\ u(\theta_i - P) & \text{if } t_i \neq I \end{cases}, \qquad (1)$$

where ϕ is a strictly monotone increasing function, with $\phi(\theta) > \theta$, and u is a concave, von Neumann-Morgensten utility function with $u(0) = 0$. Also, the pay-off for buyer i who pays P and gets no object is

$$U\left[\emptyset, P; (t_i, \theta_i)\right] = u(-P). \qquad (2)$$

For the moment, we assume that a buyer cannot buy both objects (we will see in Section 6 that this is without loss of generality). We do not then need to define preferences for that case. Also, we will assume that $\phi(0) > 1$, so that a buyer of type $t_i = I$ is always willing to pay more for the object I than a buyer of type $t_i \neq I$, whatever their valuations. This will greatly simplify the analysis. Finally, we assume $\phi\prime(\theta) \geq 1$, that is, the difference in willingness to pay for the two objects does not decrease with the valuation. This will be for the sake of simplicity, as we will argue in Section 6.

Notice that we are portraying a situation in which, although the two objects can be identified (are distinct, as two apartments in the same condominium would be), they are symmetric from an ex-ante point of view. That is, neither of them is ex-ante more valuable than the other.

We next analyze the equilibrium outcomes and the seller´s revenue under two different, oral ascending, selling mechanisms. In the first, the *standard sequential auction*, one object, say A, is auctioned first in a standard English auction. Then object B is auctioned in the same fashion. In the alternative mechanism, the *right-to-choose auction*, the seller conducts an English auction whose winner chooses what object to buy at the hammered down price. Then the object left is offered, again in a standard English auction.

3 Standard sequential auction

Assume object A is offered for sale in the first place. A way to model the English (oral ascending) auction is to assume that a clock points to continuously ascending prices, and buyers keep their hands up for as long as they wish. Once they lower their hands they abandon the auction and cannot reenter (the auction for object A) later on. The good is sold to the last buyer to keep her hand up, and the price is the one the clock shows at the moment the last buyer to drop out lowers her hand. A strategy[1] for a buyer in the first auction is then a multidimensional mapping. It maps the set of parameters (t_i, θ_i), the set of integers $n \in \{0, N-2\}$, and the set of vectors $(p_1, .., p_n)$ in R^n, with $p_1 \leq .. \leq p_n$, into the set $[p_n, \infty)$.[2] We interpret the image of this mapping as the drop out price for a buyer with such a type and valuation (t_i, θ_i) when n other buyers (and only they) have dropped out at prices $p_1, .., p_n$. A bidding strategy for the second object would

[1]Given the symmetry of buyers, we will only consider "anonymous" bidding strategies, in which behavior does not depend on who took an action, but only on the nature of this action.

[2]To avoid difficulties with what is "the next moment" in continuous time, we assume that whenever one buyer drops out, the clock stops and there is time for other buyers to drop out at that same price.

be a similar function for each possible history observed in the first auction (a sequence $p_1, .., p_{N-1}$, and an index that identifies at which of these prices the buyer herself has dropped out). We now look for an (symmetric) equilibrium bidding strategy.

We should expect bidders to bid differently for different types and different valuations. Moreover, in the second auction a buyer's dominant strategy is to keep the hand up until the clock reaches a price equal to her true willingness to pay for object B. Then, the second auction will be won by the buyer of type B with the highest valuation, if there is still any of them in the auction. Otherwise, the winner will be the buyer of type A with highest valuation that bids in that auction.

Now consider the auction for object A. Any bidder of type A has higher willingness to pay for the object than any other bidder of type B. Moreover, for each hammered down price, the former expects lower utility in the second auction, the one for object B. Thus, in equilibrium we should have buyers of type A bidding more aggressively (staying longer) in the first auction than buyers of type B.

Keeping this in mind, let $b_n^t(\theta; p_1, .., p_n)$ be the drop out price in the first auction for a buyer of type t with valuation θ when n other buyers have dropped out at prices $p_1, .., p_n$. One would expect these functions to be monotone and one would also expect $b_n^B() < b_n^A()$. Also, to make things simpler, assume that if the last two buyers drop out at the same time, then the object is assigned to the lower valuation buyer. In Section 6 we will discuss the implications of dropping this assumption. Under these conditions:

Proposition 1 *In the standard sequential auction, the following strategy*

$-b_n^B(\theta; p_1, .., p_n) = \theta \qquad \forall n \leq N-3,\ p_n \leq \theta,$

$-b_{N-2}^B(\theta; p_1, .., p_{N-2}) = p_{N-2} \qquad \forall p_{N-2} \leq \theta,$

$-b_n^A(\theta; p_1, .., p_n) = \phi(\theta) \qquad \forall n,\ p_1 \leq 1,\ p_n \leq \phi(\theta),$

$$\text{and } \forall n \leq N-3,\ p_n \leq \phi(\theta),$$

$-b^A_{N-2}(\theta; p_1, .., p_{N-2}) = \phi(\theta) - \theta + \phi^{-1}(p_{N-2}) \qquad \forall p_1 \geq 1,\ p_{N-2} \leq \phi(\theta)$

−In the second auction, bid up to willingness to pay,

is a symmetric, equilibrium bidding function.

Proof. See Appendix. ■

Corollary 1: *The object t is won by the buyer of type t with the highest valuation, if there is any, and by the buyer with the second highest valuation if there are no buyers of type t.*

When a bidder drops out of the first auction at a low price, she signals herself as a type B bidder. Then all type A bidders know that they will be outbid in the auction for object B, and then the opportunity cost of loosing the auction for object A is their own willingness to pay. Thus, they are in a regular English auction, and they bid up to their willingness to pay for the object. However, if no bidder has dropped out before the clock points to 1, then they know that all buyers are of type A, and then their own opportunity cost of loosing the first auction is below their willingness to pay for object A; they still have a chance to win object B. As the auction develops, they eventually discover whether they really have a chance to win in the second auction (no more than one bidder has a higher valuation) or not.

Bidders of type B, on the other hand, can make the same inferences. If the clock has not reached a price equal to her willingness to pay for object A and there is more than one other bidder left, a bidder knows that either at least one of the remaining bidders is of type A, in which case nothing will change if she stays longer in the auction, or all of them are of type B. In this final case, the bidder's chances of getting object B are nil unless all but one drop out before the clock points at her own willingness to pay for object B (since in any of the other cases at least one of the remaining bidders will show up in the second auction and will have a higher willingness to pay for object B). Thus, the bidder will prefer to remain active unless either the clock reaches her willingness to pay for object A or all but at most one drop out before. In this final case, her best choice is also to drop out and try to get object B.

To conclude this section, notice that:

Corollary 2: *Both the allocation of the objects and the prices obtained by the seller are independent of the risk attitude of the buyers. In particular, the outcome of the auctions is the same as the one obtained when buyers are risk neutral, that is, when $u()$ is the identity.*

4 Right-to-choose auctions

Now assume that the seller conducts an oral, ascending auction for the right to choose one of the objects. That is, the clock raises the price continuously as in the previous section, and buyers keep their hands up for as long as they wish, also as in the standard sequential auction. The rules of the auction, and then who wins and what price she pays, are as in the standard sequential auction too, but now the winner chooses whether to take object A or object B.

Here, again, the buyers' bidding behavior may in principle depend on their type. However, given all the symmetry that we have assumed, one would not expect this to be the case. Moreover, one would expect that buyers with higher valuation would stay longer than buyers with lower valuation. Finally, in the second auction it is still true that bidding up to the willingness to pay (whether θ or $\phi(\theta)$) is a dominant strategy. Thus, we propose a common bidding strategy in the first auction for buyers of both types. Such a strategy is a collection of $N-1$ functions B_n for $n = 0, 1, ..., N-2$, where B_n maps the set of buyer's valuations and vectors of positive real numbers $(p_1, ..., p_n)$ with $p_1 \leq ... \leq p_n$, into the set $[p_n, \infty)$. We look for a symmetric, monotone equilibrium, that is, one in which $B_n(\theta; p_1, ..., p_n)$ is non-decreasing in θ. If buyers behave in this fashion, the fact that one drops out at a given price in the first auction does not convey information with respect to her type. Also, even the first buyer to drop out expects to obtain one object (in fact, the one she most prefers) with positive probability in the second auction. Indeed, since no information about the type is revealed, when a buyer drops out she still assigns positive probability to the event that all remaining buyers are of a type different to hers. Then:

Proposition 2 *The following strategy*

– $B_n(\theta; p_1, ..., p_n) = \theta \qquad \forall n \leq N-3,$

– $B_{N-2}(\theta; p_1, ..., p_{N-2}) = \phi(\theta) - u^{-1}(\pi_2(\theta; p_1, ..., p_{N-2})),$

where

$$\pi_2(\theta; p_1, ..., p_{N-2}) = \left(\frac{1}{2}\right)^{N-1} u(\theta - p_{N-2}) \tag{3}$$

$$+\sum_{k=2}^{N-1}\left(\frac{1}{2}\right)^{k} u(\phi(\theta) - \phi(p_{N-k})) + \left(\frac{1}{2}\right)^{N-1} u(\phi(\theta) - p_{N-2})$$

– *In the second auction bidding up to willingness to pay for the object sold; is an equilibrium strategy for the right to choose auction.*

Proof. See Appendix. ■

Corollary 3: *The buyer with highest valuation obtains the object she most prefers. The remaining object is won by the buyer with highest willingness to pay for it among the remaining buyers.*

Under monotonicity, observing $(p_1, ..., p_n)$ is equivalent to observing the lowest n realizations of the buyers' valuations, $(\theta_1, ..., \theta_n)$. Then, $\pi(\theta; p_1, ..., p_{N-2})$ is the utility a bidder with valuation θ expects if she quits after observing that $N-2$ other bidders have dropped out at prices $(p_1, ..., p_{N-2})$. The first term on the right hand side of (3) is the utility if all other buyers happen to be of her type. The second (sum of) terms represent the buyer's utility when the winner of the first auction (the other remaining bidder in the first auction) is not of her type, and the bidder who dropped out in $(N-k)th$ place is the bidder of her type with highest valuation among those that have already dropped out. The last term represents the utility when nono of the rivals is of her type.

As opposed to what was the case in the sequential auction, here the attitude towards risk does play a role. In particular:

Corollary 4: *The seller expects higher revenue when the buyers are risk averse than when they are risk neutral.*

Proof. The price in the second auction is not affected by risk attitudes. For any realization of the buyers' valuations $(\theta_1 \leq \theta_2 \leq ... \leq \theta_N)$, the price in the first auction is $\phi(\theta_{N-1}) - u^{-1}(\pi(\theta_{N-1}; \theta_1, ..., \theta_{N-2}))$. Now, if u is strictly concave,

$$\pi(\theta_{N-1}; \theta_1, ..., \theta_{N-2}) < u\left(\left(\frac{1}{2}\right)^{N-1}(\theta - p_{N-2}) + \sum_{k=2}^{N-1}\left(\frac{1}{2}\right)^{k}(\phi(\theta) - \phi(p_{N-k})) + \left(\frac{1}{2}\right)^{N-1}(\phi(\theta) - \phi(p_{N-2}))\right)$$

and then the price for the seller is higher than

$$\phi(\theta_{N-1}) - \left(\left(\frac{1}{2}\right)^{N-1}(\theta - p_{N-2}) + \sum_{k=2}^{N-1}\left(\frac{1}{2}\right)^{k}(\phi(\theta) - \phi(p_{N-k})) + \left(\frac{1}{2}\right)^{N-1}(\phi(\theta) - \phi(p_{N-2}))\right),$$

which is the price in the case of risk neutral buyers. ■

5 Comparing the two mechanisms

We could directly compare the revenues for the seller in both mechanisms. However, we will proceed in the tradition of revenue equivalence for risk neutral agents. Once this is done, the corollaries to Propositions 1 and 2 will directly imply that risk aversion makes the right-to-choose auction a more attractive one for a risk neutral seller.

Thus, for any (anonymous, for economy of notation) mechanism (and one of its equilibria) for assigning both objects, let us specify the 6-tuple $\{x_A^A, x_B^A, x_A^B, x_B^B, P_A, P_B\}$ where $x_I : \Theta \rightarrow [0,1]$, and $P_I : \Theta \rightarrow R$, $I = A, B$, where the value $x_I^J(\theta)$ represents the probability that a buyer of type I obtains obtains the object J when her valuation is θ (and everybody behaves as the equilibrium predicts), and $P_I(\theta)$ is the expected payment in such a case. Then, if buyers are *risk neutral*, incentive compatibility (a necessary condition for equilibrium behavior) requires

$$\theta = \arg\max_{z\in[0,1]} \; x_A^A(z)\;\phi(\theta) + x_A^B(z)\;\theta - P_A(z),$$

for all θ, for a buyer of type A. Under differentiability, the first order condition for this problem is the following differential equation

$$\frac{dP_A(\theta)}{d\theta} = \frac{dx_A^A(\theta)}{d\theta}\phi(\theta) + \frac{dx_A^B(\theta)}{d\theta}\;\theta,$$

which integrating (by parts) gives

$$R_A(\theta) - R_A(0) = \int_0^\theta \left(x_A^A(z)\;\frac{d\phi(z)}{dz} + x_A^B(z) \right)\; dz,$$

where $R_A(z) = x_A^A(z)\;\phi(z) + x_A^B(z)\;z - P_A(z)$ represents the equilibrium rents of a buyer of type A and valuation z. Similarly,

$$R_B(\theta) - R_B(0) = \int_0^\theta \left(x_B^A(z) + x_B^B(z)\;\frac{d\phi(z)}{dz} \right)\; dz.$$

That means that the revenue equivalence holds in this situation, i.e., any two mechanisms that assign the objects in the same fashion and for which a buyer with valuation 0 obtains the same rents, for both type A and type B, give the seller the same expected revenue. We have seen that the allocation of the objects is the same in both mechanisms analyzed. Now, notice that in either of them,

$$R_I(0) = \left(\frac{1}{2}\right)^{N-1} \left[\phi(0) - \int_0^1 z(N-1)f(z)F(z)^{N-2}dz\right].$$

That is, a buyer with valuation 0 makes positive rents only when none of the other buyers is of her type, in which case she gets the object she most prefers for a price equal to the first order statistic of all other buyers' valuations. Then, we conclude that the two mechanisms, the sequential auction and the right-to-choose auction, give the seller the same expected revenue when buyers are risk neutral. On the other hand, the price obtained by the seller for each object does not depend on risk attitudes, in the case of a sequential auction, whereas the revenue for the seller in the right-to-choose auction is higher when buyers are risk averse. So we come to our main conclusion:

Proposition 3 *Under risk neutrality, the seller expects the same revenue whether he uses a right-to-choose or a sequential auction. However, if buyers are risk averse, the former raises higher expected revenue than the latter.*

6 Comments on assumptions and extensions

6.1 Breaking ties

In the sequential auction, we have assumed that, when two buyers tie (and are of the same type), the good is allocated to the buyer with higher valuation if they are of type A, and to the one with lower valuation if they are of type B. This has two advantages. First, an equilibrium exists, one which is simple enough to understand. Second, the equilibrium implies that both auction formats, sequential and right-to-choose, result in the same allocation of the objects. The second fact makes the results depend solely on the risk attitudes of buyers (efficiency is not a question). Without this result, the total surplus under the two auction formats would not only be different; neither could they be compared in general. Indeed, one would tend to think that the right-to-choose auction guarantees an efficient allocation, but this is not necessarily the case. When buyers of both types are present, an efficient mechanism would allocate object A to the buyer of type A with highest valuation and object B to the buyer of that type with highest valuation. However, if only buyers of one type are present, say buyers of type A, the two objects would be allocated to the two highest valuation bidders, θ_{N-1}, θ_N, but which object each one would get would depend on whether $\phi(\theta_{N-1}) + \theta_N$ or $\phi(\theta_N) + \theta_{N-1}$ is higher, and this in general depends on the particular realizations of θ_N, θ_{N-1}, and on the function $\phi()$.

There is a case in which neither the relative efficiency of the mechanisms nor the validity of Proposition 1 depends on the assumption mentioned above: the case in which $\phi(\theta_N) + \theta_{N-1} = \phi(\theta_{N-1}) + \theta_N$ for all $\theta_{N-1}, \theta_{N2}$. That is, when $\phi(\theta) - \theta$ is constant (how much a buyer is willing to pay to get the object she most prefers instead of the other does not depend on the buyer's valuation). In this case, one can check that Proposition 1 still holds when the allocation of an object is random in the case of a tie. Then both mechanisms would assign the two object to the two buyers with the highest valuation when all of them were of the same type. For what we just said, this means the same surplus. Still, buyers' rents would be the same under both mechanisms for risk neutral buyers ($\phi\prime(\theta) = 1$, and then the rents would also depend only on the sum of probabilities of obtaining *one* object), which means equal revenues for the seller. Thus, both mechanisms would raise the same revenue under risk aversion, and again the right-to-choose auction would raise more revenue under risk aversion.

6.2 Buying more than one object; unit and multi-unit demands

Assume now that a buyer can buy both objects, in which case her utility is the same as when she buys the object she most prefers. That is, buyers are still unit-demand buyers, but could purchase both objects. This could be in the interest of a buyer with valuation θ who has already bought the object she prefers least but could also get the other for an additional price lower than $\phi(\theta) - \theta$. Notice that this cannot happen in the right-to-choose auction, since there the winner of the first auction already gets the object she prefers the most, whatever strategies are used. In the sequential auction, if a buyer of type B wins the first auction (and then object A), then she could still be tempted to bid for the second object (and should actually do so!). However, if buyers are using the strategies we proposed in Proposition 1, a buyer of type B will win the first auction only if all other buyers are of type B and one of them has actually a valuation above hers. That is, the price she would have to pay for object B exceeds what she is willing to pay for it. Then, as long as buyers have unit demands, the assumption that a buyer cannot participate in the second auction if she has won the first is without loss of generality.

A simple way of introducing the possibility of multi-unit demands is to assume that willingness to pay is additive, that is, to assume that a buyer has utility $u(\phi(\theta) + \theta - P)$ when she wins both objects and pays P. Assume, for instance, that $N = 3$. One can check that an equilibrium in the sequential auction is for each buyer to stay put in the first auction until the clock reaches a price equal to her willingness to pay for object A. Then, if there are buyers of both types, the buyer with highest valuation among those of type A gets the first object, and the one with highest valuation among the type B buyers gets the other. If there are buyers of only one type, the highest-valuation buyer gets both objects (and there is no need for the tie-breaking rule here).

In the right-to-choose auction, the following strategy constitutes an (symmetric) equilibrium:

– $B_0(\theta) = \phi(\theta) - u^{-1}\left[\left(\frac{1}{2}\right)^2 u(\phi(\theta) - \theta)\right]$

– $B_1(\theta; p_1) = \phi(\theta) - u^{-1}\left[\left(\frac{1}{2}\right)^2 u(\phi(\theta) - B_0^{-1}(p_1))\right]$,

– In the second auction bid up to willingness to pay for the object sold.

The result once again is the same allocation of the objects: the buyer with the highest valuation gets the object she most prefers. If there are buyers of both types, then the buyer with the highest valuation among those whose type is different from that of the highest valuation buyer gets the second object. If there are only buyers of one type, then the buyer with the highest valuation gets both objects. Again, we can check the revenue equivalence result for risk neutral buyers, and observe that rents are independent of attitudes toward risk in the standard sequential auction. In the right-to-choose auction, however, the drop out prices in the first auction are higher when buyers are risk averse, as before, and we then get the same conclusion.

6.3 Overlapping in willingness to pay for different types

The model we have analyzed is particularly simple due to the assumption that $\phi(0) > 1$. This in particular was the reason why so much information was released in the sequential auction. And the fact that the sequential auction was less uncertain, on the other hand, is the reason we have proposed for why the sellers use right-to-choose auctions rather than sequential ones when buyers are risk averse. Still, if the willingness to pay is not "lexicographically" ranked, observing other buyers' bidding behavior in a sequential auction conveys information to buyers with respect to types, something which is absent in the right-to-choose auction as long as we maintain the assumption that ex-ante the goods are totally symmetric. Then, the basic intuitions gained with the present paper should be of relevance for these more general settings. To the effects of risk aversion, however, one should add effects coming from different allocations that the two auction formats would probably imply.

6.4 Declining prices

A tendency for the sequence of prices of repeated sales to decline has been widely observed. In particular, this has been observed by Ashenfelter and Genesove (1992) in the auctioning of condominium units when sellers used right-tochoose auctions. The authors are quick to point out that such price behavior should be expected if the units are not homogeneous. However heterogeneity of the units does not entirely explain the downward drift, according to the authors. Here, we have been considering a case of "symmetric" (from an ex ante point of view, of course) objects. From Proposition 2 it is straightforward to observe that the expected price of the second object is equal to the actual price of the first, if buyers are risk neutral. However, risk aversion leaves the price for the second object unchanged for each realization of the valuations and types, whereas the price for the first object increases with this risk aversion. Then, a tendency to falling prices in right-to-choose auctions would also be caused by buyers' risk aversion[3].

7 Concluding remarks

I have presented buyers' risk aversion as an explanation for the use of right-to-choose auctions instead of standard auctions when selling ex-ante symmetric substitutes. Sequential auctions reveal too much information to bidders, so that, if they loose in early rounds, they face less uncertain future competition. In contrast, auctioning the right to choose can be considered as an instrument to reduce this flow of information in early rounds. By using this auction format, the seller is able to make buyers more willing to bid aggressively in these early rounds to avoid the risk associated with losing them.

[3]For regular, sealed bid sequential auctions, McAfee and Vincent (1992) have already found that, if a pure strategy equilibrium exists, prices should decrease under risk aversion.

References

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[3] Ashenfelter, O. and D. Genesove (1992) "Testing for Price Anomalies in Real-Estate Auctions" *American Economic Review*, 82-2, pp. 501-505

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A Appendix

Next we offer the proof of Proposition 1:

Proof. Consider a buyer of type B with valuation θ, and let us compare the buyer's utility when using the strategy stated in the Proposition and when using any other one. Let us investigate what the possible differences are in utility for the buyer. First, it could be that the buyer stays in the auction after the clock points to θ, in which case the buyer cannot get utility above $u(0)$, which is guaranteed by the proposed equilibrium strategy. It could also be that the buyer ends up dropping out at a price below θ when there are still some two or more buyers in the auction. If any of these remaining buyers is of type A, this would not make any difference, since this buyer will not drop out before the price gets to θ. Thus, the only case we need to analyze is that in which all buyers are of type B. Again, if at least two of the remaining buyers have valuations above θ, the two strategies would give the same utility to the buyer: she will end up dropping out of the first auction, and the winner of this auction would not be altered. If only one of the remaining bidders has a valuation above θ, then the buyer would obtain the object A and pay the second highest valuation of all other buyers, ($\theta_{N-2} \leq \theta$) when using the proposed equilibrium strategy, which means utility $u(\theta - \theta_{N-2})$. When using the alternative strategy (which implies dropping out before), the utility would drop to $u(0)$, since the buyer with valuation θ_{N-2} would win object A and the buyer with valuation above θ would win the object B in the second auction.[4] Finally, if there is no buyer with a valuation above θ the two strategies are equivalent: the buyer would win object B whether she drops out before θ or not. Finally, assume that N-2 buyers drop out at prices $p_1 \leq p_2 \leq ... \leq p_{N-2} < \theta$. Assume the buyer does not drop out inmediatly at p_{N-2}. Then, if the other remaining buyer is of type B, that buyer will drop out and our buyer will win object A at a price p_{N-2}, with utility

[4] Here, we use the tie-breaking rule that we have mentioned above.

$u(\theta - p_{N-2})$. The outcome would be the same if our buyer drops immediately and the other remaining buyer happened to have higher valuation. However, if our buyer had higher valuation she would not be assigned object A and would instead compete for object B, win that object, and pay a price $\phi(p_{N-2})$, having utility $u(\phi(\theta) - \phi(p_{N-2}))$. Since we assumed $\phi\prime(\theta) \geq 1$, this utility is no lower than $u(\theta - p_{N-2})$, which concludes the proof for buyers of type B.

Now, consider a buyer of type A with valuation θ, and let us again compare the buyer's utility when using the strategy stated in the Proposition and when using any other strategy. If a buyer has dropped out at some price $p_1 < 1$, then this means that this buyer is of type B, and so our buyer will not win object B in the second auction. Thus, it is dominant to stay in the auction for object A until the clock points to $\phi(\theta)$. If no buyer drops out before the clock points to the price 1, then that means that all the buyers are of type A. Then let us compare our proposed equilibrium strategy with any other the buyer may use. It may be that the buyer ends up dropping out before the clock points to $\phi(\theta)$ when no more than $N-3$ other buyers have dropped out. In this case the buyer will win object B in the second auction only if she has a valuation higher than the second highest of the other buyers' valuations, θ_{N-2}, in which case her utility is $u(\theta - \theta_{N-2})$. If instead she had used the proposed equilibrium strategy, she would have won either that same object B at the same price (if her valuation was above θ_{N-2} but below the highest of all the other buyers' valuations, θ_{N-1}) or object A for a price equal to $\phi(\theta_{N-1}) - \theta_{N-1} + \theta_{N-2}$ (if her valuation had been also higher than the highest of all the other buyers' valuations). But

$$u(\phi(\theta) - \phi(\theta_{N-1}) + \theta_{N-1} - \theta_{N-2}) \geq u(\theta - \theta_{N-2}),$$

since $\phi(\theta) - \phi(\theta_{N-1}) \geq \theta - \theta_{N-1}$. Thus, the deviation is not profitable for the buyer. Finally, when $N-2$ other buyers have dropped out, the buyer knows that she would win the auction for object B at a price $\phi^{-1}(p_{N-2})$, if she loses the auction for object A. That means her reservation value is now $\phi(\theta) - \theta +$

$\phi^{-1}(p_{N-2})$. Then the proposition follows. ■

Next we turn to the proof of Proposition 2:

Proof. First, consider any other strategy for the buyer that results in her dropping out of the first auction before $N-2$ buyers have dropped out and the price has not reached θ. It may be that the buyer does not have the highest of all valuations, in which case she would not have won the first auction even following the equilibrium strategy and she would not have affected who won that auction. It may also be that she does have the highest of all valuations.

Then, assume the buyer has the highest of all valuations. If the second highest is of her type (an event with probability $\left(\frac{1}{2}\right)^{N-1}$), dropping out means winning the less prefered object tomorrow only in the case where all other buyers are also of the same type, thus obtaining utility $u(\theta - \theta_{N-2})$, where again θ_{N-2} represents the second highest of the rival buyers' valuations. If the second highest is not of her type, then she will win her most prefered object tomorrow. The price she will pay will depend on the type of all the other buyers: it will be equal to the highest willingness to pay for the object among the remaining buyers. Thus, given the realized buyers' valuations, $(\theta_1, \theta_2, ..., \theta_{N-2})$, the expected utility for the buyer will be $\pi_2(\theta; \theta_1, ..., \theta_{N-2})$, where, if other buyers are using the prosposed strategy, $\theta_n = p_n$ for all $n = 1, 2, ..., N-2$.

If the buyer follows the proposed strategy instead, she will win her most prefered object for a price equal to

$$B_{N-2}(\theta_{N-1}; p_1, ..., p_{N-2}) = \phi(\theta_{N-1}) - u^{-1}(\pi_2(\theta_{N-1}; p_1, ..., p_{N-2})),$$

which is the price at which the buyer with the highest valuation $\theta_{N-1} < \theta$ among all the other buyers drops out. The buyer's utility would then be

$$u\left[\phi(\theta) - \phi(\theta_{N-1}) + u^{-1}(\pi_2(\theta_{N-1}; p_1, ..., p_{N-2}))\right] >$$
$$u[u^{-1}(\pi_2(\theta_{N-1}; p_1, ..., p_{N-2}))] = \pi_2(\theta_{N-1}; p_1, ..., p_{N-2}),$$

since $\phi(\theta) > \phi(\theta_{N-1})$. Then, the deviation is not profitable for the buyer.

Assume that the deviation results in the buyer dropping out after some other $N-2$ buyers have dropped out at prices $(p_1, p_{2,}..., p_{N-2})$, but before the clock point to a price $B_{N-2}(\theta; p_1, ..., p_{N-2})$. Again, if the remaining buyer has a higher valuation, this deviation is of no consequence. However, if the buyer's valuation is the highest, the deviation has the same consequence as that analyzed before, and then it is not in the buyer's interest. Finally, and for similar but opposite reasons, staying in the auction after the price $B_{N-2}(\theta; p_1, ..., p_{N-2})$ has been reached is not in the interest of the buyer and then the Proposition follows. ■